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THERMOSOLUTAL CONVECTION IN A WALTER'S (MODEL B') NANOFLUID IN A POROUS MEDIUM: RIGID-RIGID AND RIGID-FREE BOUNDARY CONDITIONS

TERMORASTVORLJIVA KONVEKCIJA U VALTEROVOM (MODEL B') NANOFLUIDU SA POROZNOM SREDINOM, GRANIČNIH USLOVA: KRUTO-KRUTO I KRUTO-SLOBODNO

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- thermosolutal convection
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- porous media

Abstract

This study investigates the stability of thermosolutal convection in Walter's (model B') nanofluid saturated in a porous medium under two different boundary conditions: rigid-rigid and rigid-free. Linear stability analysis is conducted using perturbation theory and normal mode analysis to evaluate the system's stability. The effects of Brownian motion and thermophoresis are incorporated into the nanofluid model. The nanoparticle Rayleigh number, thermosolutal Lewis number and solutal Rayleigh number have a destabilising effect, enhancing the onset of convection. In contrast, parameters such as porosity, thermo-nanofluid Lewis number, modified diffusivity ratio, Dufour parameter and Soret parameter have a stabilising effect, delaying the onset of convection. Effects of these parameters are presented graphically using MATLAB® software. This study improves the understanding of thermosolutal convection in nanofluid-saturated in a porous medium, with implications for optimising heat and mass transfer in applications like chemical reactors, oceanic convection, material synthesis, food processing and biological systems.

Ključne reči
- termorastvorljiva konvekcija
- nanofluid
- Valterov (model B') nanofluid
- Rejlejev broj
- porozna sredina

Izvod

U ovom radu se bavimo stabilnošću termorastvorljive konvekcije u Valterovom (model B') nanofluidu koji je zasićen u poroznoj sredini, graničnih uslova: kruto-kruto i kruto slobodno. Analiza linearne stabilnosti se izvodi primenom teorije poremećaja analize u normalnom mod, radi procene stabilnosti sistema. Uticaji Braunovog kretanja i termoforeze su sadržani u modelu nanofluida. Rejlejev broj za nanočestice, Luisov broj za termorastvorljivost i Rejlejev broj za rastvorljivost imaju destabilizujući efekat, čime se povećava konvekcija. Nasuprot tome, parametri kao što su poroznost, termo-nanofluidni Luisov broj, modifikovani odnos difuzivnosti, Dufurov parametar i Soretov parametar imaju efekat stabilizacije, čime se odlaže početak konvekcije. Uticaji ovih parametara su predstavljeni grafički, korišćenjem softvera MATLAB®. Ovim radom daje se bolje razumevanje termorastvorljive konvekcije u nanofluidu - zasićen u poroznoj sredini, sa posledicama u optimizaciji prenosa toplote i mase u raznim primenama, kao kod hemijskih reaktora, konvekcija u okeanima, sintezi materijala, preradi hrane i u biološkim sistemima.

INTRODUCTION

Thermosolutal convection in porous media is a complex phenomenon with significant implications for numerous applications in fields of geophysics, soil sciences, bioengineering, cancer therapy, oceanography, etc. When nanofluids, which are engineered colloidal suspensions containing nanoparticles, are introduced into such porous environments, they exhibit enhanced thermal and solutal properties due to the high thermal conductivity and large surface area-to-volume ratio of nanoparticles. Thermosolutal instability has been thoroughly researched in various fluids and different geometrical arrangements. Chandrashekhar /1/ studied thermal convection in Newtonian fluids, examining hydrodynamic and hydromagnetic effects, and contributed key insights into convective instability and heat transfer. Veronis /2/ studied thermohaline convection in a fluid layer heated from below with a constant salinity gradient, advancing our understanding of convection driven by temperature and salinity differences. Bhatia and Steiner /3/ analysed the thermal instability of a Maxwellian viscoelastic fluid under a magnetic field, enhancing the understanding of magnetic effects on fluid stability. The ongoing advancement of nanotechnology in the current era has captivated the attention of several researchers and innovators. Choi /4/ was the first to introduce nanofluids, suspensions of nanoparticles in a base fluid. Typically, nanoparticles smaller than 100 nm are used, as their small size prevents channel blockage and wall erosion, allowing easy fluidization. Common nanoparticle materials include oxides (Al_2O_3, CuO), metal carbides (SiC), nitrides (AlN, SiN) and metals (Al, Cu), while base fluids include water, ethylene glycol, oils, coolants, lubricants, biofluids and polymer solutions. Buongiorno /5/ proposed that the absolute velocity of nanoparticles is the sum of the base fluid velocity and a relative slip velocity, improving the modelling of nanoparticle motion in nanofluids. Nanofluids in porous materials have attracted significant research due to their applications in areas such as steam engines, fuel cells, medical devices, refrigeration, heat exchangers, nuclear reactors, and biomedical equipment. Sheu /6/ studied oscillatory instability in nanofluid-saturated porous media using a viscoelastic fluid model, shedding light on the dynamic behaviour of these systems.

Tzou /7/ studied the onset of convection in a nanofluid-saturated horizontal layer heated from below, enhancing the understanding of convective behaviour in nanofluids. Alloui et al. /8/ studied natural convection in a nanofluid-filled shallow cavity heated from below, advancing understanding of heat transfer in such systems. Kuznetsov and Nield /9, 10/, along with Nield and Kuznetsov /11-13/, have investigated thermosolutal instability in nanofluid-saturated porous media through numerical and analytical methods, providing valuable insights into the heat and mass transfer processes and the onset of convection in these systems. Toms et al. /14/ found that the behaviour of a methyl methacrylate solution in n-butyl acetate matches Oldroyd's model /15/ but noted that many elastoviscous fluids do not adhere to Maxwell's or Oldroyd's constitutive relations.

