

STOKES FLOW AROUND STAGGERED CIRCULAR CYLINDERS IN RECTANGULAR MICROCHANNEL

STOKSOVO STRUJANJE OKO ŠAHOVSKOG RASPOREDA KRUŽNIH CILINDARA U PRAVOUGAONOM MIKROKANALU

Originalni naučni rad / Original scientific paper

Rad primljen / Paper received: 15.04.2026

<https://doi.org/10.69644/ivk-2026-siB-0053>

Adresa autora / Author's address:

¹⁾ Department of Mathematics, School of Basic and Applied Sciences, K.R. Mangalam University, Gurugram, Haryana, India
V. Chhabra <https://orcid.org/0009-0000-5612-0245>

²⁾ Department of Science, and Mathematics, International Institute of Information Technology, Naya Raipur, Chhattisgarh, India
C.S. Nishad <https://orcid.org/0000-0002-8941-4079>

³⁾ Department of Mathematics, School of Technology, Pandit Deendayal Energy University, Gandhinagar, Gujarat, India
M. Sahni <https://orcid.org/0000-0001-7949-9147>

*email: Manoj.Sahni@sot.pdpu.ac.in

Keywords

- boundary element method
- staggered circular cylinders
- microchannel
- slip patterning
- Stokes equation

Abstract

The present work formulates a two-dimensional hydrodynamical model for steady, viscous, incompressible, pressure driven fluid around circular cylinders of fixed radii configured in staggered arrangement within a rectangular horizontal microchannel. To enhance our understanding towards the hydrodynamics of microchannel flows, we consider surface roughness of both horizontal surfaces of the microchannel which is mathematically possible with the utilisation of Navier slip boundary conditions in alternative manner. The Stokes equations govern the considered fluid flow at low Reynolds number, which is solved numerically by using boundary element method (BEM). The MATLAB[®] software is used to implement the BEM. The hydrodynamic quantities are systematically analysed to study in-depth dynamics of Stokes flow. We analysed a noticeable change in the pressure gradient at the slip regime, which occurs due to the reduced wall resistance that alters the near-wall velocity and modifies the overall pressure distribution. The proposed model holds practical significance in various fields, including hemodynamic flow analysis and the design of lab-on-chip diagnostic devices.

INTRODUCTION

When a fluid is highly viscous and incompressible, viscous effects dominate inertial effects. In this regime, the flow behaves as Stokes flow. Extensive research has been carried out on creeping flow in several types of conduits, including micro- and nanochannels /1, 2/, as well as tubes, cavities, and other geometrical arrangements /3-6/. This research has wide practical relevance and finds applications in several areas, including processing of polymeric fluids /7/, mixers actuated by combined translational and rotational motion /8/, and microfluidic drug delivery framework /9/. Jeong /10/ presents an analytical investigation of two-dimensional Stokes flow through a slit in a microchannel, incorporating slip boundary conditions relevant to micro- and nano-scale

Ključne reči

- metoda graničnih elemenata
- šahovski raspored kružnih cilindara
- mikrokana
- oblikovanje polja klizanja
- Stoksova jednačina

Izvod

U ovom radu se formuliše dvodimenzionalni hidrodinamički model kod ravnornog, viskoznog, nestišljivog fluida pod pritiskom, oko kružnih cilindara fiksnih poluprečnika, šahovski raspoređenih unutar pravougaonog horizontalnog mikrokana. Kako bismo produbili razumevanje hidrodinamike mikrokanskog strujanja, razmatramo hrapavost površina za obe horizontalne površine mikrokana, a što je matematički izvodljivo primenom Navijeovih graničnih uslova klizanja u alternativnom smislu. Stoksove jednačine opisuju razmatrano strujanje fluida za mali Reynoldsov broj, a što se numerički rešava korišćenjem metode graničnih elemenata (BEM). Koristi se softver MATLAB[®] za implementaciju BEM. Hidrodinamičke veličine se sistematički analiziraju radi detaljnijeg proučavanja dinamike Stoksovog strujanja. Zapaža se značajna promena gradijenta pritiska u režimu klizanja, koja se pojavljuje usled smanjenog otpora zida, čime se utiče na brzine strujanja uz zid, a menja se i čitava raspodela pritiska. Predloženi model ima praktičnog značaja u raznim oblastima, uključujući analize hemodinamičkog strujanja i kod razvoja dijagnostičkih uređaja tipa laboratorija-na-čipu.

fluid systems. The work analyses the influence of surface slip on the flow dynamics, pressure distribution, and hydrodynamic forces, providing insight into microscale transport phenomena. Georgiou et al. /11/ investigated the flow of continuum fluids through a channel restricted by cylinders and demonstrate that fluid elasticity and complex geometry significantly affect flow behaviour and resistance, with experimental observations supported by numerical simulations. Sumner et al. /12/ experimentally studied the flow around staggered cylinders focusing on vortex shedding characteristics. Using wind tunnel experiments over a subcritical Reynolds number range, they systematically varied the pitch ratio and incidence angle to examine their influence on drag, lift, and Strouhal number for both upstream and downstream cylinders. According to their study, the flow behaviour can

be classified into minimal, moderately, and widely spaced configurations, each exhibiting distinct force characteristics and vortex shedding patterns.

The study of fluid motion in impermeable micro- and nanochannels naturally brings attention to the influence of the channel boundaries. The physical response of the fluid is governed by the wall surface properties, making the identification of hydrophobic and hydrophilic behaviour an important consideration. These surface effects are commonly represented in mathematical models by employing slip or no-slip boundary conditions to describe hydrophobic and hydrophilic walls, respectively. Over recent decades, this topic has attracted considerable scholarly attention [13-16]. It is widely recognised that surface properties, including wall roughness and contamination, play a crucial role in determining the slip length [17, 18]. The slip length refers to an idealised location beyond the physical boundary where the fluid velocity extrapolates to a value lower than that at the solid surface. Within this framework, Hendy et al. [19] analysed Newtonian fluid flow in micro- and nanochannels by employing patterned slip boundary conditions and further explored the effects of surface wettability using molecular dynamics simulations.

