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A NOVEL SPHERICAL FUZZY LOGIC APPROACH FOR STRUCTURAL INTEGRITY AND HEALTH MONITORING WITH ENTROPY ANALYSIS

NOVI FAZI LOGIČKI PRISTUP ZA PRAĆENJE STANJA I INTEGRITETA KONSTRUKCIJA SA ANALIZOM ENTROPIJE

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- structural health monitoring
- spherical fuzzy sets
- entropy measure
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- infrastructure safety

Abstract

This research introduces a novel framework for structural health monitoring technique selection by integrating spherical fuzzy sets with multi-criteria decision-making methods. The approach enhances traditional Multi-Criteria Decision-Making methods by addressing uncertainty and vagueness in expert judgments. A case study validates its effectiveness, showing more reliable outcomes compared to conventional methods. The framework offers a robust tool for optimising structural health monitoring techniques, improving infrastructure safety and sustainability.

Ključne reči
- praćenje stanja konstrukcija
- sferni fazi skupovi
- mera entropije
- upravljanje neizvesnošću
- bezbednost infrastrukture

Izvod

U ovom istraživanju se uvodi novi okvir za izbor metode praćenja stanja konstrukcije integrisanjem sfernih fazi skupova sa multi-kriterijumskim metodama odlučivanja. Pristup poboljšava tradicionalne multi kriterijumske metode odlučivanja svođenjem neizvesnosti i nedorečenosti u ekspertskom odlučivanju. Data studija slučaja služi za proveru efikasnosti, gde su uočljivi pouzdaniji ishodi u poređenju sa konvencionalnim metodama. Dati okvir pruža robusni alat za optimizaciju metoda praćenja stanja konstrukcija, sa unapređenjem bezbednosti infrastrukture i održivosti.

INTRODUCTION

Structural health monitoring has become an essential area of focus in maintaining the safety, reliability, and durability of civil infrastructure systems. As structures like bridges, buildings, and offshore platforms deteriorate over time, the ongoing surveillance is crucial to identify and mitigate possible damages prior to severe failures /1/. Conventional methods for structural health monitoring typically depend on deterministic models; nonetheless, the intrinsic uncertainties and complexities associated with structural behaviour demand more advanced strategies capable of addressing imprecision and ambiguity /2/. In recent years, the application of artificial intelligence and meta-heuristic algorithms has seen a significant rise, aimed at improving the effectiveness and efficiency of SHM systems /3/. The utilisation of sophisticated computational techniques allows for the processing and examination of extensive structural data, thereby supporting the precise detection and localisation of damages /4/. Among these AI techniques, fuzzy logic has attracted considerable interest because of its capacity to model uncertainties and integrate expert knowledge into structural health monitoring frameworks /5/. Fuzzy pattern recognition has proven effective in identifying damage in structures by categorising damage patterns according to fuzzy similarity criteria /4, 5/. The combination of fuzzy logic and genetic algorithms has significantly improved the development of intelligent base isolation systems, boosting their flexibility and effectiveness in response to dynamic loading scenarios /6/. In addition to fuzzy logic, the integration of model updating techniques with fuzzy systems has demonstrated efficacy in creating resilient structural health monitoring approaches for intricate structures like offshore jacket platforms /7/. The methodologies employed utilise partial modal data along with sensitivity-based strategies to enhance structural models, thereby guaranteeing precise damage identification and localisation /8/. Additionally, fuzzy control models have been suggested for overseeing civil infrastructure systems, showcasing their capability in handling and analysing various structural data streams /9/. The utilisation of genetic fuzzy systems is pertinent to the online surveillance of composite helicopter rotor blades, where the processing of real-time data and decision-making are essential for ensuring operational safety /10/. Health monitoring systems utilising fuzzy logic are capable of forecasting residual life and offering valuable insights into the degradation mechanisms that impact composite materials /11/. Simultaneously, stochastic methods that integrate Artificial Neural Networks with Adaptive Neuro-Fuzzy Inference Systems have emerged to forecast the fracture toughness of polymer nanocomposites, underscoring the adaptability of fuzzy techniques in the field of materials science, /12/. Recent developments in fuzzy logic applications encompass the forecasting of mechanical properties in quaternary nanocomposites and rubberised mortars, demonstrating how fuzzy logic models adeptly manage the complexity and variability that characterise material behaviours /13, 14/. The ability to predict outcomes is crucial for the development of robust and long-lasting construction materials, which in turn influences the overall efficiency of structural health monitoring systems /14/. The management of uncertainty is a crucial element in structural health monitoring, with fuzzy logic providing strong frameworks for addressing the probabilistic characteristics of structural responses across different loading scenarios /15/. Advanced systems that incorporate intelligent fuzzy control techniques have been created to oversee reinforced concrete structures exposed to collision-type forces, demonstrating the real-world utility of fuzzy logic in practical applications /16/. The significance of interdisciplinary training in enhancing Structural Health Monitoring (SHM) technologies is highlighted by educational and foundational contributions to the field, including graduate programmes centred on civil engineering and SHM, /17/. Furthermore, methods for decision fusion that employ fuzzy logic have been investigated to improve the precision and dependability of SHM systems, facilitating the amalgamation of various data sources and decision-making parameters, /18/.

