

NUMERICAL INVESTIGATION OF SYMMETRY BREAKING AND STABLE SOLUTIONS IN LID-DRIVEN CAVITY FLOWS AT HIGH REYNOLDS NUMBERS

NUMERIČKO ISTRAŽIVANJE NARUŠAVANJA SIMETRIJE I STABILNA REŠENJA STRUJANJA ZATVORENOG TOKA SA POKRETNIM POKLOPCEM SA VELIKIM REYNOLDS BROJEVIMA

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Keywords

- finite volume technique
- lid-driven cavity
- boundary value problems
- incompressible flow
- Navier-Stokes equations
- partial differential equations

Abstract

This study presents a numerical investigation of Newtonian two-dimensional lid-driven square cavity fluid flow up to Reynolds numbers of 5000 using a grid size of 501×501 . Building on earlier research, the work explores two-sided and four-sided wall-motion scenarios to analyse multiple and unique flow solutions. For the two-sided case, multiple solutions occur when both the left and upper walls move downward and rightward at equal speed, while unique solutions arise when the left wall moves at speed 1.5 and the upper wall at speed 1, with the right and bottom walls fixed. For the four-sided case, multiple solutions are observed when all walls move at equal speed in opposing directions, whereas unique solutions emerge when either the left or bottom wall moves at speed 1.5 while the others move at speed 1. The flow field exhibits symmetry along cavity diagonals, bifurcating to asymmetric patterns at Reynolds numbers of 1466 (two-sided) and 175 (four-sided). Between Reynolds numbers 175 and 385, two stable solutions exist, and beyond 385, three stable solutions are observed for the four-sided equal-speed case. Increasing one wall's speed to 1.5 transitions the flow from multiple to unique stable solutions. Velocity profiles and streamline contours are analysed to characterise these behaviours. Computations are performed using the PARAM Shavak HPC system.

INTRODUCTION

The phenomenon of cavity flow driven by moving boundaries is of substantial scientific and engineering relevance. Although geometrically simple, lid-driven cavity flow reveals complex fluid dynamical processes such as vortex formation, hydrodynamic instability, symmetry breaking, and flow bifurcation. These flows appear in numerous engineering operations including short-dwell coating, species removal from patterned surfaces, polymer processing, and mixing

Ključne reči

- metoda konačnih zapremina
- zatvoreni tok sa poklopcem
- problemi sa graničnim uslovima
- nestišljivo strujanje
- Navije-Stoksove jednačine
- parcijalne diferencijalne jednačine

Izvod

U radu istražujemo dvodimenzionalno strujanje Njutnovskog fluida u zatvorenom toku s poklopcem sve do Reynoldsovog broja 5000, korišćenjem mreže veličine 501×501 . Kao nadogradnja ranijeg istraživanja, u radu razmatramo scenarije kretanja dvostranih i četverostranih zidova radi analize višestrukih i jedinstvenih rešenja strujanja. Kod dvostranog slučaja se javljaju višestruka rešenja kada se i levi i gornji zid pomeraju nadole i nadesno istom brzinom, dok se jedinstvena rešenja javljaju kada se levi zid pomera brzinom 1,5, a gornji zid brzinom 1 pri fiksiranom desnom i donjem zidu. Za četverostrani slučaj, primećuju se višestruka rešenja kada se svi zidovi pomeraju istom brzinom u suprotnim pravcima, a jedinstvena rešenja se javljaju kada se ili levi ili donji zid pomera brzinom 1,5, a ostali zidovi brzinom 1. Polje strujanja ima simetriju duž dijagonala šupljine sa račvanjem ka asimetričnom šablonu za Reynoldsove brojeve 1466 (dvostrano) i 175 (četverostrano). Između Reynoldsovih brojeva 175 i 385 postoje dva stabilna rešenja, a iznad 385 tri stabilna rešenja kod četverostranog kretanja istom brzinom. Povećanjem brzine jednog zida na 1,5 izaziva promenu strujanja sa višestrukim rešenjima ka jedinstvenim stabilnim rešenjima. Analiziraju se brzinski profili i strujne konture radi karakterizacije ponašanja. Računske operacije se izvode primenom PARAM Shavak HPC sistema.

and drying devices. Classical studies have provided extensive insights into one-sided lid-driven cavity behaviour /1-4/, however, recent investigations continue to expand the understanding of cavity flows across nonstandard geometries and multi-sided driving conditions. For example, flows in triangular, trapezoidal, and semicircular cavities have revealed new vortex topologies and transition mechanisms, /5-7/. Moreover, contemporary research has demonstrated complex bifurcation structures and symmetry-breaking scenarios in multi-lid configurations, /8, 9/.

The numerical simulation of incompressible cavity flow fundamentally relies on Navier-Stokes equations. Patankar's seminal SIMPLE algorithm /10/ remains a cornerstone method for solving these equations within the finite volume framework. U. Ghia et al. provide benchmark high-Reynolds-number results for the square cavity using central differences and multigrid techniques /11/. Subsequent studies, such as E. Erturk, have illustrated that cavity flow exhibits multiple physical features including corner-driven secondary vortices and shear-layer separation /12/. The backward-facing step, a related canonical geometry, exemplifies separation, reattachment, and shear-layer evolution, all of which are central to many engineering systems such as condensers, automotive aerodynamics, and heat-transfer devices.

The SIMPLE family of algorithms has seen continuous refinement and widespread application /13-15/. More recent computational work has extended these formulations to deep cavities, non-orthogonal grids, and multi-lid configurations, highlighting the increasing interest in accurately capturing complex vortex structures and transition behaviour.

Demirdžić et al. /14/ use the SIMPLE technique to solve the problem of sloping lid-driven flow with side walls. In their study, Albensoeder and Kuhlmann /16/ conduct a linear stability analysis to demonstrate that the instability of the two-dimensional flow varies depending on the cross-sectional aspect ratio and Reynolds number. Blohm and Kuhlmann /17/ utilise laser-doppler and hot-film techniques to measure both steady and time-dependent vortex flows occurring at higher Reynolds numbers. Yang and Luo /18/ explore the presence of multiple two-dimensional steady flow solutions in cavity flow with heat transport, as well as the transitions between these stable solutions. They also establish a boundary line, resembling a thumb shape, which delineates a restricted region of stable solutions based on the Grashof and Reynolds numbers. Lid-driven flow across staggered grids has seen significant use in recent years. The main advantage of the finite volume method is that it only requires evaluating the flow on cell borders /19/. The finite volume method is one of the most effective techniques for finding solutions to incompressible Newtonian fluid flow /20/. When improving a square cavity driven by an oscillating lid, a two-dimensional model is developed using the finite volume method on a staggered grid /21/. This method has many applications in heat transfer and fluid movement /22/.

