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DYNAMICS OF SH-WAVE PROPAGATION IN FIBRE-REINFORCED MEDIUM WITH POWER LAW MODEL

DINAMIKA PROSTIRANJA SH TALASA U SREDINI OJAČANOJ VLAKNIMA PO MODELU EKSPONENCIJALNE PROMENE

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- SH-waves
- fibre reinforcement
- initial stress
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- dispersion equation

Abstract

This study helps us to find the propagation behaviour of horizontally polarised shear waves (SH) within a fibre-reinforced viscoelastic composite medium subjected to initial stress. A comprehensive analytical framework is developed by emphasising a structural configuration in which the elastic moduli exhibit as a power law model with the depth parameter. A closed-form dispersion relation is rigorously derived, capturing the combined effects of viscoelasticity, fibre-reinforcement, and pre-stress. A rigorous numerical analysis is made to survey the behaviour of phase velocity (SH wave propagation). A numerical investigation is conducted to examine the behaviours of phase wave velocities, which are subsequently anticipated through graphical representations. The results not only show the significance of inhomogeneity parameters but also clarify the deviation from classical wave behaviour. The study may be helpful for the design and analysis of advanced engineering structures, seismic wave interpretation, and materials used in aerospace and geotechnical applications.

Ključne reči
- SH talasi
- ojačanje vlaknima
- inicijalni napon
- poluprostor
- fazna brzina
- jednačina disperzije

Izvod

U ovom radu izučavamo ponašanje prostiranja horizontalnih polarizovanih transverzalnih talasa (sh) unutar vlaknasto ojačane sredine viskoelastičnog kompozita, opterećena naponom. Razvijen je sveobuhvatni analitički okvir razmatranjem modela konfiguracije strukture, u kojoj moduli elastičnosti pokazuju eksponencijalnu promenu sa parametrom dubine. Relacija disperzije zatvorenog oblika je izvedena u rigoroznim uslovima, kombinovanjem uticaja viskoelastičnosti, ojačanja vlaknima i prednaponskog stanja. Izvedena je rigorozna numerička analiza kako bi se ispratilo ponašanje fazne brzine (prostiranje sh talasa). Numerički se istražuju ponašanja brzina faznih talasa, sa odgovarajućim grafičkim predstavljanjem. Rezultati ne pokazuju samo značaj nehomogenih parametara, već razjašnjavaju razliku u odnosu na klasično talasno ponašanje. Istraživanje može poslužiti za projektovanje i analizu naprednih inženjerskih konstrukcija, za interpretaciju seizmičkih talasa, i za materijale koji imaju primenu u aerokosmotehnici i geotehnici.

INTRODUCTION

The current discussion explores the nature of shear-horizontal waves (SH-waves) propagating through a layered medium, e.g., crucial for understanding the Earth's internal structure. These investigations have a very wide range of applications in geophysics, civil engineering, and material science. Fibre-reinforced composites draw significant interest in electrodynamics due to their anisotropic mechanical behaviour. These materials consist of polymer matrices embedded with highly oriented fibres typically of the same type that form an anisotropic system as long as the material remains within its elastic limit. Also, when these components bond properly, they act as a single anisotropic unit without relative displacement. Common examples include reinforced light alloys, fibreglass, and alumina composites. These materials offer an excellent platform for studying the propagation of seismic waves.

In the field of engineering, the SH-wave has vast areas of application which helps to evaluate the shear strength and stiffness of materials used in buildings, walking roads, and foundations of large infrastructures. Also, the SH-wave propagation provides important insights into the internal structure of anisotropic, composite, and viscoelastic materials, especially in advanced systems such as fibre-reinforced composites, functionally graded medium and orthotropic plates, where direction-dependent behaviour is clearly observable.

Before the twentieth century, fibre-reinforced composite natural materials like wood or bamboo served as basic reinforcements. The study of artificial reinforcement has grown rapidly since the nineteenth century. Love /1/ developed the fundamental mathematical structure of elasticity and Kumar et al. /2/ have described the important points on how materials respond when forces are applied to them.

Later, the research proceeds into the theoretical aspects and focuses on SH-waves in reinforced materials. Chattopadhyay and Chaudhury /3/ explore the transmission of SH-waves in magneto-elastic internally strengthened medium.

