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TIME PERIOD ANALYSIS OF ORTHOTROPIC PLATE HAVING 1-D CIRCULAR POISSON'S RATIO

ANALIZA VREMENSKIH SERIJA ORTOTROPNE PLOČE SA 1-D KRUŽNIM POASONOVIM ODNOSOM

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Keywords
- vibration
- parallelogram plate
- Poisson's ratio
- circular thickness
- parabolic temperature

Abstract

In this study, authors calculate the time periods for first two modes of vibration of orthotropic parallelogram plate having 1-D (one-dimensional) circular Poisson's ratio at clamped edge condition. The authors use the assumption that the thickness valuation and temperature on the plate varies 1-D circular and bi-parabolic respectively. The frequency equation is solved using a variational method known as the Rayleigh-Ritz method. The time period of the modes of frequency is then determined from the solution of the frequency equation. A convergence study of orthotropic parallelogram is perceived at clamped edge conditions. The authors conduct a comparative review of the time period and modes of frequency of various plates, including parallelograms and rectangles, at various edge conditions with data that has been published in the literature. These days, the main aim of the researcher/scientist is to reduce the valuation in vibrational frequency of the plate so that structures made by these plates perform better, as large vibrational frequency directly affects the performance of the system. This motivates author to conduct this study. Our study demonstrates that using a variable (circular) Poisson's ratio is a better choice than varying the density parameter because the time periods obtained during circular variation in Poisson's ratio and variation in time periods and frequency modes are less than those obtained when using a circular variation in density parameter.

Ključne reči
- vibracija
- paralelogramska ploča
- Poasonov odnos
- kružna debljina
- parabolička temperatura

Izvod

U ovom radu autori proračunavaju vremenske serije za prva sva moda vibracija ortotropne paralelogramske ploče sa 1-D (jednodimenzionim) kružnim odnosom Poasona u uslovima uklještene ivice. Autori koriste pretpostavku da procenjene debljina i temperatura na ploči variraju respektivno kao 1-D kružno i bi-parabolički. Frekventna jednačina se rešava primenom varijacione metode poznate kao Rejlej-Ric metoda. Vremenska serija frekvencijskih modova se zatim određuje rešavanjem frekventne jednačine. Istraživanje konvergencije ortotropnog paralelograma se podrazumeva u uslovima uklještenja na ivicama. Izvodi se uporedni pregled vremenske serije i frekventnih modova raznih ploča, uključujući paralelograma i pravougaonika, u raznim uslovima na ivicama sa podacima koji već objavljeni u literaturi. U današnje vreme, glavni cilj istraživača je u smanjenju obima procene frekvencije vibracija ploče, tako da konstrukcije izvedene sa ovakvim pločama imaju bolje karakteristike, jer velike frekvencije vibracija direktno utiču na performanse sistema. Ovim se autor motiviše za ova istraživanja. U radu se pokazuje da primena promenljivog (kružnog) Poasonovog odnosa je pogodnije u odnosu na variranje parametra debljine, jer su vremenske serije, dobijene pri kružnoj varijaciji Poasonovog odnosa i varijacija vremenskih serija i frekventnih modova, manje u odnosu na one dobijene kružnom varijacijom parametra gustine.

INTRODUCTION

Vibration is an important consideration in engineering design, especially for machines and structures. As technology continues to progress, comprehending the vibration traits of plates with diverse parameters becomes of paramount importance. Tapered plates having consistent as well as non-uniform thickness, and subjected to fluctuations in temperature, find widespread application across various domains such as aeronautics, architecture, and underwater constructions. Researchers have studied the vibration characteristics of tapered plates with both uniform and non-uniform thickness, under different temperature conditions. There is a significant body of literature on the study of vibrational characteristics of plates.