Walters' (model B') elastoviscous fluid, commonly used in chemical science and industry, effectively demonstrates thermal and thermosolutal instability in porous media. Gupta and Aggarwal /16/, Rana and Sharma /17/, Rana et al. /18, 19/, Kumar et al. /20, 21/, Sharma et al. /22-33/, Malashetty et al. /34/, Wang and Tan /35/, Shivakumara et al. /36/, Sheu /6/, Lata and Kumar /37/ and Pundir et al. /38/ provide comprehensive insights into these phenomena. Rana /39/ specifically studied the onset of thermosolutal instability in an elastoviscous nanofluid layer in a porous medium with free-free boundaries, showing that the Soret and Dufour parameters delay convection onset.

The increasing use of nanofluids in various applications, particularly in medical fields like cancer therapy, has driven the motivation for this study. The primary objective is to examine the thermosolutal convection of an elastoviscous Walter's (model B') nanofluid under two different boundary conditions: rigid-rigid and rigid-free. As far as the authors are aware, this specific problem has not been addressed in the existing literature.

MATHEMATICAL FORMULATION

Here, we consider a horizontal layer with thickness d in the presence of Walter's (model B') nanofluid situated between the plates $z = 0$ and $z = d$ (Fig. 1). The fluid layer is heated below and working in the upward direction with gravity force $g(0,0,-g)$. Temperature, concentration and volumetric fraction of nanoparticles, at lower and upper boundaries are taken to be T_0 and T_1, C_0 and C_1, ϕ_0 and ϕ_1, respectively, where $T_0 > T_1$, $C_0 > C_1$ and $\phi_0 > \phi_1$.

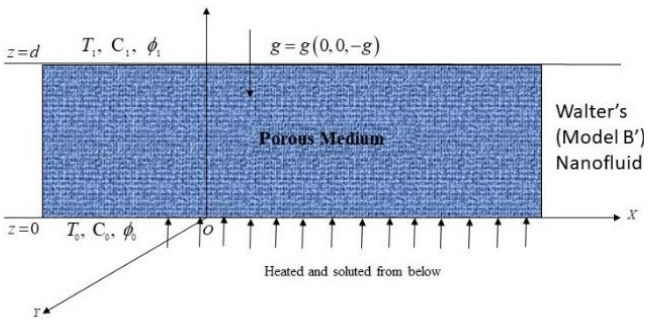

Figure 1. Modelling the Walter's (model B') nanofluid layer.

GOVERNING EQUATIONS

Equations of continuity and motion for Walter's (model B') elastic-viscous nanofluid in porous medium as given by Chandrasekhar /1/, Nield and Kuznetsov /11-13/ are:

$$\nabla . q_D = 0 , \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\frac{dq_D}{dt} = -\nabla p + \rho g - \frac{1}{k_1}\left(\mu - \mu' \frac{\partial}{\partial t}\right) q_D , \tag{2}$$

where: ρ, μ, p, ε, g, k_1, q_D, μ', ρ_p, ρ_f, and α denote in respect, the density, viscosity, pressure, medium porosity, acceleration due to gravity, coefficient of thermal conductivity, Darcy velocity vector, kinematic viscoelasticity, density of nanoparticles, density of base fluid and coefficient of thermal expansion.

The density ρ of nanofluid can be written as, /5/,

$$\rho = \phi \rho_p + (1-\phi)\rho_f , \tag{3}$$

where: ϕ is volume fraction of nanoparticles; ρ_p is density of nanoparticles; and ρ_f is density of base fluid. Following Tzou /7/ and Nield and Kuznetsov /11-13/ , we approximate the density of nanofluid by that of the base fluid, that is, we consider $\rho = \rho_f$. Now, introducing the Boussinesq approximation for the base fluid, the specific weight, ρg in Eq.(2) becomes

$$\rho g \cong \left[\phi \rho_p + (1-\phi)\{\rho(1-\alpha_T(T-T_1) - \alpha_C(C-C_1))\}\right] g , \tag{4}$$

where: α_T is coefficient of thermal expansion; and α_C is analogous to solute concentration.

If one introduces a buoyancy force, the equation of motion for Walter's (model B') nanofluid by using Boussinesq approximation and Darcy model for porous medium /9, 10/ is given by

$$0 = -\nabla p + \left[\phi \rho_p + (1-\phi)\{\rho(1-\alpha_T(T-T_1) - \alpha_C(C-C_1))\}\right] g - \\ -\frac{1}{k_1}\left(\mu - \mu' \frac{\partial}{\partial t}\right) q_D . \tag{15}$$

The equation of conservation of mass for the nanoparticles /5/ is

$$\frac{\partial \phi}{\partial t} + q_D \nabla \phi = D_B \nabla^2 \phi + \frac{D_T}{T_1}\nabla^2 T . \tag{6}$$

The thermal energy equation for a nanofluid /9, 10/ is

$$(\rho c)_m \left[\frac{\partial T}{\partial t} + q_D . \nabla T\right] = k_m \nabla^2 T + \varepsilon (\rho c)_p \times \\ \times \left[D_B \nabla \phi . \nabla T + \frac{D_T}{T_0}\nabla T . \nabla T\right] + (\rho c)_f D_{TC} \nabla^2 C , \tag{7}$$

where: $(\rho c)_m$ is heat capacity of fluid in porous medium; $(\rho c)_p$ is heat capacity of nanoparticles; and k_m is thermal conductivity and is a diffusivity of Dufour type.

The conservation equation for solute concentration /9, 10/ is

$$\frac{\partial c}{\partial t} + \frac{1}{\varepsilon} q_D . \nabla c = D_{SM} \nabla^2 C + D_{CT} \nabla^2 T , \tag{8}$$

where: D_{SM} and D_{CT} are the solute diffusivity of porous medium and diffusivity of the Soret type, respectively.