Turkylmazoglu /23/ investigated the impact of radial heat conduction on a cone-plate viscometer, revealing significant effects on heat transfer rates and identifying optimal cooling conditions. In another study, Turkylmazoglu /24/ analyses free convection flow in a porous medium with a vertical permeable flat surface or cone subjected to nonconstant heat flux and internal heat generation/absorption. Analytical solutions reveal unique or multiple solutions depending on cooling or heating conditions, with explicitly determined thresholds. Turkylmazoglu /25/ explores the flow characteristics of a lid-driven cavity with a butterfly-shaped cross-section, investigating its mixing phenomenon. Numerical simulations show that adjusting the nonparallel walls of the butterfly cavity allows controlled flow and efficient particle separa-

tion in industrial applications. Turkylmazoglu /26/ further explores a novel cavity wall mechanism for chemical mixing in lid-driven cavity flow, examining its effects on pressure distribution and vortex flow patterns. He then focuses /27/ on heat transfer and mixing through fluid convection in a cavity. Using a finite element technique, the effects of non-uniform wall heating on heat flow, streamlines, and Nusselt number are analysed, revealing enhanced heat transfer when the heating position is closer to the upper horizontal wall.

Hammami et al. /28/ numerically investigate transition to an unsteady state at lower Reynolds numbers in a two-sided lid-driven cubical cavity with a central obstacle compared to one-sided configurations. Hammami et al. /29/ present three-dimensional numerical simulations investigating transition to unsteady flow in lid-driven cavity phenomena with a central circular cylinder at Reynolds numbers up to 2000. Souayeh et al. /30/ analyse the flow characteristics of an incompressible fluid inside a cavity driven by upper-lid motion, considering both two- and three-dimensional cases, Reynolds number effects, and inner spherical shapes. Souayeh et al. /31/ investigate the effect of velocity ratio and radius size of an inner semicircle on bifurcation occurrence in a lid-driven cavity using the finite volume method. Souayeh et al. /32/ computationally demonstrate that altering the radius size of a cylinder in a cuboidal enclosure affects flow instability, with larger radii promoting cell distortion and decreasing the critical Reynolds number. Souayeh et al. /33/ show numerically that a centrally located cylindrical shape inside a two-sided lid-driven cubical cavity accelerates the transition to unsteadiness. Souayeh et al. /34/ numerically simulate natural convection of air in a cubic enclosure with an inclined inner cylinder, revealing the effects of cylinder inclination and Rayleigh number on fluid flow and heat transfer.

The presence of heated triangular ribs as obstacles in a flowing liquid stream increases the drag coefficient, especially when installed on both upper and lower channel walls, as studied by Rehman et al. /35/. Rehman et al. /36/ explore computational analysis of a partially heated corrugated enclosure with a Y-shaped fin and non-Newtonian liquid, showing increased heat transfer rate and direct proportionality between Casson fluid temperature and Rayleigh number. Rehman et al. /37/ study drag-force variation due to different installations of heated rectangular ribs in a rectangular domain with a partially heated circular cylinder. Mahmood et al. /38/ use non-conforming finite elements to simulate incompressible fluid flows, demonstrating good agreement with benchmark cavity-flow data. Rehman et al. /39/ numerically simulate flow fields in a convergent-divergent channel with infinite cylinder obstacles, showing higher drag coefficients for triangular and square-shaped obstacles.

Two- and four-sided lid-driven incompressible laminar fluid flow within a square cavity is numerically simulated in /40/. In the two-sided case, the fluid flow remains symmetric about one cavity diagonal, whereas in the four-sided case symmetry appears about both diagonals. The critical Reynolds numbers, 1073 and 129, correspond to two-sided and four-sided driven cavity flows, respectively, indicating bifurcation from stable symmetric to asymmetric flow fields.

In order to calculate stable solutions for lid-driven square and rectangular cavities, Dass et al. /41/ employ a single-relaxation-time lattice Boltzmann model. Cadou et al. /42/ examine two- and four-sided lid-driven flows using perturbation techniques and observe a second stable bifurcation at Reynolds number 867 without Hopf bifurcation, together with three Hopf bifurcations at Reynolds numbers 2922, 3212, and 4078. Chen et al. /43/ explore bifurcated fluid-flow behaviour in a four-sided cavity using Keller's continuation technique at Reynolds number 250.

The non-uniqueness of steady-state flow over a wide range of Reynolds numbers and aspect ratios is demonstrated using the finite volume approach by Albensoeder et al. /44/. Their computational study of two-sided lid-driven flow identifies cavity regions where multiple steady solutions exist. In such cases, two perpendicular cavity side walls moving tangentially at constant rates drive the fluid motion, and the resulting flow states depend on both aspect ratio and Reynolds numbers. Azzouz et al. /45/ examine high-Reynolds-number square cavity flow for Reynolds numbers from 100 to 50,000, demonstrating feasibility of numerical computations in this range. Faridzadeh et al. /46/ investigate the effects of multiple parameters on flow paths, isotherm lines, and average Nusselt number using the finite volume SIMPLE algorithm for laminar nanofluid flow inside an inclined square cavity.

Although one-sided cavity flows have been extensively investigated, comparatively fewer studies have examined two-sided and four-sided lid-driven configurations. In the present work, these scenarios are analysed in detail. For the two-sided case, the top wall moves rightward while the left wall moves downward at equal velocity, producing multiple stable flow solutions. In the four-sided case, the top and bottom walls move leftward and rightward, respectively, while the left and right walls move upward and downward, all at equal speed, resulting in multiple solutions. Numerical simulations are performed for Reynolds numbers up to 3000 in the two-sided cavity and up to 1000 in the four-sided cavity. The existence of multiple solutions and critical Reynolds numbers for bifurcation are identified. Furthermore, when one wall velocity is increased to 1.5 while the others remain at unity, the flow transitions from multiple to unique stable solutions. Velocity profiles, streamline contours, and symmetry-breaking behaviour are presented graphically, with computations carried out using the finite volume SIMPLE algorithm.

The lid-driven cavity problem is solved under conditions of incompressible, steady-state fluid flow using the finite volume approach. In this study, both two-sided and four-sided cavity boundary conditions are considered. Section 2 presents the governing equations, numerical methodology, and validation of the finite volume SIMPLE technique against the classical benchmark solution of Ghia et al. /11/. Section 3 discusses the two-sided cavity flow, highlighting the occurrence of multiple and unique stable solutions and associated bifurcation behaviour. Section 4 extends the analysis to four-sided cavity flow, where symmetry breaking and stability transitions are examined in detail. Finally, Section 5 summarises the conclusions, emphasising the identification

of critical Reynolds numbers, the transition from multiple to unique solutions, and the broader implications of symmetry breaking in lid-driven cavity flows at high Reynolds numbers.