A model is presented by Gupta et al. /1/ to analyse the vibrations of a parallelogram shaped, viscoelastic, orthotropic plate with linear thickness variations in both directions. Farag and Ashour /2/ propose an innovative adaptation of the finite element method that combines the Kantorovich technique with transition matrix methods. This modification is intended for the examination of vibration patterns in thin orthotropic skew plates. Sharma et al. /3/ explore the behaviour of a viscoelastic orthotropic parallelogram plate. They consider varying thickness and thermal effects, utilising mathematical methods to ascertain its vibration frequency. Addressing an orthotropic parallelogram plate with bi-linear thickness variation and parabolic temperature distribution in both directions at the SSSS edge condition is focused by Sharma et al. /4/. Gupta et al. /5/ investigate the influence of linear thickness variation on the vibration of a viscoelastic orthotropic parallelogram plate that has clamped boundary conditions. They employ the separation of variables method and the Rayleigh-Ritz technique. Sharma and Dhiman /6/ investigate plate vibration considering non-uniform thickness changes and temperature distribution in both directions. The use of quasi-Green function method (QGFM) in analysing the vibration characteristics of parallelogram-shaped thin plates with clamped support on a Winkler foundation presented by Li and Yuan /7/. Lather et al. /8/ employ the method of Rayleigh-Ritz to survey the time period of vibration of a 2-D circular-thickness orthotropic parallelogram plate subjected to 2-D parabolic temperature conditions. Khanna and Singhal /9/ apply the Rayleigh Ritz method to study frequency modes of a viscoelastic isotropic rectangular plate. This plate has bi-parabolic thickness and temperature variation, and it is analysed under five boundary conditions. Gupta et al. /10/ employ the Rayleigh-Ritz technique and Kelvin model to investigate vibration characteristics of a clamped rectangular plate. This plate has viscoelastic properties and exponential thickness variation in two directions. Srinivasa et al. /11/ present experimental and finite element studies on free vibration of isotropic and laminated composite skew plates, analysing the effects of skew angle, aspect ratio, fibre orientation angle, and laminate stacking sequence on natural frequencies. The study done by Rana and Robin /12/ delve into damped vibrations of an orthotropic rectangular plate resting on an elastic foundation. The plate has a thermal gradient and variable thickness. A classical plate theory and the Levy approach is utilised by Gupta /13/ to investigate the transverse motion of an elastic rectangular plate with nonlinear thickness variation and thermal gradient, subject to simple support on two parallel edges. Considering the impact of thermal gradient, Gupta et al. /14/ investigate the vibration of a non-homogeneous parallelogram plate by linearly varying thickness in both directions.

Literature shows that a negligible amount of work is reported on vibrational analysis of orthotropic plate with variable Poisson's ratio. This key point makes the authors curious about the impact of variable Poisson's ratio on the time period of orthotropic plate. To know the impact, authors investigate the influence of 1-D circular Poisson's ratio on time period of frequency modes of non-uniform orthotropic parallelogram-shaped plate at clamped edge. Additionally, the study explores the repercussions of 1-D circular variation in thickness as well as 2-D parabolic temperature changes, on the vibrational frequency modes. The outcomes are documented in Tables 1-3. To affirm the credibility and claim of the findings, the authors carried out a comparative analysis involving time periods and vibrational frequency modes. This comparison includes an orthotropic parallelogram plate under CCCC, CCCF, CFCF, CSCF, and SFSF edge conditions, where S, C, and F represent the simply supported, clamped, and free edges of the plate, respectively. It also involves a rectangular plate with CCCC edge conditions, and an orthotropic parallelogram plate with linear and parabolic thickness variations, at CCCC edge conditions. The results are then contrasted with previously published data, as indicated in Tables 6-8.

ANALYSIS

Model description

Crafted from non-uniform material properties, the orthotropic parallelogram plate possesses variable thickness denoted as l, accompanied by an inclination angle θ. The skew coordinates for the parallelogram plate are $\xi = x - y\tan\theta$ and $\eta = y\sec\theta$.

Boundaries of the plate in skew coordinates are $\xi = 0, a$ and $h = 0, b$.

