

DYNAMIC RESPONSE OF A CRACKED STRUCTURE INFLUENCED BY MICROMECHANICAL PARAMETERS OF SAND SUPPORTING

DINAMIČKI ODZIV KONSTRUKCIJE SA PRSLINOM POD UTICAJEM MIKROMEHANIČKIH PARAMETARA PESKOVITOG OSLOMCA

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Keywords

- structure
- dynamics
- crack
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- micromechanics parameters
- discrete element method

Abstract

Response of cracked structure under dynamic loading depends on several parameters such as the stiffness of structure and the nature of the soil implanted under the structure. The paper deals with dynamic response of a damaged structure implanted on a sand deposit. The sand profile is modelled using discrete element approach. Micromechanics parameters of sand such as density, granular contact forces, elastic and viscous damping parameters, are taken into account in the modelling of wave propagation through sand profiles studied up to the free surface with the aim of using a vertical dynamic component to excite the damaged structure. The state of the structure in terms of damage presence is treated in a rough way using a damage model of approved effectiveness; the degree of damage has remarkable influences on the evolution of displacements, velocities and accelerations.

INTRODUCTION

The dynamic actions on structures are generally studied through the results of recordings of displacements, speeds and accelerations caused by the presence of a vibration machine or other equipment that generates this type of excitement. The definition of specific responses to the particular conditions of the site where they are located /1-3/ is a fairly important parameter. However, it has been observed that site conditions can in some cases amplify dynamic surface movement and cause significant damage to structures /4, 5/.

The ground, and in particular the sand profile, is a medium that conducts dynamic amplification; before reaching the structure, dynamic waves propagate from the source of rupture towards the free surface by crossing the geological layers. The prediction of the dynamic response of the sand profile, which is a very complex material, requires on one hand a good modelling of its microstructure with the determination of various parameters that characterise its dy-

Ključne reči

- konstrukcija
- dinamika
- prslina
- peskoviti oslonac
- mikromehanički parametri
- metoda diskretnih elemenata

Izvod

Ponašanje konstrukcije sa prslinom pod dinamičkim opterećenjem zavisi od nekoliko parametara, kao što su krutost konstrukcije i priroda tla ispod konstrukcije. U radu istražujemo dinamički odziv oštećene konstrukcije na peskovitom tlu. Model profila tla se izvodi diskretnim elementima. Mikromehanički parametri peska, na pr. gustina, kontaktne sile između zrna, parametri elastičnosti i viskoznog prigušenja se uzimaju u obzir za modeliranje prostiranja talasa kroz profil peska, sve do slobodne površine, sa ciljem uvođenja vertikalne dinamičke komponente za pobudu oštećene konstrukcije. Stanje konstrukcije u smislu prisustva oštećenja se daje kao gruba procena, primenom modela oštećenja dokazane efikasnosti; stepen oštećenja ima izuzetne uticaje na razvoj pomeranja, brzina i ubrzanja.

dynamic behaviour, and on the other hand, the development of a behaviour model which reproduces as reliably as possible the real behaviour of materials constituting it.

The evaluation of the effect of damage and nature of the soil at the same time on the dynamic response of structures is rarely carried out. However, the influence of damage on the dynamic behaviour of structures is investigated in several papers /6-10/. On the other hand, analysis of wave propagation has been treated at the macromechanical scale in the works of Seed and Idriss /11/. These works take into account the variation of the shear modulus in the soil profile.

There are several approaches to analyse the macromechanical behaviour by returning to the understanding of what is happening at the micromechanical scale of a soil deposit subjected to dynamic loading at its base. As a topical subject, the most widely used model is based on the discrete element approach /11-14/. The first analyses of the seismic response of soils thus began with the development of the SHAKE programme /15/, based on the implementation of the same approach in the frequency domain. Subsequently, several

other programmes are developed and applied with increasing success for the dynamic analysis of soils using nonlinear behaviour models or frequency-dependent models, /16/.

The present work aims to study the dynamic responses of a damaged frame structure subjected to vertical ground motion excitation. Based on concepts of fracture mechanics, the damage section is modelled by an equivalent spring stiffness model /17-19/. On the other hand, the layer deposit implanted in this structure is also modelled as a micromechanics model. The constant average acceleration method is employed to determine dynamic responses for the cracked structure, however, the grains making up the profile of the sand are governed by Newton's second law which translates the equations of translational and rotational movements of grains. A programme is developed with the aim of conducting a parametric study to better understand the factors influencing the damaged structure implanted on amplified soil.

SAND MODELLING WITH DISCRETE ELEMENT METHOD

The discrete element technique simulates the grains of a granular material using independent components; at contact sites, each element communicates with its neighbours. The relative movements of grains, believed to be rigid bodies, are the primary cause of the medium's overall deformation. As a result, the integration of dynamic equations applied to each element can be used to characterise the behaviour of the medium. These equations are constructed by accounting for all external forces, including hydrodynamic, gravitational, and contact forces. Integration should be carried out incrementally with short enough time increments because these pressures can alter suddenly over time /12-14/.

$$m_i \ddot{x}_i = \sum_j \vec{F}_{ij}^{contact} + \vec{F}_i^{hi} + m_i \vec{g},$$

$$I_i \ddot{\phi}_i = \sum_j \vec{M}_{ij}^{contact} + \vec{M}_i^{hi}, \quad (1)$$

where: \ddot{x}_i are accelerations of translation; $\ddot{\phi}_i$ are accelerations of rotation; $\vec{F}_{ij}^{contact}$ and $\vec{M}_{ij}^{contact}$ are torque interaction and force contact between grains; \vec{M}_i^{hi} and \vec{F}_i^{hi} are hydrodynamic torque and force; m_i is mass of grain; I_i is moment of inertia of grain; and \vec{g} is acceleration of gravity.