The boundary conditions are:

$$w=0,\ T=T_0,\ C=C_0,\ D_B\frac{\partial\phi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{T_1}=0 \quad \text{at} \quad z=0,$$

$$w=0,\ T=T_1,\ C=C_1,\ D_B\frac{\partial\phi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{T_1}=0 \quad \text{at} \quad z=d\,. \tag{9}$$

We introduce dimensionless variables are follows:

$$(x',y',z')=(x,y,z/d),\quad (u',v',w')=(u,v,w/\kappa_m)d,$$

$$t'=t\kappa_m/\sigma d^2,\quad P'=pk_1/\mu\kappa_m,\quad \phi'=\frac{\phi-\phi_0}{\phi_1-\phi_0},$$

$$T'=\frac{T-T_1}{T_0-T_1},\quad C'=\frac{C-C_1}{C_0-C_1}.$$

We obtain the Eq.(1), Eqs.(5)-(8) in non-dimensional form after eliminating the dashes (′) for simplicity in the form

$$\nabla.q_D=0, \tag{10}$$

$$0=-\nabla p-\left(1-F\frac{\partial}{\partial t}\right)q_D-R_m\hat{e}_z+R_aT\hat{e}_z-R_n\phi\hat{e}_z+\frac{R_S}{L_e}C\hat{e}_z \tag{11}$$

$$\frac{1}{\sigma}\frac{\partial\phi}{\partial t}+\frac{1}{\varepsilon}q_D.\nabla\phi=\frac{1}{L_n}\nabla^2\phi+\frac{N_A}{L_n}\nabla^2T, \tag{12}$$

$$\frac{\partial T}{\partial t}+q_D.\nabla T=\nabla^2T+\frac{N_B}{L_n}\nabla\phi.\nabla T+\frac{N_AN_B}{L_n}\nabla T.\nabla T+N_{TC}\nabla^2C \tag{13}$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}+\frac{1}{\varepsilon}q_D\nabla C=\frac{1}{L_e}\nabla^2C+N_{CT}\nabla^2T, \tag{14}$$

where: $L_e=\kappa_f/D_S$ is thermosolutal Lewis number; $F=\mu'\kappa_m/\mu\sigma d^2$ is kinematic viscoelasticity parameter; $R_a=\rho g\alpha dk_1\cdot(T_0-T_1)/\mu\kappa_m$ is thermal Rayleigh number; $R_m=\rho_p\phi_0+\rho(1-\phi_0)gk_1d/\mu\kappa_m$ is basic density Rayleigh number; $L_n=\kappa_m/D_B$ is thermo-nanofluid Lewis number; $R_n=(\rho_p-\rho)(\phi_1-\phi_0)\cdot gk_1d/\mu\kappa_m$ is nanoparticle Rayleigh number; $R_S=\rho g\alpha dk_1\cdot(T_0-T_1)/\mu\kappa_m$ is solutal Rayleigh number, $N_A=D_T\cdot(T_0-T_1)/D_BT_1(\phi_1-\phi_0)$ is modified diffusivity ratio; $N_B=\varepsilon(\rho c)_p(\phi_1-\phi_0)/(\rho c)_f$ is modified particle-density ratio $N_{CT}=D_{TC}(C_0-C_1)/\kappa_m(T_0-T_1)$ is Soret parameter; and $N_{TC}=D_{CT}(T_0-T_1)/\kappa_m(C_0-C_1)$ is Dufour parameter.

The dimensionless boundary conditions are

$$w=0,\quad T=1,\quad C=1,\quad \frac{\partial\phi}{\partial z}+N_A\frac{\partial T}{\partial z}=0 \quad \text{at} \quad z=0, \quad \text{and}$$

$$w=0,\quad T=0,\quad C=0,\quad \frac{\partial\phi}{\partial z}+N_A\frac{\partial T}{\partial z}=0 \quad \text{at} \quad z=1. \tag{15}$$

The steady state is given as

$$u=v=w=0,\quad p=p_b(z),\quad T=T_b(z),\quad \phi=\phi_b(z)\,. \tag{16}$$

Putting Eq.(16) into Eqs.(11)-(14), these equations become

$$0=-\frac{dp_b}{dz}-R_m\hat{e}_z+R_aT_b\hat{e}_z-R_n\phi_b\hat{e}_z+\frac{R_S}{L_e}C_b\hat{e}_z\,. \tag{17}$$

$$\frac{d^2\phi_b}{dz^2}+N_A\frac{d^2T_b}{dz^2}=0, \tag{18}$$

$$\frac{d^2T_b}{dz^2}+\frac{N_B}{L_n}\frac{d\phi_b}{dz}\frac{dT_b}{dz}+\frac{N_AN_B}{L_n}\left(\frac{dT_b}{dz}\right)^2+N_{TC}\frac{d^2C_b}{dz^2}=0, \tag{19}$$

$$\frac{1}{L_e}\frac{d^2C_b}{dz^2}+N_{CT}\frac{d^2T_b}{dz^2}=0\,. \tag{20}$$

Using Eq.(15) in Eq.(18), we get

$$\frac{d\phi_b}{dz}+N_A\frac{dT_b}{dz}=0\,. \tag{21}$$

Substituting Eq.(21) into Eq.(19), we get

$$\frac{d^2T_b}{dz^2}+N_{TC}\frac{d^2C_b}{dz^2}=0\,. \tag{22}$$

Substituting Eq.(15) in Eq.(20), we get

$$C_b=(1-T_b)L_eN_{CT}-(1+N_{CT}L_e)z+1\,. \tag{23}$$

Substituting Eq.(23) in Eq.(22), we get

$$T_b=1-z\,. \tag{24}$$

Substituting Eq.(24) in Eq.(23), we get

$$C_b=1-z\,. \tag{25}$$

Substituting Eq.(25) in Eq.(21), we get

$$\phi_b=N_AZ+\phi\,. \tag{26}$$

These basic solutions are identical with solutions obtained by Nield and Kuznetsov /11-13/.

