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STUDY OF SURFACE FATIGUE CRACKS

ISTRAŽIVANJE POVRŠINSKIH ZAMORNIH PRSLINA

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Izvod

U analizi je izložena primena linearno elastične mehanike loma na predviđanje rasta zamorne prsline na primeru površinskih prslina kod čelika BS 4360-50D i u metalu šava njegovog sučeonog zavarenog spoja pri savijanju u 4 tačke. Predviđanje se bazira na dostupnim eksperimentalnim podacima i na zakonu rasta zamorne prsline koji su predložili Branko i saradnici. Za proračun raspona faktora intenziteta napona korišćena su rešenja u analizi Skota i Torpa. Razvijeni su računarski programi za ovo predviđanje. Krive predviđanja rasta prsline i zavisnosti dužine prsline po celoj debljini i duž površinskih pravaca se dobro slažu sa eksperimentalnim podacima.

Uticaj početnih konfiguracija prsline na njen oblik, profil prsline, kao i varijacije faktora intenziteta napona u tačkama na najvećoj dubini i na površini imaju jedinstvenu karakteristiku sličnu drugim objavljenim rezultatima. Predviđanje profila prsline je pokazalo da se konačni lom epruvete javlja kada je prsline duž površina dostigla ivične strane epruvete. Oblici profila zavise od debljine, širine i početne konfiguracije prsline.

Abstract

This study presents the application of linear elastic fracture mechanics to the prediction of fatigue crack growth of surface cracks in BS 4360-50D steel and in weld metal of its butt-welded joint under 4-point bending. The prediction is based on the available experimental data and the fatigue crack growth law proposed by Branco et al. To calculate stress intensity factor ranges, the solutions reviewed by Scott and Thorpe have been used. Computer programmes have been developed for this prediction. The predicted crack growth curves and relationships between the crack length in the through thickness and along the surface directions correlate well with the experimental data.

The effects of initial crack configurations on the crack shape, crack profile and variation of stress intensity factors at the deepest and surface points have a unique characteristic similar to other published results. The prediction of crack profiles has shown that the final failure of the specimen is caused when the crack along the surface reached the sides of the specimen. The shapes of the profiles depend on the thickness, width and initial crack configuration.

UVOD

Bez obzira na obimnu literaturu o zamoru metala, zamor je i dalje veliki problem i najčešći uzrok otkaza inženjerskih konstrukcija. Ovi otkazi se najčešće javljaju zbog inicijacije potkritičnih prslina i njihovog rasta do kritične veličine pri kojoj se javlja nestabilan lom. Ove potkritične prsline se mogu uneti tokom izrade, posebno kod ukleštenih zavarenih konstrukcija ili se razvijaju tokom eksploatacije. Praktično čitav zamorni vek se odnosi na rast zamorne prsline, stoga je od suštinskog značaja razumevanje brzine rasta zamorne prsline u konstrukcijskom materijalu radi predviđanja radnog veka različitih konstrukcija, kao i radi utvrđivanja sigurnih intervala za inspekcije.

Novi koncept projektovanja korišćenjem mehanike loma je dokazao da je mnogo pogodniji za opisivanje i analizi ponašanja pri zamornom rastu prsline u odnosu na koncept tradicionalnog projektovanja. Mehanika loma pretpostavlja postojanje prslina ili grešaka u realnoj konstrukciji i koristi principe mehanike kontinuuma za uvođenje parametara koji karakterišu naponsko polje ispred čela prsline.

INTRODUCTION

Despite the voluminous literature on metal fatigue, fatigue continues to be a major problem and is the most common cause of failures in engineering structures. These failures are usually caused by the initiation of subcritical cracks and their propagation to the critical size at which unstable fracture occurs. These subcritical cracks may be introduced during fabrication, particularly in constrained welded structures, or developed during the service. Virtually the total fatigue life is occupied by fatigue crack growth, thus, to predict the service life of many structures and to establish safe inspection intervals, an understanding of the fatigue crack growth rate in structural material is essential.

The new design concept using fracture mechanics has proved to be more suitable for the description and analysis of fatigue crack growth behaviour than the traditional design concept. Fracture mechanics assumes the pre-existence of cracks or flaws in a real structure and utilizes continuum mechanics principles to develop parameters which characterize the stress field ahead the crack front.

Kada je materijal linearno elastičan, analize pokazuju da su naponi (deformacije) singularni u toj oblasti, a obrnuto su proporcionalni kvadratnom korenu rastojanja od vrha prsline ili njenog čela.

FAKTOR INTENZITETA NAPONA ZAMORNE PRSLINE

Danas je dobro poznato da prisustvo grešaka u komponenti skraćuje zamorni vek konstrukcije. U mnogim razmatranjima projektovanja realno je pretpostaviti postojanje grešaka tipa prslina, posebno kod zavarenih konstrukcija, gde se greške obično nalaze u zoni prelaza između metala šava i osnovnog materijala.

Površinske prsline ili delimično prolazne prsline su najčešće greške u mnogim komponentama konstrukcija. Pretpostavlja se da imaju polueliptične oblike. Utvrđeno je da na brzinu rasta prsline utiče naponsko stanje na vrhu prsline, koje se u linearno elastičnoj mehanici loma (LEML) opisuje veličinom „faktora intenziteta napona K". Prema tome, primena LEML na ove prsline zahteva poznavanje faktora intenziteta napona za polueliptičnu prslinu, što ustvari podrazumeva meru intenziteta napona.

Primena LEML zavisi isključivo od koncepta faktora intenziteta napona. Potrebna je tačna K-kalibracija (mera) za polueliptičnu prslinu radi pouzdanog predviđanja zamornog veka kod komponente sa površinskom prslinom. Mnogi istraživači su upotrebom raznih metoda pokušali da dođu do kalibracija koje bi mogle da daju najbolju inženjersku procenu faktora intenziteta napona na vrhu prsline.

U slučajevima gde je geometrija relativno jednostavna mogu se primeniti analitički pristupi kao što su složene analitičke metode i metode integralne transformacije. Eksperimentalne metode su takođe praktične u nekim jednostavnim slučajevima. Međutim, faktor intenziteta napona se mora odrediti posredno preko merljive veličine kao što su popustljivost ili deformacija. Prostorna geometrija i granični uslovi problema sa prslinom su često komplikovani u praksi, pa se moraju upotrebiti numeričke metode. Rešenja faktora intenziteta napona se mogu dobiti numerički metodama kao što su metoda konačnih elemenata, granična integralna jednačina i metode granične kolokacije.

OPŠTI OBLIK REŠENJA

Ranija analiza elastičnog naponskog polja oko vrha prsline je pokazala da je u slučaju beskonačne ploče sa centralnom prslinom dužine $2a$, pod dejstvom udaljenog napona σ, sl. 1, faktor intenziteta napona, K_I, definisan u obliku

$$K_I = \sigma\sqrt{\pi a} \qquad (1)$$

Pre nego što je Irvin /1/ izveo opšte rešenje za polueliptičnu površinsku prslinu, Snedon /2/ je rešavao problem sa kružnim oblikom prsline u beskonačnom telu pod dejstvom ravnomernog zatezanja. Rešenje po Snedonu je

$$K_I = \frac{2\sigma}{\pi}\sqrt{\pi a} \qquad (2)$$

Njegov rad pruža osnovu za odgovarajuća rešenja kod polueliptične površinske prsline. Irvin /1/ je iskoristio rešenje Grina i Snedona /3/ za eliptičnu prslinu unutar beskonačnog čvrstog tela pod dejstvom jednoosnog zatezanja i izračunao faktor intenziteta napona kao

When the material is linearly elastic, analyses show that stresses (strains) are singular in that region, and are inversely proportional to the square root of the distance from the crack tip or front.

STRESS INTENSITY FACTOR FOR FATIGUE CRACKS

It is now well established that the presence of flaws within a component reduce the fatigue life of a structure. For many design considerations it is quite realistic to assume that crack-like defects are always present, especially in welded structures where flaws are generally situated at the weld/parent metal interface.

Surface cracks or part-through cracks are the most common flaws in many structural components. They are assumed to have semi-elliptical shapes. It is found that the rate of propagation of the crack is controlled by the state of stress at the crack tip which is described in linear-elastic fracture mechanics (LEFM) terms by "the stress intensity factor K". Therefore, the application of LEFM to these cracks requires knowledge of the stress intensity factor for a semi-elliptical crack, that in fact refers to the stress intensity calibration.

Application of LEFM depends solely on the concept of stress intensity factor. For reliable prediction of fatigue lifetimes of a surface-cracked component, accurate K-calibration for a semi-elliptical crack is necessary. Many investigators have attempted to find calibrations which could give the best engineering estimate of the crack tip stress intensity factor by using various techniques.

In cases where the geometry is relatively simple, analytical approaches such as complex analytical methods and integral transform methods may be used. Experimental methods are also feasible in some simple cases. However, the stress intensity factor has to be determined indirectly through a measurable quantity such as compliance or strain. The physical geometry and the boundary conditions of crack problems are usually complicated in practical situations, so numerical methods have to be used. Stress intensity factor solutions can be obtained numerically by methods such as Finite Element Methods, the Boundary Integral Equation and Boundary Collocation Methods.

