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OTVORENI PROBLEMI MODELIRANJA PROCESA LOMA U BETONU

OPEN PROBLEMS IN MODELLING FRACTURE PROCESS OF CONCRETE

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Izvod

Modeliranje procesa loma u betonu do sada nije dalo tačno i jedinstveno rešenje. Postoje tri opšte poznate teorije koje se bave opisivanjem fenomena loma u betonu: mehanika loma, teorija plastičnosti i mehanika neprekidnog oštećenja. Međutim, bez obzira koja se od ovih teorija primenjuje, sve se suočavaju sa mnogim problemima od kojih su četiri od njih istaknuta u radu kao najznačajnija. Namera autora nije bila da obeshrabri, već ima za cilj da usmeri pažnju istraživača na ključne tačke u analizi loma u betonu.

Abstract

Modeling fracture process in concrete has not given so far exact and unique solution. There are three common theories used to describe the fracture phenomenon in concrete: fracture mechanics, theory of plasticity and continues damage mechanics. However, regardless which of this theory is applied, they are faced with many problems, but four of them are pointed out in the paper as the most disputable. Yet, author's intention was not to discourage but it is aimed to focus researching attention to the key points in fracture analysis of concrete.

UVOD

I pored ogromnog napretka u rešavanju problema loma konstrukcija i materijala /1/, koji je ostvaren tokom 20-og veka, mnogi problemi otkaza konstrukcija u eksploataciji su ostali nerešeni. Zbog toga su istraživanja problema loma nastavljena, što je omogućilo da su na početku 21-og veka, nauka o materijalima uopste, a teorije loma u materijalu posebno, dostigli značajan nivo u svom razvoju. Ovaj je razvoj baziran prvenstveno na napretku u kompjuterskoj simulaciji procesa i stanja, ali takođe i na stalnom i brzom razvoju i usavršavanju tehnika ispitivanja bez razaranja materijala.

Međutim, teorija loma betona, uprkos velikim naporima istraživača iz celog sveta, nije još uvek pružila konačan odgovor na pitanje modeliranja procesa loma u betonu. Postoje tri teorije koje se bave opisivanjem fenomena loma u betonu: mehanika loma, teorija plastičnosti i mehanika neprekidnog oštećenja.

INTRODUCTION

In spite of tremendous development in solving the problem of fracture of structures and materials /1/, achieved during 20th century, many problems of service failures of structures were not solved. For that the investigation of fracture problem was continued, enabling at the beginning of the 21st century material science in general and theory of material fracture in particular, to achieve a significant level in their development. The development is based primary on the advance in computer simulation of the processes and states, but on the continuous and fast development and more sophisticated non-destructive techniques as well.

However, the fracture theory of concrete, in spite of the tremendous efforts of the researchers from all over the world, has not yet given the final answer to the modeling of fracture process. There are three theories used to describe the fracture phenomenon in concrete: fracture mechanics, theory of plasticity and continuous damage mechanics.

Imajući u vidu visoko idealizovane osnovne pretpostavke sve tri pomenute teorije, problem postaje tim složeniji i ni jedna od njih ne uspeva da pruži konačno i odlučujuće rešenje. Uviđajući da problem prevazilazi predpostavke i mogućnosti pomenutih teorija, neki istraživači su skloni primeni složenijih postupaka poznatih pod opštim imenom: teorija kompleksnih sistema (npr. statististička fizika, teorija dinamičkih sistema, bifurkacije i samo-organizacije).

Odavno je već uočeno da samo kombinovani prilaz može da obuhvati problem u celini, ali pokušaji učinjeni do sada u tom pravcu pružili su samo komplikovana i maglovita rešenja.

U radu su istaknuti i ukratko analizirani najznačajniji problemi u modeliranju procesa loma u betonu.