In this paper, the molecular dynamics approach first put forth by Cundall and Strack /12/ is used. This approach permits a small amount of grain overlap when calculating the contact forces through a contact law. In this approach, it is assumed that grains are round.

By evaluating the typical distance between grains D_n at each time step, contacts are found (Fig. 1), where:

$$D_n = \|\vec{x}_j - \vec{x}_i\| - r_i - r_j. \quad (2)$$

It is possible to separate a contact force into normal and tangential components when applied by a grain j to a grain i . Here, the viscoelastic linear model is used to derive the normal force, these forces are given by:

$$\vec{F} = (-k_n D_n - v_n V_n) \vec{n}, \quad (3)$$

$$v_n = -\frac{2 \log(\varepsilon_n) \sqrt{k_n m_{eff}}}{\sqrt{\pi^2 + (\ln \varepsilon_n)^2}}, \quad (4)$$

$$m_{eff} = \frac{m_i m_j}{m_i + m_j}, \quad (5)$$

where: V_n is normal velocity; k_n are elastic constants; v_n are viscous damping constants; ε_n are ratios of normal velocities at the start and the end of the contact.

The straightforward frictional Coulomb model is used to calculate the tangential force given by:

$$\vec{F}_s = \mu_d F_n \frac{\vec{V}_s}{\|\vec{V}_s\|}, \quad (6)$$

where: \vec{V}_s is tangential velocity of grain j relative to grain i ; and μ_d is dynamic coefficient of friction.

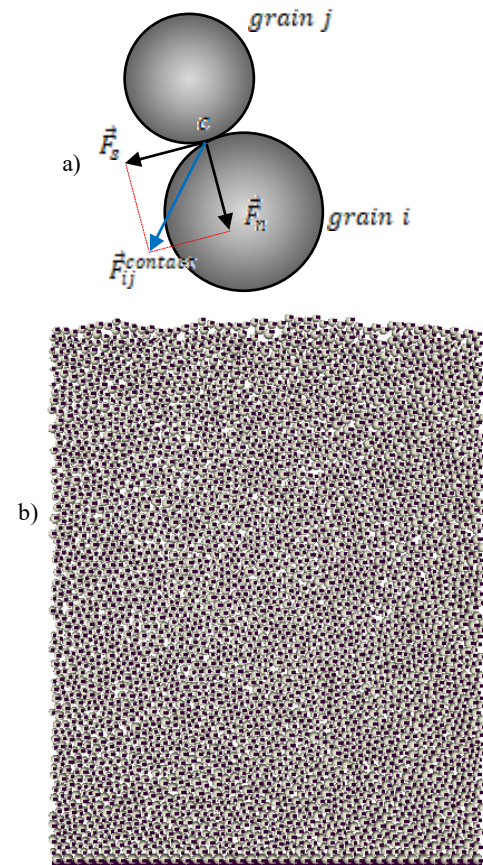


Figure 1. Contact force on: a) discrete element method; and b) sand deposit model.

MODEL FOR CRACKED STRUCTURE

The paper should contain a description of methods and research approach in sufficient detail to enable other researchers to perform similar analyses or link their research to the reported results. Well-established methods can be briefly elaborated and adequately cited.

For a continuous column subjected to a vertical seismic excitation, the differential equation corresponds to longitudinal motion is given by Doyle /20/,

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 u(x,t)}{\partial t^2} = F_i = -\bar{m} \ddot{x}_s(x,t), \quad (7)$$

where: $c = \sqrt{E/\rho}$ is wave celerity; $u(x,t)$ corresponds to the displacement in x -direction; and $\ddot{x}_s(x,t)$ is ground acceleration.

In order to introduce the damage at the any position of the column, this latest is discretised in N axial spring k . The stiffness of connecting springs is $k = N(EA/L)$. Each element has a modulus of elasticity E_i , gross sectional area A_i , density ρ_i , and length L_i , for $i = 1, 2, \dots, n$.

The crack is modelled by a translational axial spring with stiffness k_x at the position of damage, as shown in Fig. 1.

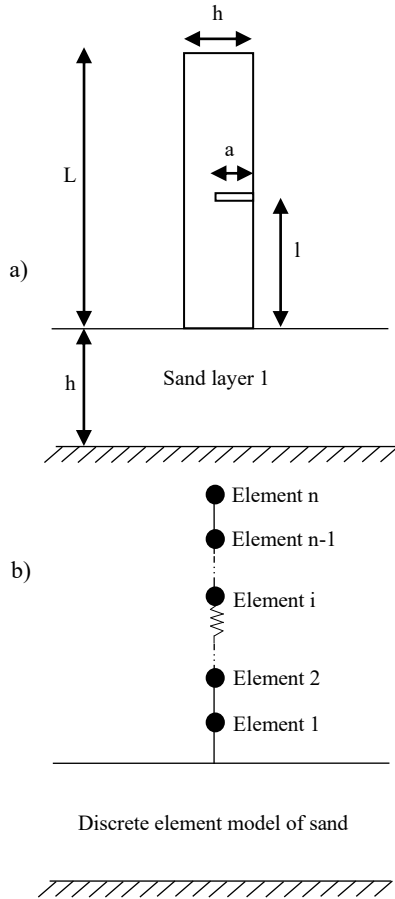


Figure 2. Model of cracked column implanted on soil layers: a) distributed mass system; b) lumped mass system.

