Pushap Lata Sharma[*], *Praveen Lata*, *Ajit Kumar*

STATIONARY CONVECTION ON THERMOSOLUTAL CONVECTION IN JEFFREY NANOFLUID SATURATED IN A POROUS MEDIUM FOR RIGID-RIGID AND RIGID-FREE BOUNDARY CONDITIONS

STACIONARNA KONVEKCIJA KOD TERMORASTVORLJIVE KONVEKCIJE U JEFFREY NANOFLUIDU SA POROZNOM SREDINOM, GRANIČNIH USLOVA: KRUTO-KRUTO I KRUTO-SLOBODNO

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Adresa autora / Author's address:
Department of Mathematics & Statistics, Himachal Pradesh University, Summer Hill, Shimla-171005, India

P.L. Sharma https://orcid.org/0000-0001-5848-9214
P. Lata https://orcid.org/0009-0002-9726-8814
A. Kumar https://orcid.org/0000-0002-5888-3761 ;
[*]email: pl_maths@yahoo.in

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Abstract

This work examines the stationary convection on thermosolutal convection in a Jeffrey nanofluid saturated in a porous medium, surrounded by rigid-rigid and rigid-free surfaces. Brownian motion effects and thermophoresis assumptions are now included in the governing equations. The impacts of numerous variables are derived by the application of normal mode techniques and linear stability analysis. Thermo-nanoparticle Lewis number, Soret parameter, modified diffusivity ratio and porosity have a stabilising impact on both boundary conditions, that delays the beginning of convection, while the other parameters like solutal Rayleigh number, Dufour parameter, thermosolutal Lewis number, concentration Rayleigh number, Jeffrey parameter have a destabilising impact on both boundary conditions, that enhances the onset of convection. The study of thermosolutal convection in Jeffrey nanofluid can improve the design of nanofluid-based cooling systems for electronics, energy storage and other industrial applications.

Izvod

U ovom radu se bavimo stacionarnom konvekcijom kod termorastvorljive konvekcije u Jeffrey nanofluidu koji je zasićen u poroznoj sredini, u okruženju graničnih površina: kruto-kruto i kruto-slobodno. U osnovnim matematičkim izrazima su uvrštene pretpostavke efekata Braunovog kretanja i termoforeze. Uticaji brojnih promenljivih veličina su izvedeni primenom metoda normalnog moda i analize linearne stabilnosti. Luisov broj termo-nanočestica, Soretov parametar, modifikovani odnosi difuzivnosti i poroznosti imaju stabilizujući uticaj u oba granična uslova, čime se odlaže početak konvekcije, dok drugi parametri, kao što su Rejlejev broj za rastvorljivost, Dufurov parametar i Soretov parametar, Luisov broj termorastvorljivosti, Rejlejev broj koncentracije, Jeffrey parametar, imaju destabilizujući efekat kod oba granična uslova, čime se podstiče početak konvekcije. Razmatranjem termorastvorljive konvekcije kod Jeffrey nanofluida ima za cilj unapređenje projektovanja rashladnih sistema na bazi nanofluida u elektronici, skladištenju energije, i u drugim industrijskim primenama.

INTRODUCTION

A nanofluid is a fluid that exhibits nanoparticles, with sizes in the range of 1-100 nm. These fluids have unique properties that differ from those of the base fluid. Choi /1/ was the first who introduced the term nanofluid. Nanofluids have many potential heat transfer applications, including microelectronics, fuel cells, pharmaceutical processes and hybrid-powered engines. Bejan /2/ revealed that fluid flow, heat transfer and porous structure interact to govern convective behaviour in porous media. Buongiorno /3/ introduced the two-component nanofluid model which accounts for the effects of nanoparticle size, shape and concentration on the thermal conductivity and viscosity of nanofluids. Nield and Bejan /4/ provided a detailed analysis of physical processes and mathematical models used to study convection in porous media. Tzou /5-6/ concluded that the presence of nanoparticles in a fluid can significantly affect its thermal behaviour and stability in natural convection, leading to changes in the onset of convection and heat transfer rates. Nield and Kuznetsov /7-8/ proposed that the nanofluid's properties and porous medium's characteristics significantly influence the onset of convection. Kuznetsov and Nield /9/ examined the onset of convection in porous media saturated with a nanofluid with modified Brinkman model. Sheu /10-11/ investigated the thermal instability of a viscoelastic nanofluid saturating a horizontal porous medium layer. Shivakumara et al. /12/ showed in their study that magnetic and rotational forces interact to significantly impact convective flow in a ferrofluid-saturated porous layer, highlighting the importance of rotation rate, magnetic field strength and porous layer properties, while Chand and Rana /13/ demonstrated that the incorporation of nanoparticles in a rotating porous medium significantly alters thermal convection dynamics, with rotational effects further influencing convective onset. Chand and Rana /14/ used a linear stability analysis to derive the governing equations and find the critical Rayleigh number for the onset of convection and show that the hall effect has a significant impact on the thermal instability of the nanofluid. Convective transport in a Maxwell nanofluid-saturated porous layer is significantly influenced by nanoparticle concentration, magnetic field and porous medium properties was examined by Umavathi and Mohite /15/. A macroscopic filtration model is developed to investigate natural convection in a Darcy-Maxwell nanofluid saturated porous layer with zero nanoparticle flux at the boundaries and is studied by Singh and Tyagi /16/. Kumar et al. /17/ studied the effect of a magnetic field on the onset of thermal convection in a Jeffery nanofluid layer saturated by a porous medium for distinct boundary conditions. A mathematical model to analyse the thermal convection in various nanofluids, considering variable gravity was employed by Sharma et al. /18, 19/, Devi et al. /20/, Bains and Sharma /21/, Bains et al. /22/, Kopp and Yanovsky /23/, and Sharma et al. /24-26/.

