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ANALYSIS OF WELDED JOINTS BY APPLYING THE FINITE ELEMENT METHOD

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INTRODUCTION

Welded joints are usually critical points in determining the service life of a structure. If the welding process is known in detail, there is a possibility to extend the life of a welded joint, achieving significant savings. Numerical methods, especially the finite element method, applied in solving corresponding structural mechanics equations can significantly contribute to the welding process and welded joint analysis.

From the structural mechanics point of view, the problem of welding and welded joint analysis are considered as non-linear problems in general, for they include significant thermal stresses caused by thermal shocks, melting and solidification with a large temperature gradient. They also include significant mechanical property changes during the welding process. Taking all this into account, it is clear that during welded joint analysis, large displacements and strains, i.e. geometric and material non-linearity, must be allowed. It is also clear that when defining the problem it is necessary to start with fundamental continuum mechanics equations, i.e. the coupled displacement and heat transfer equations, using the thermoelastic visco-plastic (TEVP) model of material behaviour. Use of TEVP causes many complications with practical calculation, for it is a complex and yet not fully developed procedure, but is necessary to apply in welding process analysis.

STRUCTURAL MECHANICS EQUATIONS AND CONSTITUTIVE FUNCTIONS

Welded joint analysis by finite element method is based on corresponding general continuum mechanics equations, i.e. coupled movement and heat conduction equations. Movement equations and their limiting conditions will be considered in the following form:

$$\left(z_a^i \sigma^{ab}\right)\big|_b + \rho f^i = \ddot{z}^i \ \text{ in } B \tag{1}$$

$$\sigma^i - z^i \sigma^{ab} n_b = 0 \ \text{ on } \partial B_\sigma \tag{2}$$

$$z^i - \bar{z}^i = 0 \ \text{ on } \partial B_z \tag{3}$$

where $z^i(\xi^a,t)$ represents Cartesian coordinates of a random element of object B at instant t; ξ^a are convective coordinates of the same element i, a = 1; 2; 3; $z_a^i = \partial z^i / \partial \xi^a$ are the base vectors; σ^{ab} is the stress tensor; ρ–density; f^i–volume force vector; σ^i–surface force vector; n_b–outer unit normal to the contour element ∂B_σ; $\bar{z}^i$–movements given to the contour element ∂B_z; ∂B_σ and ∂B_z are object B contours for which corresponding limit conditions (2) and (3) are defined, respectively; the | symbol signifies the covariant or countervariant differentiation along ξ^a coordinates; the point above letters indicates the time derivative, e.g. $\ddot{z}^i$ is the acceleration vector.

The heat transfer equation and its limit conditions will be considered in the following form:

$$\rho \mathbf{T} \dot{\eta} - q^b\big|_b - \rho r - \partial = 0 \ \text{ in } B \tag{4}$$

$$q - q^b v_b = 0 \ \text{ on } \partial B_q \tag{5}$$

$$\mathbf{T} - \bar{\mathbf{T}} = 0 \ \text{ on } \partial B_T \tag{6}$$

where $\mathbf{T}$ is the absolute temperature; η is the specific entropy; q is the heat flux; r is the heat source; ∂ is the inner dissipation; v_b is vector of the outer unit normal to the contour element ∂B_q; $\bar{\mathbf{T}}$ is the given temperature for the contour element ∂B_T.

In order to establish the relation between certain constitutive functionals (free energy, stress entropy and inner dissipation on one side, and heat flux and temperature on the other side), we must know the constitutive functions for free energy ψ and heat flux q^a. During the welding process, the material behaves in a thermo-elastic-visco-plastic manner.

It is very difficult to determine the free energy function that can in a satisfying way describe such material behaviour. Here, we will use the function based on the derivation which was given by Nowacki /1/, for linear thermo-elastic material. In accordance with his approach, paper /7/ gives the explicit form of the free energy equation, using convective coordinates. The equivalent free energy expression is given in /2/. Also given in this paper is the generalization in case of thermo-plastic materials. The free energy equation given in /7/ is expanded in a similar way in /2/, in case of TEVP material. Generally, the free energy function can be written as:

$$\psi = \psi\left(\gamma_{ab}, \gamma_{ab}^{HP}, \theta\right) \tag{7}$$

where ψ denotes free energy, while γ_{ab} is the strain tensor given by:

$$\gamma_{ab} = \frac{1}{2}\left(g_{ab} - G_{ab}\right) = \frac{1}{2}\delta_{ij}\left(z_a^i u_b^j + z_b^i u_a^j + u_a^i u_b^j\right) \tag{8}$$

Here, g_{ab} and G_{ab} are fundamental metric tensors in both current and referent configuration, in respect; $u_a^i = \partial u^i/\partial\xi^a$ is the displacement gradient; γ_{ab}^{HP} is the visco-plastic component of the strain tensor; θ is the relative temperature given by:

$$\theta = \mathbf{T} - \Theta \tag{9}$$

where Θ represents the referent temperature.

In case of an isotropic body and a linear elastic part of strain (which is an acceptable assumption for metals), the free energy function $\psi = \psi(\gamma_{ab}, \gamma_{ab}^{HP}, \theta)$ can be represented in the following form:

$$\begin{aligned}\rho_0\psi = {}& \frac{1}{2}E^{abcd}(\theta)\left(\gamma_{ab} - \gamma_{ab}^{HP}\right)\left(\gamma_{cd} - \gamma_{cd}^{HP}\right) - \\ & -E^{ab}(\theta)\left(\gamma_{ab} - \gamma_{ab}^{HP}\right)\theta + \\ & +c(\theta)\theta\left[\frac{\theta}{\Theta} - \left(1 + \frac{\theta}{\Theta}\right)\ln\left(1 + \frac{\theta}{\Theta}\right)\right]\end{aligned} \tag{10}$$

$$\begin{aligned}E^{abcd}(\theta) = {}& \frac{\mu(\theta)}{1 - 2\nu(\theta)}\left\{\nu(\theta)G^{ab}G^{cd} + \right. \\ & \left. +\left[1 - 2\nu(\theta)\right]G^{ab}G^{cd}\right\}\end{aligned} \tag{11}$$

$$E^{ab}(\theta) = 2\nu(\theta)\frac{1 + \nu(\theta)}{1 - 2\nu(\theta)}\alpha(\theta)G^{ab} \tag{12}$$

where $\mu(\theta)$, $\nu(\theta)$, $\alpha(\theta)$ and $c(\theta)$ are temperature–dependent material properties, and they are: shear modulus, Poisson's ratio, thermal expansion coefficient, and specific heat per unit volume, respectively, and G^{ab} is the counter-variant fundamental metric tensor in a referent configuration.