Under longitudinal solicitation and based on fracture mechanics, Rizos et al. /21/ modelled the cracked section of a beam by an axial spring stiffness k_x with the expression as

$$k_x = \frac{EA}{(1-\mu^2)2\pi h f(a/h)}, \quad (8)$$

where: $f(a/h)$ is an expression depending on crack depth ratio and on column dimensions. Depending on the limit conditions and the nature of loading, this expression can be evaluated as follows /22/,

$$f(a/h) = 0.627(a/h)^2 - 0.172(a/h)^3 + 5.921(a/h)^4 - 10.700(a/h)^5 + 31.564(a/h)^6 - 67.445(a/h)^7 + 139.048(a/h)^8 - 146.577(a/h)^9 + 92.306(a/h)^{10}. \quad (9)$$

The equivalent stiffness of the damaged column is:

$$k^D = k_{eq} = \frac{k \times k_x}{k + k_x}. \quad (10)$$

The Eq.(7) can be expressed in the following form:

$$k^D = k \times \frac{\alpha}{\alpha + 1}, \quad (11)$$

with,
$$\alpha = \frac{1}{2\pi(1-\mu^2)\frac{h}{L}f(a/h)}. \quad (12)$$

If the crack is located at clamped end of the column, the stiffness matrix for the cracked element is given by:

$$[K^D] = \begin{bmatrix} \frac{\alpha+2}{\alpha+1}k & -k & 0 & \dots & 0 & 0 \\ -k & 2k & -k & \dots & 0 & 0 \\ 0 & -k & 2k & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2k & -k \\ 0 & 0 & 0 & \dots & -k & k \end{bmatrix}, \quad (13)$$

and when the crack is located at distance x of the column, the stiffness matrix for the cracked element is given by:

$$[K^D] = \begin{bmatrix} 2k & -k & 0 & \dots & 0 & 0 & 0 \\ -k & 2k & -k & \dots & 0 & 0 & 0 \\ 0 & -k & \ddots & \vdots & \vdots & 0 & 0 \\ \vdots & \vdots & \dots & \frac{\alpha+2}{\alpha+1}k & \frac{-\alpha}{\alpha+1}k & \vdots & \vdots \\ 0 & 0 & \dots & \frac{-\alpha}{\alpha+1}k & \frac{\alpha+2}{\alpha+1}k & \dots & \vdots \\ 0 & 0 & 0 & \vdots & \vdots & \ddots & -k \\ 0 & 0 & 0 & \dots & \dots & -k & k \end{bmatrix}. \quad (14)$$

Considering no change in mass of cracked element, the diagonal mass matrix is given by:

$$[K^D] = \begin{bmatrix} \frac{m}{N} & 0 & 0 & \dots & 0 \\ 0 & \frac{m}{N} & 0 & \dots & 0 \\ 0 & 0 & \frac{m}{N} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \dots & \frac{m}{N} \end{bmatrix}, \quad (15)$$

where: $m = \rho AL$.

SOLUTION METHOD

The evaluation of behaviour of the damaged structure implanted on dense sand is carried out through the numerical resolution of the following equation of motion:

$$[M]\ddot{x}_t + [C]\dot{x}_t + [K^D]x_t = \{F_t\}, \quad (16)$$

where: $[M]$, $[C]$, and $[K^D]$ are mass matrix, viscous damping matrix and stiffness matrix of cracked element, respectively; $\{F_t\}$ is seismic load vector. Equation (7) is solved numerically using the constant average acceleration method.

Rayleigh damping model is assumed with

$$C = \beta_1[M] + \beta_2[k],$$

where: β_1 and β_2 are Rayleigh damping coefficients.

For one degree freedom of structure, the equilibrium equation at instant $(t + \Delta t)$ is given by:

$$M\ddot{x}_{t+\Delta t} + C\dot{x}_{t+\Delta t} + K^D x_{t+\Delta t} = F_{t+\Delta t}. \quad (17)$$

We assume that acceleration is constant in the interval (t) and $(t + \Delta t)$,

$$\ddot{x}_{moy} = \frac{\ddot{x}_t + \ddot{x}_{t+\Delta t}}{2} \quad (18)$$

Based on average acceleration step-by-step integration method, the velocity and the displacement are given by the following expressions:

$$\dot{x}_{t+\Delta t} = \dot{x}_t + \frac{1}{2}(\Delta t) \times (\ddot{x}_t + \ddot{x}_{t+\Delta t}), \quad (19)$$

$$x_{t+\Delta t} = x_t + (\Delta t) \times \dot{x}_t + \frac{1}{4} \times (\Delta t)^2 (\ddot{x}_t + \ddot{x}_{t+\Delta t}). \quad (20)$$