Thermosolutal convection arises from the relationship between thermal and solutal gradients which collectively influence fluid dynamics and convective transport. This phenomenon occurs when a fluid is exposed to simultaneous temperature and concentration gradients, resulting in density fluctuations that induce convective flows. Veronis /27/ investigated the significance of finite amplitude instability in shaping thermohaline convection patterns, with important implications for understanding oceanic processes. Wang and Tan /28/ worked on double diffusive convection on Maxwell fluid in a porous medium. The instability of double diffusive convection in a viscoelastic binary fluid was examined by Malashetty et al. /29-30/. Aggarwal /31/ looked into the influence of thermosolutal Rivlin-Ericksen fluid convection that was permeable with suspended particles in a porous medium. Kuznetsov and Nield /32/ studied double diffusive convection of Rivlin-Ericksen elastico-viscous nanofluid with porous medium. Yadav et al. /33/ explained the convective instability threshold in a binary nanofluid-saturated porous layer, revealing the intricate interaction between nanoparticle concentration, porous medium properties and thermophysical parameters. The thermosolutal convection phenomenon in a horizontal nanofluid layer, incorporating oscillatory motion dynamics was studied by Gupta et al. /34/, whereas Rana et al. /35/ and Rana and Chand /36/ investigated that thermosolutal instability in an elastico-viscous nanofluid-saturated porous medium is significantly influenced by the interplay of viscoelastic properties, nanoparticle concentration and porous medium characteristics, which collectively govern the onset of convective instability. Thermosolutal convection has various applications in different fields, including chemical engineering, geothermal systems, biological, biomedical applications, environmental engineering and aerospace engineering. Because of its huge applications, Sharma et al. /25, 37-40/ and Lata and Kumar /41/ looked into the impacts of rotation and magnetic field on thermosolutal convection on Jeffrey nanofluid in a porous medium. Kumar et al. /42/ worked on thermosolutal convection in porous medium in the presence of a Jeffrey nanofluid. This study aims to address this gap in knowledge by conducting a comprehensive stability analysis of thermosolutal convection of Jeffrey nanofluids saturating a porous medium under different boundary conditions. This work is original and has not been previously published. I hereby declare that we have gone through the detailed review of literature and have not found any evidence of prior publication.

MATHEMATICAL FORMULATIONS

Consider a horizontal layer of fluid enclosed between two planes $z^* = 0$ and $z^* = d$. The fluid layer receives heat from below and moves upwards with a gravitational force $g(0,0,-g)$. The temperature, volumetric fraction and concentration at the lower wall are T_0^*, ϕ_0^* and C_0^* while at the upper wall be T_1^*, ϕ_1^* and C_1^*, respectively, Fig. 1.

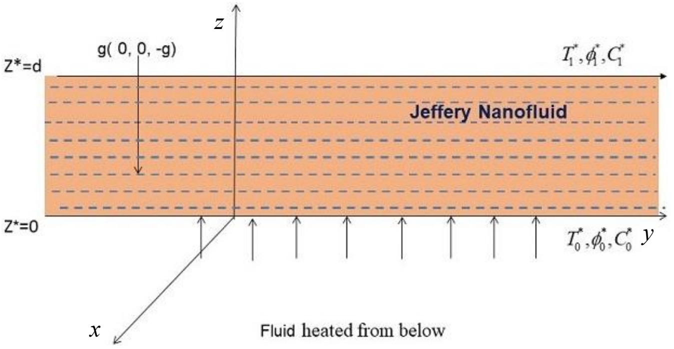

Figure 1. Physical configuration of the problem.

GOVERNING EQUATIONS

According to Kuznetsov and Nield /32/, Yadav et al. /33/, Rana and Chand /36/, Kumar et al. /42/, and Sharma et al. /40/, the governing equations for a thermosolutal convection are

$$\nabla . \mathbf{v}_D^* = 0 \,, \tag{1}$$

$$\frac{\rho_f}{\varepsilon} \frac{d\mathbf{v}_D^*}{dt^*} = -\nabla^* p^* + \rho g - \frac{\mu}{K(1+\lambda)} \mathbf{v}_D^* \,, \tag{2}$$

where: K, ρ_f, $\mathbf{v}_D^*$, p^*, t^*, μ and ε are permeability, fluid density, Darcy velocity, pressure, time, viscosity of the fluid, and porosity, respectively.

According to Buongiorno /3/, the density of nanofluid can be written as

$$\rho = \phi^* \rho_P + (1-\phi^*) \rho_f \,. \tag{3}$$

We assume the density of the nanofluid to be similar to that of the base fluid, we consider $\rho = \rho_f$. Now applying the Boussinesq approximation for the base fluid, the specific weight, ρg in Eq.(2) becomes

$$\rho g \approx \phi^* \rho_P + (1-\phi^*) \{ \rho_f (1 - \alpha_T (T^* - T_1^*) - \alpha_C (C^* - C_1^*)) \} \,. \tag{4}$$

By introducing a buoyancy force, the equation of motion Eq.(2) for Jeffrey nanofluid, by using Boussinesq approximation and Darcy model for porous medium, Kuznetsov and Nield /9/, and Nield and Kuznetsov /7-8/, becomes

$$0 = -\nabla^* p^* + \Big[\phi^* \rho_P + (1-\phi^*) \{ \rho_f (1 - \alpha_T (T^* - T_1^*) - \\ - \alpha_C (C^* - C_1^*)) \} \Big] g - \frac{\mu}{K(1+\lambda)} \mathbf{v}_D^* \,. \tag{5}$$

where: ρ_P is nanoparticle mass density.

For nanoparticles, the equation of continuity is

$$\frac{\partial\phi^*}{\partial t^*}+\frac{1}{\varepsilon}\mathbf{v}_D^*.\nabla^*\phi^*=D_B\nabla^{*2}\phi^*+(D_T/T_c^*)\nabla^{*2}T^*, \qquad (6)$$

where: D_T is thermophoresis diffusion coefficient; and D_B is Brownian diffusion coefficient.