PROBLEM br. 1 - MIKROMEHANIKA OŠTEĆENJA

Beton se može smatrati kompozitnim materijalom koji se sastoji od tri glavne komponente: cementne matrice (mikroporozni materijal), agregata i prelaznog sloja - interfejsa nazvan "transition halo". Zbog svoje visoko orijentisane, kristalaste strukture, beton se u ovoj prelaznoj zoni, koja je najporozniji deo kompozita, smatra najslabijim. Nije sporno, da se inicijalna oštećenja u betonu pojavljuju baš u ovoj zoni, ali osobenost procesa loma u betonu je pojava unutrašnjih mikroprslina, čak pre nego sto je spoljnje opterećenje primenjeno. Ove mikroprsline, nazvane "već postojeće", nastaju kao posledica skupljanja u betonu i razvoja toplote tokom procesa hidratacije cementa ("tehnološko oštećenje") zavisno od mnogih uticajnih faktora: v/c faktora, vrste cementa, gradacije frakcija agregata, proces negovanja betona. Već postojeće mikroprsline igraju važnu ulogu u procesu loma u betonu. Velika većina ovih prslina je slučajno raspoređena u masi betona i locirana u graničnoj zoni zrna agregata i cementne mase, delujući na taj način kao svojevrsni potencijalni koncentratori napona. Nakon što je spoljnje opterećenje prvi put primenjeno, ova koncetracija napona dovodi do propagiranja mikroprslina, koje se odigrava na dva načina, zavisno od pravca delovanja glavnih napona i orijentacije i pravca pružanja mikroprslina. Ustvari, lom tipa I i II nastupaju istovremeno sve dok proces propagacije ne bude zustavljen kada se vrh prsline suoči sa površinom zrna agregata, pošto cementna masa ima veću čvrstoću od zone interfejsa i dalji porast opterećenja je potreban da bi prslina nastavila da propagira kroz cementnu masu. Izgleda da ove mikroprsline deluju međusobno, stvarajući mreže unutrašnjeg oštećenja u betonskom materijalu. Ova mreža postaje nestabilana što dovodi do lokalizacije deformacije (deformacijsko omekšavanje) i do pojave makroskopskog loma.

Iz gore pomenutog postaje jasno da nepremostiv problem za primenu mehanike loma za modeliranje procesa loma u betonskom materijalu pretstavlja činjenica da sastavne komponente betona imaju značajno različitu žilavost, odnosno, značajno različite faktore intenziteta napona za sve tri komponente kompozita. Neki istraživači /2/ smatraju i predlažu sledeću hijerarhiju kritičnih faktora intenziteta napona za komponente:

Keeping in mind the highly idealized basic assumptions of all three mentioned theories, problem appears too be more complex and neither of them is able to give the final and decisive solution. Realizing that the problem overcomes the assumtions and the abilities of the mentioned three theories some researchers are inclined to use more advanced techniques known under the general name: theory of complex systems (e.g. statistical physics, theory of dynamical systems, bifurcations and self-organization).

It was early realized that only a combined approach is able to encompass the entire problem, but the attempts made so far have brought no more than complicate and vague proposed solutions.

In the paper, the most significant problems in modeling fracture process in concrete are briefly pointed out.

PROBLEM No.1 - MICROMECHANICS OF DAMAGE

Concrete may be considered as a composite material consisting of the three main components: the cement matrix (microporous material), the aggregates and the transition layer - interface named "transition halo". Due to highly oriented crystallized structure, concrete in this transition zone which is the most porous part of the composite is considered as the weakest one. There is no dispute that concrete damage initiation appears just in this zone, but the particularity of the fracture process in concrete is the appearance of internal microcracks even before loads applied. These microcracks known as "preexisting", are formed as the consequence of shrinkage in concrete and the heat development in process of cement hidratation ("technological damage") depending of the many influencing factors: w/c ratio, type of cement, sieve gradation, curing process. Preexisting microcracks play an important role in the failure process in concrete. The majority of these microcracks are randomly distributed in concrete mass and located on the aggregate-cement past interface, acting so as the particular potential stress concentrators. When external load is first applied the stress concentration makes this microcrack to propagate which took place in two ways, depending of the direction of principal stresses and microcracks orientation and growth direction . Actually, fracture mode I and mode II are taking place simultaneously until the propagation process is arrested when crack tip meets the surface of aggregate grain since the cement past has a higher strength then the interface zone and next increase of loads is necessary for its propagation in cement matrix. It seams that the microcracks interact with one another subsequently, forming the network of internal damage in concrete material. This network become unstable leading to strain localization (strain softening) and macroscopic failure mode take place.