Since the free energy function is established, it is now possible to determine the remaining constitutive functions in the following way:

$$\sigma^{ab} = \rho D_{\gamma_{ab}}\psi = \frac{\rho}{\rho_0}\left[E^{abcd}(\theta)\left(\gamma_{cd} - \gamma_{cd}^{HP}\right) + E^{ab}(\theta)\right] \tag{13}$$

$$\rho_0\eta = \rho_0 D_\theta\psi \tag{14}$$

$$\partial = \sigma^{ab}\dot{\gamma}_{ab}^{HP} = -D_{\gamma_{ab}^{HP}}\psi\dot{\gamma}_{ab}^{HP} \tag{15}$$

where symbol D signifies the partial derivative, and its index indicates along which quantity.

The constitutive function for the heat flux q^a can be represented in the form Fourier heat conduction law:

$$q^a = \kappa(\theta)G^{ab}T_b \tag{16}$$

where $\kappa(\theta)$ is the heat conductivity coefficient, yet another temperature–dependent material property, and where $T_b = \partial T/\partial\xi_b$ is the temperature partial derivative along the convective coordinates.

The visco-plastic strain rate $\dot{\gamma}_{ab}^{HP}$ remains to be determined. Taking into account the fact that there is not a generally established way of determining this quantity, we will show two possibilities. According to the first, /3/, the visco-plastic strain rate is determined from the so-called over-stress function:

$$\dot{\gamma}_{ab}^{HP} = \lambda < \Phi\left[\frac{f\left(S^{ab}, \gamma_{ab}^{HP}, \xi\right) - 1}{\kappa(\xi)}\right] > \frac{\partial t}{\partial\sigma^{ab}} \tag{17}$$

where $f(\cdot)$ is the quasi-static flow (yield) function; $S^{ab} = (\rho/\rho_0)\sigma^{ab}$ is the stress tensor in a referent configuration; ξ is the scalar measure of cavity concentration (so-called imperfection parameter); Φ is the visco-plastic over-stress function; $\kappa(\theta)$ is the isotropic hardening parameter; λ is the viscosity constant; and symbols < > signify:

$$<\Phi> = \begin{cases} 0 & \text{for } r\ f(\cdot) < \kappa \\ \Phi & \text{for } f(\cdot) > \kappa \end{cases} \tag{18}$$

There are several expressions for the flow function $f(\cdot)$ and hardening parameter κ, some of which include certain effects that were neglected in previous cases, such as the presence of cavities and their concentration /3/.

According to the second approach which is more classical, plastic strain rates according to Masubushi and Muraki /4/ are combined with the viscous strain rate according to Brunet and Boyer /5/, which gives us:

$$\dot{\gamma}_{ab}^{HP} = \frac{3}{2\bar{S}}\left[\frac{1}{H}\left(\dot{\bar{S}} + \frac{\partial f}{\partial\theta}\dot{\theta}\right) + B\bar{S}^n\right]S_{ab}^D \tag{19}$$

In this expression, the equivalent stress is:

$$\bar{S} = \sqrt{\frac{3}{2}S_{ab}^D S^{abD}} \quad \text{(do not sum along } D\text{)} \tag{20}$$

while S_{ab}^D is the deviator part of the stress tensor. It is assumed that the stress on the yield level S_T depends on plastic strain and temperature, hence the expression for the yield function is:

$$f = \bar{S} - S_T\left(\gamma_{ab}^p, \theta\right) \tag{21}$$

Further, we have:

$$H = -\frac{\partial f}{\partial\bar{\gamma}^p} = \frac{\partial S_T}{\partial\bar{\gamma}} \tag{22}$$

where $\gamma^{\bar{p}}$ is the equivalent plastic strain /4/. Effectively, H can be determined from the following expression:

$$H = \frac{EE_t}{E - E_t} \tag{23}$$

where E_t is the tangent elasticity modulus which is determined for the adequate equivalent strain value /15/ given by:

$$\bar{\varepsilon} = \frac{3}{2} e^{ab} e_{ab} \tag{24}$$

where

$$e^{ab} = \gamma_{ab} - \frac{1}{3} G_{ab} G^{cd} \gamma_{cd} \tag{25}$$

represents the strain deviator, i.e. for the adequate uniaxial strain:

$$\varepsilon = \frac{2}{3} \left\{ \bar{\varepsilon} + \frac{S_T(\varepsilon,\theta)}{E(\theta)} \left[1 - 2v(\theta) \right] \right\} \tag{26}$$

It should be noticed that, since the stress on its yield level S_T is the function of strain ε, the strain ε from Eq. [26] must be determined iteratively from data given in S–ε diagram (Fig. 1). $\varepsilon = \bar{\varepsilon}$ is accepted as the initial (zero) iteration.