The microchannel flows are closely connected to the advancement of microfluidic framework with widely used practical applications in the fields of chemical, biomedical and aerospace engineering. In this regard, Mour et al. [20] examined a large-scale parametric study to examine the interplay of wall roughness and friction factors to gain insight into the intricate flow processes in microfluidic devices. They found that the friction factor enhances substantially with increasing surface roughness. Nizkaya et al. [21] analysed flow through a microchannel exhibiting Navier slip boundary conditions in the presence of 1D trapezoidal surface texture at one of the walls. They found noticeable drag reduction in slender channels. Later, Ranjith et al. [22] utilise periodic Navier slip configurations on the channel walls to analyse 2D volumetric flux. They note that anti-symmetric strips give rise to a velocity component with finite magnitude directed toward the channel centreline. Later, the circulation around circular cylinders has been much of interest to researchers due to its significance in various fields of use like heat exchange apparatus, transportation in stems of plants, and aquatic life, tidal turbine farms and offshore engineering [23-27]. Canuto and Taira [28] examine wake formation in a 2D compressible flow past a cylinder by systematically varying the Reynolds number. Jeong and Yoon [29] utilise a least-squares approach to investigate the flow at low Reynolds limit past a circular cylinder through a microchannel. Their study led to an enhanced understanding of the advancement of bio-MEMS applications and targeted drug transport. Afterwards, Jeong analysed the development of toroidal vortex structures in the assumed flow past a moving sphere, which provides a key physical step towards the design and optimisation of capsule-based drug delivery systems. On the other hand, Rajalingam and Chakraborty [30] examine three-dimensional incompressible flow inside a microchannel featuring micro-pillars and micro-blind holes using numerical simulations. Their results indicate that ellip-

tical pillars provide higher thermal efficiency than the other shapes studied. Further, Lobasov et al. [31] analysed the development of cavitation near the micro cylinder within a microchannel using numerical and experimental approaches. They observed that cavitation increases the fluid velocity behind the micro-cylinder. Rodrigues et al. [32] investigated complex blood analogue polymeric flow dynamics. Further, they perform the channel blockage analysis by considering both circular and elliptical cylinders. Analytical approaches, including perturbation method and eigenfunction expansion method [29] are commonly applied in fluid flow analysis; however, their efficiency decreases when handling complex geometry as well as the boundary conditions. In such scenarios, the numerical approaches, including boundary element method (BEM) [33-35], finite element method (FEM) [36], and finite difference method (FDM) [37, 38]. Among the above-mentioned methods, BEM provides several advantages for solving linear governing equations, [39].

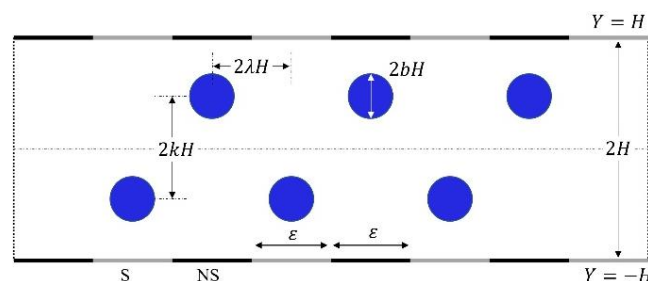


Figure 1. Schematic of considered Stokes flow problem.

In existing literature, several researchers studied fluid flow around cylinders within microchannels which have several direct practical applications in microfluidic devices but the impact of surface roughness of the channel is not yet been taken care of with other important parameters. In this context, the Stokes flow is considered around staggered cylinders inside a horizontal channel with rough surface conditions. To achieve this physically, we utilise Navier's slip boundary conditions. Further, to solve the complex boundary conditions appearing in mathematical modelling, we use boundary element method that has significant advantages in case of linear boundary value problems, [40]. The paper is structured as follows: The following section introduces mathematical modelling of the considered Stokes flow problem and solution methodology in detail. Then, we provide a detailed hydrodynamics fluid flow problem. Finally, we summarise the main findings of the present study.

MATHEMATICAL DESCRIPTION OF THE PROBLEM

Dimensional formulation of governing equations and assumptions

The following physical and geometrical assumptions are used to formulate the problem:

- the fluid flow is presumed to be steady, incompressible and without inertia; thus, it is mathematical modelled by the Stokes equations;
- the Poiseuille flow domain is a 2D microchannel with a width of $2H$ units;
- the Navier's slip conditions are applied periodically on the horizontal walls in the alternative manner;

- in the main schematic of the considered flow problem (Fig. 1), the no-slip regions are shown by black boundary segments, whereas slip regions are depicted by grey segments. Moreover, the alternate slip segments have a characteristic length ε , which is a periodic length scale of the wall patterning;
- the identical circular cylinders are arranged in staggered positions within the flow domain to create the impediment of the fluid movement in a narrowed space.

Dimensional governing equations are given by /42-44/:

$$\mu \tilde{\nabla}^2 \tilde{\mathbf{V}} = \tilde{\nabla} \tilde{P}, \quad (1)$$

$$\tilde{\nabla} \cdot \tilde{\mathbf{V}} = 0. \quad (2)$$

Scaled variables

Introducing the scaled variables /43, 44/:

$$\mathbf{V} = \frac{\tilde{\mathbf{V}}}{\tilde{U}_0}, \quad X = \frac{\tilde{X}}{H}, \quad Y = \frac{\tilde{Y}}{H}, \quad P = \frac{\tilde{P}}{\tilde{P}_0}, \quad \tilde{P}_0 = \frac{\mu \tilde{U}_0}{H}, \quad \tilde{\nabla} = \frac{\nabla}{H}. \quad (3)$$

The non-dimensional form of dimensional governing equations is given as follows:

$$\nabla^2 \mathbf{V} = \nabla P, \quad (4)$$

$$\nabla \cdot \mathbf{V} = 0. \quad (5)$$

Reduction of governing equations

The continuity equation, /41/, allows the velocity profiles to be formulated as a function of stream function ψ , resulting in:

$$U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X}. \quad (6)$$

The simplified form of above Eqs.(4) and (5) using stream function ψ is given by:

$$\nabla^4 \psi = 0, \quad (7)$$

in the above equation, $\nabla^4 \equiv \frac{\partial^4}{\partial X^4} + 2 \frac{\partial^2}{\partial X^2} \frac{\partial^2}{\partial Y^2} + \frac{\partial^4}{\partial Y^4}$.