Chandler et al. /4/ investigate attenuation in rock layers; and Kumawat and Vishwakarma /5/ investigate the circumferential SH wave propagation in piezo-reinforced composite structure. The effect of gravity has a significant influence on SH-wave modelling, studied by Sahu et al. /6/. Vishwakarma et al. /7/ and Vishwakarma and Panigrahi /8/ have given two theories on SH wave propagation, i.e., the influence of the wave in fibre-reinforced viscoelastic medium and in cross-anisotropic medium having exponential heterogeneity. Singh et al. /9/ analysed wave displacement behaviours of SH-wave propagation in heterogeneous layers, and later by Borcherdt /10/, and Alekseev and Mikhailenko /11/ highlighted the consequence of geological factors on wave transmission. These studies show the importance of seismic wave propagation, as discussed by Dholey et al. /12/, Kaur et al. /13/, Panigrahi et al. /14/, and field observations by Baroi et al. /15/. Biot's /16/ analysis on the effects of initial stress on elastic wave propagation, together with Verma /17/ proposed early models of continuous self-reinforcement, giving an essential theoretical foundation for investigations of wave behaviour.

Shear horizontal (SH) wave propagation has been a subject of wide-ranging investigation for its wide applications in seismology, geophysics, and engineering. In several advanced materials and sensing devices, SH waves provide an essential tool for analysing mechanical behaviour, detecting defects, and understanding waves in material interactions. Earlier studies have shown that imperfect or defective interfaces significantly influence the propagation behaviour of SH waves, particularly in piezoelectric, piezomagnetic, and magneto-electro-elastic media. Li and Lee /18/ are among the first to bring attention to how a defective contact affects SH wave propagation in cylindrical piezoelectric sensors. Further contributions by Liu et al. /19/, Sun et al. /20/, Otero et al. /21/, and Nie et al. /22/, have demonstrated the crucial role of imperfect bonding, magneto-electro-elastic stiffening, and multilayer interactions in controlling dispersion behaviours and interface dynamics.

At the same time, various research work is done in viscoelastic and heterogeneous medium, which expands the theoretical foundation of SH wave propagation. Chattopadhyay et al. /23/ studied the SH-wave vibrations in irregular viscoelastic layers, while Červený /24/ analysed diffraction behaviour in anisotropic viscoelastic structures. Sahu et al. /25/ developed the dispersion relation for SH-wave propagation in non-uniform viscoelastic media resting on a semi-infinite layer. Moreover, Subhash and Gaur /26/ studied Love-wave propagation in anisotropic viscoelastic materials, giving valuable insights into surface-wave behaviour. Belfield et al. /27/ studied the use of reinforcement to improve the structural performance of elastic solids.

The propagation of seismic waves, particularly surface waves, is very important for understanding Earth's internal structure. The study of wave motion in cylindrical and layered structures has therefore received significant attention. As Kumar et al. /28/ find shear-wave characteristics in a triple-layered cylindrical system, illustrating the complexity introduced by layered geometries.

In theoretical models, the interfaces are assumed to be perfectly bonded. Because of this perfect bonding, the displacements and internal forces (stresses) pass smoothly from one layer to the other without any break or discontinuity. However, in realistic geophysical and engineering systems, layers are not always perfectly interconnected like weak interfacial bonding and microcracks. Therefore, understanding SH wave behaviour in the presence of defective or imperfect interfaces is essential for accurately finding the dynamic response of multi-layered Earth structures and in engineered materials.

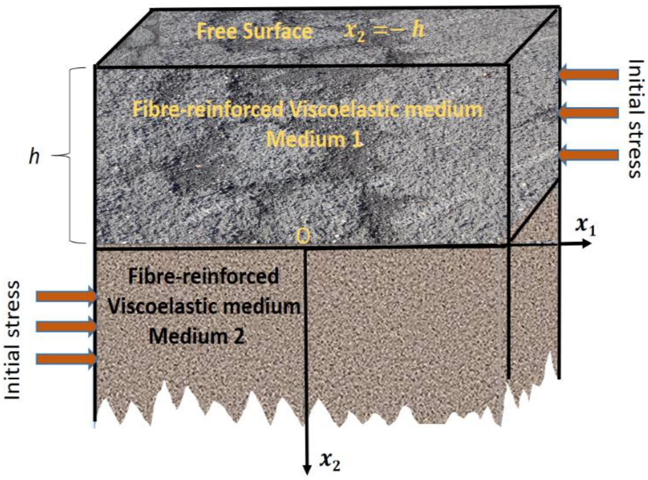

Figure 1. Geometry of the problem.