MATHEMATICAL MODELLING

Physical description

The physical behaviour of two-dimensional incompressible fluid flow within a unit square cavity under steady-state conditions is strongly influenced by the direction of wall motion, as illustrated in Fig. 1. In the classical one-sided lid-driven configuration, the top wall moves rightward with a unit velocity ($u = 1$) /11, 12/, while the remaining walls are stationary with no-slip boundary conditions. This setup generates a laminar flow field characterised by a dominant primary vortex centred within the cavity and smaller secondary vortices near the corners. The size and location of these vortices such as the upper-left, lower-left, and right-corner vortices are governed by Reynolds number which determines the strength and structure of the recirculating zones.

Boundary conditions for two-sided and four-sided lid-driven cavity flows are depicted in Fig. 2. In the two-sided configuration, the top wall moves rightward, and the right wall moves downward, while the other walls remain stationary. In the four-sided configuration, all walls are in motion: the top and bottom walls move leftward and rightward, in respect, while left and right walls move upward and downward. These lid motions induce complex flow structures composed of major vortices - upper primary vortex (UPV), lower primary vortex (LPV), right primary vortex (RPV), and bottom primary vortex (BPV) - as well as secondary vortices such as the bottom side vortex (BSV) and right-side vortex (RSV). The formation and evolution of these vortices are highly sensitive to the Reynolds number, defined as:

$$Re = \frac{\rho u L}{\mu},$$

where u corresponds to lid velocity. Importantly, the numerical solution of the flow field is robust and does not depend on the initial velocity estimates within the cavity, allowing accurate simulation of symmetry breaking and bifurcation phenomena.

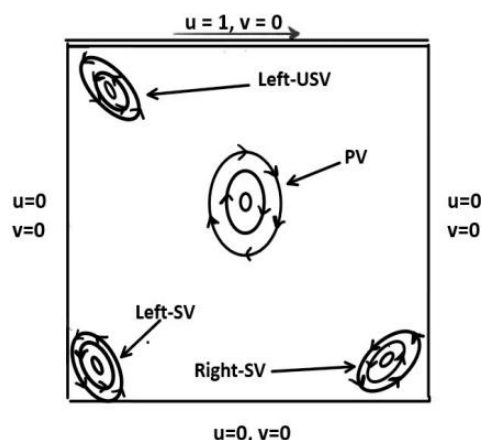


Figure 1. One sided flow diagram.

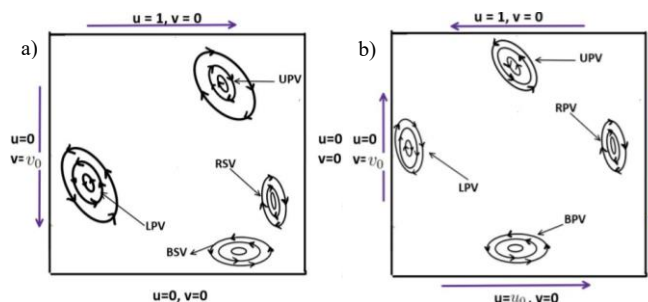


Figure 2. Flow diagrams: a) two-sided and; b) four-sided lid driven.

Numerical analysis

Consider the Newtonian internal flow of a square cavity with a lid-driven mechanism under two-dimensional viscous incompressible conditions. The incompressible two-dimensional Navier-Stokes equations consist of three governing equations: continuity, x -momentum, and y -momentum. In Cartesian coordinates, the continuity and momentum equations explain incompressible fluid-flow behaviour [47, 48/],

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0, \quad (1)$$

$$\frac{\partial(\rho u \cdot u)}{\partial x} + \frac{\partial(\rho u \cdot v)}{\partial y} = \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) - \frac{\partial p}{\partial x}, \quad (2)$$

$$\frac{\partial(\rho u \cdot v)}{\partial x} + \frac{\partial(\rho v \cdot v)}{\partial y} = \frac{\partial}{\partial x} \left(\mu \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu \frac{\partial v}{\partial y} \right) - \frac{\partial p}{\partial y}. \quad (3)$$

Here, u and v denote the velocity components in the x - and y -directions, respectively; μ is dynamic viscosity; ρ is density; and p is pressure.

The well-established one-sided lid-driven cavity case up to Reynolds number 5000 is simulated to validate the present numerical method. Figure 3 shows computed streamlines. Tables 2 and 3 provide maximal absolute values of the primary vortex and allow comparison with numerical data available in literature. Results of Ghia et al. [11/ are compared using vertical centrelines u -velocity profile passing through the geometric centre of the cavity, as shown in Fig. 4.

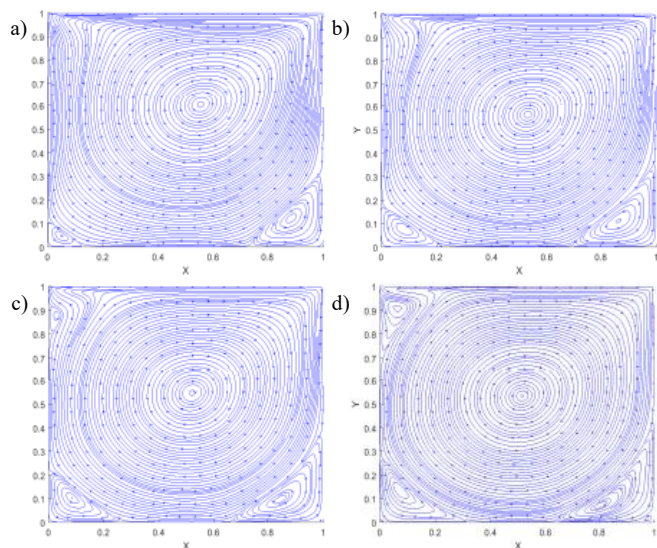


Figure 3. One-side flow - streamlines contour: a) Re = 400; b) Re = 1000; c) Re = 2000; d) Re = 5000, 501 by 501 grid.

The data presented in Tables 1 and 2 together with Figs. 3 and 4 show that the present numerical method agrees quan-

titatively well with previously reported numerical results. Increasing grid density by 85 % from 51×51 to 501×501 demonstrates that the results obtained using the 201×201 grid are grid independent. This is further confirmed in Tables 1, 2, and 3.

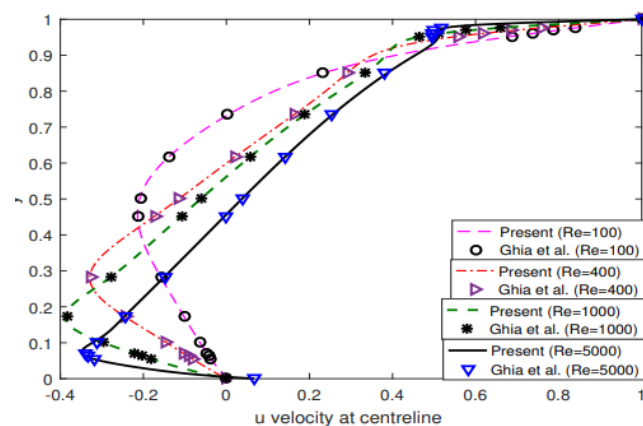
Figure 4. Centreline flow u of one-sided square cavity.