From Eqs.(19) and (20) we can write the acceleration $\ddot{x}_{t+\Delta t}$ as a function of $x_{t+\Delta t}$,

$$\frac{1}{4} \times (\Delta t)^2 (\ddot{x}_t + \ddot{x}_{t+\Delta t}) = x_{t+\Delta t} - x_t - (\Delta t) \times \dot{x}_t, \quad (21)$$

$$\ddot{x}_t + \ddot{x}_{t+\Delta t} = \frac{4}{(\Delta t)^2} [x_{t+\Delta t} - x_t - (\Delta t) \times \dot{x}_t], \quad (22)$$

$$\ddot{x}_{t+\Delta t} = \frac{4}{(\Delta t)^2} [x_{t+\Delta t} - x_t - (\Delta t) \times \dot{x}_t] - \ddot{x}_t. \quad (23)$$

The incremental velocity Eq.(19) can be represented as

$$\dot{x}_{t+\Delta t} = \dot{x}_t + \frac{1}{2}(\Delta t) \left[\ddot{x}_t + \left(\frac{4}{(\Delta t)^2} [x_{t+\Delta t} - x_t - (\Delta t) \times \dot{x}_t] - \ddot{x}_t \right) \right], \quad (24)$$

$$\ddot{x}_{t+\Delta t} = \dot{x}_t + \frac{1}{2}(\Delta t) \left(\ddot{x}_t + \frac{4}{(\Delta t)^2} x_{t+\Delta t} - \frac{4}{(\Delta t)^2} x_t - \frac{4}{(\Delta t)} \dot{x}_t - \ddot{x}_t \right), \quad (25)$$

$$\ddot{x}_{t+\Delta t} = \dot{x}_t - 2\dot{x}_t + \frac{2}{(\Delta t)} x_{t+\Delta t} - \frac{2}{(\Delta t)} x_t + \frac{1}{2} \ddot{x}_t (\Delta t) - \frac{1}{2} (\Delta t) \ddot{x}_t, \quad (26)$$

$$\ddot{x}_{t+\Delta t} = \frac{2}{(\Delta t)} (x_{t+\Delta t} - x_t) - \dot{x}_t. \quad (27)$$

Substituting Eqs.(23) and (27) into incremental Eq.(17), the following expression can be obtained

$$\left[\frac{4M}{\Delta t^2} + \frac{2C}{\Delta t} + K^D \right] x_{t+\Delta t} = F_{t+\Delta t} + C \left[\frac{2}{\Delta t} x_t + \dot{x}_t \right] + M \left[\frac{4}{\Delta t^2} x_t + \frac{4\dot{x}_t}{\Delta t} + \ddot{x}_t \right]. \quad (28)$$

Incremental displacement and acceleration can also be expressed as

$$x_{t+\Delta t} = \frac{1}{\left[\frac{4M}{\Delta t^2} + \frac{2C}{\Delta t} + K^D \right]} \left[F_{t+\Delta t} + C \left[\frac{2}{\Delta t} x_t + \dot{x}_t \right] + M \times \left[\frac{4}{\Delta t^2} x_t + \frac{4\dot{x}_t}{\Delta t} + \ddot{x}_t \right] \right], \quad (29)$$

$$\ddot{x}_{t+\Delta t} = \frac{1}{m} \left[F_{t+\Delta t} - C\dot{x}_{t+\Delta t} - K^D x_{t+\Delta t} \right]. \quad (30)$$

Depending on which kind of soil and according to parameters of cracked structure (stiffness, mass and viscous damping), the displacement, velocity, and acceleration responses of the cracked column are obtained by numerical integration of differentials equations in both systems (soil layers and cracked column).

NUMERICAL RESULTS

The first numerical example shows dynamic responses of a cracked column implanted on four layers sand deposit. The system is subjected to the vertical dynamic component of harmonic excitation. In this example the sand and the parameters of excitation are considered for calculation (see

Table 1), the dimensions of the column are 0.60 m × 0.60 m × 3 m. The modulus of elasticity used in this study is $E = 32\,164\,195.1 \text{ kN/m}^2$ and corresponds to concrete.

Table 1. Values of excitation and sand parameters.

Sand parameters	Density of grains (kg/m ³)	Normal stiffness k_n (n/m)	Tangential Stiffness k_s (N/m)	Sand profile of 4000 grains height = 1.9 m
Value	2600	12·10 ⁵	96·10 ⁴	
Excitement parameters	$A \sin(f \cdot t)$	$A = 4 \cdot 10^{-4}$	$F = 20 \text{ Hz}$	$t = 3 \text{ s}$

The other coefficients of micromechanics parameters of sand, in particular the coefficient of restitution ϵ_n , dynamic friction coefficient μ_d and the intervening coefficients in the rolling resistance are chosen so as to ensure good behaviour of the deposit to build /23-25/.