The two-term deflection function that complies with all the edge conditions is presented as

$$\Phi(\xi,\eta)=\left[\left(\frac{\xi}{a}\right)^{e}\left(\frac{\eta}{b}\right)^{f}\left(1-\frac{\xi}{a}\right)^{g}\left(1-\frac{\eta}{b}\right)^{h}\right]\times \times\left[\sum_{i=0}^{n}\Omega_i\left\{\left(\frac{\xi}{a}\right)\left(\frac{\eta}{b}\right)\left(1-\frac{\xi}{a}\right)\left(1-\frac{\eta}{b}\right)\right\}^{i}\right]. \quad (1)$$

This outcome arises from the multiplication of two distinct functions. The initial function encapsulates the boundary conditions, contingent on the magnitudes of e, f, g and h. The selection of values for these parameters is contingent upon the particular support edge scenario. Notably, values of 0, 1, and 2 correspond to free edge, simply supported, and clamped edges, respectively. The secondary function pertains to the count of frequency modes, with Ω_i signifying a set of arbitrary constants for $i = 0, 1, 2, \ldots, N$.

The expressions defining the kinetic energy T_s and strain energy V_s in the context of natural transverse vibration of a non-uniform orthotropic parallelogram are derived as in /15/:

$$T_s=\frac{1}{2}\omega^2\int_0^a\int_0^b \rho l\Phi^2\cos\theta\, d\xi\, d\eta\,, \quad (2)$$

$$V_s=\frac{1}{2}\int_0^a\int_0^b\left[D_\xi\left(\frac{\partial^2\Phi}{\partial\xi^2}\right)^2+D_\eta\left(\frac{\partial^2\Phi}{\partial\xi^2}\tan^2\theta-2\frac{\partial^2\Phi}{\partial\xi\partial\eta}\tan\theta\sec\theta+\frac{\partial^2\Phi}{\partial\eta^2}\sec^2\theta\right)^2+2D_1\left(\frac{\partial^2\Phi}{\partial\xi^2}\right)\times \left(\frac{\partial^2\Phi}{\partial\xi^2}\tan^2\theta+2\frac{\partial^2\Phi}{\partial\xi\partial\eta}\tan\theta\sec\theta+\frac{\partial^2\Phi}{\partial\eta^2}\sec^2\theta\right)+4_{\xi\eta}\left(-\frac{\partial^2\Phi}{\partial\xi^2}\tan\theta+\frac{\partial^2\Phi}{\partial\xi\partial\eta}\sec\theta\right)^2\right]\cos\theta\, d\xi\, d\eta\,, \quad (3)$$

where: ρ, l represent density and thickness of the plate.

The flexural rigidities D_ξ, D_η and torsional rigidity $D_{\xi\eta}$ of the plate are taken as in /4/,

$$D_\xi = \frac{E_\xi l^3}{12(1-\nu_\xi \nu_\eta)},\ \ D_\eta = \frac{E_\eta l^3}{12(1-\nu_\xi \nu_\eta)},\ \ D_{\xi\eta} = \frac{G_\xi l^3}{12},\ \ D_1 = \nu_\xi \nu_\eta = \nu_\eta D_\xi\,, \tag{4}$$

where: ν_ξ and ν_η are Poisson's ratios; E is Young's modulus.

Assumptions required for the model

Since vibration is a broad field of research, this study requires some limitations through specific assumptions.

The thickness of the plate, l, is considered to demonstrate circular characteristics in one-dimension, which is also illustrated in Fig. 1 as:

$$l = l_0 \left[1 + \beta \left(1 - \sqrt{1 - \frac{\xi^2}{a^2}} \right) \right]. \tag{5}$$

Here, l_0 represents the plate thickness at the origin. Additionally, the parameter β (within the range $0 \le \beta \le 1$) is indicative of the tapering of the plate.

The Poisson ratio ν is considered to possess circular attributes in one dimension as:

$$\nu_\xi = \nu_{\xi_0} \left[1 - m \left(1 - \sqrt{1 - \frac{\xi^2}{a^2}} \right) \right]. \tag{6}$$

Here, $\nu_{\xi 0}$ represents Poisson's ratio at the origin. Additionally, m (satisfying $0 \le m < 1$) is indicative of the non-homogeneity of the plate.

Plate's two-dimensional steady-state temperature fluctuations are modelled in a parabolic manner Eq.(7), as previously outlined in /4/,

$$\tau = \tau_0 \left(1 - \frac{\xi^2}{a^2} \right) \left(1 - \frac{\eta^2}{b^2} \right). \tag{7}$$

Here, τ represents the temperature increase above reference temperature at any given point on the plate, while τ_0 stands for the temperature excess at the origin.