Preliminaries

Definition 1: let S represents a Spherical Fuzzy Set, and U denotes the universal set. The act of representing an element within the universal set U involves defining its membership in S as follows: $0 \leq T_S^2(x) + F_S^2(x) + H_S^2(x) \leq 1$, where the function $T(x){:}U \rightarrow (0,1)$ defines the degree of truth for an $x \in U$; $F(x){:}U \rightarrow (0,1)$ defines the degree of falsity for $x \in U$; and $H(x){:}U \rightarrow (0,1)$ defines the degree of hesitancy or uncertainty associated with $x \in U$; the sum of the squares of the truth degree, falsity degree, and hesitancy degree must satisfy the following condition:

$$0 \leq T_S^2(x) + F_S^2(x) + H_S^2(x) \leq 1, \quad (1)$$

also, $\pi = \sqrt{1-(T_S^2(x)+F_S^2(x)+H_S^2(x))}$ is called a Spherical fuzzy indeterminacy degree of x to the set U.

Definition 2: let $S = (T_S(x), F_S(x), H_S(x))$, where $T_S(x)$, $F_S(x)$, $H_S(x) \in (0,1)$ are represented as a Spherical fuzzy number (SFN), and

$$0 \leq T_S^2(x) + F_S^2(x) + H_S^2(x) \leq 1 \cdot \quad (2)$$

Definition 3: the score function for spherical fuzzy numbers: let $S = (T_S(x), F_S(x), H_S(x))$ be a spherical fuzzy number, the score function of $S(U)$ is obtained as follows:

$$S(U) = T_S^2 - F_S^2 - H_S^2 \quad \text{where} \quad S(U) \in (-1,1) \cdot \quad (3)$$

Definition 4: the accuracy function for spherical fuzzy number: let $S = (T_S(x), F_S(x), H_S(x))$ be a spherical fuzzy number, the Accuracy function of $A(U)$ is defined as follows:

$$A(U) = T_S^2(x) + F_S^2(x) + H_S^2(x) \cdot \quad (4)$$

The accuracy function measures how close the spherical fuzzy number is to a fully certain value.

Definition 5: basic operations for Spherical fuzzy numbers $S^c = (T,F,H)$:

$$S_1 + S_2 = \Big(\sqrt{(T_{M1}^2+T_{M2}^2-T_{M1}^2T_{M2}^2)}, F_{M1}F_{M2}, \sqrt{(1-T_{M2}^2)H_{M1}^2+(1-T_{M1}^2)H_{M1}^2-H_{M1}^2H_{M1}^2}\Big), \quad (5)$$

$$S_1.S_2 = \Big(T_{M1}T_{M2}, \sqrt{(F_{M1}^2+F_{M2}^2-F_{M1}^2F_{M2}^2)}, \sqrt{((1-F_{M2}^2)H_{M1}^2+(1-F_{M1}^2)H_{M2}^2-H_{M1}^2H_{M2}^2)}\Big), \quad (6)$$

$$\lambda_M = \Big((\sqrt{1-(1-T_{M1}^2)^{\lambda}}, (F_{M1})^{\lambda}), \sqrt{(1-T_{M1}^2)^{\lambda}-(1-T_{M1}^2-H_{M1}^2)^{\lambda}}\Big), \ \lambda > 0, \quad (7)$$

$$S^{\lambda} = \Big(T_M^{\lambda}, \sqrt{1-(1-F_M^2)^{\lambda}}, \sqrt{(1-F_M^2)^{\lambda}-(1-F_M^2-H_M^2)^{\lambda}}\Big), \ \lambda > 0 \cdot \quad (8)$$

Definition 6: Spherical weighted arithmetic mean (SWAM): let $W = (W_1, W_2, \ldots, W_n)$ be s set of weights, where $W_i \in (0,1)$, and $\sum_{i=1}^{n} W_i = 1$, the SWAM for n spherical fuzzy numbers $(A_{S1}, A_{S2}, \ldots, A_{Sn})$ is defined as:

$$\text{SWAM}(A_{S1}, A_{S2}, \ldots, A_{Sn}) = W_1A_{S1} + W_2A_{S2} + \ldots + W_nA_{Sn}$$

$$\text{SWAM} = \Big(\sqrt{1-\prod_{i=1}^{n}(1-T_M^2)^{W_i}}, \prod_{i=1}^{n} F_M^{W_i}, \sqrt{\prod_{i=1}^{n}(1-T_M^2)^{W_i}-\prod_{i=1}^{n}(1-T_M^2-\pi_{AS_i}^2)^{W_i}}\Big) \cdot \quad (9)$$

ENTROPY MEASURE OF SPHERICAL FUZZY SET

In the context of fuzzy set theory, entropy applied to spherical fuzzy sets represents an underexplored area. This mathematical tool which measures the level of uncertainty or disorder between two entities has garnered little attention in current research. Fundamentally, entropy serves as a guide in decision-making by highlighting the inherent ambiguities in complex situations, /19/.