The equation of thermal energy for nanoparticles is

$$(\rho c)_m\frac{\partial T^*}{\partial t^*}+(\rho c)_f\,\mathbf{v}_D^*.\nabla^*T^*=k_m\nabla^{*2}T^*+\varepsilon(\rho c)_p\times$$
$$\times\left[D_B\nabla^*\phi^*.\nabla^*T^*+(D_T/T_c^*)\nabla^*T.\nabla^*T^*\right], \qquad (7)$$

where: k_m; $\sigma=(\rho c)_m/(\rho c)_f$; $\alpha_m=k_m/(\rho c)_f$ are effective thermal conductivity of the porous medium, heat capacity ratio, and thermal diffusivity of porous medium, respectively.

The equation of conservation for solute concentration is

$$\frac{\partial C^*}{\partial t^*}+\frac{1}{\varepsilon}\mathbf{v}_D^*.\nabla^*C^*=D_{SM}\nabla^{*2}C^*+D_{CT}\nabla^{*2}T^*, \qquad (8)$$

where: D_{SM} is porous medium's solute diffusivity; and D_{CT} is Soret-type diffusivity.

The following are the boundary conditions

$$w^*=0,\ C^*=C_0^*,\ \phi^*=\phi_0^*,\ T^*=T_0^*,\ D_B\frac{\partial\phi^*}{\partial z^*}+\frac{D_T}{T_1}\frac{\partial T^*}{T_1^*}=0$$
$$\text{at}\quad z^*=0,$$
$$w^*=0,\ C^*=C_1^*,\ \phi^*=\phi_1^*,\ T^*=T_1^*,\ D_B\frac{\partial\phi^*}{\partial z^*}+\frac{D_T}{T_1}\frac{\partial T^*}{T_1^*}=0$$
$$\text{at}\quad z^*=d. \qquad (9)$$

The following dimensionless quantities are introduced

$$(u,v,w)=\frac{(u^*,v^*,w^*)d}{\alpha_m},\ (x,y,z)=\frac{(x^*,y^*,z^*)}{d},\ P=\frac{p^*k}{\mu\alpha_m},$$
$$t=\frac{t^*\alpha_m}{\sigma d^2},\ C=\frac{C^*-C_1^*}{C_0^*-C_1^*},\ \phi=\frac{\phi^*-\phi_0^*}{\phi_1^*-\phi_0^*},\ T=\frac{T^*-T_1^*}{T_0^*-T_1^*}.$$

We obtain Eq.(1) and Eqs.(5)-(8) in non-dimensional form as follows

$$\nabla.\mathbf{v}=0, \qquad (10)$$

$$0=-\nabla p-\frac{\mathbf{v}}{(1+\lambda)}-R_n\phi\hat{e}_z-R_m\hat{e}_z+R_aT\hat{e}_z+\frac{R_S}{L_e}C\hat{e}_z, \qquad (11)$$

$$\frac{1}{\sigma}\frac{\partial\phi}{\partial t}+\frac{1}{\varepsilon}\mathbf{v}.\nabla\phi=\frac{1}{L_n}\nabla^2\phi+\frac{N_A}{L_n}\nabla^2T, \qquad (12)$$

$$\frac{\partial T}{\partial t}+\mathbf{v}.\nabla T=\nabla^2T+\frac{N_B}{L_n}\nabla\phi.\nabla T+\frac{N_AN_B}{L_n}\nabla T.\nabla T+N_{TC}\nabla^2C, \qquad (13)$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}+\frac{1}{\varepsilon}\mathbf{v}.\nabla C=\frac{1}{L_e}\nabla^2C+N_{CT}\nabla^2T\,. \qquad (14)$$

The parameters, $L_e=\alpha_m/D_B$ is the Lewis number, $R_a=\rho\beta kdg(T_0^*-T_1^*)/\mu\alpha_m$ is thermal Rayleigh-Darcy number, $R_m=[\rho_P\phi_1^*+\rho(1-\phi_1^*)]kdg/\mu\alpha_m$ is basic density Rayleigh number, $R_n=(\rho_P-\rho)(\phi_1^*-\phi_0^*)kdg/\mu\alpha_m$ is concentration Rayleigh number, $L_n=\alpha_m/D_B$ is thermo-nanofluid Lewis number, $N_A=D_T(T_0^*-T_1^*)/D_BT_1^*(\phi_1^*-\phi_0^*)$ is the modified diffusivity ratio, $R_S=\rho c(C_0^*-C_1^*)kdg/\mu D_S$ is the solutal Rayleigh number, $N_B=(\rho c)_p(\phi_1^*-\phi_0^*)/(\rho c)_m$ is modified particle-density increment, $N_{CT}=D_{CT}(T_0-T_1)/\alpha_m(C_0-C_1)$ is Soret parameter, and $N_{TC}=D_{TC}(C_0-C_1)/\alpha_m(T_0-T_1)$ is the Dufour parameter.