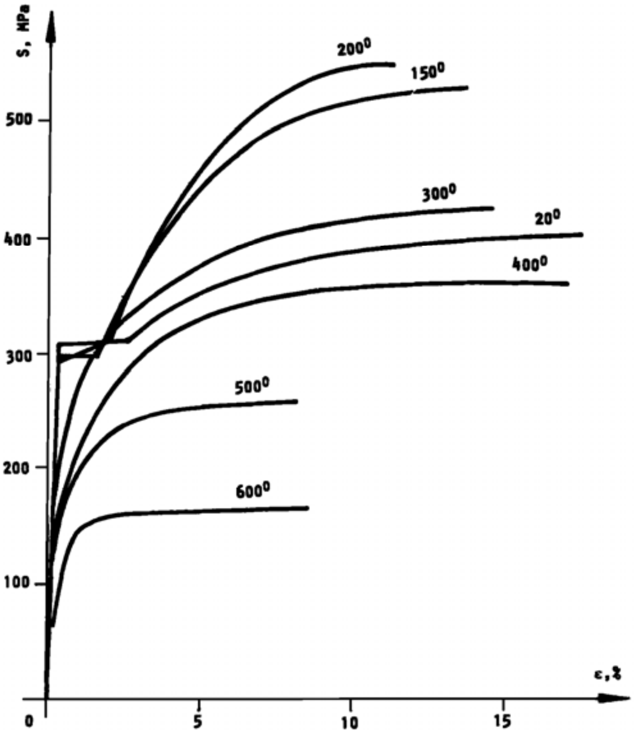

Figure 1. S–ε diagram dependence of temperature for steel St37 according to DIN.

The remaining problem in determining the equivalent stress rate, $\dot{\bar{S}}$, can be solved in accordance with /4/, using the following expression:

$$\dot{\bar{S}} = \frac{3}{2\bar{S}} S^D_{ab} \left\{ \left[E^{abcd} + \left(\frac{3\mu}{\bar{S}} \right)^2 \frac{1}{H + 3\mu} S^{ab}_D S^{cd}_D \right] \dot{\gamma}_{cd} + \right.$$
$$\left. + \left[E^{ab} + \frac{\partial E^{ab}}{\partial \theta} + \frac{1}{H + 3\mu} \left(\frac{H}{\mu} \cdot \frac{\partial \mu}{\partial \theta} + \frac{3\mu}{\bar{S}} \cdot \frac{\partial f}{\partial \theta} \right) S^{ab}_D \right] \dot{\theta} \right\} \tag{27}$$

For known S–ε diagrams in case of different temperatures, (Fig. 1), the derivative $\partial f/\partial \theta$ can be approximately determined by using the finite difference method:

$$\frac{\partial f}{\partial \theta} = -\frac{\partial S_T}{\partial \theta} = \frac{S_T(\theta_1) - S_T(\theta_2)}{\theta_2 - \theta_1} \tag{28}$$

Finally, we should notice that B, the temperature–dependent creep coefficient, and n, the creep exponent, represent parameters in the following relations:

$$\dot{\gamma}^V_{ab} = B\sigma^n_{ab} \tag{29}$$

measured for uniaxial creep.

FINITE ELEMENT EQUATIONS

Once the movement equations are determined, as shown in the previous chapter, it is possible to apply adequate procedures, i.e. that of Galerkin, in order to obtain discretized movement equations:

$$M^{IJ} \ddot{z}^j_I = R^{Jj} - S^{Jj} \tag{30}$$

Upper case indexes are related to the element node, I,J = 1,2,...,N, where N is the number of element nodes; lower case indexes are related to the problem dimension, i,j = 1,2,3 for a three-dimensional problem.

Taking into account that the discrete equations like [30] are obtained by interpolating and integrating certain quantities within a finite element, and then summing along all elements /6/, the matrices and vectors given in Eq. (30) can be written as:

- mass matrix $$M^{IJ} = \sum_e \int_{B_e} \rho P^I P^J dv \tag{31}$$

- external forces vector

$$R^{Jj} = \sum_e \int_{B_e} \rho f^j P^J dv + \sum_e \int_{\partial B_e} \sigma^j P^J ds \tag{32}$$

- internal forces vector $$S^{Jj} = \sum_e \int_{B_e} \rho z^j_a P^J_b D_{\gamma_{ab}} \psi dv \tag{33}$$

where P^I is the interpolation function; B_e is the element domain, and ∂B_e is its contour. Interpolation is carried out in such a way that variables are being separated: $\Phi(\xi^a, t) = P^I(\xi^a)\Phi(t)$.

In a similar way, we can discretize the heat conductivity equations, given by expressions [4]–[6], from which we have:

$$U^{IJ} \dot{T}_I = Q^J + \partial^J - G^J - D^J - J^J \tag{34}$$

Matrices and vectors in Eq. [34] have the following meaning and forms:

- heat capacity matrix

$$U^{IJ} = -\sum_e \int_{B_e} \rho T \delta T P^I P^J D_e \eta dv \tag{35}$$

- heat source and flux vector

$$Q^J = \sum_e \int_{B_e} \rho r P^J dv + \sum_e \int_{\partial B_e} q P^J ds \tag{36}$$

- inner dissipation vector

$$\partial^J = -\sum_e \int_{B_e} \rho \dot{\gamma}^{HP}_{cd} D_{\gamma^{HP}_{cd}} \psi dv \tag{37}$$

- inner flux vector

$$G^J = \sum_e \int_{B_e} q^b P_b^J dv \tag{38}$$

- thermo-elastic coupling vector

$$D^J = -\sum_e \int_{B_e} \rho T \delta_{ij} z_a^i \dot{z}_b^j P^J D_{\gamma_{ab}} D_\theta \eta dv \tag{39}$$

- thermo-highly-plastic coupling vector

$$J^J = -\sum_e \int_{B_e} \rho T \dot{\gamma}_{cd}^{HP} D_{\gamma_{cd}^{HP}} D_\theta \eta dv \tag{40}$$

It should be noticed that in Eq. [34], temperature change rate appears as the main variable which is of great practical significance for explicit numerical procedures in solving such equations /7/.

It should also be mentioned that while calculating the above matrices and vectors, the integration domain may be transformed into a referent configuration by using the following continuity equations:

$$\rho dv = \rho_0 dV \tag{41}$$

where ρ_0 represents the density, and dV is the volume element, and both quantities are in a referent configuration.