The modulus of elasticity for engineering structures is obtained from /4/, which provides a temperature-dependent relationship for this property,

$$E_\xi(\tau) = E_1(1-\gamma\tau),\ \ E_\eta(\tau) = E_2(1-\gamma\tau),\ \ G_{\xi\eta}(\tau) = G_0(1-\gamma\tau), \tag{8}$$

where: E_ξ and E_η are Young's moduli in ξ and η directions respectively; $G_{\xi\eta}$ is shear modulus and γ is taken as slope variation of moduli with temperature.

Substituting Eq.(7) in Eq.(8), we get the following expressions:

$$E_\xi(\tau) = E_1 \left[1 - \alpha \left(1 - \frac{\xi^2}{a^2} \right) \left(1 - \frac{\eta^2}{b^2} \right) \right],$$
$$E_\eta(\tau) = E_2 \left[1 - \alpha \left(1 - \frac{\xi^2}{a^2} \right) \left(1 - \frac{\eta^2}{b^2} \right) \right],$$
$$G_{\xi\eta}(\tau) = G_0 \left[1 - \alpha \left(1 - \frac{\xi^2}{a^2} \right) \left(1 - \frac{\eta^2}{b^2} \right) \right], \tag{9}$$

where: $\alpha = \gamma\tau_0$, $(0 \le \alpha < 1)$ is called temperature gradient.

Using Eqs.(5), (8), and (9) in Eq.(4), we get

$$D_\xi = \frac{E_1 h_0^3}{12(1-\nu_{\xi_0}(1-m\Upsilon_1)\nu_\eta)} \left[\left\{ 1 - \alpha \left(1 - \frac{\xi^2}{a^2} \right) \left(1 - \frac{\eta^2}{b^2} \right) \right\} \{(1+\beta\Upsilon_1)\}^3 \right],$$
$$D_\eta = \frac{E_2 h_0^3}{12(1-\nu_{\xi_0}(1-m\Upsilon_1)\nu_\eta)} \left[\left\{ 1 - \alpha \left(1 - \frac{\xi^2}{a^2} \right) \left(1 - \frac{\eta^2}{b^2} \right) \right\} \{(1+\beta\Upsilon_1)\}^3 \right],$$
$$D_{\xi\eta} = \frac{G_0 h_0^3}{12} \left[\left\{ 1 - \alpha \left(1 - \frac{\xi^2}{a^2} \right) \left(1 - \frac{\eta^2}{b^2} \right) \right\} \{(1+\beta\Upsilon_1)\}^3 \right],$$
$$D_1 = \frac{E_1 h_0^3 \nu_\eta}{12(1-\nu_{\xi_0}(1-m\Upsilon_1)\nu_\eta)} \left[\left\{ 1 - \alpha \left(1 - \frac{\xi^2}{a^2} \right) \left(1 - \frac{\eta^2}{b^2} \right) \right\} \{(1+\beta\Upsilon_1)\}^3 \right], \tag{10}$$

where: $\Upsilon_1 = 1 - \sqrt{1 - \frac{\xi^2}{a^2}}$. Now, introducing the non-dimensional variable as:

$$E_1^* = \frac{E_1}{1-\nu_{\xi_0}(1-m\Upsilon_1)\nu_\eta},\ \ E_2^* = \frac{E_2}{1-\nu_{\xi_0}(1-m\Upsilon_1)\nu_\eta},\ \ E^* = \nu_{\xi_0}(1-m\Upsilon_1)E_2^* = \nu_\eta E_1^*, \tag{11}$$

and components of E_1^*, E_2^*, E^* and G_0 are E_1^*, $E_2^* \sec\theta$, and $G_0 \sec\theta$, respectively in ξ and η directions.