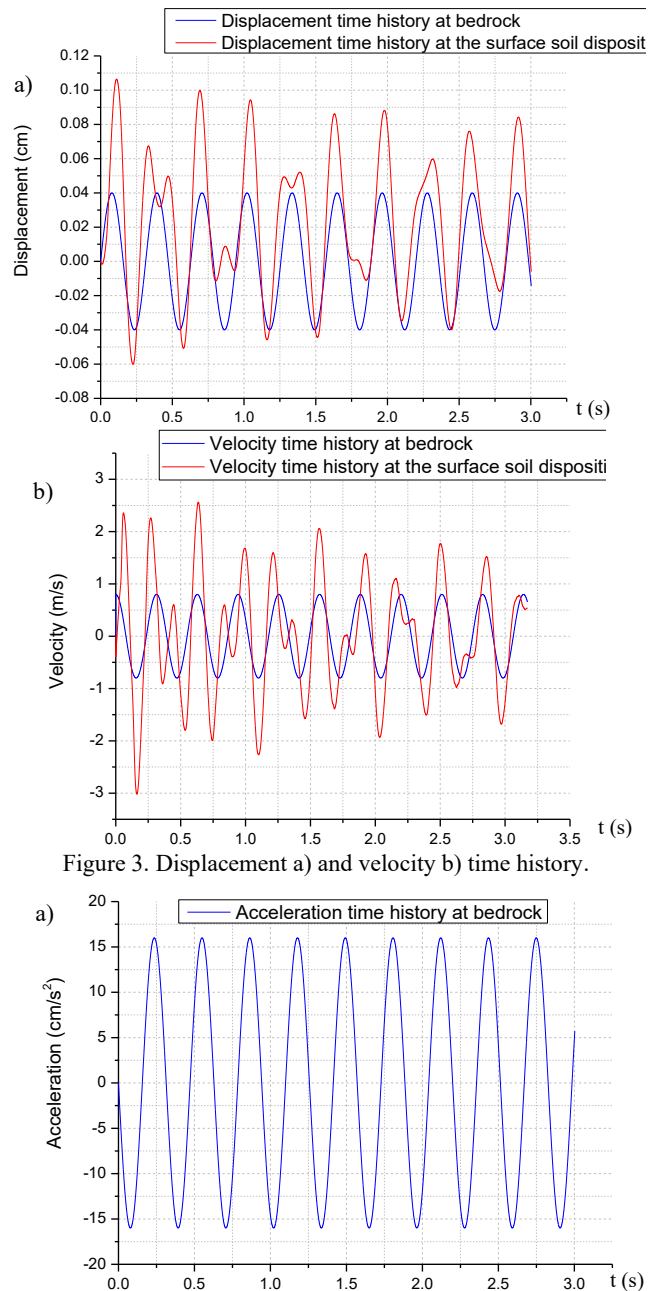


Figure 3. Displacement a) and velocity b) time history.

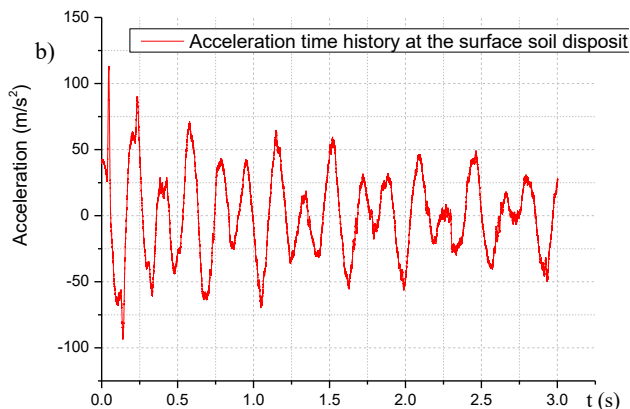


Figure 4. Acceleration time history at: a) bedrock (accelerogram 1); b) surface of soil deposit (accelerogram 2).

These figures clearly illustrate progressive amplification of motion, velocity, and acceleration from the base (bedrock) to the surface of granular deposit, highlighting the dynamic effect of soil profile on wave propagation mechanisms.

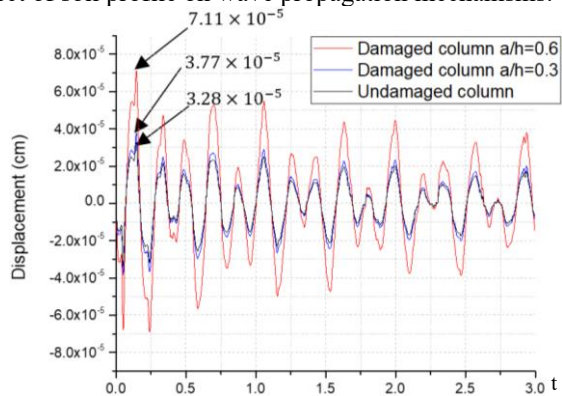


Figure 5. Influence of crack depth ratio on dynamic response of a damaged column (accelerogram 2).

Figure 5 shows the variation of longitudinal displacement vs. crack depth ratio (a/h) when the crack is located at the clamped end of the column. It can be observed that longitudinal displacement of cracked column depends essentially on crack depth. As shown in this figure, displacement increases with increase of crack depth because the presence of the crack leads to decrease in column stiffness.