Next, the vorticity variable ω is introduced to decompose the Eq.(7) into Eqs.(8) and (9), to formulate the boundary element formulation of the Eq.(7), given by:

$$\nabla^2 \psi = -\omega, \quad (8)$$

$$\nabla^2 \omega = 0. \quad (9)$$

Boundary conditions

At upper horizontal wall, the no-slip boundary condition is given by, /45/:

$$\psi = 1, \quad \frac{\partial \psi}{\partial Y} = 0. \quad (10)$$

At upper horizontal wall, the slip boundary condition is given by, /46/:

$$\psi = 1, \quad \frac{\partial \psi}{\partial Y} + l_s \omega = 0. \quad (11)$$

At lower horizontal wall, the no-slip boundary condition is given by, /45/:

$$\psi = -1, \quad \frac{\partial \psi}{\partial Y} = 0. \quad (12)$$

At lower horizontal wall, the slip boundary condition is given by, /46/:

$$\psi = -1, \quad \frac{\partial \psi}{\partial Y} - l_s \omega = 0. \quad (13)$$

At downstream, /47/:

$$\psi = \frac{3}{2} \left(Y - \frac{Y^3}{3} \right), \quad \omega = 3Y. \quad (14)$$

At upstream, /47/:

$$\frac{\partial \psi}{\partial X} = 0, \quad \frac{\partial \omega}{\partial X} = 0. \quad (15)$$

Numerical solution of governing equations using BEM

Let $\alpha(X_0, Y_0) \in \Omega \cup \Gamma$ and $\beta(X_1, Y_1) \in \Gamma$ are the field point and integration point, respectively. Further, we assume $G^L = \frac{1}{2\pi} \ln \sigma$ and $G^B = \frac{1}{8\pi} \sigma^2 (\ln \sigma - 1)$ are the free space Green's function of Laplacian and the Biharmonic operator, respectively, /47/. Also, the distance between the points α and β is denoted by σ . Using the Green's second identity to convert the governing equations stated by Eqs.(8) and (9), substituting the free space Green's functions, we obtain:

$$\begin{aligned} \zeta(\alpha) \psi(\alpha) = & \int_{\Gamma} \left[\psi(\beta) \frac{\partial G^L(\alpha, \beta)}{\partial n_{\beta}} - G^L(\alpha, \beta) \frac{\partial \psi(\beta)}{\partial n_{\beta}} \right] ds(\beta) - \\ & - \int_{\Gamma} \left[\omega(\beta) \frac{\partial G^B(\alpha, \beta)}{\partial n_{\beta}} - G^B(\alpha, \beta) \frac{\partial \omega(\beta)}{\partial n_{\beta}} \right] ds(\beta), \end{aligned} \quad (16)$$

$$\zeta(\alpha) \omega(\alpha) = \int_{\Gamma} \left[\omega(\beta) \frac{\partial G^L(\alpha, \beta)}{\partial n_{\beta}} - G^L(\alpha, \beta) \frac{\partial \omega(\beta)}{\partial n_{\beta}} \right] ds(\beta). \quad (17)$$

In above Eqs.(16) and (17), the term $\zeta(\alpha)$ is called free term coefficient, defined by:

$$\zeta(\alpha) = \begin{cases} 1, & \text{if } \alpha \in \Omega \\ 1/2, & \text{if } \alpha \in \Gamma \\ 0, & \text{if } \alpha \notin \Omega \cup \Gamma. \end{cases} \quad (18)$$

The integrals appearing in the above-mentioned integral equations exhibit weak and strong singular behaviour which hinders analytical solution procedures, especially in domains with complicated geometries and boundary specifications. This necessitates the use of an appropriate numerical method for the fluid flow formulation.

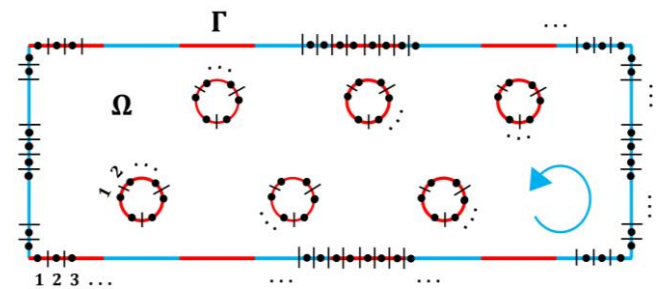


Figure 2. Discretization scheme of physical domain.

We discretise the boundary of microchannel using linear line elements which are continuously connected by their end points. Moreover, the boundary variables are considered to remain constant along each boundary segment. Accordingly, the boundary Γ is discretised into N constant elements (refer to Fig. 2). The variables along the boundary such as ψ , $\partial \psi / \partial n_{\beta}$, ω and $\partial \omega / \partial n_{\beta}$ are approximated by piecewise constant representations ψ_j , $\partial \psi_j / \partial n_{\beta}$, ω_j and $\partial \omega_j / \partial n_{\beta}$, $j = 1: N$.

$$\zeta_i \psi_i = \sum_{j=1}^N \left[\psi_j \int_{q \in \Gamma_j} \frac{\partial G^L(\beta_i, \beta)}{\partial n_\beta} ds(\beta) - \frac{\partial \psi_j}{\partial n_\beta} \int_{\beta \in \Gamma_j} G^L(\beta_i, \beta) ds(\beta) - \omega_j \int_{q \in \Gamma_j} \frac{\partial G^B(\beta_i, \beta)}{\partial n_\beta} ds(\beta) + \frac{\partial \omega_j}{\partial n_\beta} \int_{\beta \in \Gamma_j} G^B(\beta_i, \beta) ds(\beta) \right], \quad (19)$$

$$\zeta_i \omega_i = \sum_{j=1}^N \left[\omega_j \int_{\beta \in \Gamma_j} \frac{\partial G^L(\beta_i, \beta)}{\partial n_\beta} ds(\beta) - \frac{\partial \omega_j}{\partial n_\beta} \int_{\beta \in \Gamma_j} G^L(\beta_i, \beta) ds(\beta) \right]. \quad (20)$$

Now, it is therefore convenient to express Eqs.(19) and (20) in the form of a coupled system of linear algebraic equations:

$$A_{ij} \psi_j + B_{ij} \frac{\partial \psi_j}{\partial n_\beta} + C_{ij} \omega_j + D_{ij} \frac{\partial \omega_j}{\partial n_\beta} = 0, \quad (21)$$

$$A_{ij} \omega_j + B_{ij} \frac{\partial \omega_j}{\partial n_\beta} = 0. \quad (22)$$

In above equations, $\psi = (\psi_1, \psi_2, \dots, \psi_N)^T$ and $(A)_{ij} = A_{ji}$. The analysis of the velocity distribution and other hydrodynamic quantities depends on the solution of the above influence coefficients. In the present work, they are evaluated through the numerical Gaussian quadrature method.