PERTURBATION SOLUTIONS

Let the initial steady state solutions as described by Eq. 16, Eqs.(24)-(26) be slightly perturbed so that the perturbed state is given by

$$q_D(u,v,w)=0+q_D^*(u,v,w),\quad T=T_b+T^*=1-z+T^*,$$

$$p=p_b+p^*,\quad C=C_b+C^*=1-z+C^*,$$

$$\phi=\phi_b+\phi^*=N_AZ+\phi_0+\phi^*. \tag{27}$$

Using Eq.(27) into Eqs.(10)-(14), eliminating the star (*) for simplicity of the form, we get the following equations,

$$\nabla.q_D=0, \tag{28}$$

$$0=-\nabla p-\left(1-F\frac{\partial}{\partial t}\right)q_D+R_aT\hat{e}_z-R_n\phi\hat{e}_z+\frac{R_S}{L_e}C\hat{e}_z, \tag{29}$$

$$\frac{1}{\sigma}\frac{\partial\phi}{\partial t}+\frac{N_A}{\varepsilon}w=\frac{1}{L_n}\nabla^2\phi+\frac{N_A}{L_n}\nabla^2T, \tag{30}$$

$$\frac{\partial T}{\partial t}-w=\nabla^2T+\frac{N_B}{L_n}\left(N_A\frac{\partial T}{\partial z}-\frac{\partial\phi}{\partial z}\right)-\frac{2N_AN_B}{L_n}\frac{\partial T}{\partial z}+N_{TC}\nabla^2C, \tag{31}$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}-\frac{1}{\varepsilon}w=\frac{1}{L_e}\nabla^2C+N_{CT}\nabla^2T, \tag{32}$$

boundary conditions for the given equations are

$$w=0,\quad T=0,\quad C=0,\quad \frac{\partial\phi}{\partial z}+N_A\frac{\partial T}{\partial z}=0,$$

$$\text{at} \quad z=0 \quad \text{and at} \quad z=1. \tag{33}$$

The seven unknowns u, v, w, p, T, C and ϕ can be reduced into four unknowns by operating Eq.(29) with $\hat{e}_z$.curl.curl which yields

$$\nabla^2w=F\frac{\partial}{\partial t}\nabla^2w+R_a\nabla_H^2T-R_n\nabla_H^2\phi+\frac{R_S}{L_e}C\nabla_H^2\,. \tag{34}$$

NORMAL MODE ANALYSIS

Analysing the disturbances into the normal modes and assuming that the perturbed quantities /9, 11, 18, 35, 38/ are of the form

$$[w,T,\phi,C]=[W(z),\Theta(z),\Phi(z),\Psi(z)]\exp(ilx+imy+\omega t)\,. \tag{35}$$

Substituting Eq.(35) into Eq.(34) and Eqs.(30)-(32), we get

$$(D^2-a^2)(1-\omega F)W+a^2R_a\Theta-a^2R_n\Phi+\frac{R_S}{L_e}k^2\Psi=0, \tag{36}$$

$$\frac{W}{\varepsilon}+N_{CT}(D^2-a^2)\Theta+\frac{1}{L_e}\left(D^2-a^2-\frac{\omega}{\sigma}\right)\Psi=0, \tag{37}$$

$$W+\left(D^2+\frac{N_B}{L_n}D-\frac{2N_AN_B}{L_n}D-a^2-\omega\right)\Theta-\frac{N_B}{L_n}D\Phi+$$

$$+N_{TC}(D^2-a^2)\Psi=0\,, \tag{38}$$

$$\frac{N_A}{\varepsilon}W-\frac{N_A}{L_n}(D^2-a^2)\Theta-\left(\frac{1}{L_n}(D^2-a^2)-\frac{\omega}{\sigma}\right)\Phi=0\,, \tag{39}$$

where: $D = d/dz$ and $a^2 = l^2 + m^2$ is the dimensionless wave number.

The relevant boundary conditions of the problem are:

$$W=0,\ \ \Theta=0,\ \ \Phi=0,\ \ \Psi=0,\ \ D\Phi+N_A D\Theta=0$$
$$\text{at}\ \ z=0\ \ \text{and}\ \ z=1. \tag{40}$$

LINEAR STABILITY CONVECTION

Galerkin Weighted Residuals techniques are applied to evaluate the approximate solutions of ordinary differential Eqs.(36)-(39) satisfying boundary conditions Eq.(40). Choice of trial function depends on type of boundary conditions. Accordingly, W, Θ, Φ, Ψ are written in following format,

$$W=\sum_{i=1}^{N}A_iW_i\,,\ \ \Theta=\sum_{i=1}^{N}B_i\Theta_i\,,\ \ \Phi=\sum_{i=1}^{N}C_i\Phi_i\,,\ \ \Psi=\sum_{i=1}^{N}D_i\Psi_i\,, \tag{41}$$

where: A_i, B_i, C_i, D_i are constants; i = 1, 2, …, N. Using Eq.(41) in Eqs.(36)-(39) and multiplying the equations by W_i, Θ_i, Φ_i, Ψ_i, respectively, and integrating between limits 0 and 1, i.e., $0 < z < 1$, using boundary conditions Eq.(33), a system of $4N$ homogenous equations in $4N$ unknowns A_i, B_i, C_i, D_i is obtained. Using the condition of orthogonality and making the determinant of the coefficient matrix of set of equations to vanish for non-trivial solutions, gives the characteristic equation with thermal Rayleigh number as eigenvalue.