From the mentioned above becomes clear that the unbridgeable problem for fracture mechanics application in fracture process modeling in concrete is a fact that toughness of constitutive components is different, that is, significantly different critical stress intensity factors for all three components of the composite. Some researchers /2/ propose the following hierarchy of the critical intensity factors for the components:

$K_{Ic}^{a} >> K_{Ic}^{c} > 2K_{Ic}^{if}$

Ovde su navedene veličine:

$K^{a}{}_{Ic}$ - kritični faktor intenziteta napona za agregat

$K^{c}{}_{Ic}$ -kritični faktor intenziteta napona za cementnu. pastu

$K^{if}{}_{Ic}$ - kritičnifaktor intenziteta napona za interfejs

Propagacija i spajanje mikroprslina i šupljina u betonskom materijalu su već opisani ranije. Plastično tečenje je posledica dislokacionih procesa duž preferentnih ravni klizanja, indukovanih delovanjem smičućih napona. Proces je objašnjen analogno ponašanju metalnih materijala, ali budući da je kristalna struktura betona sasvim različita od one kod metala isto razmatranje ovde ne može biti uspešno primenjeno.

Navedeno postaje jasnije kada se posmatraju fotografije načinjene elektronskim mikroskopom sa uvećanjem x100 (sl. 1) i x1000 (sl. 2.). Zapaža se bitno drugačija kristalna struktura od one kod metalnih materijala. Očigledno da kod ovakve strukture materijala nema smisla govoriti o formiranju kliznih ravni u izrazito heterogenoj strukturi. Kako onda objasniti uzrok pojavi nelineranosti na σ-ε dijagramu - ostaje otvoreno pitanje.

Slika 1. Mikrostruktura betona, ×100

$K_{Ic}^{a} >> K_{Ic}^{c} > 2K_{Ic}^{if}$

Here the mentioned values are:

$K^{a}{}_{Ic}$ - critical stress intensity factor for agreggate

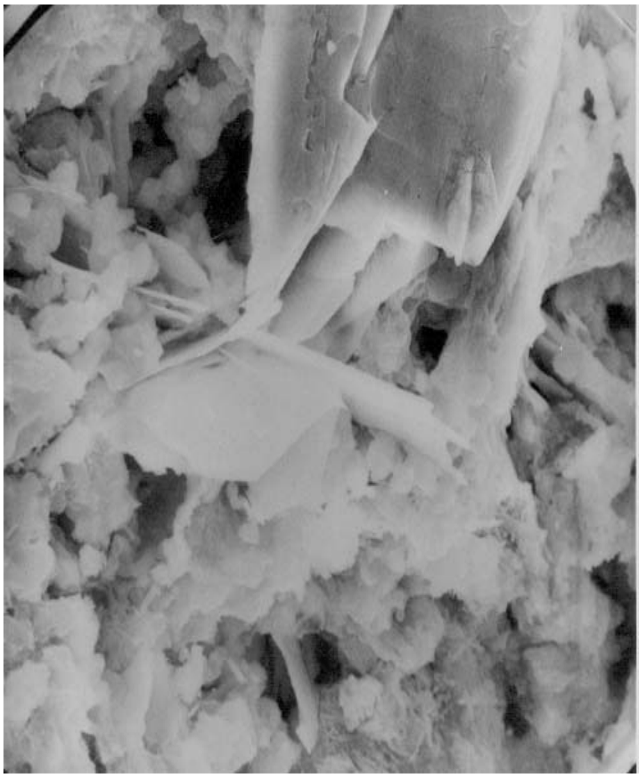
$K^{c}{}_{Ic}$ - critical stress intensity factor for cement paste

$K^{if}{}_{Ic}$ - critical stress intensity factor for interface

The microcracks propagation and coalescence in the concrete material is already described earlier. Plastic flow is the consequence of dislocation processes along preferential sliding planes, induced by the action of shear stresses. The process is described in analog way as in the behaviour of metallic materials, but since the crystal structure of concrete is quiet different than that in metals, the same consideration cannot be applied here successfully.