GENERAL FORM OF SOLUTION

The early analysis of elastic stress field around the crack tip has shown that, for an infinite plate with central crack of length $2a$, under remotely applied stress σ, Fig. 1, stress intensity factor, K_I, is defined in the form

$$K_I = \sigma\sqrt{\pi a} \qquad (1)$$

Before a general solution for a semi-elliptical surface crack was derived by Irwin /1/, Sneddon /2/ had tackled the problem on an embedded penny-shape in an infinite solid subjected to uniform tension. Sneddon's solution is

$$K_I = \frac{2\sigma}{\pi}\sqrt{\pi a} \qquad (2)$$

His work provides the foundation for the appropriate solutions for a semi-elliptical surface crack. Irwin /1/ utilized the solution of Green and Sneddon /3/ for an elliptical crack embedded in an infinite solid under uniaxial tension and he derived the stress intensity factor as

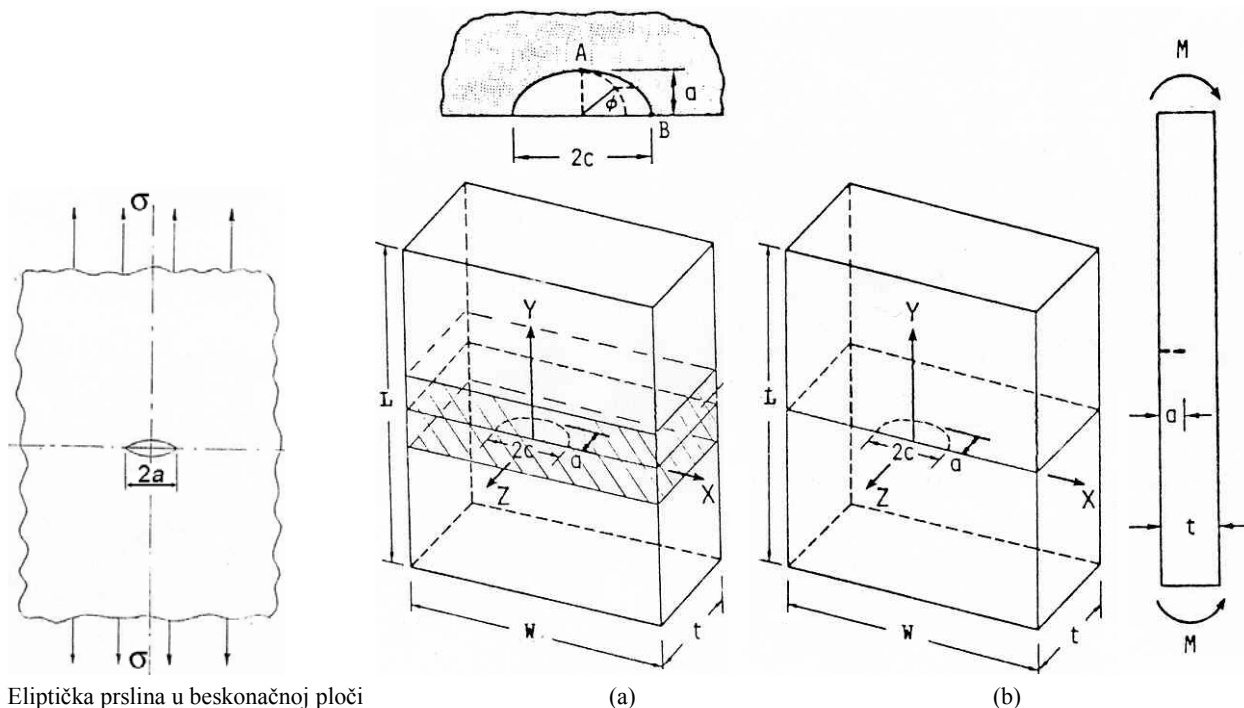$$K_I = \frac{\sigma\sqrt{\pi a}}{E(k)}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{0.25} \quad (3)$$

gde je korišćen eliptički integral druge vrste, *E(k)*,

$$E(k) = \int_0^{\pi/2}\left[1 - \frac{c^2 - a^2}{c^2}\sin^2\phi\right]^{0.5} d\phi \quad (4)$$

$$K_I = \frac{\sigma\sqrt{\pi a}}{E(k)}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{0.25} \quad (3)$$

where the elliptical integral of the second kind, *E(k)*, is used

$$E(k) = \int_0^{\pi/2}\left[1 - \frac{c^2 - a^2}{c^2}\sin^2\phi\right]^{0.5} d\phi \quad (4)$$

Slika 1. Eliptička prslina u beskonačnoj ploči pod dejstvom udaljenog ravnomernog zatezanja
Figure 1. Elliptical crack in infinite plate under remote uniform tension.

(a)   (b)
Slika 2. Shematski prikaz površinske prsline pod dejstvom zamornog savijanja u 4 tačke, (a) zavarena epruveta; (b) obična epruveta
Figure 2. Schematic diagram of surface crack under 4-point bending fatigue, (a) welded specimen; (b) plain specimen.

Uvedeni su i korekcijski faktori koji obuhvataju uticaj konačnih dimenzija prsline i tela sa prslinom na faktor intenziteta napona. Oni zavise od parametarskih odnosa *a/c*, *a/t* i *c/W*, sl. 2. Unošenjem ovih faktora u izraz [3], on može da se napiše u obliku

$$K_I = f\left(\frac{a}{c}, \frac{a}{t}, \frac{a}{W}\right)\cdot\frac{\sigma\sqrt{\pi a}}{E(k)}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{0.25} \quad (5)$$

gde je *f(a/c, a/t, a/W)* funkcija koja sadrži faktor uvećanja ivice čela i korekcijske faktore za konačnu debljinu i za konačnu površinu.

Izraz [5] se može smatrati opštim oblikom rešenja za polueliptičnu površinsku prslinu u konačnom čvrstom telu. Međutim, vrednost funkcije *f* je u velikoj meri zavisna od tipa opterećenja kojim se zadaje delujući napon *σ*. Najčešće se analiza izvodi za zatezno opterećenje ili savijanje ili njihovu kombinaciju. Da bi rezultati bili što realniji, korekcijski faktor plastičnosti vrha prsline bi takođe morao da se uvrsti u izraz [5]. Međutim, on se obično zanemaruje jer se u LEML pretpostavlja mala veličina plastične zone u poređenju sa dužinom prsline.

In order to take account of the effects of finite geometries of the crack and cracked body on the stress intensity factor, correction factors are introduced. They are dependent on the parameters *a/c*, *a/t* and *c/W* ratios, Fig. 2. Including these factors into Eqn. [3], it may be written as

$$K_I = f\left(\frac{a}{c}, \frac{a}{t}, \frac{a}{W}\right)\cdot\frac{\sigma\sqrt{\pi a}}{E(k)}\left[\sin^2\phi + \frac{a^2}{c^2}\cos^2\phi\right]^{0.25} \quad (5)$$

where *f(a/c, a/t, a/W)* is a function which consists of the front face magnification factor, and correction factors for finite thickness and finite area.

Equation [5] may be considered as the general form of solution for a semi-elliptical surface crack in a finite solid. However, the value of function *f* is very much dependent on loading types which specify the value of applied stress *σ*. Most of the analysis is concerned with either tension load or bending load or a combination of the two. To be more realistic, the correction factor for crack tip plasticity should also be included in Eqn. [5]. However, it is usually omitted since in LEFM plastic zone size is assumed to be small compared with the crack length.

KOREKCIJSKI FAKTORI ZA KONAČNE DIMENZIJE

Irvin /1/ je uveo konstantu veličine 1,1 u svoj originalni izraz (jedn. [3]) koja predstavlja uticaj dve neopterećene površine. Dobijen je rezultat

$$K_I = 1,1 \frac{\sigma \sqrt{\pi a}}{E(k)} \left[\sin^2 \phi + \frac{a^2}{c^2} \cos^2 \phi \right]^{0.25} \qquad (6)$$

Pokazalo se da se jedn. [6] u razumnoj meri slaže sa eksperimentalnim podacima za plitku prslinu ($a/t < 0,5$), sl. 2. Kod dublje prsline ($a/t > 0,5$), jedn. [6] teži ka smanjenju vrednosti faktora intenziteta napona čak i do 90 procenata. Istraživači su pokušavali da procene ili numerički odrede faktor intenziteta napona za duboku površinsku grešku da bi dobili bolja slaganja sa eksperimentalnim rezultatima. Ovaj pokušaj je doveo do predlaganja mnogobrojnih izraza za gore spomenute korekcijske faktore. Svaki od ovih korekcijskih faktora će biti posebno diskutovan u sledećim odeljcima sa ograničenjem na najdublju tačku ($\phi = \pi/2$) i tačku preseka na površini ($\phi = 0$).

FAKTOR UVEĆANJA IVICE ČELA $M_{f(\Phi)}$

Za polueliptičnu površinsku prslinu u polubeskonačnoj ploči potreban je korekcijski faktor ivice čela. Irvin /1/ je smatrao da uticaj slobodne površine omogućava veće otvaranje prsline (pomeranje otvora prsline – COD), a time se povećava vrednost faktora intenziteta napona. Predložio je izraz kao što je dat jedn. [6]. Kobaiaši i Mos /4/ su procenili faktor uvećanja ivice čela za duboku površinsku grešku primenom postojećih rešenja i takođe su predložili izraz za ivicu čela.

RAST ZAMORNE PRSLINE

Zamorna ispitivanja radi ocene svojstava materijala ili konstrukcije se mogu izvesti kao zamor pri konstantnoj amplitudi opterećenja, pri konstantnoj amplitudi deformacija i pri pozitivnom ili negativnom srednjem opterećenju. Načelno postoje dve filozofije projektovanja konstrukcija na zamor, projektovanje za sigurni vek i projektovanje sa sigurnošću od loma. Prvi je u osnovi analitički, kada se u određenom periodu rada ne očekuje pojava opasne zamorne prsline. U drugom vidu se pretpostavlja da je konstrukcija već oštećena i da će nastaviti da radi sa postojećim oštećenjem. Projektovanje sa sigurnošću od loma je danas češće.

Dugi niz godina inženjeri su primenjivali tradicionalni pristup projektovanja protiv zamora. Dopušteni zamorni naponi se zasnivaju na rezultatima ispitivanja pažljivo izvedenih glatkih ili zarezanih laboratorijskih epruveta. Rezultati su predstavljeni dijagramom napon–broj promena do loma i poznati su kao S-N kriva. Na osnovu filozofije projektovanja sa sigurnošću od loma je sagledano da većina konstrukcija, posebno zavarenih, sadrži greške, tako da se čitav zamorni vek karakteriše rastom zamorne prsline.

Prema tome, u projektovanju je neophodno poznavanje osobina rasta zamorne prsline u datom materijalu. Poteškoće se javljaju kada se primeni tradicionalni pristup jer on ne daje nikakve podatke o relativnom udelu inicijacije i rasta prsline na ukupni zamorni vek.