Linear stability analysis and dispersion relation for rigid-rigid boundaries

Applying the standard procedure of Galerkin Weighted Residuals technique, the appropriate boundary conditions for both the rigid bounding surfaces are:

$$W=0,\ DW=0,\ \Theta=0,\ \Phi=0,\ \Psi=0\ \text{ at }\ z=0\ \text{ and }\ z=1. \tag{42}$$

The trial functions satisfying the boundary conditions are chosen as:

$$W=z^2(1-z)^2W_0,\ \ \Theta=z(1-z)\Theta_0,\ \ \Phi=z(1-z)\Phi_0,$$
$$\Psi=z(1-z)\Psi_0. \tag{43}$$

Substituting Eq.(43) into Eqs.(36)-(39), integrating each equation from z = 0 to z = 1 and performing some integrations by parts, we obtain

$$\begin{bmatrix} (1-\omega F)(24+2a^2) & -9R_aa^2 & 9R_na^2 & -9\dfrac{R_S}{L_e}a^2 \\ 3N_A/\varepsilon & \dfrac{14N_A}{L_n}(10+a^2) & \dfrac{14}{L_n}\left[(10+a^2)+\dfrac{\omega L_n}{\sigma}\right] & 0 \\ 3 & -14(10+a^2+\omega) & 0 & -14N_{TC}(10+a^2) \\ 3/\varepsilon & -14N_{CT}(10+a^2) & 0 & -\dfrac{14}{L_e}\left[(10+a^2)+\omega\dfrac{L_e}{\sigma}\right] \end{bmatrix}\begin{bmatrix}W_0\\ \Theta_0\\ \Phi_0\\ \Psi_0\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}. \tag{44}$$

The linear system Eq.(44) has a non-trivial solution if and only if:

$$R_a=\frac{1}{\left(\dfrac{10+a^2}{L_e}+\dfrac{\omega}{\sigma}\right)\varepsilon-N_{TC}(10+a^2)}\left\{\frac{-R_n}{\left(\dfrac{10+a^2}{L_n}+\dfrac{\omega}{\sigma}\right)}\left[N_A(10+a^2+\omega)\left(\frac{10+a^2}{L_e}+\frac{\omega}{\sigma}\right)-N_AN_{TC}N_{CT}(10+a^2)(10+a^2)+\frac{N_A}{L_n}(10+a^2)\times\right.\right.$$
$$\left.\times\left(\frac{10+a^2}{L_e}+\frac{\omega}{\sigma}\right)\varepsilon-\frac{N_A}{L_n}N_{TC}(10+a^2)(10+a^2)\right]+\frac{28}{27a^2}(12+a^2)(1-\omega F)\varepsilon\left[(10+a^2+\omega)\left(\frac{10+a^2}{L_e}+\frac{\omega}{\sigma}\right)-N_{TC}N_{CT}(10+a^2)(10+a^2)\right]+$$
$$\left.+\frac{R_S}{Le}\left[\varepsilon N_{CT}(10+a^2)-(10+a^2+\omega)\right]\right\}. \tag{45}$$

Equation (45) is the required dispersion relation accounting for the effect of thermosolutal Lewis number, thermo-nanofluid Lewis number, kinematic visco-elasticity parameter, solutal Rayleigh number, nanoparticle Rayleigh number, modified diffusivity ratio, Soret and Dufour parameter on thermosolutal instability in a layer of Walters' (model B′) nanofluid saturating a porous medium.

Stationary convection (rigid-rigid boundaries)

For stationary convection, putting $\omega = 0$ in Eq.(45), we get

$$R_a=\frac{28\varepsilon(12+a^2)(10+a^2)(1-L_eN_{TC}N_{CT})}{27a^2(\varepsilon-N_{TC}L_e)}+\frac{R_S(\varepsilon N_{CT}-1)}{\varepsilon-N_{TC}L_e}+$$
$$+\frac{\{-N_A(L_n+\varepsilon)+N_AL_e(L_nN_{CT}+1)N_{TC}\}}{\varepsilon-N_{TC}L_e}R_n\,, \tag{46}$$

in the absence of the Dufour and Soret parameters (i.e., N_{TC}, N_{CT}), the Eq.(46) reduces to

$$R_a=\frac{28}{27a^2}(12+a^2)(10+a^2)+\frac{R_S}{\varepsilon}-\left(\frac{L_n}{\varepsilon}+N_A\right)R_n\,, \tag{47}$$

which is identical with the result derived by Kuznetsov and Nield /9, 10/.

In the absence of the stable solute gradient parameter R_S, Eq.(47) reduces to

$$R_a=\frac{28}{27a^2}(120+22a^2+a^4)-\left(N_A+\frac{L_n}{\varepsilon}\right)R_n\,. \tag{48}$$

The above result is corroboration with the earlier result of Kuznetsov and Nield /9, 10/.

Now we find the critical wave number from Eq.(46), by minimising R_a with respect to a^2, thus the critical wave number must satisfy

$$\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c}=0\quad \text{it gives}\quad a_c=3.31\,.$$

This result is validated with the original result of Nield and Kuznetsov, /11-13/.

Linear stability analysis and dispersion relation for rigid-free boundaries

The relevant boundary conditions of this case are:

$$W = DW = \Theta = \Phi = \Psi = 0 \quad \text{at} \quad z = 0 \quad \text{and}$$
$$W = D^2W = \Theta = \Phi = \Psi = 0 \quad \text{at} \quad z = 1. \qquad (49)$$

The exact solution of differential Eqs.(36)-(39), is not possible in analytical form for the set of boundary conditions Eq.(49). So, these equations are solved numerically by choosing the appropriate trial functions pacifying the boundary conditions, Eq.(50), as:

$$W = z^2(1-z)(3-2z),\ \Theta = z(1-z),\ \Phi = z(1-z),\ \Psi = z(1-z). \qquad (50)$$