Table 1. Time to converge, number of iterations, and size of grid.

Re	Grid size	Time (s) to converge	No. of iterations (conv. criteria 10^{-7})
100	51×51	10.5264	3735
	101×101	25.853	12111
	201×201	342.6613	36362
	501×501	7616.444	144838
400	51×51	10.7546	3846
	101×101	26.4664	12293
	201×201	239.134	52637
	501×501	12554.42	229908
1000	51×51	14.0277	15355
	101×101	72.0553	45396
	201×201	910.2772	138106
	501×501	30454.86	574144
2000	51×51	53.876	124145
	101×101	566.387	381595
	201×201	7555.772	1155499
	501×501	273756.5	4335278
5000	51×51	72.6275	176055
	101×101	796.118	551336
	201×201	10902.93	1590514
	501×501	402215.1	6808631

Table 2. Data of one-sided for Re = 100, 400 and 1000.

Re	100					
Vortex	PV		SY-Right		SY-Left	
Position	x	y	x	y	x	y
Present	0.6171	0.722	0.9453	0.06193	0.03146	0.03853
Ghia et al.	0.6171	0.7344	0.9453	0.0625	0.0313	0.0391
Hou et al.	0.6196	0.7372	0.9451	0.0627	0.0391	0.0352
Gupta et al.	0.6125	0.7375	0.9375	0.0625	0.0374	0.0374
Kamel et al.	0.6132	0.74	0.94	0.06	0.0333	0.0333
Re	400					
Vortex	PV		SY-Right		SY-Left	
Position	x	y	x	y	x	y
Present	0.5547	0.5937	0.89059	0.1195	0.0508	0.0471
Ghia et al.	0.5546	0.6055	0.89059	0.125	0.0507	0.0468
Hou et al.	0.5607	0.6078	0.8901	0.1255	0.0548	0.0509
Gupta et al.	0.55	0.6125	0.8874	0.125	0.05	0.05
Kamel et al.	0.555	0.6049	0.885	0.12	0.05	0.05
Re	1000					
Vortex	PV		SY-Right		SY-Left	
Position	x	y	x	y	x	y

Present	0.5311	0.55059	0.8592	0.115	0.08584	0.07674
Ghia et al.	0.5312	0.5625	0.8594	0.1094	0.0859	0.0781
Hou et al.	0.5333	0.5646	0.8667	0.1137	0.0902	0.0783
Gupta et al.	0.525	0.5625	0.8625	0.1125	0.0874	0.074
Kamel et al.	0.53	0.5649	0.8649	0.115	0.085	0.0749

Table 3. Data of one-sided flow for $Re = 2000$ and 5000 .

Re	2000							
Vortex	PY		SY-Right		SY-Left		USV-Left	
Position	x	y	x	y	x	y	x	y
Present	0.525	0.558	0.845	0.084	0.0849	0.1034	0.03	0.8595
Hou et al.	0.525	0.549	0.847	0.098	0.0902	0.1058	-	-
Gupta et al.	0.525	0.55	0.8375	0.1	0.0874	0.1	0.0374	0.8874
Kamel et al.	0.525	0.55	0.8449	0.1	0.085	0.105	0.03	0.88
Re	5000							
Vortex	PY		SY-Right		SY-Left		USV-Left	
Position	x	y	x	y	x	y	x	y
Present	0.514	0.538	0.8001	0.086	0.0757	0.1382	0.0688	0.8967
Hou et al.	0.517	0.537	0.8077	0.074	0.0783	0.1373	0.0666	0.9098
Gupta et al.	0.512	0.537	0.8	0.074	0.074	0.1313	0.0688	0.9124
Kamel et al.	-	-	-	-	-	-	-	-

Results obtained through code verification with application of finite volume simple algorithm numerical approach agree well with the current data and validate with obtained results from [11, 50-52]. This is shown in Tables 1, 2 and 3, respectively. As a result of this research, in Sections 3 and 4, we come to the conclusion that the present numerical methodology is the mostly effective for analysing the problem.

NUMERICAL RESULT OF TWO-SIDED LID-DRIVEN CAVITY FLOW

Lid-driven cavity problem for two-sided geometry diagram is described in Fig. 2a. The top and left walls are moving to the right and downward side with equal speeds, respectively, while bottom and right walls remain stationary. Two primary vortices called LPV and TPV, and also two secondary vortices called BSV and RSV, are generated in square lid-driven flow. The obtained fluid flow is symmetrical around the one of cavity diagonal.

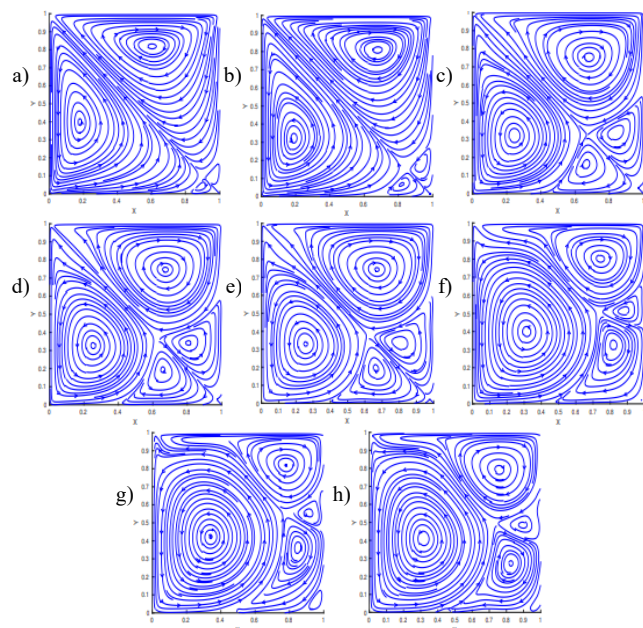


Figure 5. Streamlines contour for two-sided lid-driven for unit velocity and Re values: a) 10; b) 100; c) 500; d) 1000; e) 1466; f) 1467; g) 2000; h) 3000, on grid 201×201 .

Different streamlines of the flow can be observed in Fig. 5 for increasing Reynolds number low to high. The secondary vortices sizes expand as well as primary vortices also expand for increased Reynolds numbers. It is shown that the flow changes from a symmetrical bifurcation to an asymmetrical one inside 1466 and 1467 Reynolds numbers. As a direct consequence of this, the flow bifurcates at $Re = 1467$ which is considered as critical Reynolds number for lid-driven two sided flow. This value is very close to being accurate, with an error margin of just 0.02 points.

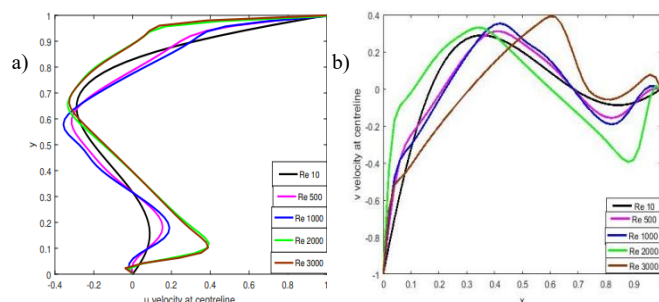


Figure 6. Midplane velocity of lid-driven flow of two-sided with equal 1 velocity: a) u -velocities; b) v -velocities.