At the end of this chapter, we should mention that the authors of this paper have not come across equations of the type [34] in references, which contain thermo-visco-plastic coupling vectors.

SOLVING A COUPLED THERMO-MECHANICAL PROBLEM

Equations [30] and [34] are first transformed into a matrix form which is more suitable for solving:

$$M\ddot{z} = R - Sz \tag{42}$$

$$U\dot{T} = Q + \partial - G - D\dot{z} - JT \tag{43}$$

In order to solve such a problem, there are two procedures available – the implicit and the explicit procedure.

The implicit procedure

This procedure requires expanding Eqs. [42] and [43] into a Taylor series in time, linearize them and approximately solve the system of linear equations obtained in this way (for example, by using the central difference method). By using the procedure described in detail in /7/, we get:

$$\begin{bmatrix} 4M/h^2 + K + S & L \\ 2D/h + F & 2U/h + H + J \end{bmatrix} \begin{bmatrix} u \\ \tau \end{bmatrix} = \begin{bmatrix} R_h + R + 4M\dot{z}/h - 2Sz - h\delta R \\ Q_h + Q - wJT - 2G + 2\partial + h\Delta \end{bmatrix} \tag{44}$$

where, except for the already known expressions, additional matrices and vectors appear, whose members are determined in an already described way:

- stiffness matrix

$$K^{IiJj} = \sum_e \int_{B_e} \rho z_a^j z_c^i P_b^J P_d^I D_{\gamma_{ab}} D_{\gamma_{cd}} \psi dv \tag{45}$$

- thermo-mechanical coupling matrix

$$L^{IJj} = \sum_e \int_{B_e} \rho z_a^j P_b^J P^I D_{\gamma_{ab}} D_\theta \psi dv \tag{46}$$

- visco-plastic force matrix

$$\delta R^{Jj} = \sum_e \int_{B_e} \rho z_a^j \dot{\gamma}_{cd}^{HP} P_b^j D_{\gamma_{ab}} D_{\gamma_{cd}^{HP}} \psi dv \tag{47}$$

- thermomechanical coupling matrix

$$F^{IiJ} = -\sum_e \int_{B_e} \rho P^J P_d^I \left(\overline{Tz_c^i} D_{\gamma_{cd}} D_\theta \psi + T z_c^i D_{\gamma_{cd}} D_\theta \dot{\psi} \right) + \\ +\sum_e \int_{B_e} \rho z_c^i \dot{\gamma}_{ab}^{HP} P_d^I P^J \left(D_{\gamma_{cd}} D_{\gamma_{ab}^{HP}} \psi - T D_{\gamma_{cd}} D_\theta D_{\gamma_{ab}^{HP}} \psi \right) dv + \\ +\sum_e \int_{B_e} \rho z_c^i P_b^J P_d^I D_{\gamma_{cd}} q^b dv \tag{48}$$

- heat conductivity matrix

$$H^{ij} = \sum_e \int_{B_e} P_b^J \left(P^I D_\theta q^b + P_a^I D_{\theta_a} q^b \right) dv - \sum_e \int_{B_e} \rho P^I P^J \times \\ \times \left[\dot{T} D_\theta^2 \psi + T D_\theta^2 \dot{\psi} + \dot{\gamma}_{ab}^{HP} \left(T D_{\gamma_{ab}^{HP}} D_\theta^2 \psi - D_{\gamma_{ab}^{HP}} \psi \right) \right] dv \tag{49}$$

- visco-plastic flux vector

$$\Delta^J = \sum_e \int_{B_e} P_b^J \dot{\gamma}_{cd}^{HP} D_{\gamma_{cd}^{HP}} q^b dv + \sum_e \int_{B_e} \rho P_b^J \dot{\gamma}_{ab}^{HP} \dot{\gamma}_{cd}^{HP} \times \\ \times \left(D_{\gamma_{ab}^{HP}} D_{\gamma_{cd}^{HP}} \psi - T D_\theta D_{\gamma_{ab}^{HP}} D_{\gamma_{cd}^{HP}} \psi \right) dv \tag{50}$$

It is important to mention here that new variables are used in Eq. [44] – movement and temperature increase τ, within a given time interval h. If these variables, in the moment $t + h$, are denoted with index h, the following relations are valid:

$$\dot{z}_h = \frac{2}{h} u - \dot{z} \tag{51}$$

$$u = z_h - z \tag{52}$$

$$\tau = \theta_h - \theta \tag{53}$$

When solving system [44], one should have in mind that the system matrix is asymmetrical and that this system is actually an approximation of the initial system [42]–[43]. Hence, it is necessary to use iteration within the time interval h in order to obtain the solution which will satisfy the initial system. For this purpose, one should solve a matrix system obtained by a time integration within interval h of the system [44]:

$$\begin{bmatrix} 4M/h^2 + K + S & L \\ 2D/h + F & 2U/h + H + J \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta \tau \end{bmatrix} = \begin{bmatrix} R_h - S_h z_h - M\ddot{z} \\ Q_h + \partial_h - G_h - D_h \dot{z}_h - J_h T_h - U_h \dot{T}_h \end{bmatrix} \tag{54}$$

Once $u = u + \Delta u$ and $\tau = \tau + \Delta\tau$ are known within time interval h, one can obtain the required quantities at the end of the time interval:

$$z_h = z + u \tag{55}$$

$$\dot{z}_h = \frac{2}{h} u - \dot{z} \tag{56}$$

$$\ddot{z}_h = \frac{4}{h^2} u - \frac{4}{h} \dot{z} - \ddot{z} \tag{57}$$

$$T_h = T + \tau \tag{58}$$

$$\dot{T}_h = \frac{2}{h}\tau - \dot{T} \tag{59}$$

The explicit procedure

This procedure is based on initial velocity and acceleration values, visco-plastic strain and its rate, temperature and its changing rate. Using these values, it is possible to determine the final values of all above quantities within a chosen time interval, and from them to determine the remaining necessary quantities. This way, the initial values for the following interval can also be determined. Significant for this is the length of the time interval h which is in case of an elastic material limited by Neumann's criterion:

$$h < \min \alpha \left(\frac{\ell}{c}\right)_{\min} \tag{60}$$

where α is the linear dimension of a finite element, and c is the normal impact wave extension velocity, given by the following expression:

$$c = \frac{2}{\rho_o} \cdot \frac{1-\nu}{1-2\nu} \tag{61}$$

and α is an empirical decrease factor, for which the recommended values are between 0.2 and 0.9, /8/.