Substituting the trial functions given by Eq.(50) in the system of Eqs.(36)-(39) and then integrating by parts using the condition of orthogonality and boundary conditions Eq.(49), we obtain

$$\begin{bmatrix} 2(216+19a^2)(1-\omega F) & -39a^2R_a & 39a^2R_n & -39\dfrac{R_S}{L_e}a^2 \\ \dfrac{13}{\varepsilon}N_A & 14\dfrac{N_A}{L_n}(10+a^2) & \dfrac{14(10+a^2)}{L_n}+\dfrac{14\omega}{\sigma} & 0 \\ 13 & -14(10+a^2+\omega) & 0 & -14N_{TC}(10+a^2) \\ \dfrac{13}{\varepsilon} & -14N_{CT}(10+a^2) & 0 & -\dfrac{14(10+a^2)}{L_e}-\dfrac{14\omega}{\sigma} \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \\ \Psi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (51)$$

The linear system Eq.(51) has a non-trivial solution if and only if:

$$R_a = \frac{1}{\left(\dfrac{10+a^2}{L_e}+\dfrac{\omega}{\sigma}\right)\varepsilon - N_{TC}(10+a^2)} \left\{ \frac{-R_n}{\left(\dfrac{10+a^2}{L_n}+\dfrac{\omega}{\sigma}\right)} \left[N_A(10+a^2+\omega)\left(\frac{10+a^2}{L_e}+\frac{\omega}{\sigma}\right) - N_AN_{TC}N_{CT}(10+a^2)(10+a^2) + \right.\right.$$
$$\left. + \frac{N_A\varepsilon}{L_n}(10+a^2)\left(\frac{10+a^2}{L_e}+\frac{\omega}{\sigma}\right) - \frac{N_A}{L_n}N_{TC}(10+a^2)(10+a^2) \right] + \frac{28}{507a^2}(216+19a^2)(1-\omega F)\varepsilon\left[(10+a^2+\omega)\left(\frac{10+a^2}{L_e}+\frac{\omega}{\sigma}\right) - \right.$$
$$\left.\left. -N_{TC}N_{CT}(10+a^2)(10+a^2)\right] + \frac{R_S}{L_e}\left[\varepsilon N_{CT}(10+a^2)-(10+a^2+\omega)\right]\right\}. \qquad (52)$$

Equation (52) is the required dispersion relation accounting for the effect of thermosolutal Lewis number, thermo-nanofluid Lewis number, kinematic visco-elasticity parameter, solutal Rayleigh number, nanoparticle Rayleigh number, modified diffusivity ratio, Soret and Dufour parameter on thermosolutal instability in a layer of Walters' (model B′) nanofluid saturating a porous medium.

Stationary convection (rigid-free boundaries)

For stationary convection, putting $\omega = 0$ in Eq.(52), we get

$$R_a = \frac{28(216+19a^2)(10+a^2)(1-L_eN_{TC}N_{CT})\varepsilon}{507a^2(\varepsilon - N_{TC}L_e)} + \frac{R_S(\varepsilon N_{TC}-1)}{\varepsilon - N_{TC}L_e} +$$
$$+ \frac{\{-N_A(L_n+\varepsilon)+N_AL_e(L_nN_{CT}+1)N_{TC}\}}{\varepsilon - N_{TC}L_e}R_n. \qquad (53)$$

In the absence of Dufour and Soret parameters (i.e., N_{TC}, N_{CT}), the Eq.(53) reduces to

$$R_a = \frac{28}{507a^2}(216+19a^2)(10+a^2) + \frac{R_S}{\varepsilon} - \left(\frac{L_n}{\varepsilon}+N_A\right)R_n, \qquad (54)$$

which is identical with the result derived by Kuznetsov and Nield /9,10/.

In the absence of the stable solute gradient parameter R_S, Eq.(54) reduces to

$$R_a = \frac{28}{507a^2}(2160+406a^2+19a^4) - \left(N_A+\frac{L_n}{\varepsilon}\right)R_n. \qquad (55)$$

Equation (55) is identical with the results derived by Nield and Kuznetsov /11-13/.

Now we find the critical wave number from Eq.(53), by minimising R_a with respect to a^2, thus the critical wave number must satisfy

$$\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c} = 0, \quad \text{it gives} \quad a_c = 3.31. \qquad (56)$$

This result is validated with the original result of Nield and Kuznetsov /11-13/.

RESULTS AND DISCUSSION

The thermal Rayleigh-Darcy number behaves as for a Newtonian fluid for both the cases of rigid-rigid and rigid-free boundary conditions, meaning that thermal Rayleigh-Darcy number does not depend on viscoelastic parameter. The dispersion relations Eq.(46) and Eq.(53) are analysed numerically, and graphs are plotted to depict the stability characteristics. We have plotted them graphically by using MATLAB® R2023a. In Figs. 2-9, we use abbreviations such as RR for rigid-rigid and RF for rigid-free boundary conditions.

Figure 2 shows the graph of R_a w.r.t. wave number a for different values of ε = 0.2, 0.4, 0.6 and by fixing the other parameters: R_S = 100, N_{TC} = 5, N_{CT} = 5, L_n = 500, N_A = 5, L_e = 500, and R_n = –1. It is clear from Fig. 2 that within increase in the value of ε, R_a goes on increasing and hence, shows the stabilising effect on stationary convection. Thus, ε delays the onset of convection.

Figure 3 shows the graph of R_a w.r.t. wave number a for different values of R_S = 10, 50, 100 and by fixing the other parameters: ε = 0.4, N_{TC} = 5, N_{CT} = 5, L_n = 500, N_A = 5, L_e = 500, and R_n = –1. It is clear from Fig. 3 that within increase in the value of R_S, R_a goes on increasing and hence shows the destabilising effect on stationary convection. Thus, R_S enhances the onset of convection.

Figure 4 shows the graph of R_a w.r.t. wave number a for different values of N_{TC} = 5, 10, 15 and by fixing the other parameters: ε = 0.4, R_S = 100, N_{CT} = 5, L_n = 500, N_A = 5, L_e = 500 and R_n = –1. It is clear from Fig. 4 that within increase in the value of N_{TC}, R_a goes on increasing and hence, shows the stabilising effect on stationary convection. Thus, N_{TC} delays the onset of convection.