Definition 7: let O and L be Spherical fuzzy sets defined over a universal set U. The entropy measure, a quantitative metric used to evaluate uncertainty in a system is given by the mapping G: SFS(U) → (0,1), for SFSs are subject to the subsequent conditions:

(P1): Minimality: $G(O) = 0$ if O is a crisp set, (10)

(P2): Maximality: $G(O) = 1$ if $T_S(x) = F_S(x) = H_S(x) = \dfrac{1}{\sqrt{3}} \quad \forall q$, (11)

(P3): Symmetry: $G(O) = (G(O^c))$ where O^c is the complement of O. (12)

(P4): Resolution: for each $x \in U$, $G(O) \leq G(L)$ if O is less than T, $F_S(x)^O \leq F_S(x)^L \leq T_S(x)^L \leq T_S(x)^O$. (13)

Definition 8: let $S_1 = (T_{M1}, T_{M1})$ and $S_2 = (T_{M2}, T_{M2})$ be the SFNs. then the distance between S_1 and S_2 is given by:

$$D(M_1,M_2) = \frac{1}{2}\Big(\left|T_{M1}^2-T_{M2}^2\right|+\left|F_{M1}^2-F_{M2}^2\right|+\left|H_{M1}^2-H_{M2}^2\right|+\left|\pi_{M1}^2-\pi_{M2}^2\right|\Big) \cdot \quad (14)$$

Theorem 1: let $G \in$ SFS(Z), based on entropy for SFSs, the function is SF-entropy measure:

$$G(O) = \frac{1}{n}\sum_{M=1}^{n}\Big(e^{(F_M^2-T_M^2)}(-T_M^2-F_M^2+1) \times (F_M^2\pi_M^2-H_M^2)\Big(-\frac{27}{2}\Big)\Big) \cdot \quad (15)$$

Proof. For this, the function, given by Eq.(8), must hold the following axioms (P1)-(P5):

(P1): since $0 \leq T_M^2 + F_M^2 + \pi_M^2 \leq 1$, obviously,

$$0 \leq G(O) \leq 1 \cdot \quad (16)$$

(P2): minimality: assuming O is a crisp set, the deduction from Eq.(5) leads to the conclusion that $G(O) = 0$

$$G(O)=\frac{1}{n}\sum_{M=1}^{n} e^{(F_M^2-T_M^2)}(-T_M^2-F_M^2+1)\times \times(F_M^2\pi_{M1}^2-T_M^2)(-27/2)=0\,. \quad (17)$$

(P3): maximality: $T(x) = F(x) = 1/\sqrt{3}$ for all $x\in U$, then:

$$G(O)=\frac{1}{n}\sum_{M=1}^{n} e^{(F_M^2-T_M^2)}(-T_M^2-F_M^2+1)\times \times(F_M^2\pi_{M1}^2-T_M^2)(-27/2)=1\,. \quad (18)$$

(P4): symmetry: for the property, we have $O = (T(x), F(x), \pi(x))$ as $O^c = (F(x), T(x), \pi(x))$. Thus, we have

$$O=G(O)=\frac{1}{n}\sum_{M=1}^{n} e^{(F_M^2-T_M^2)}(-T_M^2-F_M^2+1)\times \times(F_M^2\pi_{M1}^2-T_M^2)(-27/2)=$$

$$G(O)=\frac{1}{n}\sum_{M=1}^{n} e^{(T_M^2-F_M^2)}(-F_M^2-T_M^2+1)\times \times(T_M^2\pi_{M1}^2-F_M^2)(-27/2)=O^c\,. \quad (19)$$

(P5): resolution: to establish the fourth property, let's examine the function $G(T(x), F(x))$ such that:

$$\frac{dG}{dT}=e^{(F_M^2-T_M^2)}(-T_M^2-F_M^2+1)(F_M^2\pi_M^2-T_M^2)\left(-\frac{27}{2}\right), \quad (20)$$

$$\frac{dG}{dF}=e^{(F_M^2-T_M^2)}(-T_M^2-F_M^2+1)(F_M^2\pi_M^2-T_M^2)\left(-\frac{27}{2}\right). \quad (21)$$

We obtain $dG/dT \geq 0$: when $T \leq F$ and $dG/dT \leq 0$: when $T \geq F$; where as $dG/dF \leq 0$: when $T \leq F$ and $dG/dF \geq 0$: when $T \geq F$, thus G is increasing with respect to T when $T \leq F$, and decreasing when $T \geq F$. Moreover, G is decreasing with respect to F when $T \leq F$ and increasing when $T \geq F$.