FINITE GEOMETRIES CORRECTION FACTORS

Irwin /1/ introduced the constant of value 1.1 to his original expression (Eqn. [3]) to account for the effect of the two stress-free surfaces. The result was

$$K_I = 1.1 \frac{\sigma \sqrt{\pi a}}{E(k)} \left[\sin^2 \phi + \frac{a^2}{c^2} \cos^2 \phi \right]^{0.25} \qquad (6)$$

It was found that Eqn. [6] correlates reasonably well with the experimental data of shallow crack ($a/t < 0.5$), Fig. 2. For a deeper crack ($a/t > 0.5$), Eqn. [6] tends to underestimate the value of the stress intensity factor by as much as 90 percent. Investigators have been attempting to estimate or determine numerically the stress intensity factor for a deep surface flaw in order to obtain better correlations with experimental results. This effort has led to numbers of expressions of correction factors, mentioned earlier, being proposed. Each of these correction factors will be discussed in turn in the following sections and will be limited to the deepest point ($\phi = \pi/2$) and the surface intersection point ($\phi = 0$).

FRONT FACE MAGNIFICATION FACTOR $M_{f(\Phi)}$

For a semi-elliptical surface crack in a semi-infinite plate, a front face correction factor is necessary. Irwin /1/ suggested that the effect of the free surface allows a greater crack-opening-displacement (COD) and therefore the value of the stress intensity factor increases. He suggested the expression as Eqn. [6]. Kobayashi and Moss /4/ estimated the front face correction factor for the deep surface flaw by using the available solutions and they also proposed an expression for the front face.

FATIGUE CRACK PROPAGATION

Fatigue testing to evaluate material or structural performance can be accomplished by tests such as fatigue at constant load amplitude, constant strain amplitude and at positive or negative mean load. There are generally two types of structural design philosophies in fatigue design, safe-life and fail-safe design. The former is fundamentally an analytical one where safety depends on the absence of a serious fatigue crack for a certain service period. The latter is based on the assumption that the structure is damaged by some means and it will continue to function with the damage present. The fail-safe design is now commonly used.

For many years, the traditional approach to design against fatigue has been used by engineers. It is to base allowable fatigue stresses on the results of tests on carefully-made plain or notched laboratory specimens. The results are plotted as a stress–number of cycles to failure known as the S-N curve. Referring to the fail-safe design philosophy, it is now realized that most structures contain flaws, especially if welding is used, so that the whole fatigue life is occupied by fatigue crack growth.

Consequently, the fatigue crack growth properties of the material concerned is required in design. Difficulties arise if the traditional approach is employed since it does not give any information on the relative contribution of crack initiation and crack growth to total fatigue life.

Uložen je značajan trud da bi se okarakterisalo ponašanje rasta prsline u čeliku, legurama titana i legurama aluminijuma u različitim uslovima ispitivanja. Pristup se sastoji u iznalaženju zavisnosti brzine rasta (*da/dN*) od dužine prsline, veličine plastične zone, napona, konstanti materijala, i dimenzija epruvete. Ovde su ukratko objašnjeni i analizirani koncept rasta zamorne prsline, razvoj odgovarajućih zakona, oblik krive rasta prsline i uticajni faktori.

RAZVOJ ZAKONA RASTA ZAMORNE PRSLINE

Brojni su zakoni rasta zamorne prsline koji podjednako važe u smislu da svaki zakon predstavlja skup podataka o rastu zamorne prsline. Šenli /5/ je proizvoljno pretpostavio

$$\frac{da}{dN} = A_1 \sigma_a^n a \tag{7}$$

gde su A_1 i n određeni empirijski. Hed /6/ je zanemario uticaj debljine, mikroskopske promenljive i Baušingerov efekt; a pretpostavio beskonačan medijum, srednji napon jednak nuli, i da veličina plastične zone ispred vrha prsline ne zavisi od njene dužine. Predložio je zavisnost

$$\frac{da}{dN} = \frac{A_2 \sigma_a^3 a^{1.5}}{\left(\sigma_Y - \sigma_a\right)^2 a_0^{0.5}} \tag{8}$$

a A_2 se izračunava kao

$$A_2 = \frac{1}{12E}\left(\frac{F}{\sigma_f - \sigma_Y}\right) \tag{9}$$

Vejbul /7/ je pretpostavio da je brzina rasta proporcionalna lokalnom naponu na stepen n i da dužine prsline nema značaja za *da/dN*. Predložio je da je

$$\frac{da}{dN} = A_3 \sigma_{net}^n \tag{10}$$

a A_3 i n su dobijeni empirijski. Paris i saradnici /8/ su predložili zavisnost na osnovu linearno elastičnog modela

$$\frac{da}{dN} = f(K) \tag{11}$$

Paris je zaključio da vrednost K može da utiče na brzinu rasta prsline, kako je kasnije diskutovano, sl. 4.

Hepner i Krup /9/ su analizirali neke prethodno predložene zakone, a tri generalisana oblika zakona rasta zamorne prsline su predstavljeni. To su

$$\frac{da}{dN} = B_1\left(\sigma_a\right) \tag{12}$$

$$\frac{da}{dN} = B_2 \sigma(a)^n \tag{13}$$

$$\frac{da}{dN} = B_3 f(K)^n \tag{14}$$

Najšire je prihvaćena oblik jedn. [14]. Jedan dobro poznati oblik ovog izraza predložili su Paris i Erdogan /8/. Taj izraz je povezao brzinu cikličnog rasta zamorne prsline (*da/dN*) sa opsegom faktora intenziteta napona *ΔK*:

$$\frac{da}{dN} = C_1\left(\Delta K\right)^m \tag{15}$$

C_1 i m su konstante koje zavise od materijala i uslova ispitivanja, a određuju se eksperimentalno.

Considerable work has been done to characterize the crack growth behaviour of steel, titanium alloys and aluminium alloys under various test conditions. The approach is to relate the growth rate (*da/dN*) with crack length, plastic zone size, stress, material constants, and specimen dimensions. The concept of fatigue crack growth, the development of corresponding laws, form of crack growth curves and affecting factors are shortly presented and analyzed.

DEVELOPMENT OF FATIGUE CRACK GROWTH LAWS

There are numerous fatigue crack growth laws, equally valid in that each law represents a set of fatigue crack growth data. Shanley /5/ rather arbitrarily proposed that

$$\frac{da}{dN} = A_1 \sigma_a^n a \tag{7}$$

where A_1 and n are determined empirically. Head /6/ neglected thickness effect, microscopic variables and the Bauschinger effect; and assumed the infinite medium, the mean stress zero, and the extent of the plastic zone at the crack tip independent of crack length. He suggested the relationship

$$\frac{da}{dN} = \frac{A_2 \sigma_a^3 a^{1.5}}{\left(\sigma_Y - \sigma_a\right)^2 a_0^{0.5}} \tag{8}$$

and A_2 calculated from

$$A_2 = \frac{1}{12E}\left(\frac{F}{\sigma_f - \sigma_Y}\right) \tag{9}$$

Weibull /7/ assumed that the growth rate was proportional to some power of the peak stress and that the crack length was of no consequence in *da/dN*. He proposed that

$$\frac{da}{dN} = A_3 \sigma_{net}^n \tag{10}$$

and A_3 and n are determined empirically. Paris et al. /8/ proposed a relation based on a linear elastic model as follows

$$\frac{da}{dN} = f(K) \tag{11}$$

Paris reasoned that the value of K would control the rate of crack propagation, as discussed later, Fig. 4.

Hoeppner and Krupp /9/ discussed some already proposed laws, and three generalised forms of fatigue crack growth laws have been presented. They are

$$\frac{da}{dN} = B_1\left(\sigma_a\right) \tag{12}$$

$$\frac{da}{dN} = B_2 \sigma(a)^n \tag{13}$$

$$\frac{da}{dN} = B_3 f(K)^n \tag{14}$$

The most widely accepted form is Eqn. [14]. A well known expression of this type was proposed by Paris and Erdogan /8/. It related the cyclic crack growth rate (*da/dN*) to the range of the stress intensity factor *ΔK*:

$$\frac{da}{dN} = C_1\left(\Delta K\right)^m \tag{15}$$

C_1 and m are constants dependent upon the material and testing conditions and are determined experimentally.

KRIVE RASTA PRSLINE

Podaci o rastu zamorne prsline su obično predstavljeni u obliku dijagrama *da/dN* u zavisnosti od ΔK u logaritamskim osama, kao na sl. 3. Dijagram pokazuje linearnu zavisnost. Međutim, u praksi dijagram je kriva oblika S, ili se bar sastoji iz delova različitih nagiba, /8,12/.

Uopšteno, krivu sačinjavaju sledeće tri oblasti:

Oblast I, oblast početnog laganog rasta; u ovoj oblasti nema porasta ako je vrednost ΔK ispod praga, ΔK_{th}. U oblasti iznad praga grafik ubrzano raste u oblast II.

Oblast II, oblast stabilnog dokritičnog rasta; u ovoj oblasti grafik je prava linija i poznat je kao Parisov režim. Većina podataka ispitivanja se nalazi u ovoj oblasti.

Oblast III je oblast nestabilnog brzog rasta prsline; ovde grafik ponovo ubrzano raste jer K_{max} dostiže K_{Ic}. Potpuni lom se javlja u ciklusa u kom se K_{max} dostiže K_{Ic}.

Jednom dobijena kriva za datu konfiguraciju pri ispitivanju omogućava predviđanje rasta zamorne prsline u svakoj drugoj konfiguraciji poznatog faktora inteziteta napona.

FAKTORI KOJI UTIČU NA RAST ZAMORNE PRSLINE

Sva tri prikazana oblika zakona rasta prsline ne uzimaju u obzir uticaje faktora kao što su odnos napona, debljina, prag zamora, frekvenciju, okolinu, i drugi. Da bi zakoni postali pouzdaniji, potrebno je i uneti takve uticaje. U narednim odeljcima biće predstavljeni neki od ovih parametara i odgovarajući modeli.