The present study investigates the propagation of surface waves in a uniformly layered, fibre-reinforced, viscoelastic medium in the presence of initial stress. Also, a two-layered medium is discussed in the following section, after which the displacement for both mediums is derived and by applying the intrinsic boundary conditions, the corresponding frequency equation is obtained. The study explores the results of parameters including, shear and viscoelasticity, material gradation, and initial stress parameters, with the help of the software MATHEMATICA®.

GEOMETRY OF THE PROBLEM

This study is based upon a two-layered fibre-reinforced medium with exponential inhomogeneity. The x_1-axis is drawn in the horizontal direction, and x_2-axis in the vertical downward direction. Upper medium is of height h, and the upper surface is stress free. Initial stress is present in both mediums.

FUNDAMENTAL EQUATIONS AND DISPLACEMENTS ACROSS THE MEDIUM

Belfield et al. /27/ provided the fundamental equation along a certain direction $\vec{a}$ for the elastic fibre-reinforced composite structure as follows:

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j) + 2(\mu_L - \mu_T)\times$$
$$\times(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j \quad (i,j,k,m=1,2,3). \quad (1)$$

where: τ_{ij} are stress components; and $e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ are strains components. Here, $\vec{a} = (a_1, a_2, a_3)$ is defined as

$a_1^2 + a_2^2 + a_3^2 = 1$. Suppose we consider $\vec{a} = (1, 0, 0)$ without restricting it to any specific case. The constants λ, μ_L, μ_T, α, β are the elastic parameters of the fibre-reinforced medium, expressed in stress units. Here, μ_L and μ_T represent longitudinal and transverse shear moduli along the preferred direction of reinforcement. Material parameters for the medium-1 (M_{l1}) and the medium-2 (M_{l2}) are denoted by:

$$\bar{\lambda}_j = \bar{\lambda}_{0j}(a^{x_2}),\ \bar{\mu}_{L_j} = \bar{\mu}_{L_{0j}}(a^{x_2}),\ \mu_{T_j} = \mu_{T_{0j}}(a^{x_2}),$$
$$\alpha_j = \alpha_{0j}(a^{x_2}),\ \beta_j = \beta_{0j}(a^{x_2}),\ \rho_j = \rho_{0j}(a^{x_2}). \quad (2)$$

Here, the subscript $j = 1, 2$ denotes the medium-1 (M_{l1}) and medium-2 (M_{l2}), respectively. The constitutive relation can be expressed as

$$\bar{\lambda}_{0j} = \lambda_{0j} + \lambda'_{0j}\frac{\partial}{\partial t},\quad \bar{\mu}_{L0j} = \mu_{L0j} + \mu'_{L0j}\frac{\partial}{\partial t},$$

where: λ_{0j}, μ_{L0j} are dimensions of stress parameter; and λ'_{0j}, μ'_{L0j} are friction of the material from inside accompanied by the elastic constants λ_{0j}, μ_{L0j}, μ_{T0j}, α_{0j}, β_{0j} with dimensions of stress; ρ_{0j} indicates the constant with dimensional form of thickness; v_j are uniformly grading parameter having dimensions equivalent to the reciprocal of the scale.

On putting Eq (2) in Eq.(1), the stress change in position is connected for the upper layer (M_{l1}) could be achieved as:

$$\begin{bmatrix}\tau_{11} & \tau_{12} & \tau_{13}\\ \tau_{21} & \tau_{22} & \tau_{23}\\ \tau_{31} & \tau_{32} & \tau_{33}\end{bmatrix} = \begin{bmatrix} \left[S_1\frac{\partial u_1}{\partial x_1} + S_1\left(\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}\right)\right]a^{x_2} & \left[S_5\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)\right]a^{x_2} & \left[S_5\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right)\right]a^{x_2}\\ \left[S_5\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)\right]a^{x_2} & \left[S_2\frac{\partial u_1}{\partial x_1}+S_3\frac{\partial u_2}{\partial x_2}+S_4\frac{\partial u_3}{\partial x_3}\right]a^{x_2} & \left[S_6\left(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\right)\right]a^{x_2}\\ \left[S_5\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right)\right]a^{x_2} & \left[S_6\left(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\right)\right]a^{x_2} & \left[S_2\frac{\partial u_1}{\partial x_1}+S_4\frac{\partial u_2}{\partial x_2}+S_3\frac{\partial u_3}{\partial x_3}\right]a^{x_2}\end{bmatrix}, \quad (3)$$

