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SUSPENDED SEDIMENT MODELLING WITH EXPONENTIAL MIXING LENGTH AND HINDERED SETTLING

MODELIRANJE SUSPENDOVANOG NANOSA SA EKSPONENCIJALNOM DUŽINOM MEŠANJA I OMETENIM TALOŽENJEM

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Keywords
- sediment transport dynamics
- open channel turbulent flow
- sediment diffusivity coefficient
- mixing length
- hindered settling velocity

Abstract

This study proposes a modified version of Hunt's model for suspended sediment concentration in turbulent open-channel flows. The classical Hunt model is enhanced by replacing its original sediment diffusivity with a modified mixing length expression $l = \kappa z e^{-z/h}$ that incorporates exponential turbulence decay with height. In addition, the hindered settling effect is introduced using the Richardson-Zaki formulation, allowing the model to better represent high-concentration sediment-laden flows. The resulting governing equation is solved numerically using a shooting method combined with the classical fourth-order Runge-Kutta scheme in Mathematica 12®. Model predictions are compared with experimental datasets from both dilute and concentrated regimes, demonstrating improved agreement over the original Hunt model, particularly in the upper flow region where traditional formulations often deviate.

Ključne reči
- dinamika transporta nanosa
- turbulentno strujanje otvorenog toka
- koeficijent difuzije nanosa
- dužina mešanja
- brzina ometenog taloženja

Izvod

U ovom radu se predlaže modifikovana verzija Huntovog modela koncentracije suspendovanog nanosa kod turbulentnog otvorenog toka. Klasični Huntov model je poboljšan zamenom originalne difuzivnosti nanosa - modifikovanim izrazom za dužinu mešanja $l = \kappa z e^{-z/h}$, gde se uvodi eksponencijalni pad turbulencije sa visinom. Pored toga, uvodi se efekat ometenog taloženja korišćenjem Ričardson-Zaki formule, čime se postiže bolji model protoka kod formiranih nanosa visoke koncentracije. Rezultujuća osnovna jednačina se rešava numerički primenom metode gađanja u kombinaciji sa klasičnom Runge-Kuta shemom u Mathematica 12®. Predviđanja modela se porede sa eksperimentalnim skupom podataka iz razblaženog i koncentrisanog režima, čime se potvrđuje bolje poklapanje sa originalnim Hunt modelom, posebno u oblasti gornjeg toka, gde često ima odstupanja u tradicionalnim izrazima.

INTRODUCTION

The transport of sediment in turbulent open-channel flow constitutes a fundamental problem in hydraulic research. Understanding these processes is critical for quantifying the interactions between fluid motion and sediment particles which directly influence erosion, deposition, channel evolution, and riverbed morphology. Studying sediment movement is also important for practical uses, such as managing floods, controlling sediment, and tracking the spread of pollutants. Therefore, comprehensive knowledge of sediment transport mechanisms in both natural and engineered channels is indispensable across disciplines such as civil engineering, river and coastal hydraulics, and environmental fluid mechanics.

In sediment-laden flows, most particles are carried in suspension, where turbulent drag lifts them from the bed and counteracts their gravitational settling through upward diffusion. This process is mathematically described by the advection-diffusion equation, from which an analytical model for suspended sediment concentration was developed by Rouse /1/. Later, Hunt /2/ proposed a two-phase flow model to predict the vertical concentration profile, assuming for simplicity that the sediment diffusion coefficient is equal to the turbulent diffusion coefficient. Einstein and Chien /3/ conducted an experimental study to investigate the effect of high sediment concentration on vertical concentration distribution and flow velocity. Building on the foundational studies of Rouse and Hunt, numerous researchers /4-8/ have explored the vertical distribution of suspended sediment concentration. In sediment-laden open-channel turbulent flow, various hydrodynamic phenomena such as mixing length, hindered settling, and stratification occur and play a crucial role. Understanding these phenomena is essential for a better understanding of suspended sediment transport, which enables more accurate computation of concentration distribution in suspension. Many researchers such as /9-17/ considered these phenomena either separately or simultaneously to compute sediment concentration distribution in suspension. However, the present study on concentration distribution incorporates the effects of mixing length and hindered settling.