The dimensionless boundary conditions are

$$w=0,\ C=1,\ \phi=1,\ T=1,\ \frac{\partial\phi}{\partial z}+N_A\frac{\partial T}{\partial z}=0\quad\text{at}\quad z=0,$$
$$w=0,\ C=0,\ \phi=0,\ T=0,\ \frac{\partial\phi}{\partial z}+N_A\frac{\partial T}{\partial z}=0\quad\text{at}\quad z=1. \qquad (15)$$

STEADY STATE SOLUTIONS

Following Kuznetsov and Nield /32/, Sheu /10-11/, Chand and Rana /14/ and Lata and Kumar /41/, the expression for time-independent basic states for nanofluids is given as

$$u=v=w=0,\ p=p_b(z),\ C=C_b(z),\ T=T_b(z),\ \phi=\phi_b(z)\cdot \qquad (16)$$

Putting Eq.(16) into Eqs.(11)-(14), these equations convert into following equations

$$0=-\frac{dp_b}{dz}-R_m\hat{e}_z+R_aT_b\hat{e}_z-R_n\phi_b\hat{e}_z+\frac{R_S}{L_e}C_b\hat{e}_z, \qquad (17)$$

$$\frac{d^2\phi_b}{dz^2}+N_A\frac{d^2T_b}{dz^2}=0, \qquad (18)$$

$$\frac{d^2T_b}{dz^2}+\frac{N_B}{L_n}\frac{d\phi_b}{dz}\frac{dT_b}{dz}+\frac{N_AN_B}{L_n}\left(\frac{dT_b}{dz}\right)^2+N_{TC}\frac{d^2C_b}{dz^2}=0, \qquad (19)$$

$$\frac{1}{L_e}\frac{d^2C_b}{dz^2}+N_{CT}\frac{d^2T_b}{dz^2}=0\,. \qquad (20)$$

Using boundary conditions Eq.(15) in Eq.(18), we get

$$\frac{d\phi_b}{dz}+N_A\frac{dT_b}{dz}=0\cdot \qquad (21)$$

Using Eq.(21) in Eq.(19), we get

$$\frac{d^2T_b}{dz^2}+N_{TC}\frac{d^2C_b}{dz^2}=0\cdot \qquad (22)$$

Using boundary conditions Eq.(15), the solution of Eq. (20) becomes

$$C_b=(1-T_b)L_eN_{CT}-(1+N_{CT}L_e)z+1\cdot \qquad (23)$$

Substituting the Eq.(23) in Eq.(22), we get

$$T_b=1-z\cdot \qquad (24)$$

Putting the Eq.(24) in Eq.(23), we get

$$C_b=1-z\cdot \qquad (25)$$

Substituting the Eq.(25) in Eq. (21), we get

$$\phi_b=N_AZ+\phi\cdot \qquad (26)$$

These solutions are identical with results obtained by Rana and Chand /36/.

PERTURBATION SOLUTIONS

Let's introduce a disturbance into the initial state, which can be expressed as

$$\mathbf{v}(u,v,w)=0+\mathbf{v}'(u,v,w),\ p=p_b+p',\ C=C_b+C',$$
$$\phi=\phi_b+\phi',\ T=T_b+T'\cdot \qquad (27)$$

Using Eq.(27) into Eqs.(10)-(15), we get the following equations

$$\nabla.\mathbf{v}'=0, \qquad (28)$$

$$0=-\nabla p'-\frac{\mathbf{v}'}{(1+\lambda)}-R_n\phi'\hat{e}_z+R_aT'\hat{e}_z+\frac{R_S}{L_e}C'\hat{e}_z, \qquad (29)$$

$$\frac{1}{\sigma}\frac{\partial\phi'}{\partial t}+\frac{N_A}{\varepsilon}\mathbf{v}'=\frac{1}{L_n}\nabla^2\phi'+\frac{N_A}{L_n}\nabla^2T', \qquad (30)$$

$$\frac{\partial T'}{\partial t}-\mathbf{v}'=\nabla^2T'+\frac{N_B}{L_n}\left(N_A\frac{\partial T'}{\partial z}-\frac{\partial\phi'}{\partial z}\right)-\frac{2N_AN_B}{L_n}\frac{\partial T'}{\partial z}+N_{TC}\nabla^2C', \qquad (31)$$

$$\frac{1}{\sigma}\frac{\partial C'}{\partial t}-\frac{1}{\varepsilon}\mathbf{v}'=\frac{1}{L_e}\nabla^2C'+N_{CT}\nabla^2T'\,. \qquad (32)$$

The boundary conditions are

$$w'=0,\ C'=0,\ \phi'=0,\ T'=0,\ \frac{\partial\phi'}{\partial z}+N_A\frac{\partial T'}{\partial z}=0$$
$$\text{at}\ z=0\ \text{and}\ z=1. \tag{33}$$

The seven unknowns u', v', w', p', ϕ', T', and C' can be simplified into four unknowns by applying the operator $\hat{e}_z$.curl.curl with Eq.(29), which yields

$$\frac{\nabla^2 w'}{(1+\lambda)}-R_a\nabla_1^2 T'+R_n\nabla_1^2\phi'-\frac{R_S}{L_e}\nabla_1^2 C'=0\,. \tag{34}$$

where: $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the Laplace operator; and $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is two-dimensional Laplacian operator.

NORMAL MODE ANALYSIS

Considering that the perturbed values are of the kind as below, and analysing the disturbances into the normal mode technique

$$[w',T',\phi',C']=[W(z),\Theta(z),\Phi(z),\Sigma(z)]\exp(ilx+imy+st)\,. \tag{35}$$

Putting Eq.(35) into Eq.(34) and Eqs.(30)-(32), we get

$$\frac{(D^2-a^2)}{(1+\lambda)}W+R_a a^2\Theta-R_n a^2\Phi+\frac{R_S}{L_e}a^2\Sigma=0\,, \tag{36}$$

$$\frac{W}{\varepsilon}+\left(\frac{s}{\sigma}-\frac{(D^2-a^2)}{L_e}\right)\Sigma+N_{CT}(D^2-a^2)\Theta=0\,, \tag{37}$$

$$W-\left(s-(D^2-a^2)+\frac{N_A N_B}{L_n}\right)\Theta-\frac{N_B}{L_n}D\Phi+N_{TC}(D^2-a^2)\Sigma=0, \tag{38}$$

$$\frac{N_A}{\varepsilon}W-\left(\frac{(D^2-a^2)}{L_n}-\frac{s}{\sigma}\right)\Phi-\frac{N_A}{L_n}(D^2-a^2)\Theta=0\,, \tag{39}$$

where: $D = d/dz$; and $a^2 = l^2 + m^2$ is the dimensionless wave number.