VALIDATION OF BEM

To validate the BEM calculation for the considered flow problem, we assume the creeping flow within a rectangular parallel walled microchannel with boundaries at $Y = 1$ and $Y = -1$.

The no penetration boundary conditions are applied on both walls. The corresponding analytical expression for exact horizontal velocity U_{exact} is given by:

$$U_{exact} = \frac{3}{2}(1 - Y^2). \quad (23)$$

The present BEM solution $U_{computational}$ of the above-mentioned scenario is compared with analytically obtained velocity U_{exact} , as shown in Fig. 3Figure.

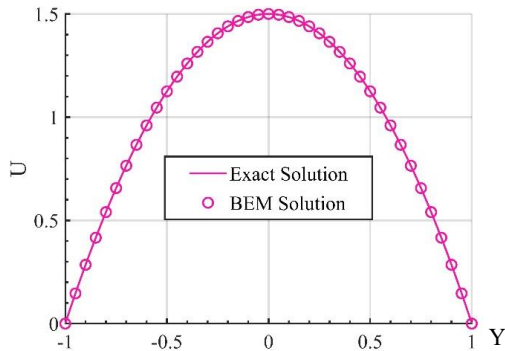


Figure 3. Comparison of U_{exact} and $U_{computational}$ in vertical direction at $X = 0$.

RESULTS AND DISCUSSION

We consider a 2D steady, incompressible, pressure driven Stokes flow around staggered circular cylinders within a rectangular microchannel exhibiting surface roughness. For BEM simulation, we considered the length of the microchannel as 20 units varying from -10 to 10 and width of the channel is 2 units varying from -1 to 1 . To study the hydrodynamics of the considered Stokes flow problem, firstly, we

have presented the streamline contour plots of viscous flow for distinct values of slip length l_s and vertical spacing between centre to centre of circular cylinders k for fixed radius of circular cylinder $b = 0.125$ in Figs. 4 and 5, respectively. The streamlines within the flow domain exhibit a uniform and orderly distribution, closely conforming to the channel configuration. The variation in fluid velocity is conveyed through the streamline spacing, where narrower gaps correspond to regions of higher velocity, while wider separations indicate regions of lower velocity.

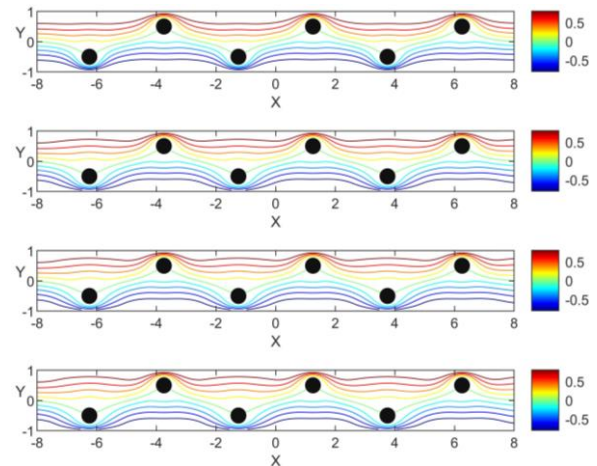


Figure 4. Streamline contour plots of viscous flow around staggered circular cylinders for distinct slip lengths $l_s = 0.1, 0.50, 1, 10$ and radius $b = 0.125$ with vertical spacing between centre to centre of cylinders $k = 0.5$.

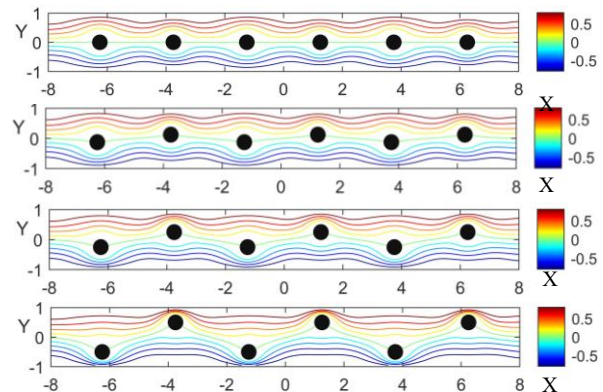


Figure 5. Streamline contour plots of viscous flow around staggered circular cylinders for distinct vertical spacing between centre to centre of cylinders $k = 0, 0.125, 0.250$ and 0.50 with radius $b = 0.125$ and slip length $l_s = 0.50$.

Figures 6 and 7 present the U and V components for varying the l_s values within the range from $l_s = 0.1$ to 10 , while keeping the other considered parameters $\varepsilon = 2.5$, $b = 0.125$, $k = 0.5$ and $\lambda = 1.250$. The results show that the U velocity field becomes distorted due to the staggered positioning of circular cylinders along the flow path. Moreover, for a given slip length l_s , the maximal value of U velocity is obtained when a no penetration condition is implemented on both horizontal boundaries. In contrast, lower values are observed under slip boundary conditions, which occurs due to the principle of mass conservation (refer to Figs. 6a and 6b). A physical explanation for this observation is that when a no penetration condition is implemented to the channel

walls, the velocity adjacent to boundaries becomes significantly lower compared to the case with slip boundary conditions. Since the fluid is incompressible, conservation of mass flux will result in velocity increase in the core-region in the no-penetration scenario. Furthermore, two specific points, located at $Y = -0.7$ and 0.6 are observed where the axial velocity U is independent of l_s values. This behaviour arises from the boundary layers in the flow, which enforce adjustments in the velocity profile such that the axial velocity at certain points remains unaffected by slip length variations.