Contrary to the first numerical example where the crack is modelled in a single column, the second example is devoted to the study of cracked structure excited by two different accelerograms from vertical ground motion (Fig. 4 a and b). The influence of sand micromechanics parameters are also considered by comparing the dynamic response of the damaged structure implanted in sand layer and the structure implanted in bedrock, the objective of this analysis is to have information on the influence of the sand layer that generates displacement in the damaged structure. The frame structure is composed of four floor levels and shown in Fig. 6, the section of columns is 60 cm \times 60 cm, the height of each level is $h = 3$ m, Young's modulus $E = 32\,164\,195.12$ kN/m², the lumped mass model is adopted in this analysis, the total weight of level one is 95 t, the other floor levels have weights of 90 t.

Accelerations at bedrock used in this study are a periodic excitation, shown in Fig. 4.

The analysis of dynamic behaviour of an intact, then damaged frame structure (Fig. 6) is carried out taking into account the amplification effect of dynamic accelerations on the surface of sand profiles (accelerogram 2). This analysis is based on the comparison of the maximum dynamic response of the structure in displacement and velocity.

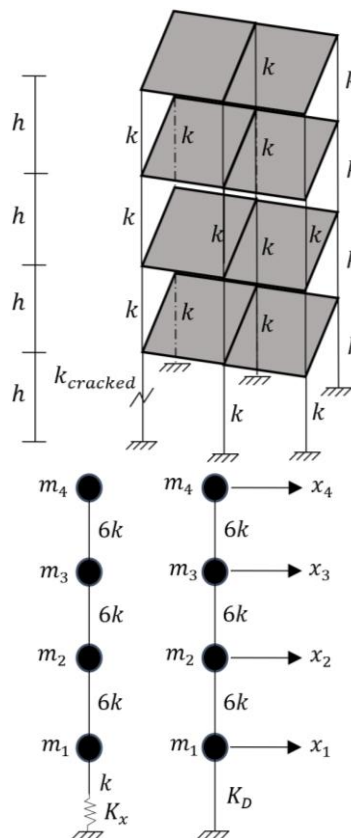


Figure 6. Model of cracked structure.

Dynamic response analysis of damaged structure

In order to determine the influence of damage on the seismic response of the structure, an analysis of the latter is made. We consider the damage structure (presence of cracks in the column) with a constant damage ratio $a/h = 0.6$, the damage is introduced by the reduced rigidity of columns, this reduction is taken into account in the global matrix of the structure.

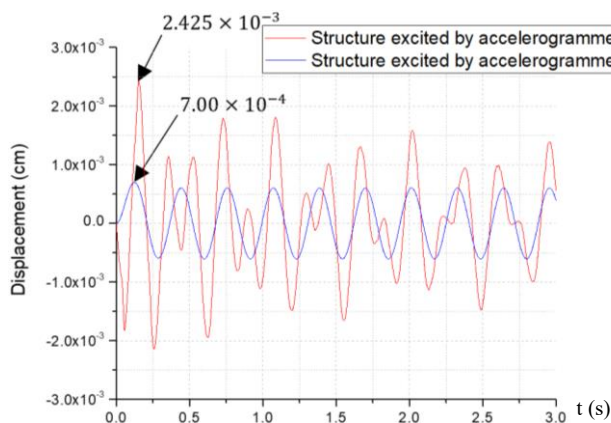


Figure 7. Time-history displacement response of damaged structure.

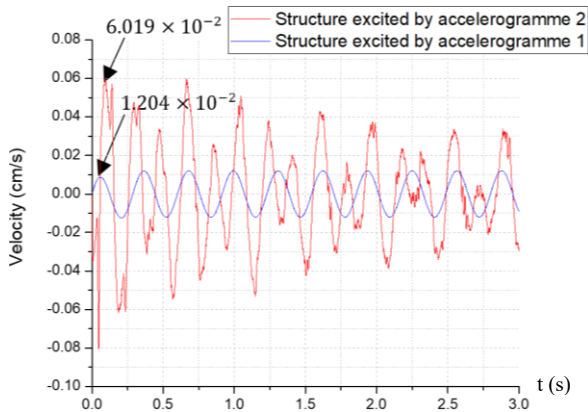


Figure 8. Time velocity response of damaged structure.

Figures 7 and 8 present the dynamic response of the damaged structure subjected to two different excitation scenarios (accelerograms 1 and 2). Results clearly show that the highest displacement and velocity values are obtained when the structure is excited by amplified signal ($x^{\max} = 2.425 \cdot 10^{-3}$ cm for displacement and $v = 6.019 \cdot 10^{-2}$ cm/s for velocity). In contrast, the lowest values are recorded under the non-amplified excitation ($x^{\max} = 7.00 \cdot 10^{-4}$ cm for displacement and $v = 1.204 \cdot 10^{-2}$ cm/s for velocity).

Dynamic response analysis of different structure type excited by the same vertical ground motion

The comparison of dynamic response for both intact and damaged structures under the effect of accelerogram 1 is carried out (see Fig. 9). It can be seen from this curve that the maximal displacement of damaged structure is greater than the displacement of the intact structure. The displacement for the damaged structure is 34.73 % more compared to the displacement of the intact structure, the same for the acceleration with a relative difference estimate of 28.25 %.