Solution of model for vibrational frequency

The Rayleigh-Ritz method is used to solve the model that is based on the principle of conservation of energy, which states that the maximal kinetic energy is equal to maximal strain energy, i.e.,

$$J = \delta(V_s - T_s) = 0\,. \tag{12}$$

Substituting Eqs. (2) and (3) in Eq.(12), we get

$$J = \frac{1}{2}\int_0^a\int_0^b \Bigg[D_\xi \left(\frac{\partial^2\Phi}{\partial\xi^2} \right)^2 + D_\eta \left(\frac{\partial^2\Phi}{\partial\xi^2}\tan^2\theta - 2\frac{\partial^2\Phi}{\partial\xi\partial\eta}\tan\theta\sec\theta + \frac{\partial^2\Phi}{\partial\eta^2}\sec^2\theta \right)^2 + 2D_1 \left(\frac{\partial^2\Phi}{\partial\xi^2} \right) \left(\frac{\partial^2\Phi}{\partial\xi^2}\tan^2\theta + 2\frac{\partial^2\Phi}{\partial\xi\partial\eta}\tan\theta\sec\theta + \frac{\partial^2\Phi}{\partial\eta^2}\sec^2\theta \right) + 4D_{\xi\eta} \left(-\frac{\partial^2\Phi}{\partial\xi^2}\tan\theta + \frac{\partial^2\Phi}{\partial\xi\partial\eta}\sec\theta \right)^2 \Bigg] \cos\theta\, d\xi\, d\eta - \frac{1}{2}\omega^2 \int_0^a\int_0^b \rho l \Phi^2 \cos\theta\, d\xi\, d\eta\,. \tag{13}$$

Using Eqs. (5), (10), and (11), Eq.(13) becomes,

$$J=\int_0^a\int_0^b\left[\left\{\left\{1-\alpha\left(1-\frac{\xi^2}{a^2}\right)\left(1-\frac{\eta^2}{b^2}\right)\right\}(1+\beta\Upsilon_1)^3\right\}\left(\cos^4\theta+\frac{E_2}{E_1}\sin^4\theta+2\nu_\eta\sin^2\theta\cos^2\theta\right)\left(\frac{\partial^2\Phi}{\partial\xi^2}\right)^2+\left\{4\frac{G_0}{E_1}(1-\nu_{\xi_0}(1-m\Upsilon_1)\nu_\eta)\sin^2\theta\cos^{2q}\right\}\times\right.$$
$$\times\left(\frac{\partial^2\Phi}{\partial\xi^2}\right)^2+\frac{E_2}{E_1}\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2 4\left\{\frac{E_2}{E_1}\sin^2\theta+\frac{G_0}{E_1}(1-\nu_{\xi_0}(1-m\Upsilon_1)\nu_\eta)\cos^2\theta\right\}\left(\frac{\partial^2\Phi}{\partial\xi\partial\eta}\right)+2\left(\frac{E_2}{E_1}\sin^2\theta+\nu_\eta\cos^2\theta\right)\left(\frac{\partial^2\Phi}{\partial\xi^2}\right)\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)-$$
$$\left.-4\left(\frac{E_2}{E_1}\sin^3\theta+2\nu_\eta\sin\theta\cos^2\theta\right)\left(\frac{\partial^2\Phi}{\partial\xi^2}\right)\left(\frac{\partial^2\Phi}{\partial\xi\partial\eta}\right)+2\left\{\frac{G_0}{E_1}(1-\nu_{\xi_0}(1-m\Upsilon_1)\nu_\eta)\sin\theta\cos^2\theta\right\}\left(\frac{\partial^2\Phi}{\partial\xi^2}\right)\left(\frac{\partial^2\Phi}{\partial\xi\partial\eta}\right)-4\frac{E_2}{E_1}\sin\theta\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)\left(\frac{\partial^2\Phi}{\partial\xi\partial\eta}\right)\right|d\xi d\eta-$$
$$-\lambda^2\int_0^a\int_0^b\left[(1-m\Upsilon_1)(1+\beta\Upsilon_1)\right]\Phi^2 d\xi d\eta\,, \tag{14}$$

where: $\lambda^2 = 12\rho_0 a^2\cos^5\theta h_0^3/E_1^* h_0^2$.