Prandtl /18/ introduced the concept of mixing length theory to estimate turbulent shear stress in clear water turbulent flows. Based on the kinetic theory of gases, the mixing length l refers to the average distance a fluid parcel moves randomly before interacting or mixing with neighbouring fluid parcels, during which the fluid parcel's properties remain unchanged /19/. In the context of clear water flow, Prandtl /18/ introduced two mathematical formulations for the mixing length. One is a linear form given as $l = \kappa z$, while the other is a parabolic form expressed as $l = \kappa z\sqrt{(l - z)}$. In addition to these two expressions, many other mixing length formulations for clear water flow can be found in literature /20-23/. Furthermore, several researchers /24-26/ modified these expressions by incorporating the aforementioned linear and parabolic forms. In this work, we introduce a modified mixing length expression, $l = \kappa z e^{-z/h}$, to the best of the authors' knowledge not previously reported, to capture turbulence suppression in sediment-laden open-channel flows.

Hindered settling is another interesting and important hydrodynamic phenomenon in sediment-laden turbulent flows. Under this phenomenon, the settling velocity of particles is influenced by the neighbouring particles in the flow. Experiments reveal that when suspended sediments are dense in water, settling particles experience greater drag compared to those in clear water. As a result, the downward settling rate of a particle is reduced by the presence of other suspended particles in the flow; this phenomenon is referred to as hindered settling. Several studies /27-29/ on sediment transport have considered the hindered settling phenomenon and investigated its influence on the vertical concentration distribution. Richardson and Zaki /30/ established a correlation between sediment concentration and sediment settling velocity. The expression proposed by Richardson and Zaki /30/ is the most widely used formulation in literature to mathematically describe the hindered settling effect. Many researchers have utilised the Richardson and Zaki expression to estimate concentration distribution in suspension. Following these studies, the present work also employs this expression to determine the vertical concentration distribution in open-channel turbulent flow.

From the aforementioned literature survey, it is observed that most studies considered either linear or parabolic mixing-length formulations to determine the concentration distribution in suspension. However, the present study proposes a new mathematical expression for mixing length in exponential form. This study simultaneously considers two critical hydrodynamic phenomena, mixing length and hindered settling, to develop a steady, one-dimensional mathematical model for predicting sediment concentration distribution in suspension. For the mixing-length expressions, this study includes a newly proposed exponential formulation, and the hindered-settling expression is taken from Richardson and Zaki /30/. For validation, the numerical solution of the proposed model is compared with experimental data from Coleman /31/ (for low-concentration sediment-laden flow), Einstein and Chien /3/ (for high-concentration flow) and Lyn /32/. Furthermore, an error analysis is conducted using the root mean square error formula. Moreover, a comparative study of different forms of the mixing length, namely linear, parabolic, and the newly proposed exponential expression, is conducted.

MATHEMATICAL MODEL

The movement of suspended sediments in open-channel flow arises from the balance between gravitational settling and upward mixing caused by turbulence. For a wide range of engineering problems, this process can be reasonably represented using a simplified one-dimensional steady model, that captures the primary vertical distribution of sediment concentration. Figure 1 provides a schematic view of suspended sediment distribution in a turbulent open-channel flow. The sketch highlights the total flow depth h, the reference level a, and associated reference concentration C_a. It also depicts the suspension region, particle entrainment from the bed, and the decay of concentration toward the free surface. In this section, a mathematical formulation is developed for such a case, incorporating the effects of hindered settling velocity and mixing length.