Erdogan /10/ je diskutovao rast zamorne prsline sa fundamentalnijeg i mikroskopskog aspekta. Izračunao je veličinu plastične zone na vrhu zamorne prsline u uslovima njene male veličine i dobio sledeći izraz,

$$\frac{da}{dN} = A\left(\frac{1}{1-R}\right)^{2\alpha_1} (\Delta K)^{2(\alpha_1+\alpha_2)} \qquad (16)$$

gde su A, α_1, α_2 konstante materijala.

Uticaj odnosa napona R je direktno povezan sa uticajem srednjeg napona, jer je R definisan kao

$$R = \frac{\sigma_{min}}{\sigma_{max}} \qquad (17)$$

a kako je
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad (18)$$

dobija se
$$\sigma_m = \frac{\sigma_{max}^{(1+R)}}{2} \qquad (19)$$

UTICAJ USLOVA PRAGA ZAMORA

Oblici formula [15] i [16] ukazuju da će prslina rasti pri bilo kojoj vrednosti ΔK, čak i pri vrlo malim vrednostima. Uprkos tome, pokazano je /11/ da postoji kritičan raspon faktora inteziteta napona ispod kojeg se rast prsline odvija ili veoma sporo, ili potpuno izostaje. Ovo je uslov praga zamora i odnosi se na oblast I.

Izraz [15] je poznat po tome što daje linearnu zavisnost između log(*da/dN*) i log(ΔK). Sada je očigledno da je u oblastima I i III ova veza nelinearna. Objašnjenje za pojavu tog odstupanja u odnosu na linearnost zahteva modele koji povezuju faktor inteziteta napona praga zamora ΔK_{th}, i žilavost loma K_{Ic}.

CRACK GROWTH CURVES

Fatigue crack growth data are usually presented in the form of the plot of *da/dN* versus ΔK on logarithmic scales, as shown in Fig. 3. The plot should show a linear relationship. However, in practice the plot is an S-shaped curve, or at least consists of parts with different slopes, /8,12/.

The curve itself is generally composed of three regions:

Region I, initial slow propagating region; in this region there is no propagation if ΔK is below the threshold, ΔK_{th}. Above the threshold the graph rises rather rapidly to region II.

Region II, stable sub-critical propagation region; in this region the graph is considered to be a straight line and known as Paris' regime. Most of the test data falls in this region.

Region III, unstable fast crack propagation region; here the graph again rises rapidly since K_{max} approaches K_{Ic}. Total failure occurs in the cycle for which K_{max} reaches K_{Ic}.

Once the curve is obtained for a particular test configuration, it is possible to predict fatigue crack propagation in any other test configuration of known stress intensity factor.

FACTORS AFFECTING FATIGUE CRACK PROPAGATION

So far presented the three forms of crack growth laws do not take into account the effects of such factors as stress ratio, thickness, threshold, frequency, environment, and the others. In order to make the laws more reliable, such effects should be included. Some of these parameters and relevant models will be presented in the following sections.

Erdogan /10/ discussed fatigue crack propagation on a more fundamental and microscopic basis. He calculated the plastic zone size at the fatigue crack tip under the small scale condition and obtained the following expression,

$$\frac{da}{dN} = A\left(\frac{1}{1-R}\right)^{2\alpha_1} (\Delta K)^{2(\alpha_1+\alpha_2)} \qquad (16)$$

where A, α_1, α_2 are material constraints.

The effect of stress ratio R, relates directly to the effect of mean stress since R is defined by

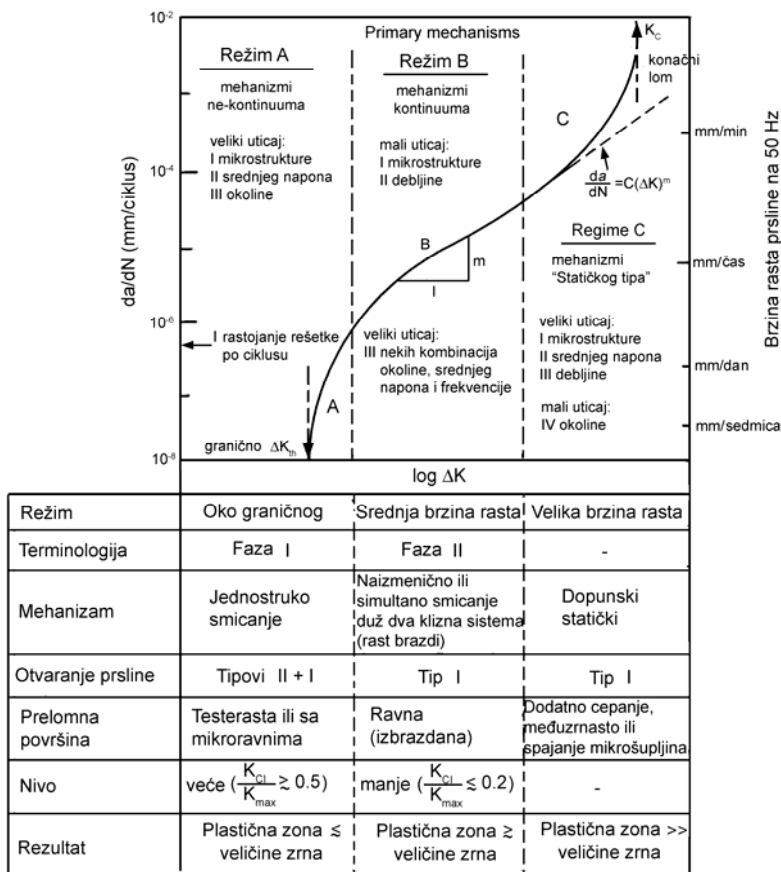$$R = \frac{\sigma_{min}}{\sigma_{max}} \qquad (17)$$

since
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad (18)$$

thus
$$\sigma_m = \frac{\sigma_{max}^{(1+R)}}{2} \qquad (19)$$

THE EFFECT OF THRESHOLD CONDITION

The forms of equations [15] and [16] indicate that crack growth will take place at any value of ΔK, even very small values. Nevertheless, it has been shown /11/ that there exists a critical range of stress intensity factor below which crack propagation occurs either extremely slowly or not at all. This is called the threshold condition and refers to region I.

Equation [15] is known to give linear relationship between log(*da/dN*) and log(ΔK). It is now evident that in regions I and III the relationship is non-linear. To account for this deviation from linearity, it is necessary to have models associated with threshold stress intensity factor range ΔK_{th}, and the fracture toughness K_{Ic}.

Režim	Oko graničnog	Srednja brzina rasta	Velika brzina rasta
Terminologija	Faza I	Faza II	-
Mehanizam	Jednostruko smicanje	Naizmenično ili simultano smicanje duž dva klizna sistema (rast brazdi)	Dopunski statički
Otvaranje prsline	Tipovi II + I	Tip I	Tip I
Prelomna površina	Testerasta ili sa mikroravnima	Ravna (izbrazdana)	Dodatno cepanje, međuzrnasto ili spajanje mikrošupljina
Nivo	veće $(\frac{K_{cl}}{K_{max}} \gtrsim 0.5)$	manje $(\frac{K_{cl}}{K_{max}} \lesssim 0.2)$	-
Rezultat	Plastična zona $\leq$ veličine zrna	Plastična zona $\geq$ veličine zrna	Plastična zona $\gg$ veličine zrna

Slika 3. Tri oblasti ponašanja pri rastu zamorne prsline i odgovarajući mehanizmi i karakteristike

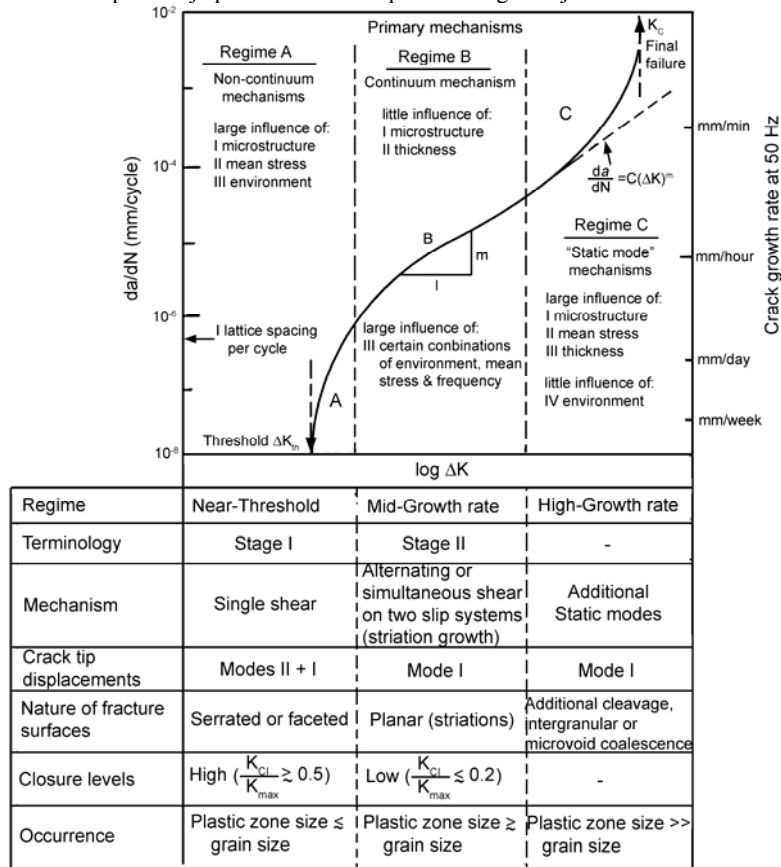| Regime	Near-Threshold	Mid-Growth rate	High-Growth rate
Terminology	Stage I	Stage II	-
Mechanism	Single shear	Alternating or simultaneous shear on two slip systems (striation growth)	Additional Static modes
Crack tip displacements	Modes II + I	Mode I	Mode I
Nature of fracture surfaces	Serrated or faceted	Planar (striations)	Additional cleavage, intergranular or microvoid coalescence
Closure levels	High $(\frac{K_{cl}}{K_{max}} \gtrsim 0.5)$	Low $(\frac{K_{cl}}{K_{max}} \lesssim 0.2)$	-
Occurrence	Plastic zone size $\leq$ grain size	Plastic zone size $\geq$ grain size	Plastic zone size $\gg$ grain size

Figure 3. The three regions of fatigue crack propagation behaviour and their corresponding mechanisms and characteristics.