In order to minimise the functional given in Eq.(14), we require the condition as follows:

$$\frac{\partial J}{\partial\Omega_i}=0,\quad i=0,1,2,\dots,N\,. \tag{15}$$

Upon solving Eq.(15), we obtain a system of equations in Ω_i that is homogeneous. The equation for the frequency is obtained by solving this system for non-zero solutions as:

$$\left|X-\lambda^2 Y\right|=0\,, \tag{16}$$

where: $X = [x_{ij}]_{N+1}$ and $Y = [y_{ij}]_{N+1}$ are square matrix of order $(n + 1)$, $i = 0,1,2,\dots, N$ and $j = 0,1,2,\dots, N$.

The time period is calculated using the following equation:

$$K=\frac{2\pi}{\lambda}\,, \tag{17}$$

where: frequency is represented by λ obtained from Eq.(16).

NUMERICAL RESULTS AND DISCUSSIONS

The time period for the first two modes of vibration of an orthotropic parallelogram plate with a fixed aspect ratio of *a/b* and skew angle $\theta = 30°$ is calculated at clamped edge for various values of plate parameters. Plate parameters include tapering parameter β, non-homogeneity m, and thermal gradient α. After reviewing, the authors assess how different plate parameters affect the time period of vibrational modes. During the calculation, the values of the following orthotropic material parameters are taken from /16/,

$$\frac{E_2^*}{E_1^*}=0.01,\quad \frac{E^*}{E_1^*}=0.3,\quad \frac{G_0}{E_1^*}=0.0333,\quad \frac{E_1^*}{\rho_0}=3.0\times10^5,$$

and $\nu_{\xi 0} = 0.345$. All the results are presented in Tables 1-3.

Table 1 delineates the time period for an orthotropic parallelogram plate for a fixed thermal gradient $\alpha = 0.2$. The non-homogeneity m varies from 0.0 to 0.8, and the tapering parameter β from 0.0 to 1.0. The following interpretations can be derived from this table.

- An increase in tapering parameter β and non-homogeneity parameter m leads to a decrease in the time period K.
- Percentage of decrement with respect to tapering is 23 % while the percentage of decrement with respect to non-homogeneity is 0.08 %.
- The rate of decrement due to the circular Poisson ratio is slow in comparison to the rate of decrement due to circular variation in thickness.

Table 2 outlines the time period K of an orthotropic parallelogram plate with a range of tapering parameter β from 0.0 to 1.0 and thermal gradient α from 0.0 to 0.8 for a constant non-homogeneity parameter $m = 0.2$. The following observations can be made based on the data.

- A significant rise in the time period K with an increasing thermal gradient α and a consequential fall in the time period K as the value of tapering parameter β increases.
- Percentage of decrement with respect to tapering is 22.7 % while the percentage increment due to thermal gradient is 37.7 %.
- The rate of decrement due to circular thickness is less in comparison to increment due to bi-parabolic variation in thermal gradient.

Table 3 portrays the time period K of an orthotropic parallelogram plate for non-homogeneity m and thermal gradient α values ranging from 0.0 to 0.8, and for variable values of tapering parameter β from 0.0 to 1.0. The following outcomes can be elucidated from Table 3.

- An increase in the value of thermal gradient α and non-homogeneity m results in increase in time period K, while an increase in tapering parameter β brings about a fall in the time period K.
- Percentage increment in time period with respect to m and α is 26.5 %, while percentage decrement in time period with respect to β is 22.7 %.
- The rate of decrement in time period due to circular variation in thickness is less in comparison to the rate of increment in time period due to circular variation in Poisson's ratio in combination with the bi-parabolic variation in temperature.

CONVERGENCE STUDY

Here, the authors present the results of a convergence study on the frequency modes λ of two types of plates:

- an orthotropic parallelogram plate with angle $\theta = 30°$, and *a/b* = 1.5 under CCCC, CSCC, CSSC, SCSS, and CCSS edge conditions, as shown in Table 4;
- an orthotropic rectangular plate *a/b* = 1.5 under the same edge conditions mentioned above, as shown in Table 5.