Figure 6 demonstrates u - and v - velocities profiles of some variable Reynolds numbers. This is done under the conditions of equal velocity lid movement for two sides square lid-driven cavity. Top wall largest velocity is denoted by 1, while bottom wall largest velocity is denoted by 0. This is due to the fact that top wall moves to right direction. When the Reynolds number of the cavity increases, then the velocity profile has linear shift that runs through cavity centre. Figure 6b illustrates different Reynolds values as well as v -midplane velocities. In the figure, left wall maximal velocity is represented with negative number, whereas the maximal velocity of the right wall is depicted as zero. This is due to the fact that left walls move in a downward manner.

Figure 7 is a representation of streamlines multiple flow solutions for the $Re = 1467$ scenario. It shows that there is one symmetric solution as well as two asymmetric solutions. When sweeping the flow domain in a left-to-right direction, one will arrive at the first asymmetric solution which is denoted by the asymmetric one, and when sweeping the flow domain in an opposite direction, denoted by asymmetric two.

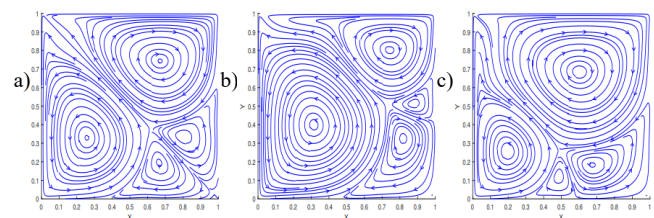


Figure 7. Streamlines contour multiple two-sided flow solutions of $Re = 1467$: a) symmetric; b) asymmetric one; c) asymmetric two, on grid 201×201 .

The u - and v - midplane velocities and v -velocities multiple solution for $Re = 1467$ are provided in Fig. 8. This is done under the conditions of equal velocity lid movement for two-sides square lid-driven cavity. Top wall largest velocity is denoted by 1, while the maximal velocity of bottom wall is represented by 0. This is due to the fact that top walls move

in the right side direction. Figure 8b demonstrate the v -midplane velocities for $Re = 1467$. On the figure, the left wall maximal velocity is represented as a negative number, whereas right wall maximal velocity is depicted by zero. This is due to the fact that left walls move in a downward manner.

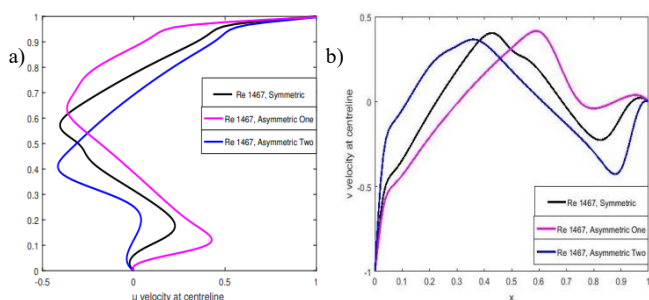


Figure 8. Midplane velocity of two-sided flow for $Re = 1467$: a) u -velocities; b) v -velocities.

Figure 9 is a representation of streamlines multiple flow solutions for $Re = 3000$ scenario. It shows that there is one symmetric solution as well as two asymmetric solutions. When sweeping the flow domain in a left-to-right direction, one will arrive at the first asymmetric solution, denoted by the asymmetric one, and when sweeping the flow domain in an opposite direction, as denoted by the asymmetric two. In multiple flow after $Re = 1467$ scenario, we observed that two secondary vortices movement are like right vertical in asymmetric one and horizontal bottom in asymmetric two.

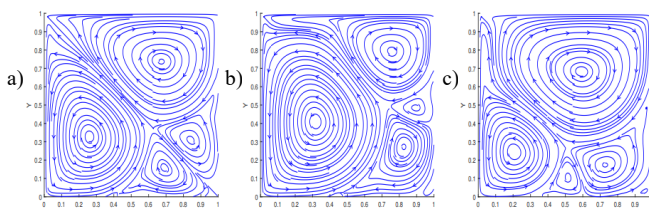


Figure 9. Streamlines contour multiple flow solutions of $Re = 3000$ in two-sided cavity flow: a) symmetric; b) asymmetric one; c) asymmetric two, on grid 201×201 .

Midplane u - and v -velocities multiple solution for $Re = 3000$ are provided in Fig. 10a. This is done under the conditions of equal velocity lid movement for two-sides square lid-driven cavity. The top wall maximal velocity is denoted as number 1, while the maximal velocity of the bottom wall is represented by number 0. This is due to the fact that top walls move in the right side direction. Figure 10b demonstrates v -midplane velocities with $Re = 3000$. In the figure, left wall maximal velocity is represented as negative number, whereas the maximal velocity of the right wall is depicted as a zero. This is due to the fact that left walls move in a downward manner.

The u - and v -midplane velocities for variable Reynolds numbers are provided in Fig. 12a. This is done under the conditions of equal velocity lid movement for two-sides square lid-driven cavity. Top wall maximal velocity is denoted as number 1, while bottom wall maximal velocity is denoted as 0. This is due to the fact that top walls move with direction toward right side. When the Reynolds number of the cavity increases then the velocity profile linearly shifts that runs through cavity centre. Figure 12b illustrates the dif-

ferent Reynolds values as well as midplane v -velocities. In the figure, the maximal velocity of left wall is represented as number -1.5 , whereas the maximal velocity of the right wall is depicted as a zero. This is due to the fact that left walls move in a downward manner.

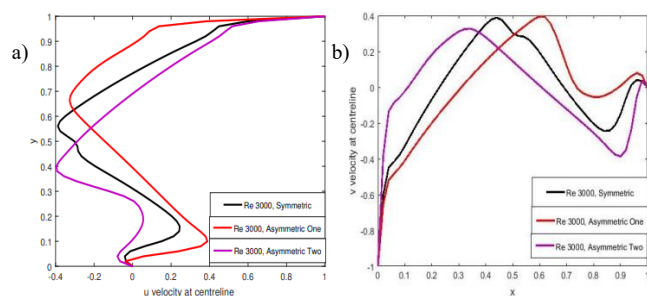


Figure 10. Midplane velocity of two-sided cavity flow for $Re = 3000$: a) u -velocities; b) v -velocities.