Once the time interval length is determined, the prediction–evaluation–correction scheme can be used.

Prediction

$$\dot{z}_h = \dot{z} + h\ddot{z} \tag{62}$$

$$T_h = T + h\dot{T} \tag{63}$$

$$\gamma_{ab_h}^{HP} = \gamma_{ab}^{HP} + h\dot{\gamma}_{ab}^{HP} \tag{64}$$

where $\dot{z}$, $\ddot{z}$, T, $\dot{\mathbf{T}}$, γ_{ab}^{HP} and $\dot{\gamma}_{ab}^{HP}$ are the initial values within the time interval h.

Evaluation

- radius vector $\quad z_h = z + \frac{h}{2}\left(\dot{z} + \dot{z}_h\right) \quad (65)$

- displacement (based on displacement and according to the given relations, the strain γ_{ab_h} and free energy ψ are calculated and then used to calculate the stress σ_{ab_h})

$$u = z_h - z \tag{66}$$

- stress deviator $\quad S_{ab_h}^D = S_{ab_h} - \frac{1}{3}G_{ab} \quad (67)$

- fon Mises stress

$$\overline{S}_h = \sqrt{\frac{3}{2}S_{D_h}^{ab}S_{ab_h}^{D}} \quad \text{(no summing along } D\text{)} \tag{68}$$

The visco-plastic strain rate can be determined in one of the above ways, e.g. from Eq. [19]. This produces all the quantities required to calculate $\ddot{z}_h$ and $\dot{T}_h$ directly from Eqs. [42] and [43]:

$$\ddot{z}_h = M^{-1}\left(R_h - S_h z_h\right) \tag{69}$$

$$\dot{T}_h = U^{-1}\left(Q_h + \partial_h - G_h - J_h T_h - D_h \dot{z}_h\right) \tag{70}$$

Correction

$$\dot{z}_h = \dot{z} + \frac{h}{2}\left(\ddot{z} + \ddot{z}_h\right) \tag{71}$$

$$T_h = T + \frac{h}{2}\left(\dot{z} + \dot{z}_h\right) \tag{72}$$

$$\gamma_{ab_h}^{HP} = \gamma_{ab}^{HP} + \frac{h}{2}\left(\dot{\gamma}_{ab}^{HP} + \dot{\gamma}_{ab_h}^{HP}\right) \tag{73}$$

Some notes on implicit and explicit procedures

First of all, it is clear that the implicit procedure is much more complicated, hence its unlikely to expect its application on extremely coupled thermo-mechanical problems /7/. This is especially the case with materials whose behaviour is described using complex constitutive functions. However, if it is possible to uncouple the problem, as it is shown in examples in the following chapter, the implicit procedure may be efficiently applied. Also should be kept in mind that the implicit procedure must be used for static problems.

The basic criterion for comparing the implicit and the explicit procedure is calculating the time, the efficiency of integration time in other words. It is clear that this in a certain way reduces the comparison to a time interval length which can be determined in various ways. In explicit procedure, the time interval length depends on material's elastic properties and can be increased by increasing the temperature, which is especially suitable for the case considered here. For an implicit procedure, the time interval length is mostly determined by the required accuracy. However, it is to notice that although this procedure is generally considered as an unconditionally stable one, this does not have to be the case for non-linear problems.

NUMERIC PROCEDURES IN WELDED JOINT FRACTURE MECHANICS

By solving the coupled thermo-mechanical problem which was described in the previous chapter, the residual stresses and strains in a welded joint can be determined. Although the residual stresses may be eliminated with additional heat treatment, such procedure is usually costly or unfeasible for real structures. Hence, it is justifiable to assume that residual stresses and strains which occur in a welded joint will have a relevant effect on the appearance and crack growth.

Taking into account that during welding the material suffers strains which go above yield stress, when speaking of welded joint fracture mechanics, one must first consider elastic-plastic fracture mechanics parameters – crack (tip) opening displacement, i.e. C(T)OD, and the J integral. Evaluation of these parameters with the use of numeric methods (e.g. the most common, finite element method) is problematic and requires additional knowledge in this area. Here, this problem will not be considered (a series of international conferences /8,9/ and /10/ as well as a recently published book /11/, are devoted to it anyway), instead, the peculiarities of finite element method application in determining COD and J-integral will be emphasized.

The procedure for COD determination is principally the same as for homogeneous materials, see /12/. Welded joint heterogeneity is involved via the input material properties data which vary for different welded joint regions, and also via the internal stress distribution which was determined in a described way.

In case of the J integral, taking into account the presence of residual stresses and strains and in order to maintain path-independence, it is necessary to compute the J integral with an area (surface) integral /13/:

$$J^* = \int_{\Gamma}\left(W dy - \sigma^{ij} n_j \frac{\partial u_i}{\partial x} ds \right) + \int_{A} \sigma^{ij} \frac{\partial \theta_{ij}}{\partial x} dA \qquad (74)$$

where θ_{ij} are the residual strains.

EXAMPLES

In this chapter two examples taken from the references will be presented. Both of these cases are about significant simplifications compared to the solving procedure given in this paper, which was justified by the examples considered.

The first example is related to rectangular steel plate joint by electric-arc butt welding /14/. The plate, along with basic welding data is given in Fig. 2, and the change of material properties (steel St37 according to DIN) with temperature is given in Fig. 3. The changes on S–ε curves are already given in Fig. 1.