The boundary conditions are

$$W=0,\ \Theta=0,\ \Phi=0,\ \Sigma=0,\ D\Phi+N_A D\Theta=0$$
$$\text{at}\ z=0\ \text{and}\ z=1. \tag{40}$$

LINEAR STABILITY ANALYSIS FOR RIGID-RIGID BOUNDARIES

According to Kuznetsov and Nield /9/, Bains and Sharma /21/, Sharma et al. /24/ and Kumar et al. /17/, the boundary conditions are

$$W=DW=\Theta=\Phi=\Sigma=0\ \text{at}\ z=0\ \text{and}\ z=1\,. \tag{41}$$

The trial functions are

$$W=z^2(1-z)^2W_0,\ \Theta=z(1-z)\Theta_0,\ \Phi=z(1-z)\Phi_0,$$
$$\Sigma=z(1-z)\Sigma_0\,. \tag{42}$$

Putting Eq.(42) into Eqs.(36)-(39), we obtain the following system of linear equations in matrix form

$$\begin{bmatrix} \frac{(24+2a^2)}{(1+\lambda)} & -9R_a a^2 & 9R_n a^2 & -9\frac{R_S}{L_e}a^2 \\ 3N_A/\varepsilon & \frac{14N_A}{L_n}(10+a^2) & \frac{14}{L_n}\left[(10+a^2)+\frac{sL_n}{\sigma}\right] & 0 \\ 3 & -14(10+a^2+s) & 0 & -14N_{TC}(10+a^2) \\ 3/\varepsilon & -14N_{CT}(10+a^2) & 0 & -\frac{14}{L_e}(10+a^2)-\frac{sL_e}{\sigma} \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \\ \Sigma_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{43}$$

A non-trivial solution exists for the system of linear Eq.(43) if and only if

$$R_a=\frac{1}{\left(\frac{10+a^2}{L_e}+\frac{s}{\sigma}\right)\varepsilon-N_{TC}(10+a^2)}\left\{\frac{-R_n}{\frac{10+a^2}{L_n}+\frac{s}{\sigma}}\left[N_A(10+a^2+s)\left(\frac{10+a^2}{L_e}+\frac{s}{\sigma}\right)-N_A N_{TC}N_{CT}(10+a^2)(10+a^2)+\frac{N_A}{L_n}(10+a^2)\times\right.\right.$$
$$\left.\times\left(\frac{10+a^2}{L_e}+\frac{s}{\sigma}\right)\varepsilon-\frac{N_A}{L_n}N_{TC}(10+a^2)(10+a^2)\right]+\frac{28}{27a^2(1+\lambda)}(12+a^2)\varepsilon\left[(10+a^2+s)\left(\frac{10+a^2}{L_e}+\frac{s}{\sigma}\right)-N_{TC}N_{CT}(10+a^2)(10+a^2)\right]+$$
$$\left.+\frac{R_S}{L_e}\left[\varepsilon N_{CT}(10+a^2)-(10+a^2+s)\right]\right\}. \tag{44}$$

Stationary convection

For stationary convection, putting $s = 0$ in Eq. (44), it becomes

$$R_a=\frac{28\varepsilon(12+a^2)(10+a^2)(1-L_e N_{TC}N_{CT})}{27a^2(1+\lambda)(\varepsilon-N_{TC}L_e)}+\frac{R_S(\varepsilon N_{CT}-1)}{(\varepsilon-N_{TC}L_e)}+\frac{\{-N_A(L_n+\varepsilon)+N_A L_e(L_n N_{CT}+1)N_{TC}\}}{(\varepsilon-N_{TC}L_e)}R_n\,. \tag{45}$$

In the non-appearance of the Dufour and Soret parameters (i.e., N_{TC}, N_{CT}), Eq.(45) reduces to

$$R_a=\frac{28}{27a^2}(12+a^2)(10+a^2)+\frac{R_S}{\varepsilon}-\left(\frac{L_n}{\varepsilon}+N_A\right)R_n\,. \tag{46}$$

This result is identical with the results derived by Kuznetsov and Nield /32/.

In the non-appearance of the solute gradient parameter R_S, Eq.(46) reduces to

$$R_a=\frac{28}{27a^2}(12+a^2)(10+a^2)-\left(\frac{L_n}{\varepsilon}+N_A\right)R_n\,. \tag{47}$$

The above result is the same with the earlier published results of Kuznetsov and Nield /9/, Kumar et al. /17/, and Sharma et al. /24/.

LINEAR STABILITY ANALYSIS FOR RIGID-FREE BOUNDARIES

According to Kuznetsov and Nield /9/, Bains and Sharma /21/, Sharma et al. /24/ and Kumar et al. /17/, the boundary conditions are

$$W = DW = \Theta = \Phi = \Sigma = 0 \quad \text{at} \quad z=0 \quad \text{and} \quad z=1,$$
$$W = D^2W = \Theta = \Phi = \Sigma = 0 \quad \text{at} \quad z=0 \quad \text{and} \quad z=1. \tag{48}$$

The trial functions are

$$W = z^2(1-z)(3-2z)W_0,\ \Theta = z(1-z)\Theta_0,\ \Phi = z(1-z)\Phi_0,$$
$$\Sigma = z(1-z)\Sigma_0. \tag{49}$$

Substituting the trial functions given by Eq.(49) in the system of Eqs.(36)-(39), we acquire the following linear system of equations in matrix form

$$\begin{bmatrix} \dfrac{2(216+19a^2)}{(1+\lambda)} & -39R_a a^2 & 39R_n a^2 & -39\dfrac{R_S}{L_e}a^2 \\ 13N_A/\varepsilon & \dfrac{14N_A}{L_n}(10+a^2) & \dfrac{14(10+a^2)}{L_n}+\dfrac{14s}{\sigma} & 0 \\ 13 & -14(10+a^2+s) & 0 & -14N_{TC}(10+a^2) \\ 13/\varepsilon & -14N_{CT}(10+a^2) & 0 & -\dfrac{14}{L_e}(10+a^2)-\dfrac{14s}{\sigma} \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \\ \Sigma_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{50}$$