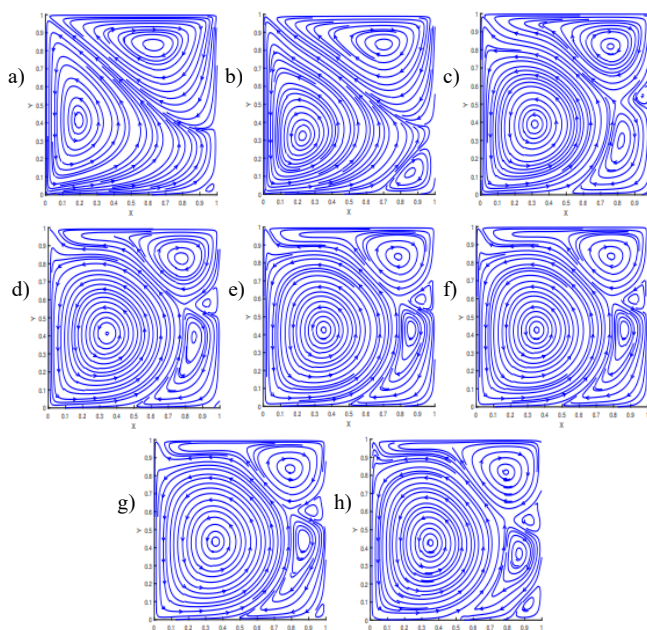


Figure 11. Streamlines contour for two-sided lid-driven under left wall velocity 1.5 for Re : a) 10; b) 100; c) 500; d) 1000; e) 1466; f) 1467; g) 2000; h) 3000, on grid 201×201 .

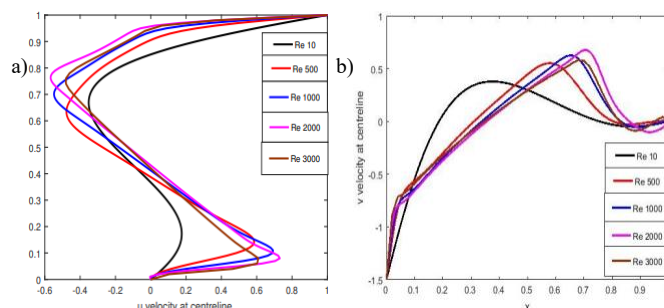


Figure 12. Midplane velocity of lid-driven two-sided cavity flow with left wall velocity 1.5: a) u -velocities; b) v -velocities.

NUMERICAL RESULT OF FOUR-SIDED LID-DRIVEN CAVITY FLOW

Lid-driven cavity problem for the two-sided geometry diagram is described in Fig. 2b. In Fig. 2, first consider that the four walls move with same speed. The flow field gets distinct

four symmetrical about both cavity diagonal vortices generated. Figure 13 demonstrates the influence on flow field for increased Reynolds number. The fluid flow bifurcates from symmetric to asymmetric state from 170 to 175 Reynolds number. Thus, 175 is the critical Reynolds number where the flow field bifurcates in four-sided.

Also, Fig. 13 demonstrates streamlines flow field pattern with 201×201 grid size on various Reynolds numbers from pre-critical numbers 10, 100, and 175, and post critical (up to $Re = 1000$). The streamlines are diagonally symmetrical about both cavity diagonals on pre-critical Reynolds number. Figures 15 and 17 show that there exist steady state multiple solutions with post critical 400 and 600 Reynolds numbers for moving four walls at equal speeds. In the four-sided lid movement with equal speed one velocity we get multiple flow solution, two multiple flow solutions for $Re = 175$ to 385, and three multiple solutions for $Re = 386$ to 1000. So $Re = 386$ as multiple flow critical Reynolds number for flow goes from two to three multiple solutions.

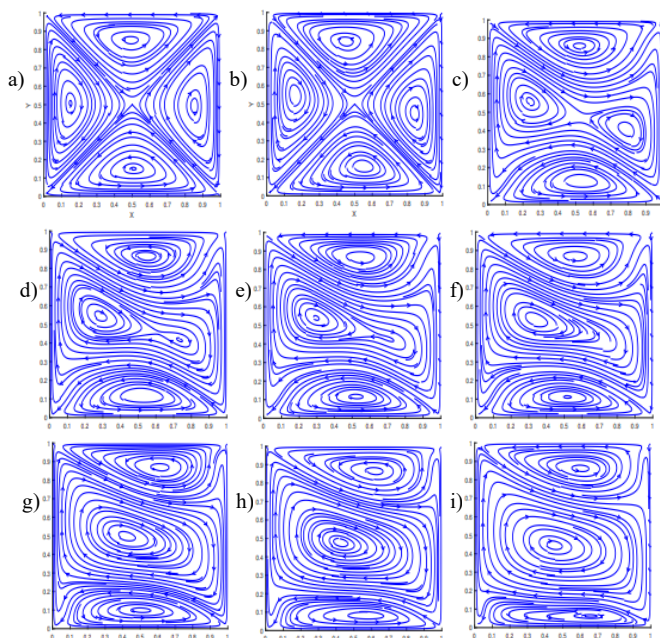


Figure 13. Streamlines contour for four-sided with all walls 1 velocity for Re: a) 10; b) 100; c) 150; d) 170; e) 175; f) 200; g) 400; h) 600; i) 1000, on grid 201×201 .

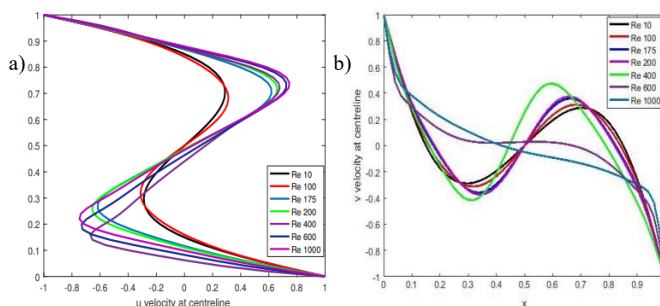


Figure 14. Midplane velocity of four-sided with all walls velocity one: a) u -velocities; b) v -velocities.

The u - midplane velocity on various Reynolds number is demonstrated in Fig. 14a. The top wall maximal velocity is denoted by -1 , while bottom maximal velocity is denoted by 1 . This is because top wall and bottom wall move to

opposite direction. As the Reynolds number of the cavity increases, there is a linear shift that takes place in the core velocity profile of the cavity. Figure 14b displays the outcomes that occur when Reynolds numbers and midplane v -velocities are altered. Due to the fact that the left and right walls travel in opposite directions, the graph displays the maximal velocities as 1 and -1 , respectively.

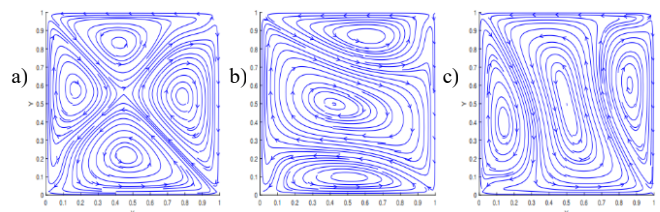


Figure 15. Streamlines contour multiple flow solutions of $Re = 400$ four-sided lid-driven cavity: a) symmetric; b) asymmetric one; c) asymmetric two, on grid 201×201 .