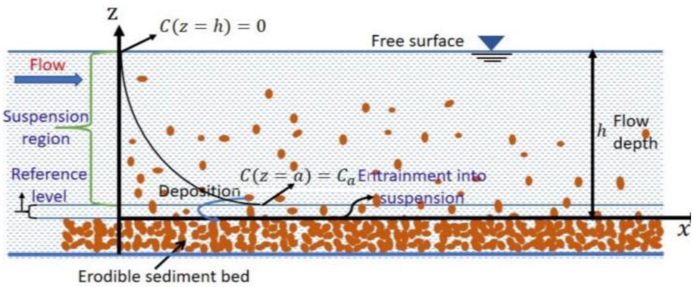


Figure 1. Illustration of the suspended sediment profile in a turbulent open channel, with flow depth h and reference concentration C_a specified at level a.

Governing equation

In steady and uniform flow conditions, the governing equation for suspended sediment concentration can be expressed as, /2/:

$$[\varepsilon_s + C(\varepsilon_t - \varepsilon_s)]\frac{dC}{dz} + C(1-C)w = 0\,, \quad (1)$$

where: C is time averaged dimensionless sediment concentration; z is vertical distance; ε_s is sediment diffusion coefficient; ε_t is turbulent diffusion coefficient; and w is settling velocity of sediment particles in clear fluid. To solve this differential equation, we need the expression of sediment diffusion coefficient, turbulent diffusion coefficient, and settling velocity.

Sediment diffusion coefficient and turbulent diffusion coefficient

The sediment diffusion coefficient ε_s is commonly related to the turbulent diffusion coefficient ε through the reciprocal of the turbulent Schmidt number β. This relationship is given as, /33/:

$$\varepsilon_s = \beta\varepsilon \quad (2)$$

Hunt /2/ assumed $\beta = 1$ to simplify the differential equation. However, several experimental studies have shown that the sediment diffusivity ε_s and the turbulent diffusivity ε are not necessarily equal under different flow conditions /34-36/. Several studies have demonstrated that sediment diffusivity ε_s and turbulent diffusivity ε are not equal under varying flow conditions /34-39/. In this study, the empirical expressions proposed by Pal and Ghoshal /36/ are considered, which are given as,

$$\beta=\begin{cases}0.033\left(\dfrac{w_0}{u_*}\right)^{0.931} C_{\hat{a}}^{-0.118}\hat{a}^{-1.196}, & \text{for dilute flow}\\[2ex] 2.2040\left(\dfrac{w_0}{u_*}\right)^{0.667} C_{\hat{a}}^{0.017}\hat{a}^{-1.178}, & \text{for non-dilute flow}\end{cases} \quad (3)$$

In turbulent flows, the Reynolds shear stress can be expressed with the Boussinesq hypothesis as, /40/,

$$\tau=\varepsilon_t\rho\frac{du}{dz},\ \ \frac{du}{dz}=\frac{u_*}{l},\ \ \tau(z)=\tau_0\left(1-\frac{z}{h}\right),\ \ \tau_0=\rho u_*^2 \cdot \quad (4)$$

Combining these relations gives the eddy diffusivity:

$$\varepsilon_t(z)=u_* l\left(1-\frac{z}{h}\right)\cdot \quad (5)$$

Finally, the sediment diffusivity follows from Eq.(2):

$$\varepsilon_s(z)=\beta u_* l\left(1-\frac{z}{h}\right)\cdot \quad (6)$$

In suspended sediment transport modelling, selecting a suitable mixing length formulation is essential for capturing turbulent diffusion. Classical Prandtl mixing lengths, whether linear or parabolic, assume uniform turbulence and are mainly applicable to clear-water flows with fully developed turbulence. In contrast, sediment-laden flows are influenced by particle-induced density stratification, which suppresses turbulence and limits vertical mixing, especially in the outer flow region. To incorporate this effect, a modified form of the mixing length is adopted:

$$l=\kappa z e^{-\frac{z}{h}} . \quad (7)$$

The detailed derivation of this expression is provided in Appendix I. It is newly introduced in this study to capture the suppression of turbulence toward the free surface in sediment-laden open-channel flows, while retaining the near-bed scaling of the classical Prandtl formulation. This expression is more consistent with experimentally observed turbulence structures and accounts for the exponential decay of turbulent kinetic energy with height. It provides a better fit for the outer region of the flow, particularly in sediment-laden environments where stratification effects are significant.