Mentioned becomes clear when considering the photos taken by electronic microscopy with the magnification x100 (Fig. 1) and x1000 (Fig. 2.). Completely different crystal structure is observed compared to metallic materials. It is evident that there is no sense to speak about sliding planes in such a heterogeneous structure. How than to explain the appearing of nonlinearity at σ-ε diagram - it stays an open question.

Figure 1. Microstrukture of concrete, ×100.



Slika 1. Mikrostruktura betona, ×1000

Figure 1. Microstrukture of concrete, ×1000.

PROBLEM br. 2 - LOKALIZACIJA DEFORMACIJE

Kako je prethodno već bilo pomenuto, oštećenja se sastoje od mikroprslina i mikrošupljina koje propagiraju kroz materijal i deluju jedna na drugu. Nakon što se opterećenje dalje poveća, oštećenje počinje da se lokalizuje u zoni, tj. nastupa fenomen poznat kao lokalizacija deformacija ili "deformacijsko omekšavanaje". Ovu lokalizaciju oštećenja ubrzo sledi formiranje makroprsline i njen rast, što dalje vodi do konačnog loma materijala.

Fizički procesi, koji se odigravaju unutar zone lokalizacije oštećenja, nisu dovoljno objašnjeni. Tehnike ispitivanja bez razaranja, na trenutnom stupnju razvoja nisu u stanju da snime, ne samo prirodu fenomena, već ni veličinu ove zone lokalizacije. U cilju objasnjenja ovog fenomena, istraživanja su uveliko usmerena ka pokušajima da se heterogenost realnog materijala sa svim mikroprslinama zameni jednim ekvivalentnim kontinuumom koji bi odgovarao makroskopskom ponašanju materijala. Postupak je poznat kao "homogenizacija" i do sada je primenjen korišćenjem više matematičkih tehnika. Međutim, veoma važan efekt interakcije mikrodefekata, ili je zanemarivan ili jednostavno nije uziman u obzir.

Izvršena je kompjuterska, numerička simulacija procesa korišćenjem prilaza zasnovanog na tri-dimenzionalnim graničnim elementima i za razumno velike uzorke, ali je za realne konstrukcije ovaj postupak još uvek izvan dometa kompjutera.

Neka sličnosti ovog procesa sa faznim transformacijama je evidentna, ali ni jedna ispravna teorija nije izvedena iz ove ideje. Pokušavajući da reše ovaj "misteriozni" fizički proces, mnogi istraživači su skloni primeni relacija napon-pomeranje (smatrajući da pomeranje nastupa kroz zonu lokalizacije).

PROBLEM br. 3 - MIKRO-MAKRO ODNOSI

Da bi se rešio vrlo složen problem procesa krto-elastično-plastične deformacije predlagani su i uvođeni različiti modeli kontinuuma sa ciljem da se ovaj proces modelira. U modeliranjuju je posebna pažnja posvećena betonu. Ipak, brzo se uvidelo da betonski materijal ne može biti smatran kontinuumom imajući na umu postojanje diskontinuiteta (kakvi su već postojeće mikroprsline). Pokušavajuči da prevaziđu teškoće, a u cilju da definišu oštećenje, istraživači su bili primorani da uvedu dve skale: makroskalu, na kojoj je čvrsto telo smatrano homogenim i mezoskalu, na kojoj je čvrsto telo nehomogeno, odnosno kod koga postoje neka polja diskontinuiteta. Međutim, razmatrano na mezoskali, pretpostavljajući "n" prslina u jedinici zapremine, oštećenje u tački je određeno sa 8xn promenljivih: 3 skalarne veličine koje definišu položaj, 3 ugla koji definišu orijentaciju i 2 skalara koja definišu veličinu svake prsline. Ovaj prilaz je ubrzo bio redukovan na samo jedan parametar - gustinu mikroprslina u jedinici zapremine čvrstog tela.

Uglavnom, teorije kontinuuma mehaničkog ponašanja materijala su podeljena na dva različita prilaza: fenomenološki i mikromehanički prilaz, oba striktno okružena svojim sopstvenim pretpostavkama i ograničenjima.

PROBLEM No.2 - STRAIN LOCALIZATION

As it was mentioned above, the damage consisting of the microcracks and microvoids, which propagate through the material interacting mutually. After load is further increased damage intensifies, becoming localized in the zone where the phenomenon called strain localization or "strain softening" occurs. This damage localization is followed soon by the macrocrack formation and propagation, leading to the final fracture of the material.