where: $S_1 = 2\alpha_{01} + \beta_{01} + \bar{\lambda}_{01} + 4\bar{\mu}_{L_{01}} - 2\mu_{T_{01}}$, $S_2 = \bar{\lambda}_{01} + \alpha_{01}$; $S_3 = \bar{\lambda}_{01} + 2\mu_{T_{01}}$; $S_4 = \bar{\lambda}_{01}$; $S_5 = \bar{\mu}_{L_{01}}$; $S_6 = \mu_{T_{01}}$, τ_{ij} $(i, j = 1, 2, 3)$ indicate stress elements of medium 1 (M_{l1}). Furthermore, the stress-displacement equation for medium 2 (M_{l2}) is

$$\begin{bmatrix}\tau'_{11} & \tau'_{12} & \tau'_{13}\\ \tau'_{21} & \tau'_{22} & \tau'_{23}\\ \tau'_{31} & \tau'_{32} & \tau'_{33}\end{bmatrix} = \begin{bmatrix} \left[T_1\frac{\partial u'_1}{\partial x'_1} + T_2\left(\frac{\partial u'_2}{\partial x'_2}+\frac{\partial u'_3}{\partial x'_3}\right)\right]a^{x_2} & \left[T_5\left(\frac{\partial u'_1}{\partial x'_2}+\frac{\partial u'_2}{\partial x'_1}\right)\right]a^{x_2} & \left[T_5\left(\frac{\partial u'_1}{\partial x'_3}+\frac{\partial u'_3}{\partial x'_1}\right)\right]a^{x_2}\\ \left[T_5\left(\frac{\partial u'_1}{\partial x'_2}+\frac{\partial u'_2}{\partial x'_1}\right)\right]a^{x_2} & \left[T_2\frac{\partial u'_1}{\partial x'_1}+T_3\frac{\partial u'_2}{\partial x'_2}+T_4\frac{\partial u'_3}{\partial x'_3}\right]a^{x_2} & \left[T_6\left(\frac{\partial u'_2}{\partial x'_3}+\frac{\partial u'_3}{\partial x'_2}\right)\right]a^{x_2}\\ \left[T_5\left(\frac{\partial u'_1}{\partial x'_3}+\frac{\partial u'_3}{\partial x'_1}\right)\right]a^{x_2} & \left[T_6\left(\frac{\partial u'_2}{\partial x'_3}+\frac{\partial u'_3}{\partial x'_2}\right)\right]a^{x_2} & \left[T_2\frac{\partial u'_1}{\partial x'_1}+T_4\frac{\partial u'_2}{\partial x'_2}+T_3\frac{\partial u'_3}{\partial x'_3}\right]a^{x_2}\end{bmatrix}, \quad (4)$$

where: $T_1 = 2\alpha_{02} + \beta_{02} + \bar{\lambda}_{02} + 4\bar{\mu}_{L_{02}} - 2\mu_{T_{02}}$, $T_2 = \bar{\lambda}_{02} + \alpha_{02}$; $T_3 = \bar{\lambda}_{02} + 2\mu_{T_{02}}$; $T_4 = \bar{\lambda}_{02}$; $T_5 = \bar{\mu}_{L_{02}}$; $T_6 = \mu_{T_{02}}$, τ'_{ij} $(i, j = 1, 2, 3)$ indicate stress elements of medium 2 (M_{l2}).

$$u_1 = u_2 = 0,\ u_3 = u_3(x_1, x_2, t)\ \text{and}\ \frac{\partial u_3}{\partial x_3} = 0. \quad (5)$$

In perspective of Eq.(5), the non-zero stress elements related with medium-1 (M_{l1}) could be achieved from Eq.(3), that is:

$$\tau_{31} = \tau_{13} = S_5\frac{\partial u_3}{\partial x_1}(a^{x_2}),\ \tau_{23} = \tau_{32} = S_6\frac{\partial u_3}{\partial x_2}(a^{x_2}). \quad (6)$$