Liu /12/ je modifikovao Parisov izraz kako bi uzeo u obzir član granice–praga kako sledi:

$$\frac{da}{dN} = C_4 \left(\Delta K - \Delta K_{th} \right)^{m_4} \tag{20}$$

Ovde su C_4 i m_4 konstante. Predložena je i jedna druga jednačina zasnovana na razmatranju otvaranja prsline koja sadrži ΔK_{th},

$$\frac{da}{dN} = \frac{C_5}{\pi \sigma_Y E} \left(\Delta K^2 - \Delta K_{th}^2 \right)$$

C_5 je konstanta. Najraniji model koji sadrži ΔK_{th} i K_{Ic} koji je predložio Erdogan je

$$\frac{da}{dN} = C_6 \frac{\left(\Delta K - \Delta K_{th} \right)^{m_5}}{(1 - R) K_{IC} - \Delta K} \tag{21}$$

gde su C_6 i m_5 konstante. Sličan izraz jedn. [21] predložili su Branko i saradnici /13/. On je upotrebljen za opisivanje brzine rasta zamorne prsline u RR58, i može se napisati kao

$$\frac{da}{dN} = C_7 \beta^{\alpha_3} \tag{22}$$

gde su C_7 i α_3 konstante koje su određene empirijski, a β se definiše kao

$$\beta = \frac{2 K_m \left(\Delta K - \Delta K_{th} \right)}{K_{IC}^2 - K_{max}^2} \tag{23}$$

Napisana drugačije jedn.[23] unosi uticaj odnosa napona R:

$$\beta = \frac{\left(1 - R^2 \right)\left(\Delta K^2 - \Delta K \Delta K_{th} \right)}{K_{IC}^2 \left(1 - R^2 \right) - \Delta K^2} \tag{24}$$

Kuk i Bivers su uočili izraženu zavisnost praga od odnosa napona. Primetili su da se vrednost praga povećava sa povećanjem napona tečenja. Grant i Galager su zatim analizirali različite podatke o pragu zamora i predložili drugi izraz koji se slaže sa eksperimentalnim rezultatima:

$$K_{th} = \left(\Delta K_{th} \right)_{R=0} \left(1 - R^2 \right) \tag{25}$$

Ovde je $(\Delta K_{th})_{R=0}$ vrednost praga za odnos napona $R = 0$.

Nedavno je Radon /14/ analizirao podatke zamora za čelik BS 4360-50D prema sledećem izrazu

$$\frac{da}{dN} = \frac{2^{1+n} (1 - 2\nu)^2 \left(\Delta K^2 eff^- \Delta K_C^2 eff \right)}{4(1 + n)\pi \sigma_{YC}^{1-n} E^{1+n} \varepsilon_f^{1+n}} \tag{26}$$

gde je n eksponent cikličnog deformacijskog ojačavanja. Ovaj izraz se zasniva na konceptu zatvaranja prsline koji je zapravo drugačiji pristup u odnosu na ostale modele. Međutim, ovaj model se može upotrebiti samo za predviđanje brzine rasta prsline u donjoj (I) i srednjoj (II) oblasti.

Svi ovi modeli rasta zamorne prsline se mogu smatrati važećim u smislu da opisuju specifičan skup podataka rasta zamorne prsline, te se mogu upotrebiti za predviđanje brzine rasta prslina u situacijama sličnim onima pri sakupljanju podataka. Zahvaljujući svojstvenom rasipanju podataka o rastu zamorne prsline, nemoguće je izabrati model koji bi bio „najpogodniji" /12/.

Liu /12/ modified Paris equation to take account of the threshold term as follows:

$$\frac{da}{dN} = C_4 \left(\Delta K - \Delta K_{th} \right)^{m_4} \tag{20}$$

where C_4 and m_4 are constants. Another equation based on crack opening displacement considerations and including ΔK_{th} was also suggested

$$\frac{da}{dN} = \frac{C_5}{\pi \sigma_Y E} \left(\Delta K^2 - \Delta K_{th}^2 \right)$$

C_5 is a constant. The earliest model which includes both ΔK_{th} and K_{Ic} suggested by Erdogan was

$$\frac{da}{dN} = C_6 \frac{\left(\Delta K - \Delta K_{th} \right)^{m_5}}{(1 - R) K_{IC} - \Delta K} \tag{21}$$

where C_6 and m_5 are constants. A similar expression to [21] was proposed by Branco et al. /13/. It was used to describe fatigue crack growth rate in RR58, and may be written as

$$\frac{da}{dN} = C_7 \beta^{\alpha_3} \tag{22}$$

where C_7 and α_3 are constants determined empirically and β defined as

$$\beta = \frac{2 K_m \left(\Delta K - \Delta K_{th} \right)}{K_{IC}^2 - K_{max}^2} \tag{23}$$

Rewritten Eqn. [23] can include effect of stress ratio R:

$$\beta = \frac{\left(1 - R^2 \right)\left(\Delta K^2 - \Delta K \Delta K_{th} \right)}{K_{IC}^2 \left(1 - R^2 \right) - \Delta K^2} \tag{24}$$

Cooke and Beevers observed a marked dependence of the threshold on the stress ratio. They noted that the threshold value increased with the increasing yield stress. Grandt and Gallagher then analysed a variety of threshold data and suggested another expression to fit the experimental data:

$$K_{th} = \left(\Delta K_{th} \right)_{R=0} \left(1 - R^2 \right) \tag{25}$$

Here $(\Delta K_{th})_{R=0}$ is the threshold value at stress ratio $R = 0$.

Recently, Radon /14/ analysed the fatigue data of BS 4360-50D steel by using the following expression

$$\frac{da}{dN} = \frac{2^{1+n} (1 - 2\nu)^2 \left(\Delta K^2 eff^- \Delta K_C^2 eff \right)}{4(1 + n)\pi \sigma_{YC}^{1-n} E^{1+n} \varepsilon_f^{1+n}} \tag{26}$$

where n is the cyclic strain hardening exponent. This expression is based on the crack closure concept which is a different approach from the other models. However, this model can only be used to predict crack growth rate in the low (I) and intermediate (II) regions.

All these fatigue crack growth models can be regarded as valid in the sense that they describe a particular set of fatigue crack growth data, and can consequently be used to predict crack growth rates in situations similar to those used to collect data. Owing to the inherent scatter in fatigue crack growth data, it is impossible to decide which model is the most "suitable" /12/.

PREDVIĐANJE RASTA ZAMORNE PRSLINE

Obavljena su ispitivanja vezana za zamorni rast prslina kod površinskih prslina u konačnim pločama pod dejstvom savijanja od strane Mogadama /17/. U okviru izrade ovog projekta dobijeni su rezultati ispitivanja čeličnih ploča od BS 4360-50D i od metala šava sučeonog zavarenog spoja čeličnih ploča od BS 4360-50D, tabele 1 i 2. Pri tom je cilj bio predviđanje, uz primenu računarske tehnike, karakteristike rasta površinskih prslina pod istim uslovima kao i u dostupnim rezultatima eksperimenata. Relevantni uslove čine dve frekvencije: 30 Hz i 0,25 Hz, i dva odnosa napona $R = 0,7$ i 0,08. Potrebno ih je uporediti na odgovarajući način sa eksperimentalnim rezultatima. Osim toga, analizirani su i uticaji debljine, širine i početnih odnosa veličina na profil i oblik prsline.

PREDICTION OF FATIGUE CRACK GROWTH

Tests concerned with fatigue crack growth of surface cracks in finite plates under bending loads have been carried out by Moghadam /17/. Results of the tests on BS 4360-50D steel plates and on the weld metal of butt-welded BS 4360-50D steel plates were made available for the purpose of this project, Tables 1, 2. The aim here was to predict, with the use of computer technique, surface cracks growth characteristics under the same conditions as for the available experimental results. The relevant conditions are of two frequencies: 30 Hz and 0.25 Hz, and two stress ratios $R = 0.7$ and 0.08. Appropriate comparisons are to be made with the experimental results. Furthermore, the effects of the thickness, width and initial aspect ratio on the crack profile and crack shape are also analysed.

Tabela 1. Hemijski sastav (maseni) čelika BS4360-50D i mehaničke osobine.
Table 1. Chemical composition (weight) of steel BS4360-50D and mechanical characteristics.

Element	C	Si	Mn	F	S	Cr	Mo	Ni	Al	Cu	Nb
%	0.180	0.36	1.40	0.018	0.003	0.11	0.020	0.095	0.035	0.16	0.039

Mehaničke osobine	Pravac		
	Podužni	Poprečni	Po debljini
Napon tečenja (MN/m^2)	386	382	388
Zatezna čvrstoća (MN/m^2)	560	551	563
% izduženje (L = 75 mm)	30	29	18
% suženje	74	68	61

Mechanical properties	Direction		
	Longitud.	Transverse	Through thickness
YS (MN/m^2)	386	382	388
UTS (MN/m^2)	560	551	563
% elong. (GL = 3 in.)	30	29	18
% reduction in area	74	68	61

Tabela 2. Hemijski sastav (maseni) elektrode za zavarivanje BS 639: E4311 R21 (3) i mehaničke osobine.
Table 2. Chemical composition (weight) of welding electrode BS 639: E4311 R21 (3) and mechanical characteristics.

Element	C	Mn	Si	S	P
%	0.06	0.58	0.43	0.02 max	0.03 max

Mehaničke osobine	
Napon tečenja (N/mm^2)	488
Zatezna čvrstoća (N/mm^2)	510
% izduženje	22
% suženje	50
Udarna žilavost KV na 20°C	80 J

Mechanical properties	
Yield strength (N/mm^2)	488
Ultimate tensile strength (N/mm^2)	510
% elongation	22
% reduction in area	50
Charpy V-notch at 20°C	80 J

U ovde prikazanoj studiji predviđanje je urađeno za sve tri oblasti korišćenjem kombinacije modela rasta prsline Parisa, jedn. [5], i Branka i saradnika, jedn. [21]. Izrazi za faktor intenziteta napona su dati prema Skotu i Torpu /15/. Analitička integracija odgovarajućih modela rasta prsline je zatim izvedena radi izračunavanja brzine rasta prsline. U sledećem odeljku sledi kratak opis procedure za dobijanje rešenja uz detaljnu diskusiju.