RESEARCH METHODOLOGY

This work intends to find the most efficient structural health monitoring method by means of a thorough multi-criteria decision-making (MCDM) method. Three well-known MCDM techniques - TOPSIS, COPRAS, and VIKOR - are used in a comprehensive assessment. The method starts with building a spherical fuzzy decision matrix wherein several SHM methods are evaluated depending on important factors like environmental sustainability, cost-effectiveness, operational feasibility, and scalability. Expert opinions helped to organise these guidelines and decisions. Therefore, the spherical fuzzy decision matrix is used to explain uncertainty or imprecision in expert judgements, therefore removing any ambiguity in the decision-making process. Spherical fuzzy logic enables one to ascertain the relative importance of professional viewpoints. The VIKOR approach ranks the answers to provide a compromise solution combining overall performance with decreasing regret for specific actions. This method enables the resolution of contradicting aspects of the decision-making process. Analysing technologies using benefit and non-benefit criteria, the COPRAS method ranks them based on their relative relevance. At last, the TOPSIS approach is used to find the alternative closest to the perfect answer, therefore generating a ranking showing the relative performance of every SHM to the ultimate result. Combining the outcomes of VIKOR, COPRAS, and TOPSIS completes the research and produces a well-rounded choice considering all relevant variables.

INTEGRATED SF-COPRA, SF-TOPSIS AND SF-VIKOR

The integration of SFSs-based TOPSIS-COPRA-VIKOR methods can be broken down into the following steps:

Step 1 - construction of SFs decision matrix: the spherical fuzzy decision matrix is constructed where the positive membership and the negative membership degree are calculated using the following structure:

$$L=\begin{pmatrix} & H_1 & H_2 & \cdots & H_j \\ A_1 & (T_{11},F_{11},H_{11}) & (T_{12},F_{12},H_{12}) & \cdots & (T_{1K},F_{1K},H_{1K}) \\ A_2 & (T_{21},F_{21},H_{21}) & (T_{22},F_{22},H_{22}) & \cdots & (T_{2K},F_{2K},H_{2K}) \\ \vdots & \vdots & \vdots & & \vdots \\ A_i & (T_{i1},F_{i1},H_{i1}) & (T_{i2},F_{i2},H_{i2}) & \cdots & (T_{iK},F_{iK},H_{iK}) \end{pmatrix} \quad (22)$$

Step 2 - determination of criteria weights: the weights X_M for each criterion are determined using the formula:

$$X_M=\frac{(1-L_M)}{\sum_{M=1}^{n}1-L_M},\quad M=1,2,\ldots,n\,, \quad (23)$$

where:

$$L_M=\frac{3}{n}e^{(F_M^2-T_M^2)}(-T_M^2-F_M^2+1)(F_M^2\pi_M^2-T_M^2)\left(-\frac{27}{2}\right). \quad (24)$$

Step 3 - application of SF-COPRAS method: the COPRAS method is applied as a robust MCDM tool that evaluates alternatives while balancing benefit and cost criteria. The steps are as follows:

Step 3.1 - construction of the weighted decision-making matrix: the weighted decision-making matrix $(J_{XY})_{P\times Q}$ is determined, where:

$$J_{XY}=X_M d_{ij}=\left(\sqrt{1-(1-T_{XY}^2}, F_{XY}^2\right). \quad (25)$$

Step 3.2 - calculate the score function $s(J_{XY})$ for each alternative $X = (1,2,\ldots,m)$ and each criterion $Y = (1,2,\ldots,n)$ the score function $s(J_{XY})$ is calculated as:

$$s(j_{xy})=T^2-F^2\,. \quad (26)$$

Step 3.3 - now, calculation of summation terms, the summation of benefit criteria and non-benefit criteria for all i is computed as follows:

$$s(Q_i)=\frac{1}{|B|}\sum_{j\in B}s(J_{XY})\,, \quad (27)$$

$$s(E_i)=\frac{1}{|NB|}\sum_{j\in B}s(J_{XY})\,, \quad (28)$$

where: B is the set of Benefit criteria and NB is the set of all Non-Benefit criteria.

Step 3.4 - calculate the relative weight of each alternative $Z_i(i = 1,2,\ldots,m)$:

$$Z_i=s(Q_i)+\frac{\sum_{i=1}^{m}e^{s(Q_i)}}{e^{s(K_i)}\sum_{i=1}^{m}e^{1/s(Q_i)}}\,. \quad (29)$$

Step 3.5 - compute the priority order $Pr_i(i = 1,2,\ldots,m)$:

$$\mathrm{Pr}_i=\frac{z_i}{\max z_i}\times 100\,. \quad (30)$$

Step 3.6 - ranking the alternatives $T_i > T_K$ if $Pr_i > Pr_k$ for all $i, k = 1,2,3\ldots,m$.

Step 4 - application of SF-TOPSIS method

Step 4.1 - determination of ideal solutions: for benefit criteria M_1 and cost criteria M_2, the PIS (positive ideal solution and NIS (negative ideal solution) are determined as:

$\lambda'^{+} = (T'^{+}_j, F'^{+}_j, H'^{+}_j, \pi'^{+}_j)$, where: $(T'^{+}_j, F'^{+}_j, H'^{+}_j, \pi'^{+}_j) = (1,0,0)$, $J \in M_1 (T'^{+}_j, F'^{+}_j, H'^{+}_j, \pi'^{+}_j) = (0,1,0)$,