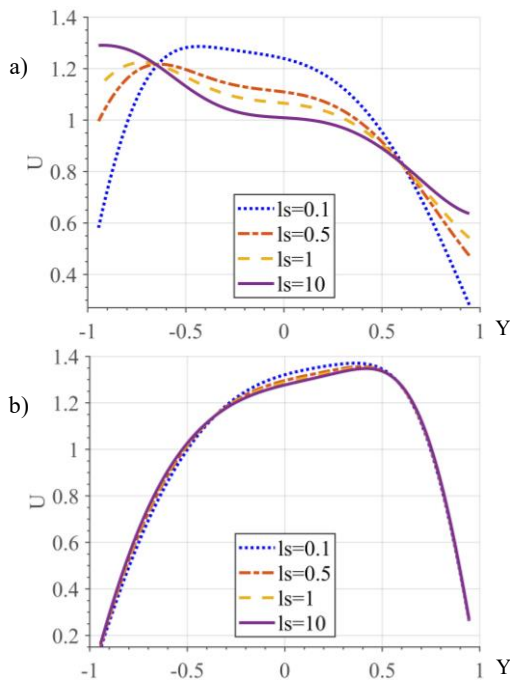


Figure 6. U vs. Y for distinct l_s values with $b = 0.125$, $k = 0.5$, $\varepsilon = 2.5$ and $\lambda = 1.250$: a) $X = -5.4$; b) $X = 5.4$.

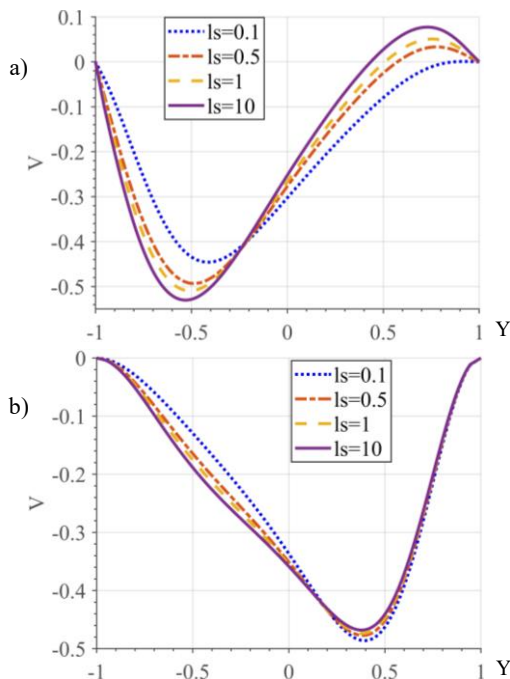


Figure 7. V vs. Y for distinct l_s values with $b = 0.125$, $k = 0.5$, $\varepsilon = 2.5$ and $\lambda = 1.250$: a) $X = -5.4$; b) $X = 5.4$.

Figure 7 indicates that at $X = -5.4$, the magnitude of the transverse velocity $|V|$ reaches its maximum below the centre-line $Y = 0$ as l_s values enhance from 0.1 to 10. On the other hand, at $X = 5.4$, the peak value of $|V|$ shifts above the centre-line as the slip length decreases from 10 to 0.1. These results highlight the strong influence of l_s value on the velocity field, especially in regions where slip effects dominate. The observed trend arises because a larger slip length enhances the velocity near the wall, and, to satisfy mass conservation, the flow adjusts through a redistribution of velocity throughout the channel cross-section.

Figure 8 illustrates the impact of l_s values on V velocity along the X -axis at $Y = 0$, with other parameters fixed at $\varepsilon = 2.5$, $b = 0.125$, $\lambda = 1.250$ and $k = 0.5$. It is observed that V velocity varies with slip length in regions where the channel surface exhibits hydrophobic behaviour, while it remains unaffected by slip length l_s in hydrophilic regions. This behaviour arises from the periodic alternation of penetration boundary conditions along the channel boundaries. The velocity field at the points where the boundary conditions change is significantly altered as the fluid adapts its movement between core flow and at circular cylindrical surfaces and then moves at the channel walls.

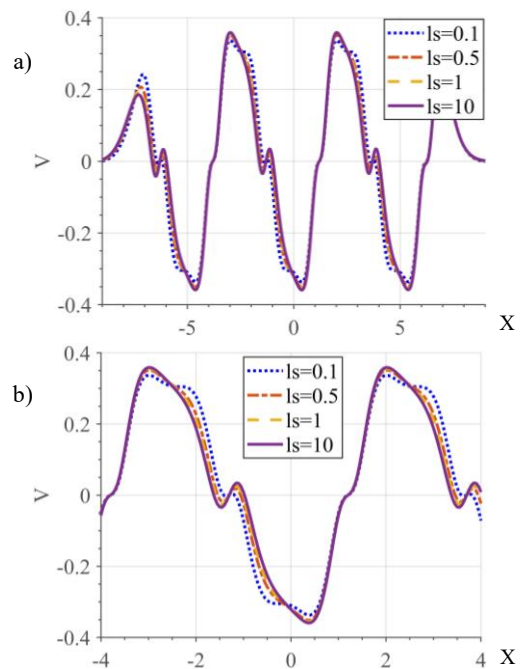


Figure 8. V vs. X for distinct l_s values at $Y = 0$ with $\varepsilon = 2.5$, $b = 0.125$, $k = 0.5$ and $\lambda = 1.250$: a) $Y = 0$; b) $Y = 0$ (zoomed-in view).

Figure 9a shows the U velocity versus X axis with respect to l_s values at $Y = 0$, when all circular cylinders of radius $b = 0.125$ are situated at the staggered positions, i.e., $k = 0.5$ in the path flow maintaining equal gap $\lambda = 1.250$ and considered length of the pattern slip regime $\varepsilon = 2.5$. We analyse that there is a significant variation in U velocity by varying l_s values on the cross sections where we have applied the penetration conditions. Further, the decrease in the U velocity observed with increasing slip length l_s at slip-treated cross-sections arises from the weakening of wall-fluid viscous coupling in the confined Stokes flow. For a fixed pres-

sure gradient, the reduced shear at the slip boundaries limits the ability of the pressure-driven flow to sustain the local streamwise motion, resulting in a localised reduction in velocity. This effect remains confined to the slip-applied regions and is further enhanced by the staggered arrangement of circular cylinders, which introduces non-uniform shear and enhanced confinement.

Next, in Fig. 9b, we present the pressure gradient versus X -axis at $Y = 0$ analysing the pressure gradient when circular cylinders are arranged in a staggered manner. In this scenario, we have considered the same parameters that we assumed in Fig. 9a. We analyse that pressure gradient shows noticeable change in the cross-sections where the penetration (slip) condition is implemented, while for the considered l_s values, there is no significant variation in pressure gradient in the no-slip regime. The localised variation in pressure gradient arises from adjustments in viscous dissipation, caused by reduced wall friction in slip-imposed regions, whereas the unchanged viscous resistance in no-slip regions leads to an almost invariant pressure gradient for all considered slip lengths.