The physical processes occurring inside the localization zone are not sufficiently explained. Non-destructive techniques, at this stage of development are not able to capture, not only the nature of the phenomenon, but neither the size of the localized zone. In order to explain this phenomena, the research is greatly directed towards the attempts of replacing the heterogeneity of the real material by one equivalent continuum which exhibits the macroscopic behavior of material. The procedure is known as "homogenization" and it is applied using many mathematical techniques. However, the very important effect of microdefect interaction is neglected or it is merely not taken in account.

Computer numerical simulation of the process is carried out using three-dimensional boundary element approach to the reasonably large samples of concrete, but for the real size structure this approach is still beyond the capacity of computers.

Some similarity of this process and the phase transformation is evident, but no any valid particular theory was derived based on that idea. Attempting to solve the "mysterious" physical process, many of the researchers are inclined to apply the stress-displacements relations (considering displacement across the localization zone).

PROBLEM No.3 - MICRO-MACRO RELATIONSHIPS

In order to solve very complex problem of brittle-elastic-plastic deformation process different kind of continuum models were proposed and introduced with the intention to model this process. In the modeling particular attention was paid the concrete. Yet, it was soon realized that concrete material cannot be considered as a continuum, keeping in mind the existence of discontinuity (such as preexisting microcracks). Trying to overcome difficulties in order to define the damage, researchers introduced two scales: macroscale, on which the solid is considered homogenous and mesoscale, on which the solid is inhomogeneous and some kind of the field discontinuous is existing. However, considering on the mesoscale, assuming "n" cracks in the units of volume, damage in the point is identified by 8xn variables: 3 scalars defining position, 3 angles defining orientation and 2 scalars which define the size of each crack. These approach is quickly reduced to only one parameter – the density of microcracks in the unit of solid volume.

Generally, continuum theories of mechanical behaviour of materials are divided into two different approaches: the phenomenological approach and micromechanics based approach strictly surrounded by their own particular assumptions and limitations.

Fenomenološki pristup je baziran na relacijama u kojima se koriste makroskopske veličine određene iz eksperimenta. Sa druge strane, u mikromehaničkom pristupu je opšta ideja de se mehaničko ponašanje materijala opiše na osnovu razmatranja uticaja i značaja raspodele mikrodefekata u jedinici zapremine, pretpostavljajući njihovu ravnomernost u materijalu.

PROBLEM br. 4 – UTICAJ VELIČINE

Poznato je da je uticaj veličine objekta i uzorka jedan on najznačajnijih inženjerskih problema u razmatranju svih vrsta materijala nazvanih kvazikrtim, kao što su: stene, morski led, vaknasti kompoziti, beton. Naime, eksperimentalna istraživanja izvršena do sada su jasno pokazala da čvrstoća kvazikrtih materijala (u koje spada i beton) zavise značajno od veličine uzorka.

Kvazikrte materijale, budući da ne poseduju svojstva plastičnosti, karakteriše postupno omekšavanje u zoni procesa loma (FPZ), a ova zona postaje značajna po veličini u poređenju sa veličinom konstrukcije. Isti problem, ali sa sasvim različitom shemom loma, zapažen je i kod naprezanja na zatezanje i na pritisak. Problem razmere je bio rano uočen u mehanici fluida, ali je ostao dugo vreme zanemaren u mehanici čvrstog tela (do odprilike 1980). Glavni uzrok ovog zanemarivanja je bio taj što su dve najvažnije teorije loma materijala bazirale svoje pristupe na konceptu kritičnih napona ili deformacija (teorija plastičnosti) i kritične dužine prsline (mehanika loma), koje su u početnom stadijumu loma nezavisne od veličine konstrukcije.

Danas, dve grupe teorija koje najviše dominiraju u pokušajima da se reši ovaj problem su:

– teorija zasnovane na energetsko-statističkim odnosima, i
– teorija zasnovane na geometriji fraktala..

Obe teorije, i ako imaju suštinski različitu polaznu osnovu, imaju izvesne prednosti, ali i izvesne mane i ograničenja.