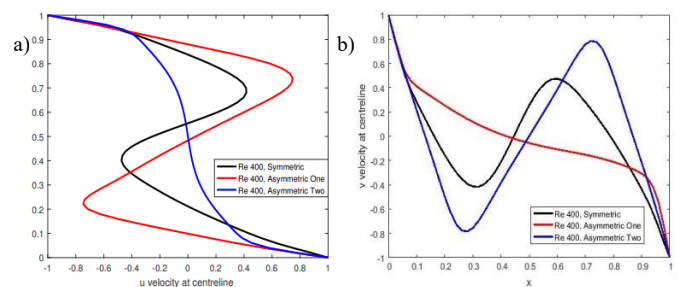


Figure 16. Midplane velocity of four-sided with all walls velocity 1 for $Re = 400$: a) u -velocities; b) v -velocities.

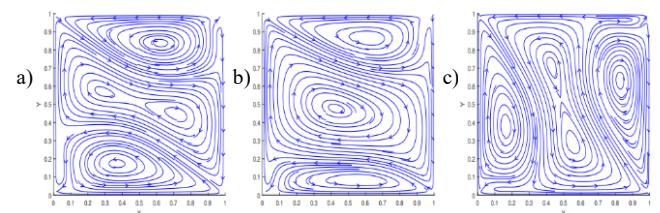


Figure 17. Streamlines contour multiple flow solutions of $Re = 600$ four-sided lid-driven cavity: a) symmetric 1; b) asymmetric; c) symmetric 2, on grid 201×201 .

Figure 15 plots streamlines multiple flow solutions on $Re = 400$ case, showing that the one symmetric and two asymmetric solutions occur. The stream function values on cavity centre are no longer zero. Furthermore, left vortex and right vortex combining form single vortex and side vortex appear incapacitated.

Figure 16a demonstrates u - midplane velocities for 400 Reynolds number. Since top and bottom walls move into opposing direction, their maximal velocities are represented by -1 and 1 , respectively on the graph. In Fig. 16b, we see the v -velocities in the midplane for Reynolds number 400. Since the walls move in different directions, the maximal velocity is shown as 1 for left wall and -1 for right wall.

Figure 17 plots the streamlines multiple flow solutions for $Re = 600$, obtaining two symmetric and one asymmetric solutions. The stream function values on cavity centre are no longer zero. Furthermore, left vortex and right vortex combining get a single vortex for asymmetric and side vortex appearing incapacitated.

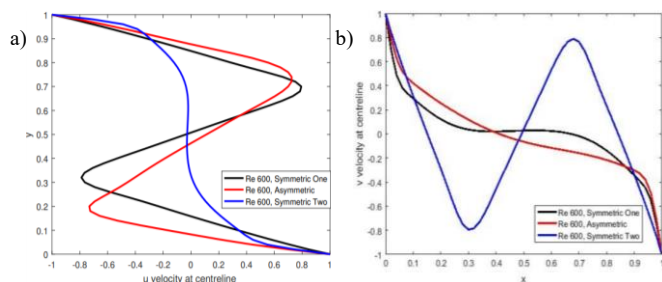


Figure 18. Midplane velocity of four-sided with all walls velocity 1 for $Re = 600$: a) u -velocities; b) v -velocities.

Figure 18a provides a visual representation of midplane u -velocities for Reynolds values of 600. On the figure, top and bottom walls maximal velocity is denoted as -1 and 1 , respectively. This is because of the fact that the top wall and bottom wall flow in reverse directions. The v -velocities in the midplane for Reynolds values of 600 are shown in Fig. 18b. On the figure, left and right walls maximal velocity is denoted as 1 and -1 , respectively. This is due to the fact that the two walls move in reverse directions.

In accordance with the results of our investigation, the top wall, bottom wall, right wall and left wall are moving to left, right, downwards and upwards with equal speed 1 , in respect. This is seen in Figs. 13 to 18, which illustrate cavity flows. This four-sided lid movement produces the multiple flows solution, which is the outcome of the movement. The transition from a multiple state solution to a unique state solution for the same Reynolds numbers is seen in Figs. 19 and 21. When the bottom side wall moves in a right direction at the speed of 1.5 with remaining three are at same speed 1 , and the left side wall moves to downward direction at the speed of 1.5 while the remaining three are at same speed 1 .

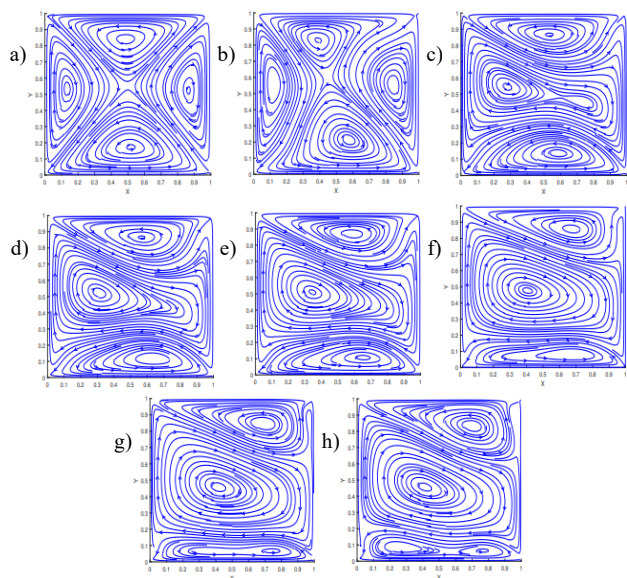


Figure 19. Streamlines contour for four-sided with bottom wall velocity 1.5 at Re : a) 10 ; b) 100 ; c) 170 ; d) 175 ; e) 200 ; f) 400 ; g) 600 ; h) 1000 , on grid 201×201 .

Keeping the view that the top wall, bottom wall, left wall and right wall are moving to left, right, upward and downward directions, respectively, in four-sided cavity flow with bottom wall moving at $u_0 = 1.5$ speed and other walls move at the speed of 1 . For different Reynolds values, when all

lid moves with equal speed 1 then we get multiple solution, but when the bottom wall moves at the speed 1.5 then we get a unique solution as asymmetric one type, which are represent in Fig. 19.

Figure 20a displays the u -midplane velocities with variety of Reynolds values. The bottom wall maximal velocity is marked by number 1.5 , whereas the top wall maximal velocity is denoted by number -1 . This occurs because the top wall and the bottom wall move in opposite directions. As the Reynolds number of a cavity increases, there is a linear shift that takes place in the internal velocity profile of the cavity. The results of changing the midplane v -velocities and Reynolds numbers are shown in Fig. 20b. The graph shows the maximal velocities as 1 and -1 , respectively, since the left and right walls move in opposite directions.