A non-trivial solution exists for the system of linear Eq.(50) if and only if

$$\begin{aligned} R_a = {} & \frac{1}{\left(\dfrac{10+a^2}{L_e}+\dfrac{s}{\sigma}\right)\varepsilon - N_{TC}(10+a^2)} \left\{ \frac{-R_n}{\dfrac{10+a^2}{L_n}+\dfrac{s}{\sigma}} \left[N_A(10+a^2+s)\left(\frac{10+a^2}{L_e}+\frac{s}{\sigma}\right) - N_A N_{TC} N_{CT}(10+a^2)(10+a^2) + \frac{N_A\varepsilon}{L_n}(10+a^2)\times \right.\right. \\ & \left. \times\left(\frac{10+a^2}{L_e}+\frac{s}{\sigma}\right)\varepsilon - \frac{N_A}{L_n}N_{TC}(10+a^2)(10+a^2)\right] + \frac{28}{507a^2(1+\lambda)}(216+19a^2)\varepsilon\left[(10+a^2+s)\left(\frac{10+a^2}{L_e}+\frac{s}{\sigma}\right) - N_{TC}N_{CT}(10+a^2)(10+a^2)\right] + \\ & \left. + \frac{R_S}{L_e}\left[\varepsilon N_{CT}(10+a^2) - (10+a^2+s)\right]\right\}. \end{aligned} \tag{51}$$

Stationary convection

For stationary convection, putting $s = 0$ in Eq.(51), it reduces to

$$R_a = \frac{28\varepsilon(216+19a^2)(10+a^2)(1-L_e N_{TC} N_{CT})}{507a^2(1+\lambda)(\varepsilon - N_{TC}L_e)} + \frac{R_S(\varepsilon N_{CT}-1)}{(\varepsilon - N_{TC}L_e)} + \frac{\{-N_A(L_n+\varepsilon) + N_A L_e(L_n N_{CT}+1)N_{TC}\}}{(\varepsilon - N_{TC}L_e)}R_n. \tag{52}$$

In the lack of the Dufour and Soret parameters (i.e., N_{TC}, N_{CT}), Eq.(52) reduces to

$$R_a = \frac{28}{507a^2}(216+19a^2)(10+a^2) + \frac{R_S}{\varepsilon} - \left(\frac{L_n}{\varepsilon}+N_A\right)R_n. \tag{53}$$

The above result agrees with the result derived by Kuznetsov and Nield /32/.

In the non-existence of the solute gradient parameter R_S, Eq.(53) reduces to

$$R_a = \frac{28}{507a^2}(216+19a^2)(10+a^2) - \left(\frac{L_n}{\varepsilon}+N_A\right)R_n. \tag{54}$$

Equation (54) matches with the results derived by Kuznetsov and Nield /9/, Kumar et al. /17/, and Sharma et al. /38/.

RESULTS AND DISCUSSIONS

In this paper, we study the stationary convection on thermosolutal convection in Jeffrey nanofluid saturated in a porous medium for rigid-rigid and rigid-free boundary conditions and we have analysed the impacts of ε, R_S, N_{TC}, N_{CT}, N_A, L_e, L_n, R_n, and λ on the stationary convection. We have plotted the graph with the help of MATLAB® software.

Figure 2 proves the graph of R_a with respect to a for numerous values of $\varepsilon = 0.2, 0.4, 0.6$. Adapting other parameters as: $R_S = 100$, $N_{TC} = 5$, $N_{CT} = 5$, $N_A = 5$, $L_e = 500$, $L_n = 500$, $R_n = -1$, $\lambda = 0.3$. It is obvious that as ε goes on increasing with the rise in the value of R_a. Thus, ε has a stabilising effect and it delays the beginning of convection of the system.

Figure 3 shows the graph of R_a with respect to a for various values of $R_S = 10, 50, 100$. Fixing other parameters as $\varepsilon = 0.4$, $N_{TC} = 5$, $N_{CT} = 5$, $N_A = 5$, $L_e = 500$, $L_n = 500$, $R_n = -1$, $\lambda = 0.3$. It is observable that as R_S goes on decreasing with the rise in the value of R_a. Thus, R_S has destabilising effect and it enhances the beginning of convection of the system.

Figure 4 clarifies the graph of R_a with respect to a for different values of $N_{TC} = 5, 10, 15$. Adjusting other parameters as $\varepsilon = 0.4$, $R_S = 100$, $N_{CT} = 5$, $N_A = 5$, $L_e = 500$, $L_n = 500$, $R_n = -1$, $\lambda = 0.3$. It is clear that as N_{TC} goes on decreasing with the rise in the value of R_a. Thus, N_{TC} has a destabilising effect and N_{TC} enhances the beginning of convection of the system.

Figure 5 demonstrates the graph of R_a with respect to a for various values of $N_{CT} = 3, 5, 7$. Taking other parameters as $\varepsilon = 0.4$, $R_S = 100$, $N_{TC} = 5$, $N_A = 5$, $L_e = 500$, $L_n = 500$, $R_n = -1$, $\lambda = 0.3$. It is obvious that as N_{CT} goes on increasing with the rise in the value of R_a. Thus, N_{CT} has a stabilising effect, and it delays the beginning of convection of the system.