Bez pretenzija da se dublje zalazi u oblast razmatranja obe od pomenutih teorija, moglo bi se reći da je problem razmere za kvazikrte materijale daleko od rešenja, odnosno, ne postoji još uvek jasno i jedinstveno rešenje.

ZAKLJUČAK

Uprkos vrlo dugoj istraživačkoj (teoretskoj i eksperimentalnoj) istoriji, lom u betonu je još uvek misteriozan fenomen i inženjeri praktičari još uvek čekaju na jednostavan, jasan, jedinstven i standardizovan model loma. Postoje tri različite teorije koje se nadmeću u razmatranju problema oštećenja i loma u betonu: teorija plastičnosti, mehanika loma i mehanika neprekidnog oštećenja. Osnovne pretpostavke, na kojima se ove teorije zasnivaju, su idealizovane i ne odgovaraju specifičnoj prirodi betonskog materijala. Tako mehanika loma, npr. razmatra uslove nestabilnosti jedne, pojedinačne prsline u "nevinom" kontinualnom čvrstom telu bez drugih oštećenja.

The phenomenological approach is based on the relations in which the macroscopic values determined from the experiments are used. On the other side, in micromechanical approach the general idea is to describe mechanical behaviour of the material based on consideration of effects and significance of microdefects distribution in volume unit, assuming their regularity inmaterial.

PROBLEM No.4 - SIZE EFFECT

This is known the size effect of objects and specimens as one of the most distinguished engineering problems considering all kind of material, termed as quasibrittle, such as: rocks, sea ice, fiber composites, concrete. Namely, experimental investigations carried out so far clearly reveal that strength of quasibrittle materials (among them also concrete), depends significantly of the size of the specimen.

These materials, having no plasticity, are characterized by gradual softening in a fracture process zone (FPZ) and this zone becomes significant in size compared with structure size. The same problem, but with the quiet different pattern is encountered for both tension and compression state of stress. The problem of scaling has been early identified in fluid mechanics, but stayed neglected in solid mechanics for a long time (until about 1980). The main cause of this neglecting was that two dominant theories of fracture of materials based their approaches on the concept of critical stresses or strains (theory of plasticity) and on the critical crack length (fracture mechanics) which is at the initial state of fracture independent of the size of structure.

Nowadays, two theories the most dominant attempting to solve this problem are:

– theory based on energetic-statistical scaling and
– theory based on the fractal geometry.

Both of them, starting from essentially different bases, have some advantages, but also some disadvantages and limitations.

Without pretending to enter deeper in the area of the two mentioned theories, it could be said that the problem of the size effect for quasibrittle materials is far to be settled and still there is no clear and unique solution.

CONCLUSION

In spite of the long researching (theoretical and experimental) history, fracture of concrete is still a mysterious phenomenon and the practical engineers have been still waiting for simple, clear, unique and codified fracture model. There are three distinguished theories to compete in describing a damage and fracture problem in concrete: theory of plasticity, fracture mechanics and continuous damage mechanics. The basic assumptions of all of these theories are idealistic and not corresponding to the real nature of concrete materials. The fracture mechanics, e.g. considers the condition of instability of one single crack in a "virgin" continuous solid with no other damages.

Na drugoj strani, mehanika kontinualnog oštećenja tretira oštećeno čvrsto telo, gde oštećenje može da se opiše kao kontinualno polje oštećenja.

Takozvani kombinovani prilaz, predložen inicijalno od strane Jansona i Hulta /7/, koji koristi kombinaciju dve ili tri teorije, je izazovan zadatak, ali je nažalost naišao na slab interes kod istraživača (M. Pavisic/8/).

Zajedničko je za pomenute teorije da su sve one suočene sa četiri značajna problema, koji su ovde izdvojeni i ukratko opisani

On the other side, continuous damage mechanics considers a damaged solid, where damage may be described in terms of continuous defects field.

So called combined approach, suggested initially by Janson and Hult, /7/ uses a combination of two or even three theories, is a challenging task, but unfortunately it found just a little interest among the researchers (M. Pavisic, /8/.

For all of three mentioned theories is common that they are all facing four important problems which are here outlined and brifly described.

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