In this present study the prediction is made for all three regions by using the combination of Paris, Eqn. [5] and Branco et al., Eqn. [21], crack growth models. Stress intensity factor expressions are those reviewed by Scott and Thorpe /15/. The analytical integration of the appropriate crack growth models is then employed to evaluate the crack growth rates. The outline of the solution procedure will be discussed in detail in the following section.

INFORMACIJE O MATERIJALU I EKSPERIMENTIMA

Osnovni materijal je konstrukcijski čelik BS 4360-50D. Sastav i mehaničke osobine materijala su dati u tabeli 1. Glatke epruvete i epruvete iz sučeono zavarenog spoja su bile oblika pravougaonih ploča i svaka je imala središnji površinski zarez izveden kružnom testerom. Konfiguracije epruveta se vide na sl. 2. i na sl. 4 /16/.
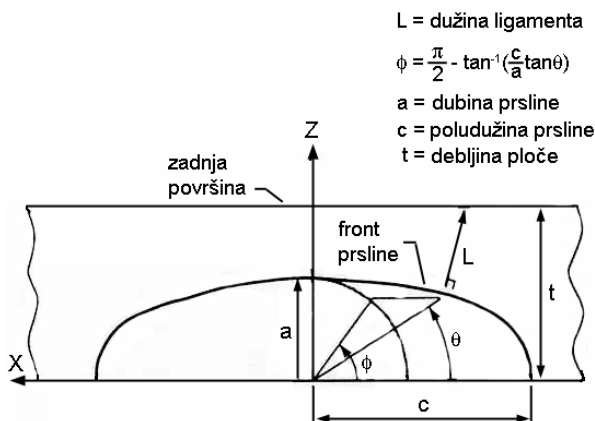
Prema rezultatima Mogadama /17/, žilavost loma pri ravnoj deformaciji čelika BS 4360-50D, K_{IC}, imala je vrednost 160 MN/m^{1,5}.

MATERIAL AND EXPERIMENTAL INFORMATION

The parent metal was structural steel, BS 4360-50D. The composition and mechanical properties of the material are given in Table 1. The plain and butt-welded specimens were rectangular plates and each contained a central surface notch which was produced by a circular cutter. The specimen configurations can be seen from Figs. 2 and 4, /16/.

According to Moghadam's results /17/, the plane strain fracture toughness of the BS 4360-50D steel, K_{IC}, was found to be 160 MN/m$^{1.5}$.

Slika 4. Shema detalja površinske prsline.

Figure 4. Schematic showing details of the surface crack.

Kako je K_{Ic} jednako 160 MN/m^{1,5}, a vrednost K_{Ic} metala šava nije bila poznata, pretpostavljena je ista vrednost kao i za osnovni metal. Vrednosti za K_{th}, međutim, su morale da se procene na osnovu eksperimentalnih krivih rasta prsline. Što se eksperimentalnih rezultata tiče, oni su se razlikovali u zavisnosti od uslova ispitivanja i materijala. Takođe je bilo teško proceniti ΔK_{th} za krive niže frekvencije (0,25 Hz) zbog poteškoća u dobijanju podataka za malu brzinu rasta prsline pri nižoj frekvenciji; prema tome, vrednosti ΔK_{th} na nižoj frekvenciji pretpostavljene su iste kao vrednosti na višoj frekvenciji (30 Hz), iako su postojali neznatni uticaji frekvencije pri vrednostima praga. Ovi uticaji su prikazani na sl. 5 /16/, i postaju više izraženi kod većih vrednosti faktora intenziteta napona, a takođe i pri većim odnosima napona.

Konfiguracija opterećenja je savijanje u 4 tačke sa konstantnom amplitudom ciklusa, što je prikazano na sl. 2.

MODELI RASTA ZAMORNE PRSLINE EPRUVETA SA POVRŠINSKOM PRSLINOM ZA SAVIJANJE (SCB)

Više modela rasta prsline je već predstavljeno u prethodnom odeljku. Ovde će se detaljnije diskutovati o modelima jedn. [15] Parisa i jedn. [21] Branka i saradnika,. koji će se koristiti za predviđanje rasta zamorne prsline. Osnovna ideja je u predviđanju krivih rasta prsline za sve tri oblasti rasta prsline (sl. 3 i 6), prema tome, za tu svrhu izabrana je jedn. [21]. Pre primene jedn. [21] potrebno je poznavanje nekih osnovnih podataka kao što su brzine rasta prsline u oblasti II, a mogu se dobiti iz jedn. [15].

S obzirom na pretpostavku da su profili prslina poluelipse, potrebno je izračunati samo brzine rasta za glavnu i pomoćnu osu. Stoga, jedn. [15] Parisovog modela može se napisati kao

$$\frac{da}{dN} = C_a \left(\Delta K_a \right)^m a \qquad (15a)$$

$$\frac{dc}{dN} = C_c \left(\Delta K_c \right)^m c \qquad (15c)$$

Slično se može napisati za model Branka i saradnika /13/

$$\frac{da}{dN} = A_a \left(\beta_a \right)^{\alpha_a} \qquad (22a)$$
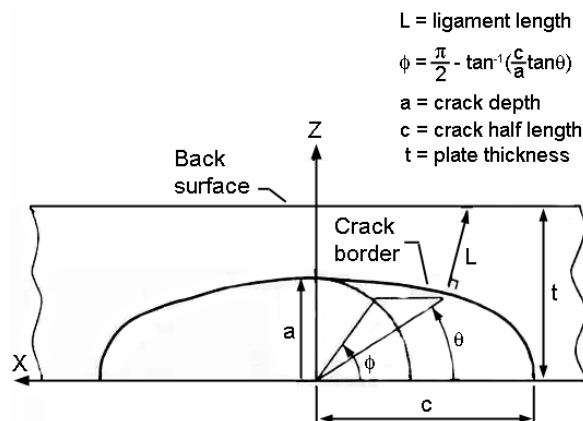
$$\frac{dc}{dN} = A_c \left(\beta_c \right)^{\alpha_c} \qquad (22c)$$

Since K_{Ic} was found to be 160 MN/m$^{1.5}$, and K_{Ic} value of the weld metal was not known, it was assumed to have the same value as the parent metal. The values of K_{th}, however, had to be estimated from the experimental crack growth curves. As far as the experimental results are concerned, they were rather different depending on the test conditions and materials. It was also difficult to estimate the ΔK_{th} for the low frequency curves (0.25 Hz) due to difficulty in obtaining data at low crack growth rate at low frequency; therefore, ΔK_{th} values at low frequency were assumed to have the same values as those at the high frequency (30 Hz), although there were slight effects of frequency on the threshold values. These effects are shown in Fig. 5, /16/, and become greater at higher stress intensity factor range and also at higher stress ratio.

The loading configuration was 4-point bending with constant amplitude cycling, as shown in Fig. 2.

FATIGUE CRACK GROWTH MODELS FOR SURFACE CRACK BENDING (SCB) SPECIMENS

A number of crack growth models have already been presented in the previous section. Now, the Paris Eqn. [15] and Branco et al. Eqn. [21] models are discussed in more detail and are used for prediction of fatigue crack growth. The basic idea is to predict the crack growth curves for all three regions of crack propagation (Figs. 3 and 6), so Eqn. [21] is chosen for this purpose. Before applying Eqn. [21] some basic information, such as crack propagation rates in region II, are required and may be obtained from Eqn. [15].

Since the crack profiles are assumed to be semi-ellipses, only the growth rates of the major and minor axes need to be calculated. Thus, the Paris model Eqn. [15] may be written as

$$\frac{da}{dN} = C_a \left(\Delta K_a \right)^m a \qquad (15a)$$

$$\frac{dc}{dN} = C_c \left(\Delta K_c \right)^m c \qquad (15c)$$
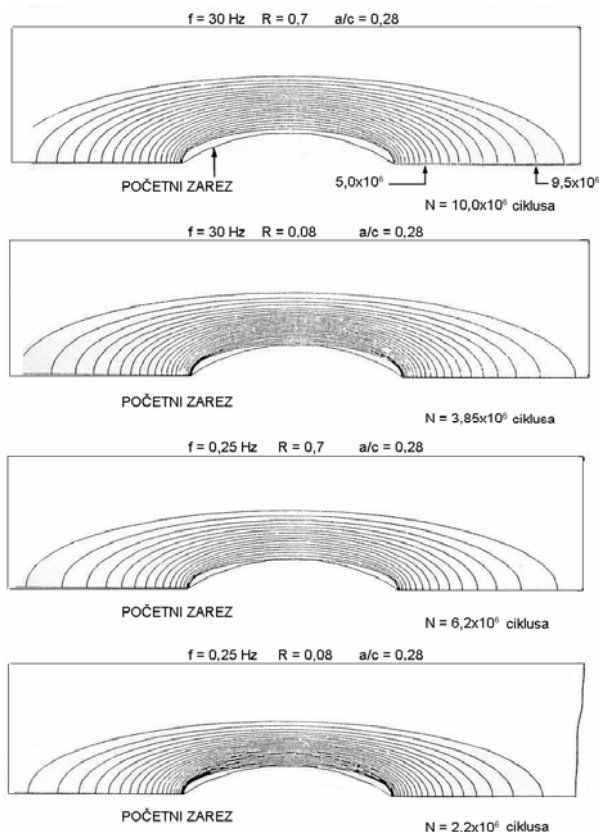
Similarly, the Branco et al /13/ model may be written as

$$\frac{da}{dN} = A_a \left(\beta_a \right)^{\alpha_a} \qquad (22a)$$

$$\frac{dc}{dN} = A_c \left(\beta_c \right)^{\alpha_c} \qquad (22c)$$

i
$$\beta_a = \frac{\left(1-R^2\right)\left(\Delta K_a^2\right) - \Delta K_a \Delta K_{tha}}{K_{IC}^2\left(1-R^2\right) - \Delta K_a^2} \qquad (23a)$$

$$\beta_c = \frac{\left(1-R^2\right)\left(\Delta K_c^2\right) - \Delta K_c \Delta K_{thc}}{K_{IC}^2\left(1-R^2\right) - \Delta K_c^2} \qquad (23c)$$

gde se indeksi *a* i *c* odnose na najdublju i na površinsku tačku prsline, respektivno. Ovi izrazi su korišćeni u čitavom ovom radu.