Figure 6 explains the graph of R_a with respect to a for distinct values of $N_A = 3, 4, 5$. Adapting another parameter as $\varepsilon = 0.4$, $R_S = 100$, $N_{TC} = 5$, $N_{CT} = 5$, $L_e = 500$, $L_n = 500$, $R_n = -1$, $\lambda = 0.3$. It is understandable that as N_A goes on increasing with the rise in the value of R_a. Thus, N_A has stabilising effect and N_A delays the beginning of convection of the system.

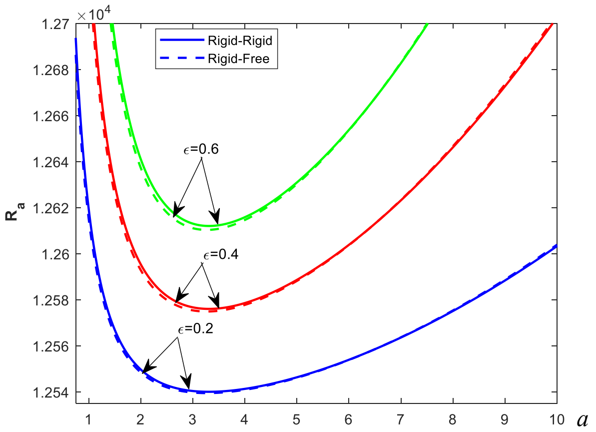

Figure 2. Variation of R_a with respect to a for various values of ε.

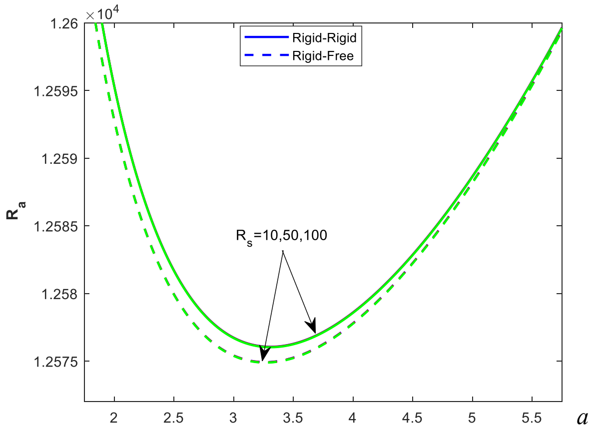

Figure 3. Variation of R_a with respect to a for various values of R_S.

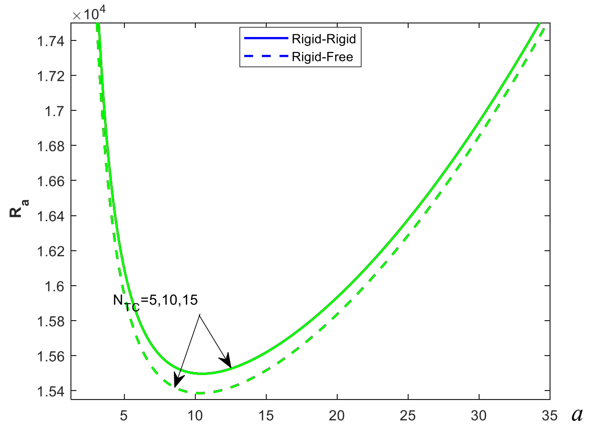

Figure 4. Variation of R_a with respect to a for various values of N_{TC}.

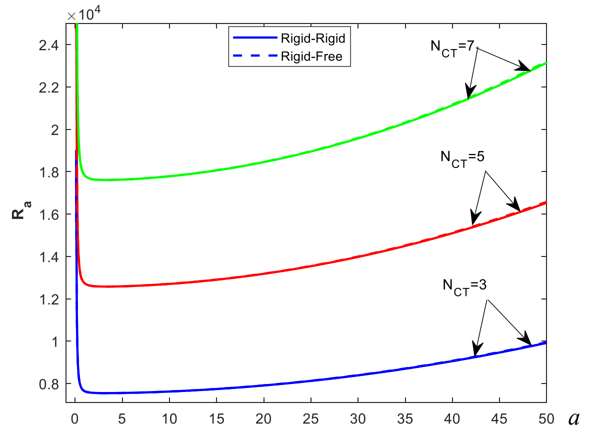

Figure 5. Variation of R_a with respect to a for various values of N_{CT}.

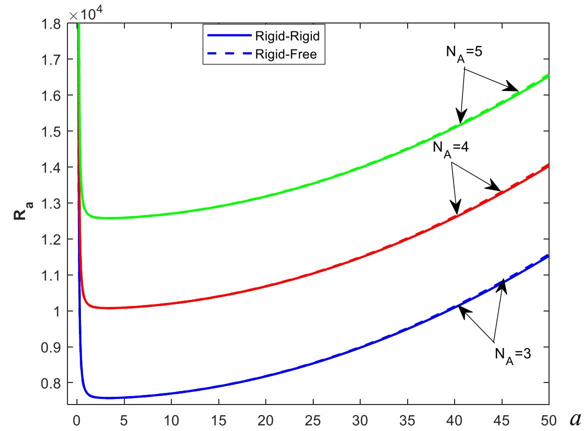

Figure 6. Variation of R_a with respect to a for various values of N_A.

Figure 7 illustrates the graph of R_a with respect to a for various values of $L_e = 500, 1000, 1500$. Changing the other parameters as $\varepsilon = 0.4$, $R_S = 100$, $N_{TC} = 5$, $N_{CT} = 5$, $N_A = 5$, $L_n = 500$, $R_n = -1$, $\lambda = 0.3$. It is clear that as L_e goes on decreasing with the rise in the value of R_a. Thus, L_e has destabilising effect and L_e enhances the beginning of convection on the system.

Figure 8 illustrates the graph of R_a with respect to a for different values of $L_n = 300, 400, 500$. Taking other parameters as $\varepsilon = 0.4$, $R_S = 100$, $N_{TC} = 5$, $N_{CT} = 5$, $N_A = 5$, $L_e = 500$, $R_n = -1$, $\lambda = 0.3$. It is understandable that as L_n goes on increasing with the rise in the value of R_a. Thus, L_n has a stabilising effect, and it delays the beginning of convection of the system.