Likewise, the non-zero stress elements for the medium-2 (M_{l2}) could be achieved from Eq.(4), that is:

$$\tau'_{31} = \tau'_{13} = S_5\frac{\partial u'_3}{\partial x_1}(a^{x_2}),\ \tau'_{23} = \tau'_{32} = S_6\frac{\partial u'_3}{\partial x_2}(a^{x_2}). \quad (7)$$

The initial stresses, P_1 and P_2, oriented along the x_1 axis, as in Fig. 1, are assumed to vary uniformly with the spatial coordinate x_2, representing a graded distribution within the medium. Accordingly, initial stress distributions in the upper and lower regions (M_{l1} and M_{l2}) are expressed as $P_{01}(a^{x_2})$ and $P_{02}(a^{x_2})$, respectively, such that:

$$P_1 = P_{01}(a^{x_2}),\ P_2 = P_{02}(a^{x_2}). \quad (8)$$

Here, P_{01} and P_{02} are constants having the same dimensional stress units. The following equation is known as the equation of motion, and is expressed as (Biot /16/)

$$\frac{\partial \tau_{31}}{\partial x_1} + \frac{\partial \tau_{32}}{\partial x_2} - P_1\frac{\partial \omega}{\partial x_1} = \rho_j\frac{\partial^2 u_3}{\partial t^2}, \quad (9)$$

where: $\omega = [(\partial u_3/\partial x_1) - (\partial u_1/\partial x_3)]/2$ is rotational component. On solving the Eqs.(5)-(8) and Eq.(9), we get

$$\left(S_5 - \frac{P_{01}}{2}\right)\frac{\partial^2 u_3}{\partial x_1^2} + S_6\frac{\partial^2 u_3}{\partial x_2^2} + S_6(\log a)\frac{\partial u_3}{\partial x_2} = \rho_{01}\frac{\partial^2 u_3}{\partial t^2}. \quad (10)$$

For solving Eq.(10), we proceed towards:

$$u_3 = W(x_2)e^{ik(x_1 - ct)}, \quad (11)$$

where: k is known as wave number; c is wave velocity; and $i = \sqrt{(-1)}$.

On putting Eq.(11) in Eq.(10), we get:

$$\frac{d^2W}{dx_2^2} + (\log a)\frac{dW}{dx_2} + k^2\frac{\mu_{L01}}{\mu_{T01}}\left\{\frac{c^2}{\beta^2} - \left(\frac{S_5}{\mu_{L01}} - \eta\right)\right\}W(x_2) = 0. \quad (12)$$

where: $\bar{\mu}_{L_{01}}$ corresponding to the term T_5 of E.(12) can be represented as: $\bar{\mu}_{L_{01}} = \mu_{L_{01}}\left(1 - \dfrac{ikc\mu'_{L_{01}}}{\mu_{L_{01}}}\right) = \mu_{L_{01}}(1 - i\gamma_1)$,

where: $\gamma_1 = kc\mu'_{L01}/\mu_{L01}$ is viscoelastic; μ_{L01} is fibre-reinforced viscoelastic layer for medium 1.

The solution of Eq.(12) is as follows:

$$u_3 = (A_1 e^{is_1x_2} + A_2 e^{-is_1x_2})e^{\frac{-p_1x_2}{2} + ik(x_1 - ct)}, \quad (13)$$

where: A_1, A_2 are arbitrary constants; and

$$p_1 = -\log a,\ s_1 = \frac{k}{2}\left[4\frac{\mu_{L01}}{T_6}\left\{\frac{c^2}{\beta^2} - \left(\frac{S_5}{\mu_{L01}} - \xi_1\right)\right\} - \frac{(\log a)^2}{k^2}\right]^{\frac{1}{2}}; \quad (14)$$

where: $\beta = (\mu_{L01}/\rho_{01})^{1/2}$ is known as phase velocity of the surface wave; and $\xi_1 = P_{01}/2\mu_{L01}$ is non-dimensional stress parameter connected with medium 1 (M_{l1}).