Slika 5. Predviđanje rasta zamorne prsline pod dejstvom savijanja u 4 tačke u konstrukcijskom čeliku BS 4360-50D.

Figure 5. Prediction of fatigue crack growth under 4 pt. bending of a structural steel BS 4360-50D.

Osim modela Parisa i Branka treća metoda za *K*-vrednosti upotrebljena u prikazanom eksperimentu je razvijena od strane Koterazave i saradnika /18/, kao što je prikazano u tabeli 3 i na sledećim slikama.

Predložena su razna rešenja faktora intenziteta napona za poluelliptične prsline u ploči pod dejstvom čistog savijanja. Skot i saradnici /15/ su analizirali i uporedili ova rešenja i zaključili da rešenje Koterazave i Minamisake /18/ sa faktorima korekcije za širinu prema Holdbruku i Doveru daju rezultate koji su najbliži eksperimentalnim podacima. Ova rešenja za dužine prsline, *a* i *c*, su data u tabeli 3.
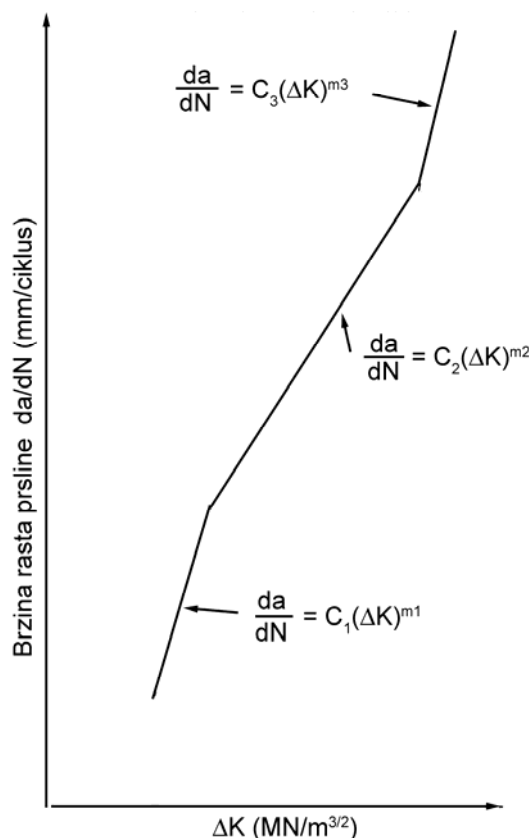
REZULTATI I DISKUSIJA

Za predviđanje brzine rasta prsline, odgovarajuće dužine prsline *a* i *c* se moraju simultano odrediti nakon izvesnog broja ciklusa opterećenja. Ove veličine se izračunavaju potprogramom CRACK računarskog programa MODEL.

and
$$\beta_a = \frac{\left(1-R^2\right)\left(\Delta K_a^2\right) - \Delta K_a \Delta K_{tha}}{K_{IC}^2\left(1-R^2\right) - \Delta K_a^2} \qquad (23a)$$

$$\beta_c = \frac{\left(1-R^2\right)\left(\Delta K_c^2\right) - \Delta K_c \Delta K_{thc}}{K_{IC}^2\left(1-R^2\right) - \Delta K_c^2} \qquad (23c)$$

where suffixes *a* and *c* correspond to the deepest and surface points of the crack respectively. These equations were used all through this present work.

Slika 6. Tipična kriva rasta prsline izražena na osnovu izraza Parisa.

Figure 6. Typical crack growth curve expressed by Paris expression.

Apart from the Paris and Branco models the third method for the *K*-values used in the present experiment was developed by Koterazawa et al. /18/, as shown in Table 3 and the following figures.

Various stress intensity factor solutions for semi-elliptic surface cracked plates under pure bending have been proposed. Scott et al. /15/ reviewed and compared these solutions and concluded that the Koterazawa and Minamisaka /18/ solution with width correction factors proposed by Holdbrook and Dover provides the result closest to experimental data. These solutions for crack lengths, *a* and *c*, are given in Table 3.

RESULTS AND DISCUSSION

To predict the crack growth rate, the subsequent crack lengths *a* and c after a certain number of loading cycles have to be evaluated simultaneously. Subroutine CRACK of computer programme MODEL calculates these values.

Predviđene su vrednosti a i c za sve stvarne uslove ispitivanja za glatke epruvete i epruvete iz zavarenih spojeva. Zavisnosti između a i c su dobijene fitovanjem proračunatih tačaka polinomima drugog reda. Korelacijski koeficijenti fitovanih polinoma bili su oko 0,99 kod glatkih epruveta i oko 0,98 kod zavarenih epruveta. Predviđene zavisnosti između a i c date su na sl. 7. i sl. 8 za glatke i zavarene epruvete, respektivno.

The values of a and c have been predicted for all actual test conditions for both plain and welded joint specimens. The relationships between a and c are obtained by fitting the computed points with 2nd order polynomials. The correlation coefficients of the polynomial fittings are about 0.99 for the plain specimens and about 0.98 for the welded specimens. The predicted relationships between a and c are given in Fig. 7 and Fig. 8 for plain and welded specimens, respectively.

Tabela 3. Korekcije za faktor intenziteta napona

$$K_a = \sigma_b M_b \left(\frac{\pi}{2}\right) M_a M_\omega \sqrt{\pi a} \,/\, E(K)$$

$$K_c = \sigma_b M_b \left(0\right) M_c M_\omega \sqrt{\pi a} \,/\, E(K)$$

gde je σ_b površinski napon savijanja

$$M_b\left(\frac{\pi}{2}\right) = 1 + 0,12\left(1 - \frac{a}{2c}\right)^2$$

$$M_a = 1 - 1,36\left(\frac{a}{t}\right)\left(\frac{a}{c}\right)^{0,1}$$

$$M_b(0)M_c = M_{bc}$$

$$M_{bc} = M_1\left(1 - 0,3\left(\frac{a}{t}\right)\right)\left(1 - \left(\frac{a}{t}\right)^{12}\right) + 0,394\phi\left(\frac{a}{t}\right)^{12}\sqrt{c/a}$$

$$M_1 = \left(1,21 - 0,1\left(\frac{a}{c}\right) + 0,1\left(\frac{a}{c}\right)^4\right)\sqrt{a/c}$$

Table 3. Corrections for stress intensity factors.

$$K_a = \sigma_b M_b \left(\frac{\pi}{2}\right) M_a M_\omega \sqrt{\pi a} \,/\, E(K)$$

$$K_c = \sigma_b M_b \left(0\right) M_c M_\omega \sqrt{\pi a} \,/\, E(K)$$

where σ_b is the surface bend stress

$$M_b\left(\frac{\pi}{2}\right) = 1 + 0,12\left(1 - \frac{a}{2c}\right)^2$$

$$M_a = 1 - 1,36\left(\frac{a}{t}\right)\left(\frac{a}{c}\right)^{0,1}$$
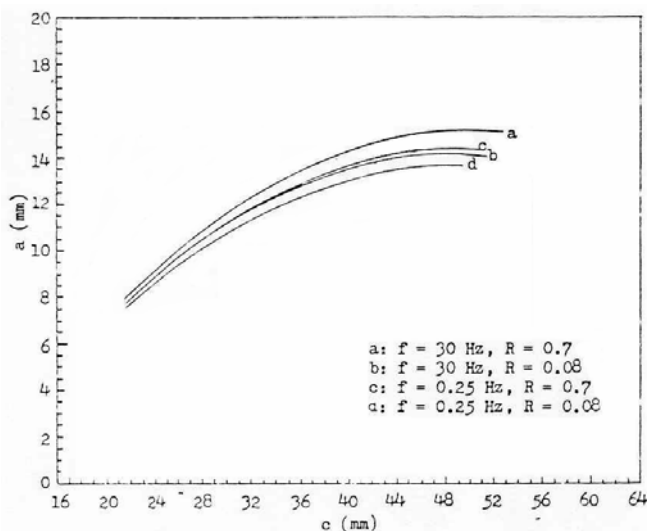
$$M_b(0)M_c = M_{bc}$$

$$M_{bc} = M_1\left(1 - 0,3\left(\frac{a}{t}\right)\right)\left(1 - \left(\frac{a}{t}\right)^{12}\right) + 0,394\phi\left(\frac{a}{t}\right)^{12}\sqrt{c/a}$$

$$M_1 = \left(1,21 - 0,1\left(\frac{a}{c}\right) + 0,1\left(\frac{a}{c}\right)^4\right)\sqrt{a/c}$$

Uticaji frekvencije i odnosa napona su jasno uočljivi na sl. 7. i 8. Predviđanje ilustruje da, ako se, pri istoj vrednosti c, povećavaju frekvencija i odnos napona, tada se uvećava i rastojanje duž kojeg prslina raste kroz debljinu. Ovaj efekt je više izražen pri većim vrednostima c. Poređenje eksperimentalnih rezultata /17/ i predviđenih zavisnosti za 30 Hz i $R = 0,7$ je prikazano na sl. 9. i 10. Slaganje je dobro iako procenjene krive, za oba materijala, imaju tendenciju da blago prevaziđu vrednosti a pri malim vrednostima c i obrnuto. Dobro slaganje između procenjenih krivih i eksperimentalnih tačaka govori u prilog opravdanosti primenjenog modela i uspešnosti dobijenih rešenja faktora intenziteta napona, /19/.

Potcenjena vrednost za a pri velikoj vrednosti c može biti usled upotrebljene metode za određivanje vrednosti a. U eksperimentu je primenjena metoda pada potencijala. Često se dešava da se ovom metodom dobijaju manje vrednosti izmerenih rezultata. Postoje i druge alternativne metode kojima se može izmeriti dužina prsline kao što su označavanje strija i termičko bojenje. Metoda označavanje strija treba da omogući tačnije merenje dužine prsline. Međutim, teškoće u postizanju jasnih tragova i istoriji opterećivanja umanjuju njihovu tačnost i primenu. Takođe je poznato i da utiču na usporavanje rasta prsline /16,19/.