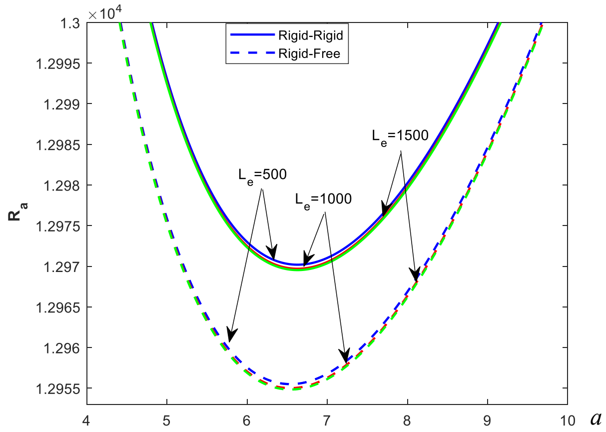

Figure 7. Variation of R_a with respect to a for various values of L_e.

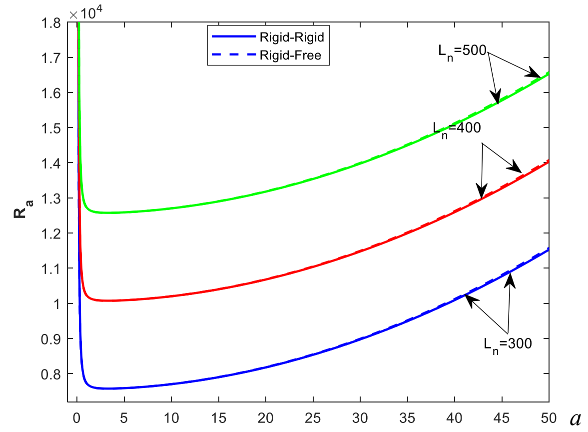

Figure 8. Variation of R_a with respect to a for various values of L_n.

Figure 9 shows the graph of R_a with respect to a for numerous values of $R_n = -1, -2, -3$. Regulating the other parameters as $\varepsilon = 0.4$, $R_S = 100$, $N_{TC} = 5$, $N_{CT} = 5$, $N_A = 5$, $L_e = 500$, $L_n = 500$, $\lambda = 0.3$. It is obvious that as R_n goes on decreasing with the rise in the value of R_a. Thus, R_n has a destabilising effect, and it enhances the beginning of convection of the system.

Figure 10 explains the graph of R_a with respect to a for different values of $\lambda = 0.3, 0.6, 0.9$. Adapting other parameters as $\varepsilon = 0.4$, $R_S = 100$, $N_{TC} = 5$, $N_{CT} = 5$, $N_A = 5$, $L_e = 500$, $L_n = 500$, $R_n = -1$. It is obvious that as λ goes on increasing with the rise in the value of R_a. Thus, λ has a destabilising effect, enhances the beginning of convection of the system.

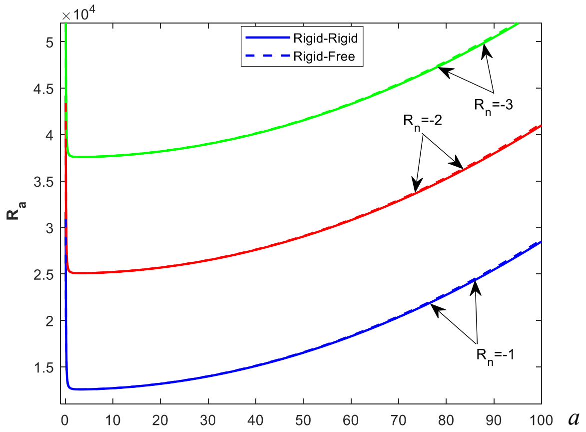

Figure 9. Variation of R_a with respect to a for various values of R_n.

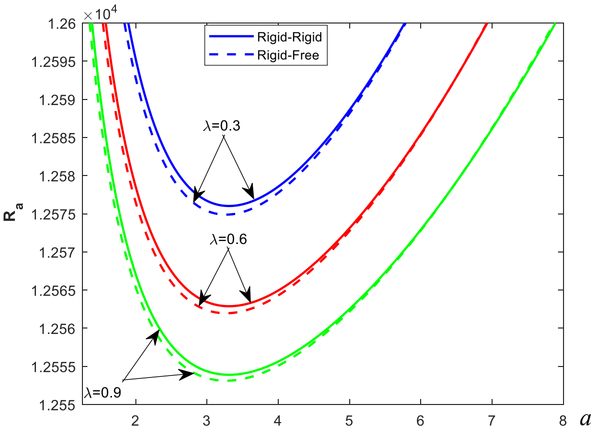

Figure 10. Variation of R_a with respect to a for various values of λ.

CONCLUSIONS

From this study we conclude that porosity, Soret parameter, modified diffusivity ratio, and thermo-nanoparticle Lewis number have stabilising effects and hence delay the onset of convection. On the other hand, the solutal Rayleigh number, Dufour parameter, thermosolutal Lewis number, concentration Rayleigh number and Jeffrey parameter have destabilising effects and enhance the onset of convection. Based on these results, it can be concluded that the stability impact is greater in the rigid free boundary as compared to the rigid-rigid boundary. This study also demonstrates that the stability of thermosolutal convection is dependent on thermal conductivity and nanoparticle concentration. We had also observed that our results reveal practical applications, vis., several industrial processes, including heat transfer, mass transport and energy efficiency are designed and optimised effectively. The research can be used to optimise chemical reactions and mass transfer in porous catalysts and reactors. The future scope of this study falls under nonlinear problems based on real-world requests and the outcomes of this study can be applied to create stable and more effective nanofluid-based systems.

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