This study delves into the convergence patterns of modes with increasing approximation orders across a range of plate parameters. These parameters encompass values such as

$$\beta=m=\alpha=0,\quad \frac{E_2^*}{E_1^*}=0.01,\quad \frac{E^*}{E_1^*}=0.3\,,$$

$$\frac{G_0}{E_1^*}=0.0333,\quad \frac{E_1^*}{\rho_0}=3.0\cdot10^5\quad\text{and}\quad \nu_0=0.345\,.$$

The outcomes are tabulated for clear presentation. The authors' deductions highlight that, with respect to the specified edge conditions, the initial two frequency modes λ for both types of plates converge up to the fourth decimal place in the fifth-level approximation.

COMPARISON OF RESULTS

The current study compares the results of time period K and modes of frequency λ with the following previously published results.

i. The time period of an orthotropic parallelogram plate /8/ at various edge conditions (CCCC, CCCF, CFCF, CSCF and SFSF) in relation to non-homogeneity parameter m.
ii. The frequency modes of a rectangular plate /17/ with CCCC edge conditions in relation to the non-homogeneity parameter m.
iii. The frequencies of an orthotropic parallelogram plate /5, 16/ with CCCC edge conditions in relation to tapering parameter β.

Table 6 compares the time period K obtained in the current study to the time period K obtained in /8/. The comparison is made for a fixed aspect ratio, skew angle, thermal gradient, i.e., $a/b = 1.5$, $\theta = 30°$, and $\alpha = 0.4$, and varying tapering parameter β from 0.0 to 1.0 and non-homogeneity m from 0.0 to 0.8 (see Table 6) at CCCC, CCCF, CFCF, CSCF, and SFSF edge conditions. The results show that the time period K in the current study is lower than that obtained in /8/ as the tapering parameter β and non-homogeneity m increase. Additionally, the rate of change in time period K is less in the current study than obtained in /8/.

Table 7 compares modes of frequency λ in the current study (orthotropic rectangular plate) to those obtained in /17/. The comparison is made for a fixed aspect ratio skew angle and varying tapering parameter and non-homogeneity, i.e., $a/b = 1.5$, $\alpha = 0.0$, $\beta = 0.2, 0.4, 0.6, 0.8$, at CCCC edge condition are displayed in Table 7. Results show that the modes of frequency λ in the current study are higher than those in /17/ as the non-homogeneity m increases, however the rate of change in modes of frequency λ in the current study is much less than in /17/ with respect to non-homogeneity m.

Table 8 compares the modes of frequency λ in the current study (orthotropic parallelogram plate) to those obtained in /5/ and /16/. The comparison is made for three different values of skew angle and a fixed aspect ratio, thermal gradient, and a varying tapering parameter, i.e., $\theta = 0°$, 45°, and 75°; $a/b = 1.5$, $\alpha = 0.0$, and $\beta = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$, at CCCC edge condition. Non-homogeneity is not considered here because it is not a parameter in /5/ and /16/. The results show that the modes of frequency λ in the current study are lower than those in /5/ and /16/ as tapering parameter increases. Also, the rate of change in the current study is less than that obtained in /5/ and /16/.

CONCLUSIONS

The authors draw the following conclusions based on the numerical data and results comparison presented above:

- Circular variation of Poisson ratio and thickness slow down the change in time period in frequency mode.
- Bi-parabolic variation in temperature increases the change in time period of frequency mode.
- The time period, as well as variation in time period, are less in case of circular variation in the Poisson ratio (current study) in comparison to circular variation in density /8/.
- The frequency mode of rectangular plate (current study) is higher in case of circular variation in Poisson's ratio in comparison to modes obtained due to circular variation in density /17/, but the variation in frequency modes is very less due to variation in Poisson's ratio in comparison to frequency modes obtained due to circular variation in density /17/.
- Frequency modes with circular variation in thickness (current research) exhibit lower values compared to the linear and parabolic variation in /5/ and /16/. Also, the rate of change in variation obtained in frequency modes for the current study is also lower compared to the variation obtained in /5/ and /16/.

Table 1. Time period of plate vs. non-homogeneity m.