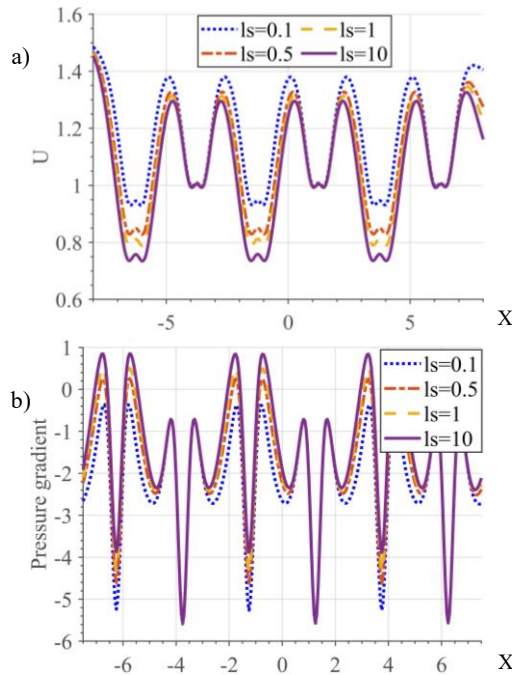


Figure 9. Influence of l_s values on: a) U profile; and b) pressure gradient, in terms of X at $Y = 0$ with $k = 0.5$, $\varepsilon = 2.5$ and $b = 0.125$.

Figure 10 presents shear stress versus X axis at $Y = 0$ when all circular cylinders of equal radius $b = 0.125$ are placed in the staggered configuration, i.e., $k = 0.5$ assuming steady values of all other parameters $\lambda = 1.250$, $\varepsilon = 2.5$. We observe that shear stress shows variation by changing l_s values at the penetration region, while at no-penetration region there are no significant variations. Moreover, Fig. 10 depicts that for maximal l_s value, the shear stress on the wall is approximately zero, which implies that the fluid flow along the channel wall will be free of resistance.

Next, Fig. 11 represents the impact of k -value on the U velocity as a function of Y with considering steady values of parameters including $\varepsilon = 2.5$, $l_s = 0.5$, $\lambda = 1.250$ and $b = 0.250$. Here, Fig. 11a depicts that U velocity is increasing

with the increase of vertical distance k , ranging from 0 to 0.50, in the domain below the centreline $Y = 0$ of the microchannel, while it drops in the domain above the centreline of the microchannel. Further, Fig. 11b demonstrates that U velocity enhances with enhanced k values in the regime above the centreline $Y = 0$ of the microchannel. Further, for $k = 0.50$ the U velocity shows maximal value in the no-penetration regime.

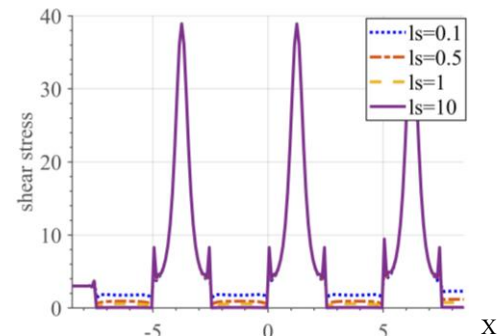


Figure 10. Shear stress vs. X for distinct l_s values at $Y = 0$ with $k = 0.5$, $b = 0.125$, $\varepsilon = 2.5$ and $\lambda = 1.250$.

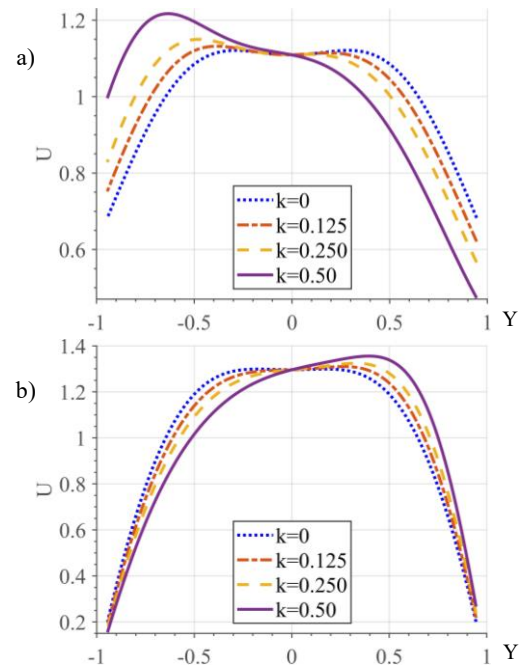


Figure 112. Impact of vertical spacing k between the circular cylinder on U velocity as a function of Y with $\varepsilon = 2.5$, $l_s = 0.5$, $\lambda = 1.250$ and $b = 0.125$: a) $X = -5.4$; b) $X = 5.4$.

CONCLUSIONS

We have investigated a 2D Stokes flow past staggered circular cylinders within a microchannel whose walls exhibit surface roughness through the alternate imposition of no-penetration and penetration boundary conditions. The boundary element method is used to solve the boundary value problem. The cylinder radius is fixed throughout the study, whereas the hydrodynamic characteristics of the flow are systematically analysed by varying two key dimensionless parameters, namely the slip length and the vertical spacing between circular cylinders. The obtained results demonstrate that the flow dynamics are strongly governed by combined

influence of slip length and vertical space of the circular cylinders. Further, the streamline contours reveal that the staggered placement of cylinders inherently distorts the flow field, leading to pronounced asymmetry in velocity distribution. This distortion becomes more prominent with increasing slip length, as enhanced wall slip weakens the wall-fluid coupling, thereby reducing local streamwise velocity in slip-treated regions. The influence of slip is further amplified by staggered geometry which modifies the interaction between adjacent cylinders and the surrounding flow.

A comparable trend is observed in the pressure gradient and wall shear stress distributions. Owing to the reduction in wall friction, variations in the pressure gradient are largely confined to slip-treated regions, whereas they remain nearly unaffected in no-slip regions across all considered slip lengths. Similarly, the wall shear stress exhibits pronounced sensitivity to slip length within slip regions, progressively decreasing and approaching zero for large slip lengths as a consequence of negligible wall resistance, while remaining almost unchanged under no-slip boundary conditions. Beyond wall slip effects, an increase in the vertical spacing between circular cylinders leads to a substantial modification of the overall flow structure. Larger cylinder spacing alleviates flow blockage and facilitates smoother fluid transport, thereby yielding higher velocity magnitudes and an asymmetric velocity distribution about the channel centreline. This influence is particularly significant under no-slip boundary conditions, where enhanced wall-fluid interaction amplifies the impact of geometric spacing on the flow field.