Hindered settling velocity

In sediment-laden flows, the settling velocity of particles decreases because the surrounding particles enhance the buoyancy effect. This effect, called hindered settling, is expressed by Richardson and Zaki, /30:

$$w_s=w(1-C)^{\eta_H} . \quad (8)$$

Here, the settling velocity reduction exponent η_H is dependent on the particle Reynolds number $\mathrm{Re_p}$. Several expressions for w_s are available in literature /30, 41-44/. In this study, the mathematical formulation proposed by Richardson and Zaki is adopted, given as:

$$n_H=\begin{cases}4.65, & \mathrm{Re_p}<0.2\\ 4.4\mathrm{Re_p}^{-0.03}, & 0.2<\mathrm{Re_p}<1\\ 4.4\mathrm{Re_p}^{-0.1}, & 1<\mathrm{Re_p}<500\\ 2.4, & \mathrm{Re_p}>500\end{cases} \quad (9)$$

The particle Reynolds number is defined as $\mathrm{Re_p}=u_s d_p/v_w$, where u_s is settling velocity, d_p is particle diameter, and v_w is kinematic viscosity of water. In this study, the clear-water settling velocity is estimated using Cheng's formula, /45/:

$$u_s=\left(\frac{v_w}{d_p}\right)\left(\sqrt{25+1.2D^{*2}}-5\right)^{1.5},\ \ D^*=\left[\frac{\left(\dfrac{\rho_s}{\rho_f}-1\right)g}{v_w^2}\right]^{1/3} d_p ,\quad (10)$$

where: g is gravitational acceleration; and ρ_s, ρ_f are sediment and fluid densities.

Final model equation

Substituting these expressions into the governing equation leads to the final steady 1D model:

$$[\beta+C(1-\beta)]\kappa u_* z\left(1-\frac{z}{h}\right)e^{-\frac{z}{h}}\frac{dC}{dz}+w_0(1-C)^{\eta_H+1}C=0 \cdot \quad (11)$$

Non-dimensional form is

$$[\beta+C(1-\beta)]\kappa\hat{z}(1-\hat{z})e^{-\hat{z}}\frac{dC}{d\hat{z}}+\hat{w}_0(1-C)^{\eta_H+1}C=0 , \quad (12)$$

where: non-dimensional variables are defined as $\hat{z}=z/h$ and $\hat{w}_0=w_0/u_*$. Equation (12) represents the final form of the modified Hunt model, expressed as a first-order nonlinear differential equation in non-dimensional variables. The formulation incorporates both the influence of hindered settling velocity and an exponential mixing length expression that captures turbulence decay with height. The nonlinear equation is solved using Mathematica 12® with the NDSolve routine.

RESULTS AND DISCUSSIONS

This section starts with a description about the experimental datasets used to validate the model. After that the derived concentration profiles obtained from Eq.(12) are compared with experimental datasets. The results show that the proposed model effectively reflects the vertical variation of sediment concentration under a range of flow and sediment conditions.

Comparison between present model, Eq.(12), with Rouse (1937) and Hunt (1954) model

To evaluate the accuracy and applicability of the present model in predicting suspended sediment concentration profiles, it is compared with two classical theoretical models: the Rouse- and the Hunt model. The validation is carried out using experimental data reported by Einstein and Chien /3/, Coleman /31/, and Lyn /32/.

Figure 2 shows the comparison of the present model with Rouse and Hunt models, along with experimental data from Einstein and Chien (Run-S12) and Coleman (Run-3). The dimensionless sediment concentration $C/C_{\hat{a}}$ is plotted against normalised vertical position $\hat{z}$. In both cases, the present model shows better agreement with experimental data compared to the Rouse and Hunt models. The better agreement is due to the improved mixing length formula and a more accurate treatment of sediment diffusion in the present model, which helps it match the concentration profiles more closely across the flow depth.