Now, u'_3 is the solution for the medium 2 (M_{l2}),

$$u_3' = A_3 e^{-\frac{p_2 x_2}{2} + (i s_2 x_2 + k(x_1 - ct))}, \quad (15)$$

for which A_3 is an arbitrary constant and

$$p_2 = -\log a,\ s_2 = \frac{k}{2}\left[4\frac{\mu_{L02}}{P_6}\left\{\frac{c^2}{\beta_1^2} - \left(\frac{T_5}{\mu_{L02}} - \xi_2\right)\right\} - \frac{(\log a)^2}{k^2}\right]^{\frac{1}{2}}. \quad (16)$$

With $\beta_1 = (\mu_{L02}/\rho_{02})^{1/2}$ as the phase velocity of SH-wave in medium 2 (M_{l2}) and the stress parameter is $\xi_2 = P_{02}/2\mu_{L02}$.

BOUNDARY CONDITIONS

The upper surface of medium 1 is stress free,

$$\tau_{23} = 0 \ \text{ at } \ x_2 = -h \,. \quad (17)$$

The interface between medium 1 and medium 2 will hold the following equation,

$$u_3 = u_3' \ \text{ at } \ x_2 = 0 \,. \quad (18)$$

Also, in the interface, stresses for both mediums are equal,

$$\tau_{23} = \tau_{23}' \ \text{ at } \ x_2 = 0 \,. \quad (19)$$

With the help of boundary conditions Eqs.(17)-(19) and displacement Eqs.(13) and (15), we obtain the following,

$$S_6\left(is_1 - \frac{p_1}{2}\right)A_1 e^{-is_1 h} - S_6\left(is_1 + \frac{p_1}{2}\right)A_2 e^{-is_1 h} = 0\,, \quad (20)$$

$$A_1 + A_2 = A_3\,, \quad (21)$$

$$S_6\left(is_1 - \frac{p_1}{2}\right)A_1 - S_6\left(is_1 + \frac{p_1}{2}\right)A_2 = T_6\left(is_1 - \frac{p_2}{2}\right)W_3 \,. \quad (22)$$

For non-zero solutions of A_1, A_2, A_3, we have as follows:

$$\begin{vmatrix} B_1 e^{-is_1 h} & B_2 e^{-is_1 h} & 0 \\ 1 & 1 & -1 \\ B_1 & B_2 & -B_3 \end{vmatrix} = 0 \,. \quad (23)$$

Equation (23) represents the dispersion relation that leads to:

$$i \tan s_1 h = \frac{B_3(B_2 - B_1)}{2B_1 B_2 - B_3(B_1 + B_2)} \,. \quad (24)$$

where: $B_1 = S_6(is_1 - (p_1/2))$; $B_2 = T_6(is_2 - (p_1/2))$.

Particular case

On considering the layers as isotropic, i.e., without gradedness parameters ($v_1 = v_2 = 0$), without initial stress parameter ($\xi_1 = \xi_2 = 0$) without viscoelastic parameters ($\gamma_1 = \gamma_2 = 0$) and free of reinforcement ($\mu_{L01} = \mu_{T01} = \mu_{01}$, $\mu_{L02} = \mu_{T02} = \mu_{02}$, $\alpha_{01} = \alpha_{02} = 0$, $\beta_{01} = \beta_{02} = 0$). The dispersion equation reduces to:

$$\tan\left[kh\sqrt{\frac{c^2}{\beta^2} - 1}\right] = \frac{\mu_{02}\sqrt{1 - \frac{c^2}{\beta_1^2}}}{\mu_{01}\sqrt{\frac{c^2}{\beta^2} - 1}} \,. \quad (25)$$

The resulting Eq.(25) simplifies the dispersion equation initially established by Love /1/, thereby confirming the validity of the current research.

Graphical presentation

Graphical presentations are carried out in this section. The impact of different parameters is shown in the graphs, where the non-dimensional wave number is taken against the dimensionless phase velocity. The study has been made for the inhomogeneity parameters a, γ_1, γ_2, ξ_1, and ξ_2. Here, Figs. 2-6 are plotted with the help of MATHEMATICA® to analyse the influence of different parameters on the velocity of the wave.

Figure 2 is plotted to show the influence of the inhomogeneity parameter a associated with medium 1. It also says about the velocity of the SH-wave through the two-layered medium. Here, the wave number kh, i.e. dimensionless, is taken in the horizontal direction and the velocity c/β in the vertical direction. It is clearly shown in the figure (medium 1), the velocity of the wave decreases when the value of the wave number increases.