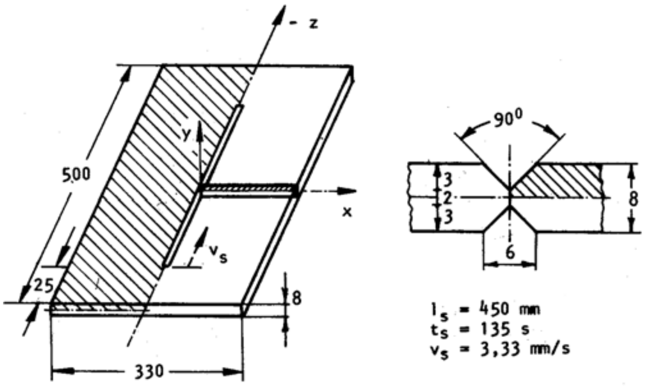

Figure 2. Arc-welding of a steel plate.

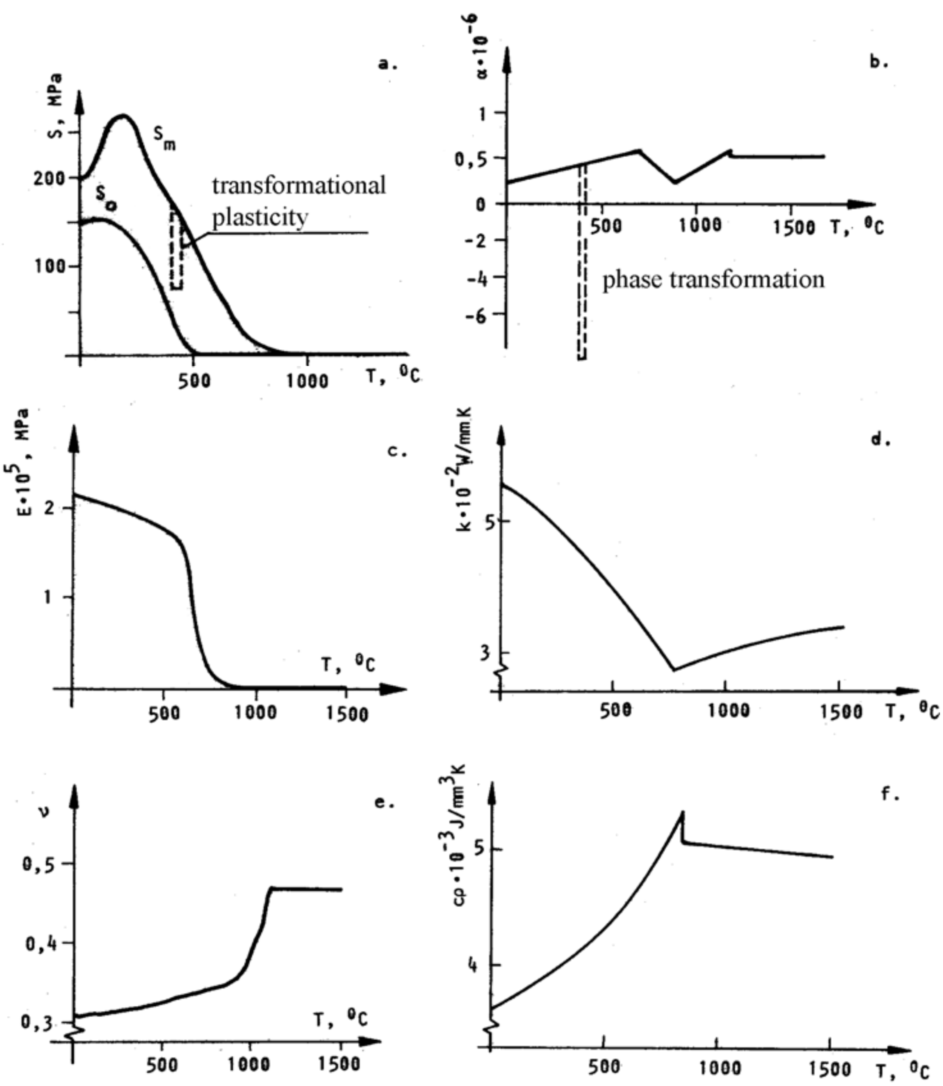

Figure 3. Change of DIN St37 steel properties with the change of temperature.
a. yield stress; b. thermal expansion coefficient; c. elasticity modulus; d. Poisson's ratio; f. specific heat.

In order to assess the accuracy of the results obtained by using the finite element method, adequate temperature field and strains were measured. The residual stresses were determined based on these measurements.

The calculation according to the finite element method is divided into two phases:
– in phase one the variable temperature field for a moving heat source (an electrode) is determined, assuming that the heat conductor is absolutely stiff. The implicit procedure was implied in solving a heat conduction problem according to the following scheme:

$$\left(\frac{1}{h}U + H\right) = \Delta Q$$

$$\dot{T}_h = \frac{1}{h}\tau - \dot{T}$$

$$\left(\frac{1}{h}U + H\right)\Delta\tau = Q_h - G_h - U_h\dot{T}_h$$

$$\tau = \tau + \Delta\tau\,, \quad T = T + \tau$$

If $\dfrac{|\Delta\tau|}{|\tau|} > 10^{-3}$, the procedure is repeated.

The temperature distribution obtained this way is shown in Fig. 4, and the corresponding weld root temperature history is shown in Fig. 5.