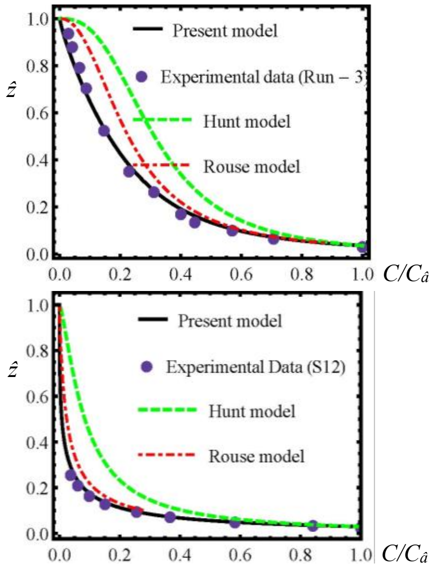


Figure 2. Comparison of the present model with the Rouse and Hunt models using experimental data from Coleman /31/ and Einstein and Chien /3/.

Variation of concentration profile with different mixing length expression

Figure 3 compares the predicted concentration profiles using various mixing length formulations for input parameters from Coleman /31/, Run-3. The results show that the proposed exponential mixing length expression yields the closest agreement with experimental data across the entire flow depth, particularly in the upper region, where conventional formulations tend to deviate. This highlights the importance of incorporating turbulence decay effects in sediment diffusivity modelling.

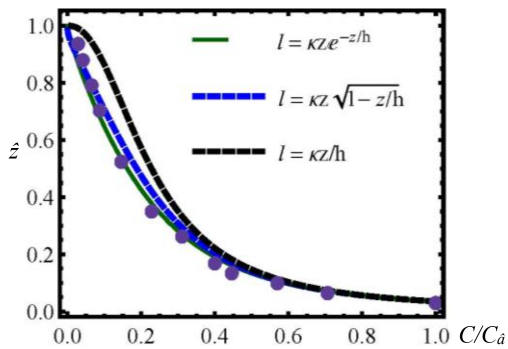


Figure 3. Concentration profiles with different mixing length expressions with the input parameters from Coleman /31/ for Run-3.

Comparison between current model solution with experimental data

Several experimental studies examined suspended sediment profiles in open-channel turbulent flows under steady and uniform conditions. Based on the same flow regime, the present study develops a one-dimensional steady model to describe the vertical sediment distribution. For validation, the model results are compared with experimental measurements reported by Einstein and Chien /3/, Coleman /31/, and Lyn /32/, covering both dilute and relatively concentrated flow cases.

Einstein and Chien /3/ carried out a series of experiments to study flow velocity and suspended sediment distribution under conditions of high near-bed sediment concentration. Tests were performed with particle sizes of D = 1.3, 0.940, and 0.274 mm, covering 16 experimental runs. Four experimental datasets are considered in this study for comparison, and the values of relevant parameters are available in Einstein and Chien /3/. As illustrated in Fig. 4, the proposed model shows good agreement with these experimental observations, thereby confirming its validity.