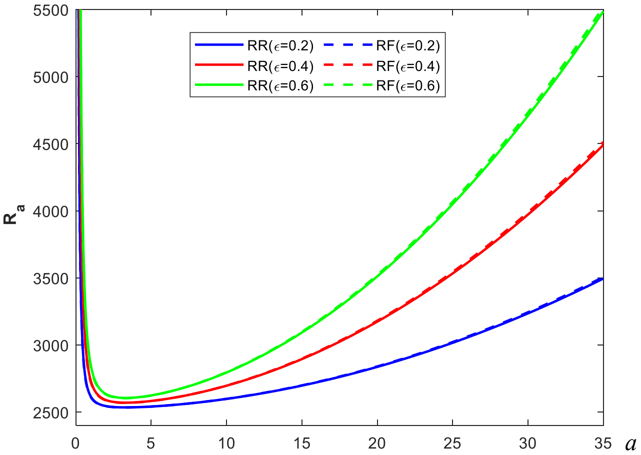

Figure 2. R_a vs. a for distinct values of ε.

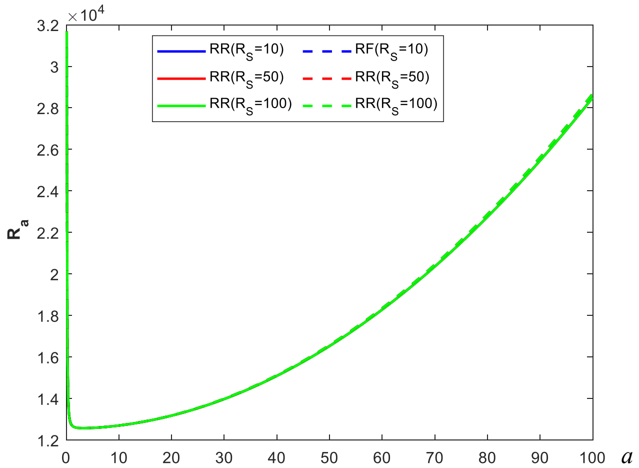

Figure 3. R_a vs. a for distinct values of R_S.

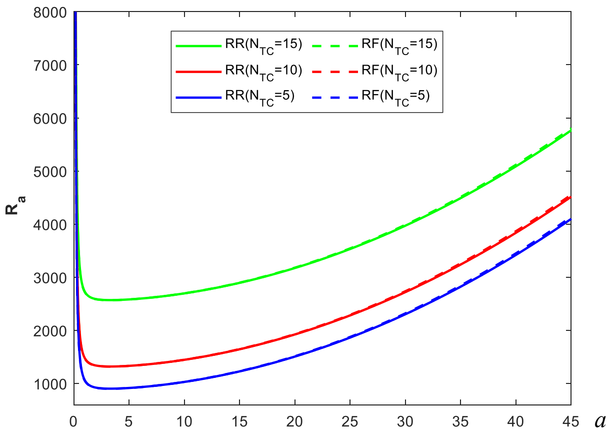

Figure 4. R_a vs. a for distinct values of N_{TC}.

Figure 5 shows the graph of R_a w.r.t. wave number a for different values of N_{CT} = 5, 10, 15, and by fixing the other parameters: ε = 0.4, R_S = 100, N_{TC} = 5, L_n = 500, N_A = 5, L_e = 500, and R_n = –1. It is clear from Fig. 5 that within increase in the value of N_{CT}, R_a goes on increasing and hence, shows the stabilising effect on stationary convection. Thus, N_{CT} delays the onset of convection.

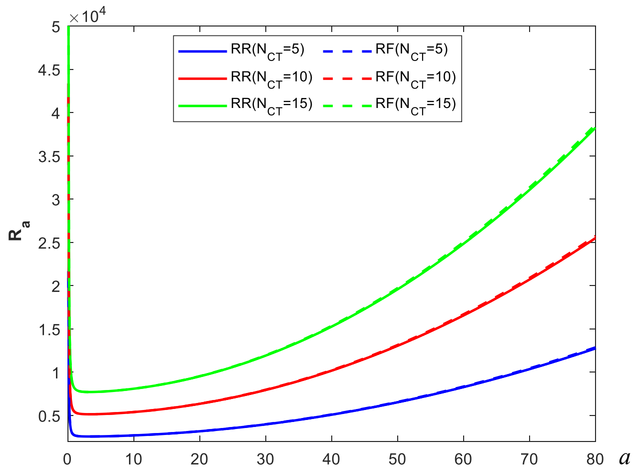

Figure 5. R_a vs. a for distinct values of N_{CT}.

Figure 6 shows the graph of R_a w.r.t. wave number a for different values of N_A = 1, 5, 10, and by fixing the other parameters: ε = 0.4, R_S = 100, N_{TC} = 5, L_n = 500, N_{CT} = 5, L_e = 500, and R_n = –1. It is clear from Fig. 6 that within increase in the value of N_A, R_a goes on increasing and hence, shows the stabilising effect on stationary convection. Thus, N_A delays the onset of convection.

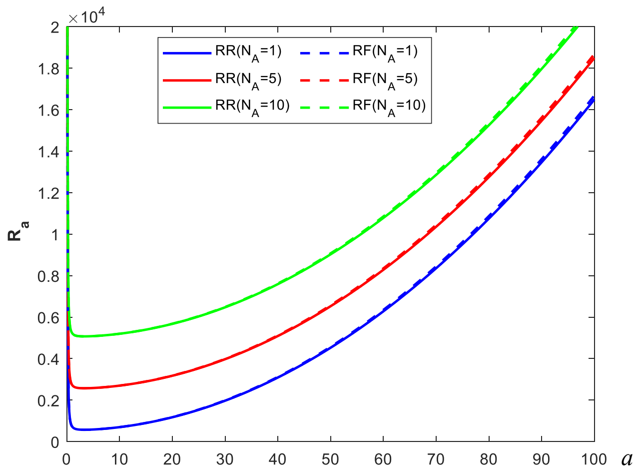

Figure 6. R_a vs. a for distinct values of N_A.