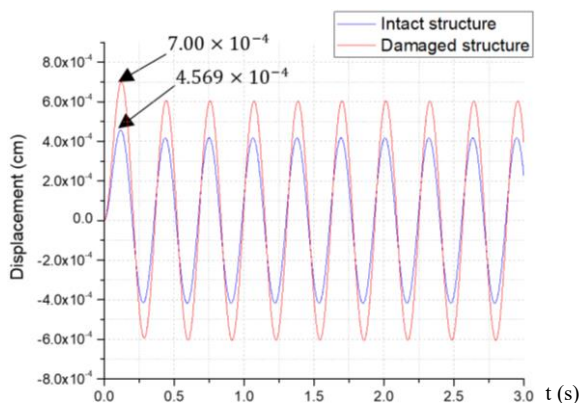


Figure 9. Time-history displacement of different structure type excited by the same vertical ground motion (accelerogram 1).

On the other hand, a comparison of dynamic response of intact and damaged structures under the effect of accelerogram 2 is made (see Fig. 10). This comparison clearly shows that seismic responses in terms of displacement and acceleration of the damaged structure has a significant change compared with seismic responses of the intact structure. The relative displacement response for the damaged structure is estimated at 32.33 % compared to the responses of the intact structure. In addition, it also noted from these figures that

the effect of the site has a greater amplifying effect ($x^{\max} = 2.425 \cdot 10^{-3}$ cm) than the effect generated by the damage of structure ($x^{\max} = 7.0 \cdot 10^{-4}$ cm).

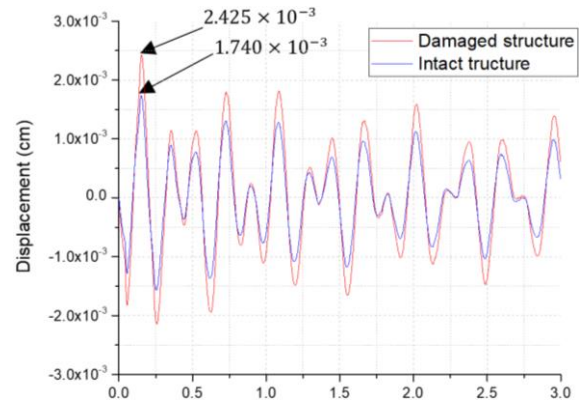


Figure 10. Time-history displacement of different structure type excited by the same vertical ground motion (accelerogram 2).

Influence of sand micromechanics parameters on seismic response of damaged structure

Figure 11 presents a comparative analysis of the influence of sand micromechanical characteristics (supporting soil) on the dynamic response of a damaged structure. Results indicate that the maximal displacement is significantly amplified when the structure is founded on dense sand, in comparison with medium and loose sand. Quantitatively, the amplification ratio between dense and loose sand reaches approximately 30.61 %, underscoring the substantial impact of soil density on wave propagation and amplification of structural motion.

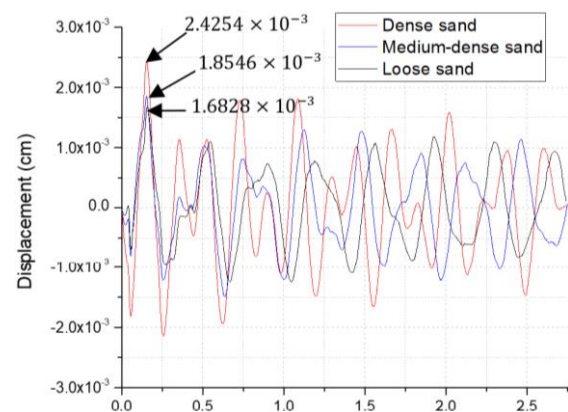


Figure 11. Effect of sand micromechanics parameters on the dynamic response of the damaged structure.

CONCLUSION

The model developed in this work made it possible to analyse the dynamic response of a structure on a sand deposit and to examine the effects of certain parameters which have influences on its response in terms of displacements and accelerations. Micromechanical parameters of the sand are taken into consideration through the use of the discrete element model for soil deposit and the local flexibility crack model for the damaged structure. In order to show at the same time both effects (amplification of the dynamic excitation and damage of structure) on the dynamic response of the real structure, two numerical examples are

considered in this paper. The results show that the maximal displacement is obtained when the intact structure is excited by the amplified signal compared with the displacement obtained when the cracked structure is excited by a non-amplified signal. It also shown in this research work that the dynamic response of a cracked structure depends essentially on the severity of damage (crack depth ratio) and the micromechanics parameters of sand layers' deposit (stiffness of soil layers).

The damage model for lumped mass structure developed in this work has confirmed its reliability in terms of presentation of damage, with the aim of exploiting it in the detection of damage in the structure under ambient vertical vibration.

The model of propagation wave in the sand profile has confirmed their effectiveness. Prospects are planned for using this model in the characterisation of soil profiles through an analysis of wave propagation velocity, variations in the shear modulus G , and the oedometer modulus E .