REFERENCES

- Ageev, A.I., Osipov, A.N. (2019), *Stokes flow in a microchannel with superhydrophobic walls*, Fluid Dyn. 54(2): 205-217. doi: 10.1134/S0015462819020010
- Datta, S., Choudhary, J.N. (2013), *Effect of hydrodynamic slippage on electro-osmotic flow in zeta potential patterned nanochannels*, Fluid Dyn. Res. 45(5): 055502. doi: 10.1088/0169-5983/45/5/055502
- Liron, N., Mochon, S. (1976), *Stokes flow for a stokeslet between two parallel flat plates*, J Eng. Math. 10(4): 287-303. doi: 10.1007/BF01535565
- Ageev, A.I., Osipov, A.N. (2021), *Pulsating flow of a viscous fluid over a cavity containing a compressible gas bubble*, Fluid Dyn. 56(6): 799-811. doi: 10.1134/S001546282106001X
- Lövgren, A.E., Maday, Y., Rönquist, E.M. (2006), *A reduced basis element method for the steady Stokes problem: Application to hierarchical flow systems*, Model. Identif. Control, 27(2): 79-94. doi: 10.4173/mic.2006.2.1
- Pozrikidis, C. (2010), *Stokes flow through a permeable tube*, Arch. Appl. Mech. 80(4): 323-333. doi: 10.1007/s00419-009-0319-9
- Canedo, E.L., Denson, C.D. (1989), *Flow in driven cavities with a free surface*, AIChE J, 35(1): 129-138. doi: 10.1002/aic.690350114
- Finn, M.D., Cox, S.M. (2001), *Stokes flow in a mixer with changing geometry*, J Eng. Math. 41(1): 75-99. doi: 10.1023/A:1011840630170
- Jeong, J.T., Jang, C.S. (2014), *Slow motion of a circular cylinder in a plane Poiseuille flow in a microchannel*, Phys. Fluids, 26(12): 123104. doi: 10.1063/1.4903477
- Jeong, J.T. (2006), *Two-dimensional Stokes flow through a slit in a microchannel with slip*, J Phys. Soc. Japan, 75(9): 7-11. doi: 10.1143/JPSJ.75.094401
- Georgiou, G., Momani, S., Crochet, M.J., Walters, K. (1991), *Newtonian and non-Newtonian flow in a channel obstructed by an antisymmetric array of cylinders*, J Non-Newton. Fluid Mech. 40(2): 231-260. doi: 10.1016/0377-0257(91)85014-A
- Sumner, D., Richards, M.D., Akosile, O.O. (2005), *Two staggered circular cylinders of equal diameter in cross-flow*, J Fluids Struct. 20(2): 255-276. doi: 10.1016/j.jfluidstructs.2004.10.006
- Navier, C.L.M.H., Darve, C. (1823), *Mémoire sur les lois mouvement des fluides*, Mem. Acad. Sci. Inst. Fr. 6: 389-440. https://www.researchgate.net/publication/246803279_Memoire_sur_les_lois_mouvement_des_fluides
- Kundt, A., Warburg, E. (1875), *Ueber Reibung und Wärmeleitung verdünnter Gase*, Ann. Phys. 231(8): 525-550. doi: 10.1002/andp.18752310803
- Maxwell, J.C. (1879), *VII. On stresses in rarified gases arising from inequalities of temperature*, Phil. Trans. R. Soc. 170: 231-256. doi: 10.1098/rstl.1879.0067
- Mooney, M. (1931), *Explicit formulas for slip and fluidity*, J Rheol. (N.Y. N.Y.), 2(2): 210-222. doi: 10.1122/1.2116364
- Bonaccorso, E., Butt, H.-J., Craig, V.S.J. (2003), *Surface roughness and hydrodynamic boundary slip of a Newtonian fluid in a completely wetting system*, Phys. Rev. Lett. 90(14): 144501. doi: 10.1103/PhysRevLett.90.144501
- Galea, T.M., Attard, P. (2004), *Molecular dynamics study of the effect of atomic roughness on the slip length at the fluid-solid boundary during shear flow*, Langmuir, 20(8): 3477-3482. doi: 10.1021/la035880k
- Hendy, S.C., Jasperse, M., Burnell, J. (2005), *Effect of patterned slip on micro- and nanofluidic flows*, Phys. Rev. E, 72(1): 016303. doi: 10.1103/PhysRevE.72.016303
- Mour, M., Das, D., Mullick, A.N. (2010), *Characteristics of fluid flow through microchannels*, AIP Conf. Proc. 1298(1): 71-79. doi: 10.1063/1.3516416
- Nizkaya, T.V., Asmolov, E.S., Vinogradova, O.I. (2013), *Flow in channels with superhydrophobic trapezoidal textures*, Soft Matter, 9(48): 11671-11679. doi: 10.1039/c3sm51850g
- Ranjith, S.K., Vedantam, S., Patnaik, B.S.V. (2015), *Hydrodynamics of flow through microchannels with hydrophobic strips*, Microfluid. Nanofluid. 19(3): 547-556. doi: 10.1007/s10404-015-1580-6
- Fornberg, B. (1980), *A numerical study of steady viscous flow past a circular cylinder*, J Fluid Mech. 98(4): 819-855. doi: 10.1017/S0022112080000419
- Drummond, J.E., Tahir, M.I. (1984), *Laminar viscous flow through regular arrays of parallel solid cylinders*, Int. J. Multiph. Flow, 10(5): 515-540. doi: 10.1016/0301-9322(84)90079-X
- Fornberg, B. (1985), *Steady viscous flow past a circular cylinder up to Reynolds number 600*, J Comput. Phys. 61(2): 297-320. doi: 10.1016/0021-9991(85)90089-0
- Bruschke, M.V., Advani, S.G. (1993), *Flow of generalized Newtonian fluids across a periodic array of cylinders*, J Rheol. 37(3): 479-498. doi: 10.1122/1.550455
- Creed, M.J., Draper, S., Nishino, T., Borthwick, A.G.L. (2017), *Flow through a very porous obstacle in a shallow channel*, Proc. R. Soc. A, 473(2200): 20160672. doi: 10.1098/rspa.2016.0672
- Canuto, D., Taira, K. (2015), *Two-dimensional compressible viscous flow around a circular cylinder*, J Fluid Mech. 785: 349-371. doi: 10.1017/jfm.2015.635
- Jeong, J.T., Yoon, S.H. (2014), *Two-dimensional Stokes flow around a circular cylinder in a microchannel*, J Mech. Sci. Technol. 28(2): 573-579. doi: 10.1007/s12206-013-1162-z
- Rajalingam, A., Chakraborty, S. (2021), *Effect of shape and arrangement of micro-structures in a microchannel heat sink on the thermo-hydraulic performance*, Appl. Therm. Eng. 190: 116755. doi: 10.1016/j.applthermaleng.2021.116755