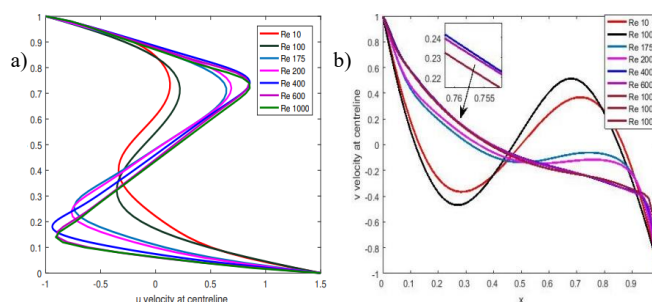


Figure 20. Midplane velocity of four-sided with bottom wall velocity 1.5 : a) u -velocities; b) v -velocities.

Keeping the view that top wall, bottom wall, left wall and right wall are moving to left, right, upward and downward directions, respectively, in four-sided cavity flow with left wall moving at $v_0 = 1.5$ speed and other walls move at the speed of 1 velocity, for different Reynolds values, when all lid moves with equal speed 1 then we get multiple solution, but left wall moving at speed 1.5 , then we get a unique solution as asymmetric two type, which are represent in Fig. 21.

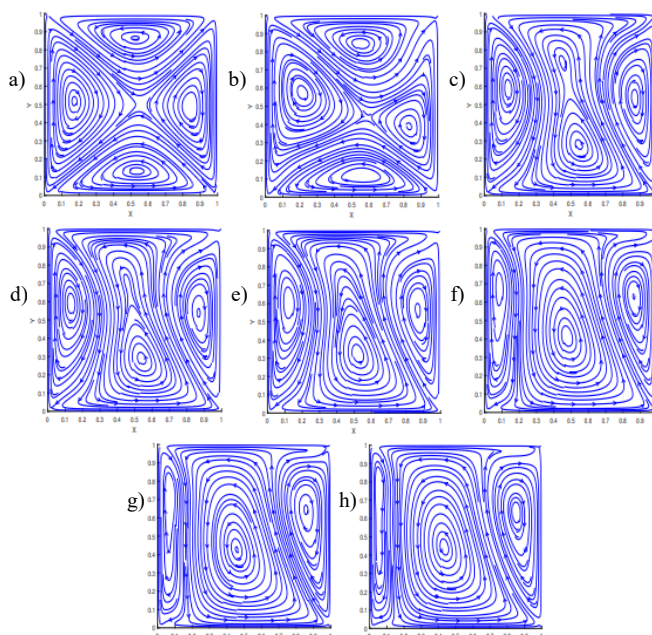


Figure 21. Streamlines contour for four-sided with left wall velocity one at Re : a) 10 ; b) 100 ; c) 170 ; d) 175 ; e) 200 ; f) 400 ; g) 600 ; h) 1000 , on grid 201×201 .

The results of changing the u -midplane velocities and Reynolds numbers are demonstrated in Fig. 22a. The graph shows the maximal velocities as 1 and -1 , respectively, since the left and right walls move in opposite directions. Figure 22b displays the v -midplane velocity for a variety of Reynolds number. Left wall maximal velocity is marked by 1.5, whereas the maximal velocity of the right wall is denoted by number -1 . This occurs when the left wall and right wall move to opposite directions. As the Reynolds number of a cavity increases, there is a linear shift that takes place in the internal velocity profile of the cavity.

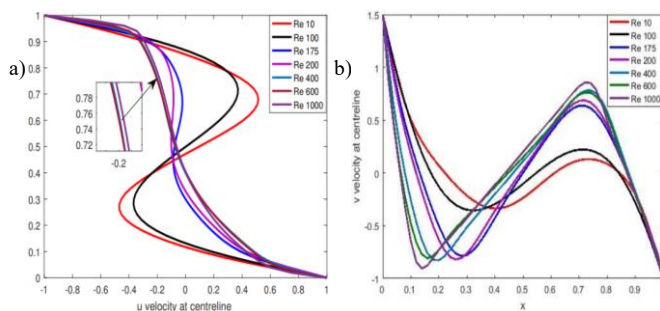


Figure 22. Midplane velocity of four-sided with left wall velocity 1.5: a) u -velocities; b) v -velocities.

CONCLUSION

This study presents a detailed numerical investigation of lid-driven cavity flows in one-sided, two-sided, and four-sided configurations, focusing on symmetry breaking and the emergence of multiple and unique stable solutions at varying Reynolds numbers. Using the finite volume SIMPLE algorithm, flow behaviour is analysed across a unit square domain under steady-state incompressible conditions.

For the one-sided case, benchmark results are validated against existing literature and confirm the accuracy of the numerical approach. In the two-sided configuration, the flow remains symmetric at low Reynolds numbers, but transitions to asymmetric states beyond a critical threshold ($Re = 1466$). Three distinct solutions are identified: two stable asymmetric and one unstable symmetric. At $Re = 1467$ and $Re = 3000$, all three solutions are successfully recovered.

In the four-sided configuration, where all walls move at equal speed, symmetry is preserved only below a critical Reynolds number ($Re = 175$). Beyond this point, instabilities lead to symmetry breaking. At $Re = 400$ and $Re = 600$, three distinct solutions are observed, while two stable solutions emerge for $Re > 175$, and three for $Re > 385$. When one wall velocity increases to 1.5, both two-sided and four-sided cases exhibit unique stable solutions, demonstrating the sensitivity of the flow field to boundary conditions.

Velocity contours, streamline plots, and centreline velocity profiles are generated for all configurations, revealing the migration of the primary vortex toward the cavity centre and the growth of secondary vortices with increasing Reynolds number. These findings confirm the role of bifurcation and symmetry breaking in lid-driven cavity flows and provide a robust reference for future studies.

The proposed finite volume framework offers a reliable tool for analysing cavity flows and can be extended to more complex geometries and flow conditions. Future work will

explore the influence of cavity aspect ratio and thermal effects on non-Newtonian fluid behaviour, further enriching the understanding of shear-driven flows in confined domains.

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Abbreviations

Symbol	Meaning
FVM	Finite volume technique
PV	Primary vortex
SIMPLE	Semi implicit method for pressure linked equation
LPV	Left primary vortex
UPV	Upper primary vortex
BPV	Bottom primary vortex
RPV	Right primary vortex
SV	Secondary vortex
BSV	Bottom secondary vortex
Left-USV	Left upper secondary vortex
Left-SV	Left secondary vortex
Right-SV	Right secondary vortex
HPC	High performance computing

Nomenclatures

Symbol	Meaning
U	x -direction velocity
u^*	approximate x -direction velocity
u'	correction x -direction velocity
v	y -direction velocity
v^*	approximate y -direction velocity
v'	correction y -direction velocity
p	pressure
p'	correction pressure
p^*	guessed pressure
i	node location in direction of x -axis
j	node location in direction of y -axis
Δx	spatial step in direction of x -axis
Δy	spatial step in direction of y -axis
Re	Reynolds number

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