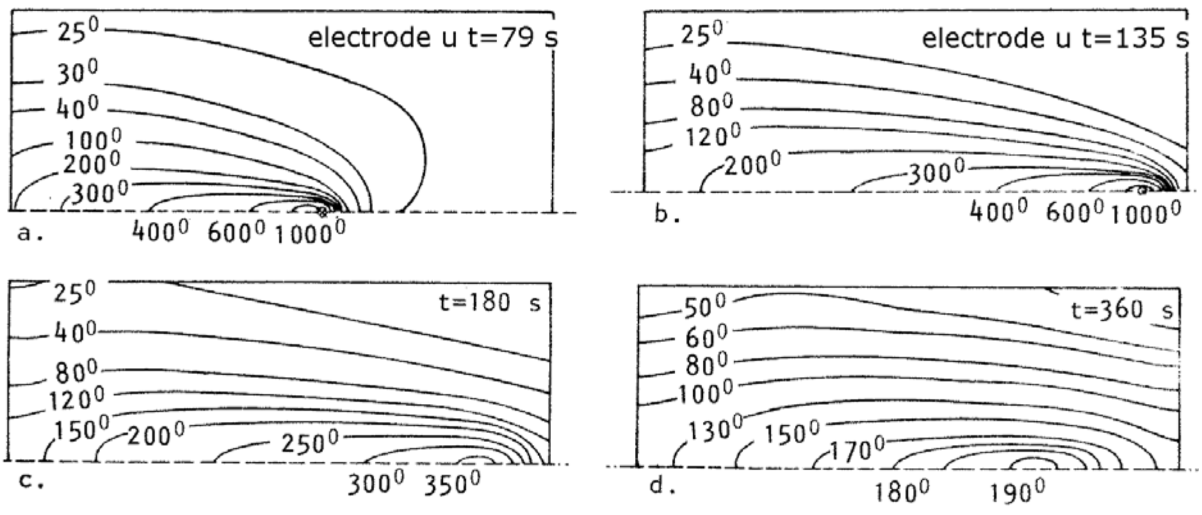

Figure 4. Temperature distribution.
a. after 79 s, b. after 135 s, c. after 180 s, d. after 360 s.

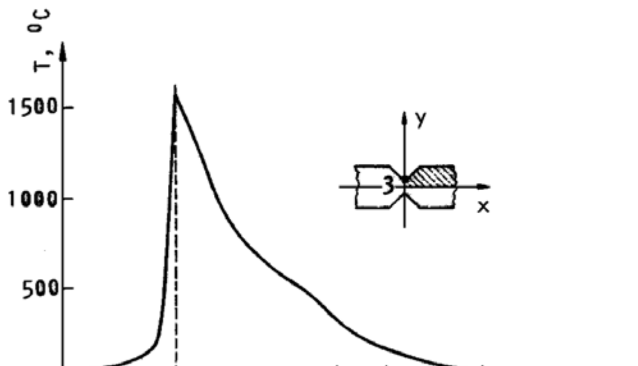

Figure 5. Weld root temperature history.

In phase two, the stresses and strains at the middle of the plate cross-section are calculated, assuming plane stress state. Also, the weld root temperature history is considered to be known. The implicit procedure applied can be schematically shown in the following way:

$$u_i = u_{i-1}$$

$$\left(\frac{1}{h}U + H\right)\tau = Q_h + Q - 2JT - 2G - 2\partial - h\Delta - \left(\frac{1}{h}D + F\right)U$$

$$\dot{T}_h = \frac{1}{h}\tau - \dot{T}$$

$$\left(\frac{1}{h}U + H\right)\Delta\tau = Q_h + \partial_h - G_h - D_h\dot{z}_h - J_hT_h - U_h\dot{T}_h$$

$$\tau = \tau + \Delta\tau\,,\ T = T + \tau$$

If $\dfrac{|\Delta u|}{|u|} \geq 10^{-3}$, the procedure is repeated starting from $z_h = z + u$.

Inertial forces and the visco-plastic part of the temperature matrix system were neglected in this procedure. The effect of strain on temperature is considered as small, hence movement and heat conductivity equations are uncoupled.

Residual stresses and strains on the upper side of the plate obtained in this way are shown in Figs. 6 and 7, in respect, along with their corresponding experimental data.

Two calculation options - incremental elastic and elastic-visco-plastic are shown in Fig. 6. The second one shows significantly better agreement with the experiment. Figure 7 shows two calculation options for residual strains - with and without the phase effect. The results show that this difference is negligible.

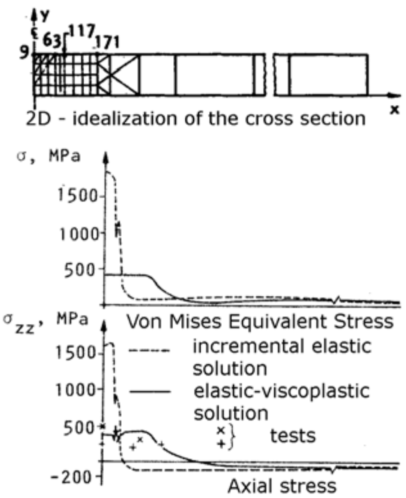

Figure 6. Residual stress distribution.

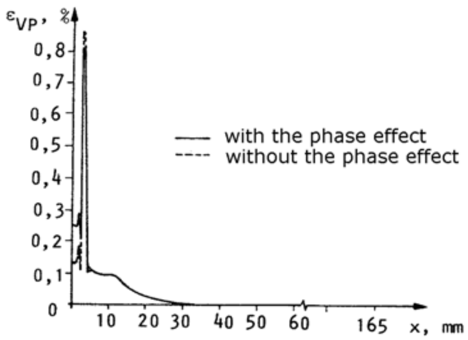

Figure 7. Residual strain distribution.

The second example is related to one-dimensional analysis of welded joint /4/. The sample shown in Fig. 8 was also experimentally tested using strain gages, thermocouples and extensometers (their distribution is shown in Fig. 8) in order to determine the temperature, strain and stress fields. The material (aluminium) property dependence on temperature is shown in Fig. 9.