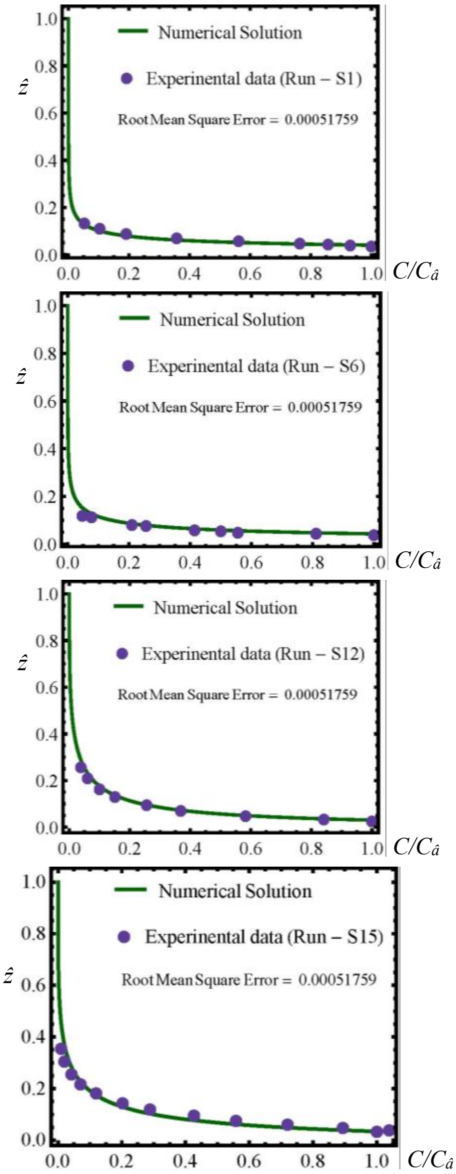


Figure 4. Comparison of computed sediment concentration profiles using Eq.(12) with experimental data from Einstein and Chien /3/ for high-concentration flow cases, demonstrating the model's performance near the bed.

Coleman /31/ performed 40 experiments in a smooth recirculating flume (15 m long and 0.356 m wide) using sediment particles of three sizes (0.105, 0.210, 0.420 mm). Four datasets are employed to validate the present model by comparing its predictions with the measured profiles. The required parameter values are available in Coleman /31/. Figures 5 show the comparison, indicating that the model captures the sediment distribution profiles well in the suspension region.

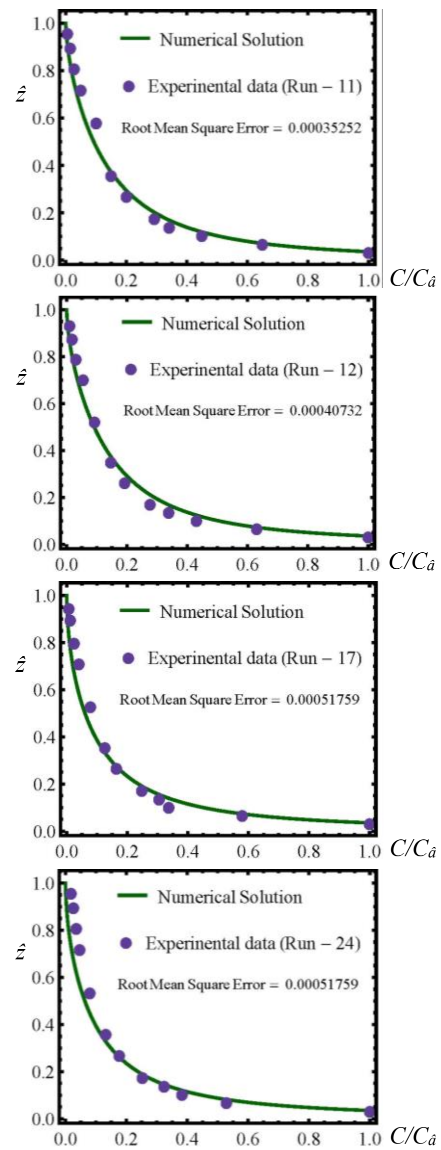
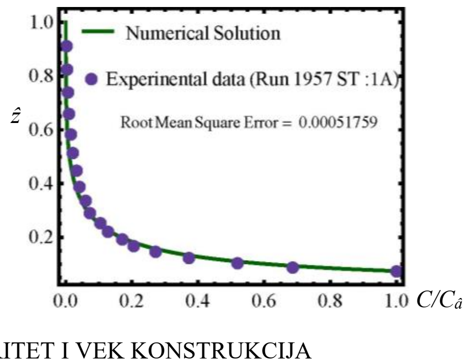


Figure 5. Sediment concentration profiles computed using Eq.(12) against experimental data from Coleman /31/ for four Runs, highlighting model performance in dilute sediment-laden flows.

Lyn /32/ conducted experiments in a 13 m long, 0.267 m wide tilting recirculating flume. The bed was coated with an epoxy layer embedded with fine sand particles $d \leq 0.15$ mm), making it not perfectly smooth. Four datasets from these experiments are used to validate the present model, as shown in Fig. 6. The comparison demonstrates good agreement between the predicted and measured concentration profiles. An error analysis is also performed for each dataset, with corresponding values displayed in Fig. 6.