31. Lobasov, A.S., Skripkin, S.G., Tsoi, M.A., Kravtsova, A.Y. (2022), *Investigation of the cavitation processes behind a cylinder located in a microchannel in a wide range of Reynolds numbers*, Case Stud. Therm. Eng. 38: 102306. doi: 10.1016/j.csite.2022.102306
32. Rodrigues, T., Galindo-Rosales, F.J., Campo-Deaño, L. (2020), *Haemodynamics around confined microscopic cylinders*, J Non-Newton. Fluid Mech. 286: 104406. doi: 10.1016/j.jnnfm.2020.104406
33. Chhabra, V., Nishad, C.S., Sahni, M. (2023), *Effect of magnetic field on viscous flow through composite porous channel using boundary element method*, J Appl. Comput. Mech. 9(4): 1016-1035. doi: 10.22055/jacm.2023.43024.4006
34. Chhabra, V., Nishad, C.S., Sahni, M. (2024), *Effect of magnetic field on viscous flow through porous wavy channel using boundary element method*, J Appl. Math. Mech. 104(6): e202300416. doi: 10.1002/zamm.202300416
35. Nishad, C.S., Chandra, A., Raja Sekhar, G.P. (2017), *Stokes flow inside topographically patterned microchannel using boundary element method*, Int. J Chem. React. Eng. 15(5): 1-17. doi: 10.1515/ijcre-2017-0057
36. Xu, W., Zhang, S., Ma, Y., et al. (2021), *A study on the FIV hydrodynamic force coefficients of two staggered flexible cylinders via an inverse method*, Ocean Eng. 219: 108272. doi: 10.1016/j.oceaneng.2020.108272
37. Dwivedi, K., Narayanan, S.R., Prithesh, G., et al. (2021), *Numerical investigation of the onset of three-dimensional characteristics in flow past a pair of square cylinders at various arrangements*, Fluid Dyn. Res. 53(4): 045508. doi: 10.1088/1873-7005/ac1998
38. Shao, M., Song, L., Li, P.-W. (2021), *A generalized finite difference method for solving Stokes interface problems*, Eng. Anal. Bound. Elem. 132: 50-64. doi: 10.1016/j.enganabound.2021.07.002
39. Chhabra, V., Nishad, C.S., Sahni, M. (2024), *Boundary element method for viscous flow through out-phase slip-patterned microchannel under the influence of inclined magnetic field*, Chem. Prod. Proc. Model. 19(5): 825. doi: 10.1515/cppm-2024-0065
40. Nishad, C.S., Chandra, A., Raja Sekhar, G.P. (2016), *Flows in slip-patterned micro-channels using boundary element methods*, Eng. Anal. Bound. Elem. 73: 95-102. doi: 10.1016/j.enganabound.2016.09.006
41. Tezduyar, T.E., Liou, J., Ganjoo, D.K. (1990), *Incompressible flow computations based on the vorticity-stream function and velocity-pressure formulations*, Comput. Struct. 35(4): 445-472. doi: 10.1016/0045-7949(90)90069-E
42. Chhabra, V., Nishad, C.S., Vijay, K.G., Sahni, M. (2025), *Stokes flow past circular cylinders in an out-phase slip patterned microchannel using boundary element method*, J Appl. Math. Mech. 105(2): e202400685. doi: 10.1002/zamm.202400685
43. Chhabra, V., Nishad, C.S., Vijay, K.G., Sahni, M. (2024), *Stokes flow past an array of circular cylinders through slip-patterned microchannel using boundary element method*, Eng. Anal. Bound. Elem. 169(Part A): 105925. doi: 10.1016/j.enganabound.2024.105925
44. Nishad, C.S., Chandra, A., Karmakar, T., Raja Sekhar, G.P. (2018), *A non-primitive boundary element technique for modeling flow through non-deformable porous medium using Brinkman equation*, Meccanica, 53(9): 2333-2352. doi: 10.1007/s11012-018-0832-4
45. Chhabra, V., Nishad, C.S., Sahni, M., Chaurasiya, V.K. (2024), *Effect of magnetic field and hydrodynamic slippage on electro-osmotic Brinkman flow through patterned zeta potential microchannel*, J Eng. Math. 148(1): 3. doi: 10.1007/s10665-024-10391-x
46. Chhabra, V., Nishad, C.S., Sahni, M., Chaurasiya, V.K. (2025), *Boundary element analysis for MHD Brinkman flow around circular cylinders inside a microchannel exhibiting wall roughness*, Eng. Anal. Bound. Elem. 176: 106235. doi: 10.1016/j.enganabound.2025.106235
47. Chhabra, V., Shekhar, C., Manoj, N. (2025), *Boundary element method for Stokes flow through wavy channel under the influence of inclined magnetic field*, Int. J Appl. Comput. Math. 11(6): 248. doi: 10.1007/s40819-025-02059-6
48. Chhabra, V., Nishad, C.S., Sahni, M. (2024), *Effect of inclined magnetic field on Stokes flow through in-phase slip-patterned microchannel using boundary element method*, In: Int. Conf. on Modeling, Simulation & Intelligent Computing - MoSICom 2024, pp.51-56. doi: 10.1109/MoSICom63082.2024.10880987

© 2026 The Author. Structural Integrity and Life, Published by DIVK (The Society for Structural Integrity and Life 'Prof. Dr Stojan Sedmak') (<http://divk.inovacionicentar.rs/ivk/home.html>). This is an open access article distributed under the terms and conditions of the [Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License](#)