S obzirom da je određivanje dužine prsline značajno pri svakom ispitivanju rasta zamorne prsline, predviđanje zavisnosti između a i c bi se moglo pokazati korisnim. Ove veličine takođe daju podatke za dalja istraživanja kao što je predviđanje karakteristika profila i oblika prsline.

The effects of frequency and stress ratio are clearly shown in Fig. 7 and Fig 8. The prediction illustrates that, at the same value of c, the higher the frequency and stress ratio the larger distance the crack propagates through the thickness. This effect is more pronounced at the larger value of c. Comparison between the experimental results /17/ and the predicted relationships at 30 Hz and $R = 0.7$ has been made as shown in Figs. 9 and 10. The agreement is good although the predicted curves, for both materials, tend to overestimate slightly the values of a at low values of c and vice versa. Good correlation between the predicted curves and the experimental points suggests that the use of the chosen model and stress intensity factor solutions are successful, /19/.

The underestimation of a at high value of c could be due to the method used in determining the value of a. The potential drop method was employed during the experiment. It is often found that this method usually underestimates the measured results. There are other alternative methods which could be used to measure the crack length such as beach marking and heat tinting. The beach marking method should give more accurate measurements of the crack length. However, difficulties due to obtaining well defined marks and load history reduce their accuracy and utilization. They are also known to retard crack growth /16,19/.

Since determination of crack length is essential in any fatigue crack growth test the prediction of the relationships between a and c could prove to be very useful. They also provide information for further investigations such as prediction of crack profile and crack shape characteristics.

Slika 7. Zavisnost između dužine prsline *a* i *c* za površinsku prslinu u čeliku BS 4360-50D pod dejstvom savijanja u 4 tačke

Figure 7. Predicted relationships between crack length *a* and *c* for surface crack in BS 4360-50D steel under 4-point bending.
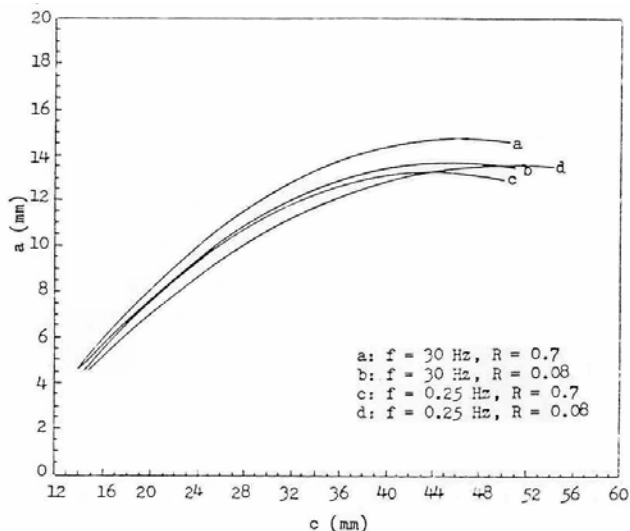

Slika 8. Zavisnosti između dužine prsline *a* i *c* za površinsku prslinu u metalu šava sučeonog zavarenog spoja od čelika BS 4360-50D pod dejstvom savijanja u 4 tačke

Figure 8. Predicted relationships between crack length *a* and *c* for surface crack in weld metal of butt-welded BS 4360-50D steel under 4-point bending.

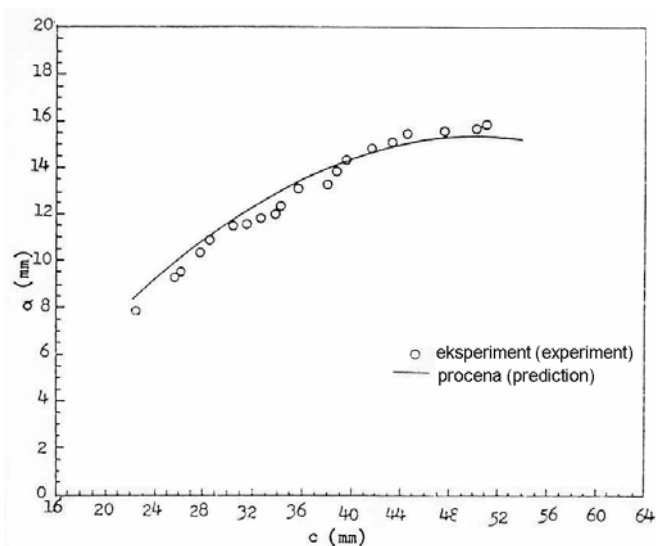Relacije između dužine prsline *a* i *c*
(Relationships between crack length *a* and *c*)

$$a = -7.6592 + 0.9211(c) - 0.92695 \cdot 10^{-2}(c)^2 \quad \text{.................. a}$$

$$a = -6.7936 + 0.8703(c) - 0.90192 \cdot 10^{-2}(c)^2 \quad \text{................. b}$$

$$a = -6.9489 + 0.8725(c) - 0.89210 \cdot 10^{-2}(c)^2 \quad \text{.................. c}$$

$$a = -5.7872 + 0.7936(c) - 0.80736 \cdot 10^{-2}(c)^2 \quad \text{............... d}$$

Relacije između dužine prsline *a* i *c*
(Relationships between crack length *a* and *c*)
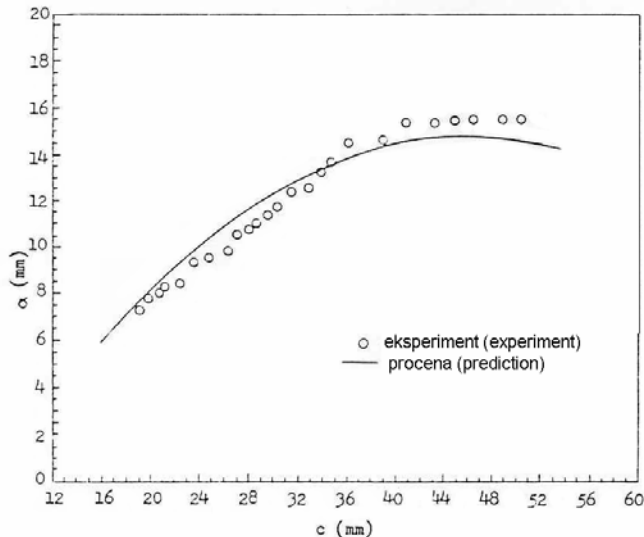
$$a = -5.7578 + 0.8887(c) - 0.95934 \cdot 10^{-2}(c)^2 \quad \text{.................. a}$$

$$a = -5.4801 + 0.2460(c) - 0.92873 \cdot 10^{-2}(c)^2 \quad \text{................ b}$$

$$a = -5.9929 + 0.8763(c) - 0.99312 \cdot 10^{-2}(c)^2 \quad \text{.................. c}$$

$$a = -4.5888 + 0.7264(c) - 0.72257 \cdot 10^{-2}(c)^2 \quad \text{............... d}$$

Slika 9. Poređenje eksperimentalne i predviđene dužine prsline *a* i *c* za površinsku prslinu u čeliku BS 4360-50D pod dejstvom savijanja u 4 tačke; 30 Hz, R = 0,7

Figure 9. Comparison between the experimental and predicted crack length *a* and *c* for surface crack in BS 4360-50D steel under 4-point bending; 30 Hz, R = 0.7.

Slika 10. Poređenje eksperimentalne i predviđene dužine prsline *a* i *c* za površinsku prslinu u metalu šava sučeonog zavarenog spoja od čelika BS 4360-50D pod dejstvom savijanja u 4 tačke; 30 Hz, R = 0,7

Figure 10. Comparison between the experimental and predicted crack length *a* and *c* for surface crack in weld metal of butt-welded BS 4360-50D steel under 4-point bending; 30 Hz, R = 0.7.

ZAKLJUČCI

Predviđanje zamornog rasta prsline pri savijanju u 4 tačke čelika BS4360-50D sa površinskom prslinom i sučeonog zavarenog spoja izvedeno je korišćenjem rešenja za faktor intenziteta napona prema Skotu i Torpu /15/ i modela zamornog rasta prsline prema Branku i saradnicima /13/. Računarski programi koji sadrže metodu analitičke integracije su razvijeni i procene se zasnivaju na ovim programima i na pristupačnim eksperimentalnim rezultatima. Procena se može svesti na sledeće zaključke:

(1) Zavisnosti za predviđanje odnosa veličine (dužine) prsline po debljinu i duž pravca razvoja na površini daju manje vrednosti za *a* pri malim vrednostima *c* i obrnuto. Što su veći frekvencija i odnos napona to vrednosti *a* postaju sve veće za svaku zadatu vrednost za *c* za isti materijal.

(2) Krive predviđenog rasta prsline se dobro slažu sa eksperimentalnim rezultatima i pokazuju da su brzine rasta prsline veće u pravcu na površini nego po debljini; veće su i pri većem odnosu napona; takođe su veće i u metalu šava nego u osnovnom metalu. Međutim, izvesne krive predviđenog rasta prsline daju manje brzine rasta prsline u oblasti praga.

CONCLUSIONS

The prediction of fatigue crack growth under 4-point bending of a surface crack in BS4360-50D steel and butt-welded steel was carried out by using stress intensity factor solutions reviewed by Scott and Thorpe /15/ and a fatigue crack growth model proposed by Branco et al. /13/. Computer programmes employing analytical integration technique have been developed and the prediction relies upon these programs and the available experimental results. The prediction may be summarised as follows

(1) The predicted relationships between the crack length in the through thickness and along the surface directions underestimate the values of *a* at low values of *c* and vice versa. The higher the frequency and stress ratio the larger the values of *a* become at any given values of *c* for the same material.

(2) The predicted crack growth curves correlate well with the experimental results and show that the crack growth rates are higher along the surface than through the thickness; higher at large stress ratio; and also higher in weld metal than in parent metal. However, some predicted crack growth curves underestimate the crack growth rates in the threshold region.

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