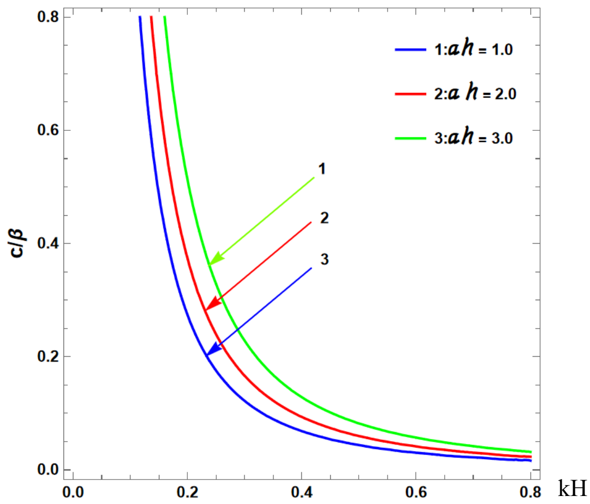

Figure 2. Effect of inhomogeneity parameter a on the phase velocity of SH wave propagation.

Again, on considering a particular wave number, i.e. 0.4, one can find the increasing nature of the curve upon increasing the value of the parameter ah = 0.1, 0.2, 0.3.

Figure 3 illustrates the effect of shear viscoelastic parameter γ_1 on the phase velocity of the surface wave, which is associated to medium 1. Here, as it is identified from the figure, the phase velocity decreases for the increasing value of the wave number. Similarly, in Fig. 4 (medium 2), the shear viscoelastic parameter γ_2 produces the same impact as medium 1.

On analysing both Fig. 3 and Fig. 4, it is clearly depicted from the figures that the influence of the parameter γ_1 for medium 1 is quite significant compared to parameter γ_2 of medium 2. In Fig. 4, the curves appear as a single line, but magnified views show that they consist of three distinct lines lying very close to each other.

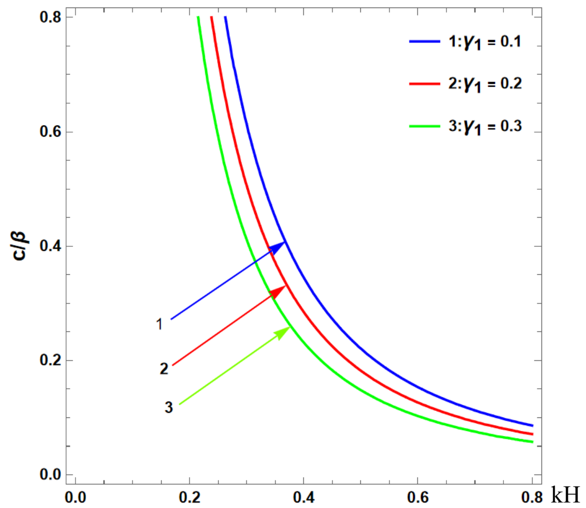

Figure 3. Effect of parameter γ_1 on the phase velocity of SH wave propagation.

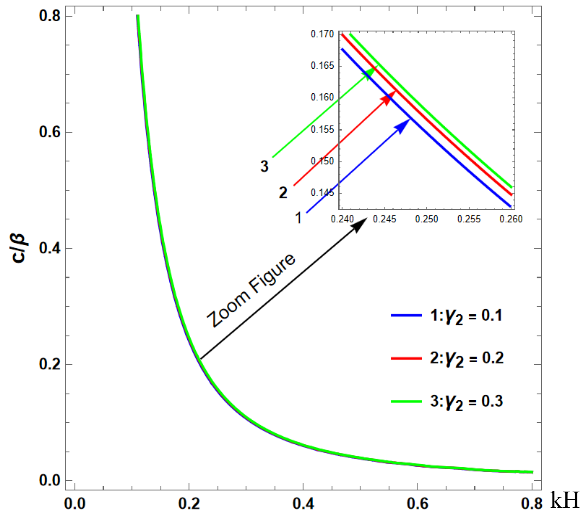

Figure 4. Effect of parameter γ_2 on the phase velocity of SH wave propagation.