$\lambda'^{-} = (T'^{-}_j, F'^{-}_j, H'^{-}_j, \pi'^{-}_j)$, where: $(T'^{-}_j, F'^{-}_j, H'^{-}_j, \pi'^{-}_j) = (0,1,0)$, $J \in M_1 (T'^{-}_j, F'^{-}_j, H'^{-}_j, \pi'^{-}_j) = (1,0,0)$. (31)

Step 4.2 - calculation of Euclidean distances: the distances from SF-PIS and SF-NIS are calculated using Spherical weighted Euclidean distance as follows:

$$D^{+}(\lambda'_1) = d_E(\lambda'_i \lambda'^{+}) = \sqrt{\frac{1}{2}\sum_{j=1}^{n} X_M ((1-T'^{2}_{ij})^2 + (F'^{2}_{ij})^2 + (1-T'^{2}_{ij} - F'^{2}_{ij})^2)}, \quad (32)$$

$$D^{-}(\lambda'_1) = d_E(\lambda'_i \lambda'^{-}) = \sqrt{\frac{1}{2}\sum_{j=1}^{n} X_M ((T'^{2}_{ij})^2 + (1-F'^{2}_{ij})^2 + (1-T'^{2}_{ij} - F'^{2}_{ij})^2)}. \quad (33)$$

Step 4.3 - calculation of closeness coefficient: the closeness coefficient for each alternative is calculated

$$K(\lambda'_i) = \frac{D^{-}(\lambda'_i)}{D^{-}(\lambda'_i) + D^{+}(\lambda'_i)} . \quad (34)$$

Step 4.4 - ranking of alternatives

Step 5 - application of SF-VIKOR method

Step 5.1 - determination of optimal solutions

- for benefit criteria

$$\text{PIS}^{+}_M = T^{+}_M, F^{+}_M, H^{+}_M = \max_M(T'_M), \min_M(F'_M), \min_M(H'_M), \quad (35)$$

$$\text{NIS}^{-}_M = T^{-}_M, F^{-}_M, H^{-}_M = \min_M(T'_M), \max_M(F'_M), \max_M(H'_M), \quad (36)$$

$$\pi^{+}_M = 1 - T^{+}_M - F^{+}_M - H^{+}_M, \ \pi^{-}_M = 1 - T^{-}_M - F^{-}_M - H^{-}_M, \quad (37)$$

- for cost criteria

$$\text{PIS}^{+}_M = T^{+}_M, F^{+}_M, H^{+}_M = \min_M(T'_M), \max_M(F'_M), \max_M(H'_M), \quad (38)$$

$$\text{NIS}^{-}_M = T^{-}_M, F^{-}_M, H^{-}_M = \max_M(T'_M), \min_M(F'_M), \min_M(H'_M). \quad (39)$$

Step 5.2 - calculation of group utility and individual regret: we define group utility I_{GU} and individual regret W_{IR} based on Eq.(23) as follows

$$I_{GU} = \left(\sum_{i=0}^{n} \left(X_M \frac{d(\text{PIS}^{+}_M, H_j)}{d(\text{PIS}^{+}_M, \text{NIS}^{-}_M)} \right)^r \right)^{1/r}, \quad (40)$$

$$W_{IR} = \max \left(\sum_{i=0}^{n} \left(X_M \frac{d(\text{PIS}^{+}_M, H_j)}{d(\text{PIS}^{+}_M, \text{NIS}^{+}_M)} \right)^r \right)^{1/r}. \quad (41)$$

Step 5.3 - determination of VIKOR index

$$E_I = \theta \frac{I_{GU} - I^{+}_{GU}}{I^{-}_{GU} - I^{+}_{GU}} + (1-\theta) \frac{W_{IR} - W^{+}_{IR}}{W^{-}_{IR} - W^{+}_{IR}}, \quad (42)$$

where: θ is a weight (often 0.5, but adjustable) representing the ‘strategy’ of maximum group utility.

Step 5.5 - determine the compromise solution.

Step 6 - ranking the alternatives.

ILLUSTRATIVE EXAMPLE

To identify the best suitable SHM for a particular infrastructure project, it is crucial to methodically assess several possibilities against established criteria.

We examine seven specific SHM methodologies: Vibration-based monitoring (A1); Acoustic emission monitoring (A2); Ultrasonic testing (A3); Fibre optic sensor systems (A4); Wireless sensor networks (A5); Infrared thermography (A6); and Electromagnetic sensing (A7). Seven essential criteria - Detection sensitivity (H1); Cost-effectiveness (H2); Ease of installation and maintenance (H3); Scalability and flexibility (H4); Data accuracy and reliability (H5); Environmental compatibility (H6); and Integration with existing systems (H7) - are used to evaluate these strategies. The first phase of the selection process entails explicitly defining the criteria and alternatives, enabling a systematic evaluation of the strengths and shortcomings of each SHM approach.