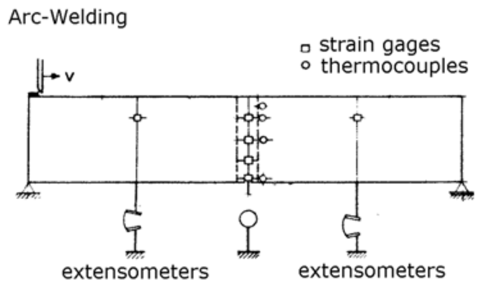

Figure 8. Welded sample.

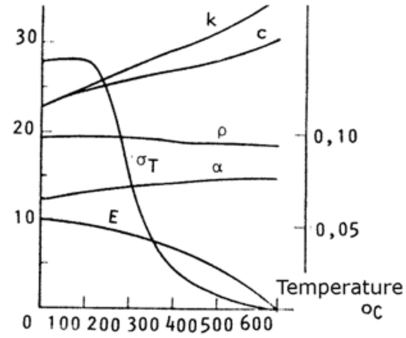

Figure 9. Aluminium properties dependence on temperature.

Calculation and experimental measurement results are shown in Fig. 10 (temperature history during welding in four typical points).

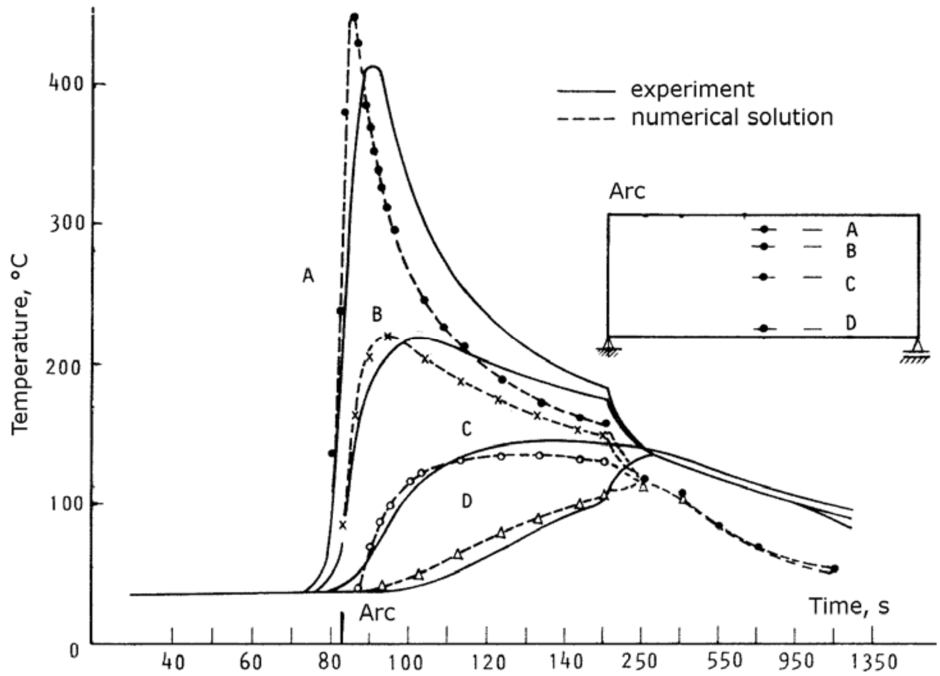

Figure 10. Temperature history during welding in four typical points.

Figure 11 (longitudinal strains during welding for three chosen points) and in Fig. 12 (residual stresses along the middle cross-section of the plate). From these results one can conclude that the temperature distribution has agreed well with the experimental measurement, while the residual stresses and strains exhibit qualitative agreement, with certain quantitative scatter. Such conclusion is generally typical for the up-to-date state of this numerical structural mechanics region.

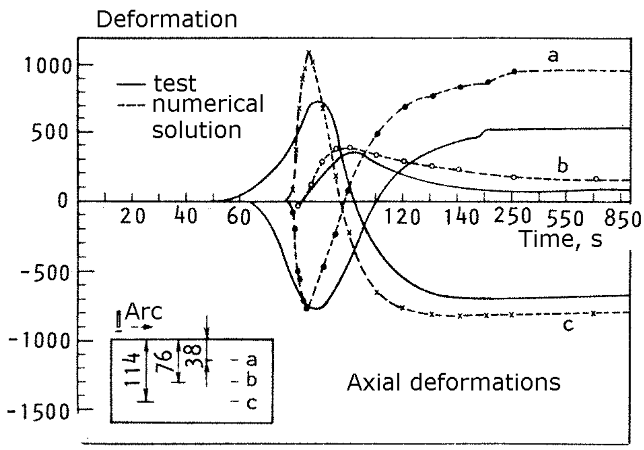

Figure 11. Longitudinal strains during welding.

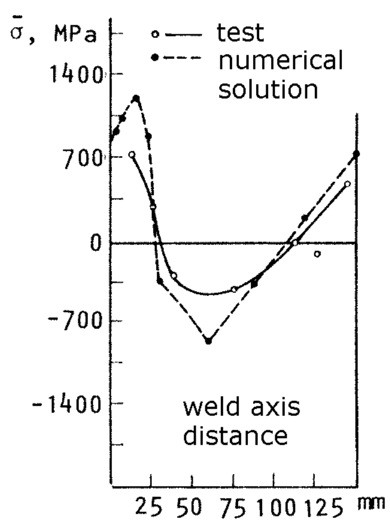

Figure 12. Residual stresses $\bar{\sigma}$.

CONCLUSION

This paper describes the application of finite elements method in welded joint and welding process analysis. General finite elements coupled thermo-mechanical behaviour equations were derived, and the procedure of solving the coupled thermo-mechanical problem was developed. Taking into account the complexity of this procedure, references only contain the examples solved by uncoupling of thermal and mechanical equations.

By analyzing some reference examples the conclusion is made that the results obtained for the temperature distribution agree with experimental results. As for strains and stresses, the values calculated agree with the experimental ones only qualitatively, with significant differences in values. In order to eliminate this disagreement it is necessary to solve a coupled problem, which requires a high-speed computer of extended memory and significant man hours as well.

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