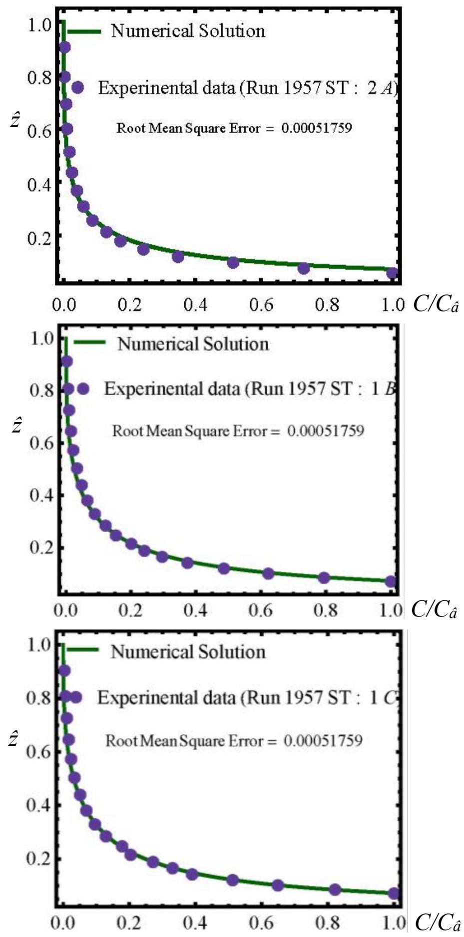


Figure 6. Comparison of predicted sediment concentration profiles from Eq.(12) with experimental data from Lyn /32/ for four runs.

Error analysis

Model performance is evaluated by comparing predicted and experimental sediment concentration profiles using the Root Mean Square Error (RMSE). RMSE measures the average difference between observed and simulated values and is given by:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(C_{i,\exp} - C_{i,\text{mod}})^2} , \qquad (13)$$

where: $C_{i,\exp}$ and $C_{i,\text{mod}}$ are the experimental and predicted concentrations at point i, and n is the number of data points. The RMSE values for different runs are presented in the figures. The results show that the model predictions are in good agreement with experimental data.

CONCLUSIONS

This work extends the Hunt model by introducing a new exponential mixing length expression, $l = \kappa z e^{-z/h}$, that represents the reduction of turbulence due to suspended particles. The approach also considers hindered settling and spatial variations in sediment diffusivity, both of which are essential for realistic simulation of sediment-laden flows. Model predictions are compared with experimental data for both low and high concentrations and show good agreement, with low RMSE values across cases.

Appendix: derivation of the modified mixing length expression

The derivation of the modified mixing length equation used in this investigation is explained in this appendix. The goal is to retain the near-bed scaling of the classical Prandtl formulation while incorporating turbulence suppression toward the free surface observed in sediment-laden and stratified open-channel flows.

Classical Prandtl form: Prandtl's original hypothesis for wall turbulence gives the linear relation

$$l = \kappa z \tag{14}$$

while appropriate in the log-layer region, this formulation tends to overpredict turbulence mixing when approaching the free surface.

Turbulence damping with height: experimental studies on open-channel flows report that turbulence intensity decays with distance from the bed, especially under conditions of sediment-induced density stratification /37, 46/. To incorporate this effect, we introduce an exponential decay factor:

$$f(z) = e^{-z/h} \,. \tag{15}$$

Modified mixing length: combining the damping function with the classical Prandtl expression, modified mixing length becomes

$$l = \kappa z e^{-z/h} \,. \tag{16}$$

This expression preserves the linear near-bed behaviour while suppressing turbulence mixing as the free surface is approached.

Remark: to the authors' knowledge, the exponential form $l = \kappa z e^{-z/h}$ has not been reported previously. It is introduced in this study as a simple, physically motivated representation of outerlayer damping in sediment-laden open-channel flows.

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