Detection sensitivity assesses the capacity of each SHM approach to recognise small or incipient problems, which is essential for preventative maintenance and the prevention of structural failures. Cost-effectiveness evaluates the initial investment and recurring operating costs to ascertain the financial viability of each method's implementation. The ease of installation and maintenance evaluates the feasibility and duration necessary to implement and maintain the SHM system, influencing project schedules and enduring efficacy. Scalability and flexibility assess the potential of the SHM approach to be enlarged or modified to support bigger structures or changing monitoring requirements, hence assuring the system's future viability. Data accuracy and reliability assess the precision and consistency of collected information, which is essential for making educated engineering choices. Environmental compatibility assesses the SHM system's efficacy over diverse environmental circumstances, guaranteeing its resilience and reducing erroneous readings caused by external influences. Finally, Integration with existing systems assesses the compatibility of the SHM approach with existing monitoring frameworks and data management platforms, enabling smooth operations and thorough data analysis. By systematically assessing each SHM methodology according to these seven criteria, decision-makers may choose the most appropriate method(s) to improve the safety, dependability, and sustainability of civil infrastructure systems. Vibration-based monitoring and Acoustic emission monitoring may provide superior Detection sensitivity and Data accuracy, making them ideal for projects where early damage identification is critical. In contrast, Wireless sensor networks and Fibre optic sensor systems may provide enhanced scalability critical. In contrast, Wireless sensor networks and Fibre optic sensor systems may provide enhanced scalability and flexibility, making them appropriate for extensive or intricate constructions. This systematic method guarantees that the chosen SHM fulfils both current monitoring needs and fits with long-term infrastructure management objectives.

Step 1 - first, we evaluate each alternative using the SFN dataset under the various criteria listed in the Table 1.

Step 2 - next, we calculate the criteria weights, which are $(0.1476, 0.1470, 0.1425, 0.1359, 0.1418, 0.1384, 0.1464)^T$ using the specified equation to determine the importance of each criterion.

Step 3.1 - then, we compute the weighted decision matrix by applying the equation to account for weights of each criterion as shown in Table 2.

Step 3.2 - after that, we compute the score function $S(r_{ij})$ using the given equation to assess how well each alternative performs under each criterion as shown in Table 3.

Step 3.3 - once the score function is calculated, we separate the criteria into benefit and non-benefit sets for all (i = 1, 2,…,m) using the respective equation as shown in Table 4.

Step 3.4 - next, we calculate the relative weight of each alternative Q_i (i 1,2,…,m), as shown in Table 5.

Step 3.5 - afterwards, we determine the priority order Pr_i (i = 1,2,…,m) by calculating the equation for priority ranking, as shown in Table 5.

Step 3.6 - then, we rank the alternatives by comparing their priority values. An alternative $T_i \prec T_K$ if $Pr_i \succ Pr_K$ for all i, k = 1,2,3,…,m, as shown in Table 5.

Step 4.1 - next, following the TOPSIS method, we determine the SF-PIS (spherical fuzzy positive ideal solution) and SF-NIS (spherical fuzzy negative ideal solution) for SFSs as shown in Table 6.

Step 4.2 - then, we use the Spherical weighted Euclidean distance to calculate the distance of alternatives from both SF-PIS and SF-NIS.

Step 4.3 - after that, we compute the relative closeness degree for each alternative as shown in Table 7.

Step 4.4 - finally, we rank the alternative based on their closeness degree, where the highest degree indicates the best alternative as shown in Table 7.

Step 5.1 - following the VIKOR method, first we identify the relative optimal solutions for both benefit and cost criteria as shown in Table 6.

Step 5.2 - then, we calculate the group utility and individual regret for each alternative as shown in Table 8.

Step 5.3 - next, we compute the VIKOR index using the given equation as shown in Table 8.

Step 5.4 - lastly, we rank the alternative based on the VIKOR index, where the lowest index corresponds to the best alternative as shown in Table 8.

Table 1. Decision-matrix for spherical fuzzy set.

	H_1	H_2	H_3	H_4	H_5	H_6	H_7
A1	0.2235 0.4012 0.541	0.2256 0.3155 0.102	0.3123 0.4143 0.321	0.3187 0.2554 0.541	0.5354 0.4874 0.5412	0.4456 0.5655 0.5412	0.3789 0.4595 0.5912
A2	0.4267 0.4056 0.621	0.2765 0.3756 0.521	0.6023 0.6455 0.2021	0.5965 0.5541 0.421	0.4299 0.6974 0.2012	0.4765 0.6756 0.512	0.4267 0.6856 0.412
A3	0.5234 0.6234 0.523	0.5621 0.3856 0.521	0.5256 0.5454 0.4013	0.5766 0.5952 0.512	0.5312 0.6234 0.541	0.4621 0.6656 0.4123	0.4234 0.6602 0.712
A4	0.3234 0.5756 0.541	0.4245 0.5456 0.521	0.6855 0.3654 0.4001	0.6751 0.2561 0.612	0.6218 0.3335 0.1002	0.6245 0.4456 0.541	0.6234 0.4756 0.612
A5	0.4991 0.4945 0.523	0.6512 0.5841 0.421	0.6564 0.5654 0.002	0.6165 0.5852 0.512	0.6574 0.5414 0.512	0.515 0.5841 0.5412	0.5991 0.4945 0.523
A6	0.5621 0.5656 0.523	0.4123 0.3455 0.5521	0.5421 0.5213 0.621	0.625 0.612 0.541	0.612 0.512 0.412	0.4267 0.6856 0.4512	0.541 0.541 0.412
A7	0.5045 0.4356 0.6123	0.4956 0.6454 0.5212	0.5165 0.5321 0.621	0.551 0.412 0.541	0.521 0.612 0.512	0.5234 0.6634 0.5123	0.412 0.412 0.452