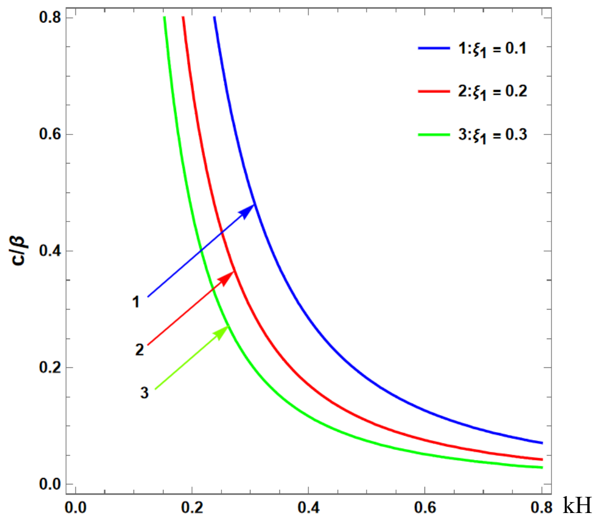

Figure 5. Effect of parameter ξ_1 on the phase velocity of SH wave propagation.

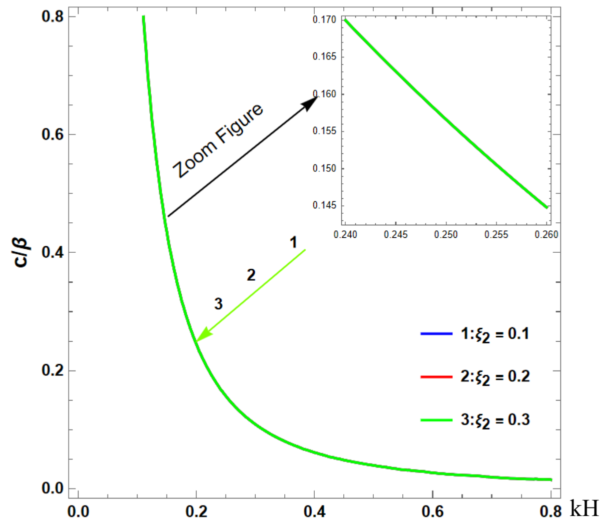

Figure 6. Effect of parameter ξ_2 on the phase velocity of SH wave propagation.

If we look at a particular wave number in both figures, then we can find that for the increasing values of the parameters, the phase velocity decreases for γ_1, whereas it increases for γ_2 (Fig. 5), which is plotted for parameter ξ_1 (medium 1) and follows the same pattern as in Fig. 3. The impact of the parameter ξ_1 (medium 1) is quite more in comparison to the parameter ξ_2 (medium 2). It has been clearly seen from Figs. 4 and 5 that the influence of parameters associated to medium 1 is quite more in comparison to medium 2. In Fig. 5, a zoomed figure is plotted to show the impact clearly, but no impact is recorded upon increasing the value of parameter ξ_2.

CONCLUSIONS

An analytical investigation is carried out to examine the transmission of SH-waves in a composite structure subjected to initial stresses in the medium 1 (M_{I1}) and in medium 2 (M_{I2}). The closed-form dispersion equation is obtained using the displacements for both layers and boundary conditions. The effects of shear viscoelastic parameters (γ_1 and γ_2), the inhomogeneity parameter a, and initial stress parameters ξ_1 and ξ_2 are illustrated through graphical representations. The principal findings of the current study are summarised as discussed below:

- The inhomogeneity parameter a exhibits a significant effect on the phase velocity of shear wave propagation. The increasing nature of the curves has been clearly visible.
- The shear viscoelastic parameters γ_1 and γ_2 exert a considerable influence on the phase velocity of surface wave propagation. The analysis is carried out for both media, i.e., for medium 1 and medium 2. It is found from the above study that the parameter γ_1 used for medium 1, has greater impact than parameter γ_2 used for medium 2.
- The initial stress parameter ξ_1 for medium 1 plays a significant role, as higher values lead to a decrease in phase velocity for a given wave number. But the parameter ξ_2 for medium 2 demonstrates no effect in phase velocity of the SH wave.

- The study may be helpful to geophysics science and engineering students in their research. Also, it may be helpful in oil exploration and damage control during earthquakes.

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