Figure 7 shows the graph of R_a w.r.t. wave number a for different values of L_e = 500, 1000, 1500, and by fixing the other parameters: ε = 0.4, R_S = 100, N_{TC} = 5, L_n = 500, N_{CT} = 5, N_A = 5, and R_n = –1. It is clear from Fig. 7 that within increase in the value of L_e, R_a goes on increasing and hence, shows the destabilising effect on stationary convection. Thus, L_e enhances the onset of convection.

Figure 8 shows the graph of R_a w.r.t. wave number a for different values of L_n = 500, 1000, 1500, and by fixing the other parameters: ε = 0.4, R_S = 100, N_{TC} = 5, L_e = 500, N_{CT} = 5, N_A = 5, and R_n = –1. It is clear from Fig. 8 that within increase in the value of L_n, R_a goes on increasing and hence, shows the stabilising effect on stationary convection. Thus, L_n delays the onset of convection.

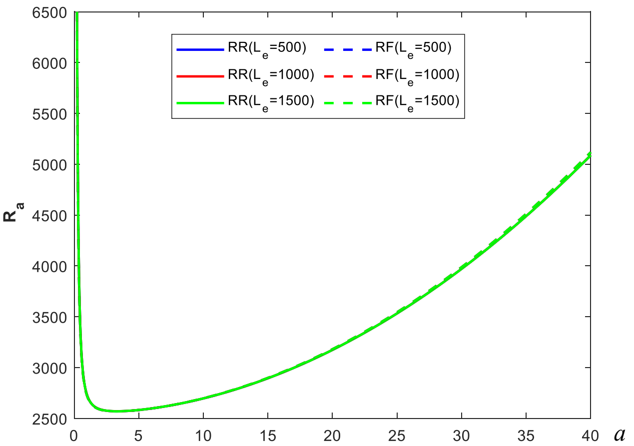

Figure 7. R_a vs. a for distinct values of L_e.

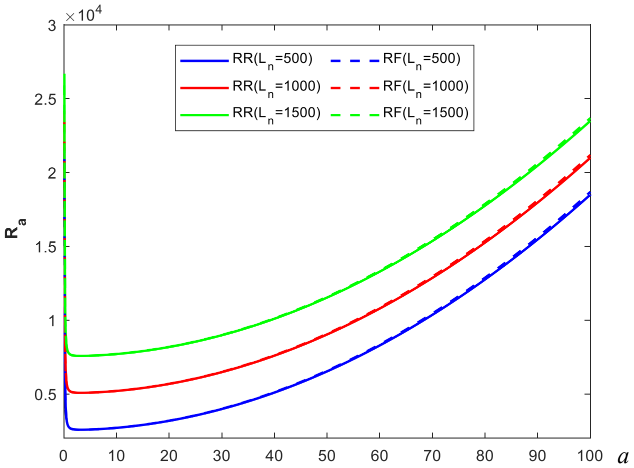

Figure 8. R_a vs. a for different values of L_n.

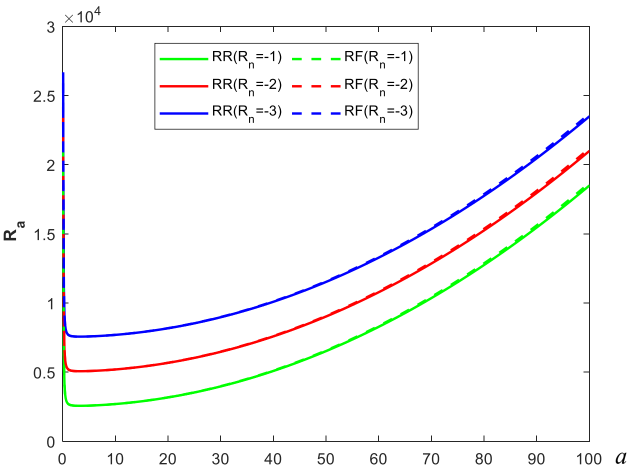

Figure 9. R_a vs. a for different values of R_n.

Figure 9 shows the graph of R_a w.r.t. wave number a for different values of $R_n = -1, -2, -3$, and by fixing the other parameters: $\varepsilon = 0.4$, $R_S = 100$, $N_{TC} = 5$, $L_e = 500$, $N_{CT} = 5$, $N_A = 5$, and $L_n = 500$. It is clear from Fig. 9 that within increase in the value of R_n, R_a goes on decreasing and hence, shows the destabilising effect on stationary convection. Thus, R_n enhances the onset of convection.

CONCLUSIONS

Thermosolutal convection on the onset of stationary convection in a layer of Walter's (model B′) nanofluid in a porous medium is investigated by using linear stability analysis. We draw the main conclusions as follows.

For the stationary convection, both the cases rigid-rigid and rigid-free, the Walter's (Model B') elastico-viscous nanofluid behaves like an ordinary Newtonian nanofluid.

Porosity ε, Soret parameter N_{CT}, Dufour parameter N_{TC}, modified diffusivity ratio N_A and thermo-nanofluid Lewis number L_n have stabilising effects on the stationary convection. Thus, these parameters delay the onset of convection for both rigid-rigid and rigid-free boundary conditions.

Nanoparticle Rayleigh number R_n, thermosolutal Lewis number L_e and solutal Rayleigh number R_S have destabilising effects on the stationary convection, which accelerates the onset of convection for both boundary conditions.

Kinematic viscosity has no impact on the onset of stationary instability.

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