Table 2. Weighted decision matrix for spherical fuzzy set.

a	H_1	H_2	H_3	H_4	H_5	H_6	H_7
A1	0.0192 0.7635	0.0195 0.7123	0.0375 0.7778	0.0391 0.6775	0.1121 0.8146	0.0769 0.8499	0.0554 0.8011
A2	0.0704 0.766	0.0294 0.7498	0.1436 0.8826	0.1406 0.8450	0.0715 0.9023	0.0882 0.8941	0.0701 0.8979
A3	0.1070 0.8697	0.1241 0.7556	0.1079 0.8412	0.1309 0.8624	0.1103 0.8739	0.0828 0.8904	0.0693 0.8883
A4	0.0402 0.8494	0.0697 0..8368	0.1901 0.7504	0.1838 0.6781	0.1537 0.7311	0.1551 0.7941	0.1545 0.8090
A5	0.0970 0.8122	0.1698 0.8537	0.1728 0.8499	0.1509 0.8583	0.1734 0.8394	0.1035 0.8578	0.1419 0.8180
A6	0.1241 0.8450	0.0657 0.7316	0.1151 0.8304	0.1554 0.8693	0.1485 0.0704	0.0704 0.8979	0.1146 0.8393
A7	0.0992 0.7823	0.0956 0.8791	0.1041 0.8353	0.1190 0.7765	0.1060 0.8693	0.1070 0.8895	0.0656 0.7765

Table 3. Score function for spherical fuzzy set.

	H1	H2	H3	H4	H5	H6	H7
A1	-0.5826	-0.507	-0.6035	-0.457	-0.651	-0.716	-0.638
A2	-0.5818	-0.5613	-0.758	-0.694	-0.809	-0.791	-0.801
A3	-0.7449	-0.555	-0.696	-0.7267	-0.751	-0.785	-0.784
A4	-0.7199	-0.6953	-0.5269	-0.426	-0.51	-0.606	-0.6306
A5	-0.6502	-0.7	-0.6925	-0.713	-0.674	-0.725	-0.649
A6	-0.6987	-0.53	-0.676	-0.7316	-0.66	-0.801	-0.691
A7	-0.6022	-0.7638	-0.686	-0.588	-0.744	-0.77	-0.598

Table 4. Benefit and non-benefit set.

	Benefit criteria	Non-benefit criteria
A1	0.328219	0.36896
A2	0.275656	0.341911
A3	0.263218	0.343022
A4	0.277862	0.357435
A5	0.252217	0.340798
A6	0.267601	0.345169
A7	0.270892	0.355347

Table 5. Priority order and ranking order.

	H_1	H_2	H_3	H_4	H_5	H_6	H_7
A1	-0.582	-0.5	-0.603	-0.457	-0.651	-0.716	-0.638
A2	-0.581	-0.561	-0.758	-0.69	-0.809	-0.791	-0.801
A3	-0.744	-0.555	-0.696	-0.7267	-0.751	-0.785	-0.784
A4	-0.719	-0.695	-0.5269	-0.426	-0.51	-0.606	-0.63
A5	-0.65	-0.7	-0.692	-0.713	-0.674	-0.725	-0.649
A6	-0.698	-0.53	-0.676	-0.731	-0.66	-0.801	-0.691
A7	-0.6	-0.763	-0.686	-0.588	-0.744	-0.779	-0.598
P_i	-0.705	-0.715	-0.716	-0.658	-0.695	-0.705	-0.715
R_i	-0.74	-0.821	-0.796	-0.624	-0.698	-0.743	-0.754
Q_i	1.15	1	1.07	1.17	1.2	1.14	1.16
$100*Q_i/\max Q_i$	115.5	100	107.05	117.62	120.05	114.74	116.97
rank	A4	A7	A6	A2	A1	A5	A3

Table 6. Best (positive ideal solution) and worst ranking (negative ideal solution).

Best ranking				Worst ranking			
T^+	F^-	H^-	P_{ie}^-	T^-	F^+	H^+	P_{ie}^+
0.315	0.16	0.273	0.063	0.049	0.388	0.385	0.496
0.424	0.099	0.01	0.057	0.05	0.416	0.304	0.839
0.469	0.133	0.004	0.048	0.097	0.416	0.385	0.627
0.455	0.065	0.177	-0.05	0.101	0.3745	0.374	0.54
0.432	0.111	0.01	0.012	0.184	0.486	0.292	0.492
0.39	0.198	0.169	0.023	0.182	0.47	0.292	0.188
0.388	0.169	0.169	-0.122	0.143	0.47	0.506	0.4562

Table 7. Relative closeness and ranking order.