REFERENCES

- Anderson, J.C., Bertero, V.V. (1974), *Effects of gravity loads and vertical ground acceleration on the seismic response of multistory frames*, In: Proc. 5th World Conf. on Earthquake Engineering, Rome, 1974, Vol.2, pp. 2914-2923.
- Galal, K., Naimi, M. (2008), *Effect of soil conditions on the response of reinforced concrete tall structures to near-fault earthquakes*, Struct. Des. Tall Spec. Build. 17(3): 541-562. doi: 10.1002/tal.365
- Kadid, A., Yahiaoui, D., Chebili, R. (2010), *Behaviour of reinforced concrete buildings under simultaneous horizontal and vertical ground motions*, Asian J Civ. Eng. (Build. Hous.), 11 (4): 463-476.
- Arslan, H., Siyahi, B. (2006), *A comparative study on linear and nonlinear site response analysis*, Environ. Geol. 50(8), 1193-1200. doi: 10.1007/s00254-006-0291-4
- Rodriguez-Marek, A., Williams, J.L., Wartman, J., et al. (2003), *Ground motion and site response*, Earthq. Spectra, 19(1 suppl.): 11-34. doi: 10.1193/1.1737246
- Ayas, H., Touati, M., Chabaat, M. (2020), *Approximate analytical solution for free axial vibration of a cracked cantilever beam*, Int. J Sustain. Build. Technol. Urban Devel. 11(4): 209-221. doi: 10.22712/subs.20200016
- Ayas, H., Amara, L., Chabaat, M. (2021), *Approximate analytical analysis of longitudinal natural frequencies of a cracked beam*, Int. J Struct. Integr. 12(4): 534-547. doi: 10.1108/IJSI-07-2020-0065
- Collins, K.R., Plaut, R.H., Wauer, J. (1992), *Free and forced longitudinal vibrations of a cantilevered bar with a crack*, J Vib. Acoust. 114(2): 171-177. doi: 10.1115/1.2930246
- Dimarogonas, A.D., Papadopoulos, C.A. (1983), *Vibration of cracked shafts in bending*, J Sound Vibr. 91(4): 583-593. doi: 10.1016/0022-460X(83)90834-9
- Ostachowicz, W.M., Krawczuk, M. (1991), *Analysis of the effect of cracks on the natural frequencies of a cantilever beam*, J Sound Vibr. 150(2): 191-201. doi: 10.1016/0022-460X(91)90615-Q
- Seed, H.B., Idriss, I.M. (1971), *Simplified procedure for evaluating soil liquefaction potential*, J Soil Mech. Found. Div. 97 (9): 1249-1273. doi: 10.1061/JSFEAQ.0001662
- Cundall, P.A., Strack, O.D.L. (1979), *A discrete numerical model for granular assemblies*, Géotechnique, 29(1): 47-65. doi: 10.1680/geot.1979.29.1.47
- Pöschel, T., Schwager, T., Computational Granular Dynamics - Models and Algorithms, Springer-Verlag: Berlin Heidelberg, 2005. doi: 10.1007/3-540-27720-X
- Mansouri, M., El Youssoufi, M.S., Nicot, F. (2017), *Numerical simulation of the quicksand phenomenon by a 3D coupled Discrete Element - Lattice Boltzmann hydromechanical model*, Int. J Numer. Anal. Meth. Geomech. 41(3): 338-358. doi: 10.1002/nag.2556
- Schnabel, P.B., Lysmer, J., Seed, H.B. (1972), *SHAKE: A computer program for earthquake response analysis of horizontally layered sites*, EERC Report 72-12, University of California, Berkeley.
- Kausel, E., Assimaki, D. (2002), *Seismic simulation of inelastic soils via frequency-dependent moduli and damping*, J Eng. Mech. 128(1): 34-47. doi: 10.1061/(ASCE)0733-9399(2002)128:1(34)
- Boltezar, M., Strancar, B., Kuhelj, A. (1998), *Identification of transverse crack location in flexural vibrations of free-free beams*, J Sound Vibr. 211(5): 729-734. doi: 10.1006/jsvi.1997.1410
- Wang, T.M. (1970), *Natural frequencies of continuous Timoshenko beams*, J Sound Vibr. 13(4): 409-414. doi: 10.1016/S0022-460X(70)80045-1
- Nandwana, B.P., Maiti, S.K. (1997), *Detection of the location and size of a crack in stepped cantilever beams based on measurements of natural frequencies*, J Sound Vibr. 203(3): 435-446. doi: 10.1006/jsvi.1996.0856
- Doyle, J.F., Wave Propagation in Structures, 2nd Ed., Springer, New York, NY, 1997. doi: 10.1007/978-1-4612-1832-6
- Rizos, P.F., Aspragathos, N., Dimarogonas, A.D. (1990), *Identification of crack location and magnitude in a cantilever beam from the vibration modes*, J Sound Vibr. 138(3): 381-388. doi: 10.1016/0022-460X(90)90593-O
- Tada, H., Paris, P.C., Irwin, G.R., The Stress Analysis of Cracks, Handbook, 3rd Ed., ASME Press, New York, 2000.
- Derbane, S., Mansouri, M., Messast, S., El Malki Alaoui, A. (2025), *On the behavior of a granular soil deposit subjected to horizontal vibration. A discrete element modeling*, Comptes Rendus Mécanique, Vol. 353: 321-338. doi: 10.5802/crmeca.282
- Derbane, S., Mansouri, M., Messast, S. (2024), *Contribution to micromechanical modeling of the shear wave propagation in a sand deposit*, Technol. Audit Prod. Res. 3(1)(77): 10-18. doi: 10.15587/2706-5448.2024.301709
- Derbane, S., Mansouri, M., Messast, S. (2024), *Micromechanical modeling for analysis of shear wave propagation in granular material*, Rom. J Transp. Infrastr. 13(1): 1-19. doi: 10.2478/rjti-2024-0003

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