	Relative closeness	Rank
1	0.337	A4
2	0.335	A7
3	0.336	A6
4	0.345	A2
5	0.348	A1
6	0.337	A5
7	0.339	A3

Table 8. Group utility, individual regret, index and ranking order.

S	R	O_i	Rank
0.444	0.096	0.302	A4
0.638	0.243	1	A7
0.406	0.150	0.394	A6
0.334	0.112	0.182	A2
0.304	0.062	0	A1
0.433	0.117	0.344	A5
0.420	0.099	0.273	A3

Table 9. Comparison between VIKOR, TOPSIS and COPRAS methods.

	VIKOR		TOPSIS		COPRAS	
	V_i	Rank	V_i	Rank	V_i	Rank
1	0.302	A4	0.337	A4	115.50	A4
2	1	A7	0.335	A7	100	A7
3	0.394	A6	0.336	A6	107.05	A6
4	0.182	A2	0.345	A2	117.62	A2
5	0	A1	0.348	A1	120.05	A1
6	0.344	A5	0.337	A5	114.74	A5
7	0.273	A3	0.339	A3	116.97	A3

COMPARATIVE ANALYSIS

In this study, we examine how Spherical fuzzy sets (SFSs) might be used to influence decision-making. As a consequence, the research becomes much more useful and reliable. To assure the reliability and consistency of our findings, we use a rigorous set of scientific methods, such as robustness testing and detailed confirmation procedures. Table 9 provides a concise summary of the study. Each component studied highlights the intricacies of the decision-making process. This lengthy overview highlights the advantages and disadvantages of a variety of strategies. The main findings of our research should be relevant to anybody making decisions concerning the proper integration of SFSs into government processes.

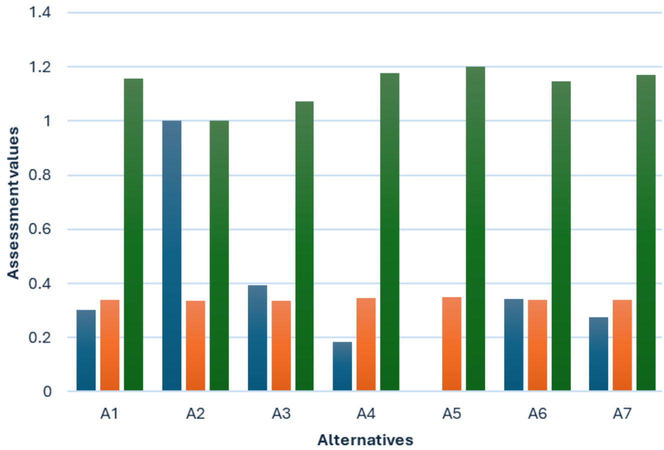

Figure 1. Visualisation of rankings for VIKOR, TOPSIS and COPRAS methods.

These discoveries not only improve strategy efficacy, but they also give a more comprehensive understanding of the SFS system's decision-making processes. Finally, our study establishes a strong foundation for making better decisions in complicated and tough situations. Figure 1 depicts the variation in review outcomes for the alternatives (A1-A7) for each of the three approaches. The green bars for each option consistently have the greatest values, showing that this technique outperforms the other two possibilities.

RESULT AND DISCUSSION

In this paper, seven options (A1 to A7) are evaluated depending on a range of criteria using three different Multi-criteria decision-making (MCDM) approaches: VIKOR, TOPSIS, and COPRAS. We are able to significantly compare their outcomes and ranks by using these techniques on a shared dataset. The results, shown in comparative tables, show a startling consistency among all three methods, for the seven choices, VIKOR, TOPSIS, and COPRAS, each a different approach. Showing a consistent and accurate review method, the last ranking places A4 at the top followed by A7, A6, A2, A1, A5, and A3. The general results stay constant even if the produced numerical outputs had varied depending on their unique computational techniques. VIKOR ratings ran from 0 to 1; lower numbers indicate improved performance. TOPSIS values fell between 0.33 and 0.35; larger values indicate more preferred options. COPRAS generated ratings between 100 and 120; larger numbers suggest better choices. While Alternative 5 emerges as the best choice across all three techniques, Alternative 2 often scores lowest. This consistency throughout the several approaches helps us to feel more confident in the authenticity and accuracy of the last rankings. The unanimity among VIKOR, TOPSIS, and COPRAS indicates that the chosen method might not have significant influence on the general decision-making process in this sense. This awareness could be useful for next research, possibly simplifying the decision-making process in such situations. Although the ranks are usually consistent, the relative differences between decisions could vary depending on the method taken. Examining these differences could enable one to acquire an understanding of the discriminating capacity and sensitivity of any approach.

CONCLUSIONS

This research presents a framework combining SFSs with COPRAS, VIKOR, and TOPSIS methods to improve decision-making in Structural health monitoring (SHM).

Seven SHM alternatives are ranked consistently, with Wireless sensor networks (A5) emerging as the top choice.

The study innovates by applying SFSs with multiple MCDM methods, offering a more comprehensive approach than in previous studies.

Differences in rankings across methods suggest further research to refine the framework's sensitivity and applicability.

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