	$\alpha = 0.2$												
	m	$\beta = 0.0$		$\beta = 0.2$		$\beta = 0.4$		$\beta = 0.6$		$\beta = 0.8$		$\beta = 1.0$	
		K_2	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2	K_1
CCCC	0.0	0.40401	0.10561	0.38306	0.09994	0.36279	0.09431	0.34360	0.08883	0.32566	0.08362	0.30899	0.07870
	0.2	0.40392	0.10559	0.38296	0.09992	0.36270	0.09429	0.34350	0.08881	0.32556	0.08359	0.30889	0.07868
	0.4	0.40382	0.10558	0.38290	0.09990	0.36260	0.09427	0.34341	0.08880	0.32547	0.08357	0.30880	0.07866
	0.6	0.40376	0.10556	0.38280	0.09989	0.36254	0.09425	0.34331	0.08878	0.32538	0.08356	0.30870	0.07864
	0.8	0.40366	0.10554	0.38271	0.09987	0.36245	0.09423	0.34322	0.08876	0.32528	0.08354	0.30861	0.07862

Table 2. Time period of plate vs. thermal gradient α.

	$m = 0.2$												
		$\beta = 0.0$		$\beta = 0.2$		$\beta = 0.4$		$\beta = 0.6$		$\beta = 0.8$		$\beta = 1.0$	
	α	K_2	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2	K_1
CCCC	0.0	0.38453	0.10065	0.36559	0.09558	0.34705	0.09048	0.32933	0.08549	0.31264	0.08070	0.29705	0.07614
	0.2	0.40392	0.10559	0.38296	0.09992	0.36270	0.09429	0.34350	0.08881	0.32556	0.08359	0.30889	0.07868
	0.4	0.42663	0.11134	0.40322	0.10493	0.38086	0.09862	0.35990	0.09255	0.34046	0.08683	0.32252	0.08148
	0.6	0.45380	0.11814	0.42729	0.11076	0.40231	0.10360	0.37919	0.09680	0.35795	0.09046	0.33851	0.08460
	0.8	0.48720	0.12635	0.45660	0.11768	0.42833	0.10942	0.40247	0.10169	0.37897	0.09458	0.35761	0.08809

Table 3. Time period of plate vs. tapering parameter β.

		$m = \alpha = 0.0$		$m = \alpha = 0.2$		$m = \alpha = 0.4$		$m = \alpha = 0.6$		$m = \alpha = 0.8$	
	β	K_2	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2	K_1
CCCC	0.0	0.38463	0.10067	0.40392	0.10559	0.42654	0.11132	0.45358	0.11809	0.48682	0.12627
	0.2	0.36568	0.09559	0.38296	0.09992	0.40313	0.10491	0.42707	0.11072	0.45622	0.11760
	0.4	0.34715	0.09050	0.36270	0.09429	0.38076	0.09860	0.40209	0.10356	0.42792	0.10934
	0.6	0.32943	0.08551	0.34350	0.08881	0.35978	0.09253	0.37897	0.09676	0.40206	0.10162
	0.8	0.31273	0.08071	0.32556	0.08359	0.34033	0.08680	0.35770	0.09041	0.37856	0.09450
	1.0	0.29714	0.07616	0.30889	0.07868	0.32242	0.08146	0.33826	0.08455	0.35723	0.08802

Table 4. An analysis of mode frequencies convergence for an orthotropic parallelogram plate under various edge conditions (CCCC, CSCC, CSSC, SCSS, and CCSS).

	CCCC		CSCC		CSSC		SCSS		CCSS	
N	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
2	16.3362	62.4150	16.6464	53.5833	14.0898	78.5438	11.1085	60.6515	14.8388	71.7236
3	16.3357	61.5851	16.6426	51.4544	14.0054	54.6287	11.1066	46.6885	14.8380	49.8194
4	16.3357	61.5829	16.6408	51.3414	7.5368	14.3622	11.1064	46.5823	14.8330	49.5550
5	16.3357	61.5829	16.6408	51.3414	7.5368	14.3622	11.1064	46.5823	14.8330	49.5550

Table 5. An analysis of mode frequencies convergence for an orthotropic rectangular plate under various edge conditions (CCCC, CSCC, CSSC, SCSS, and CCSS).

	CCCC		CSCC		CSSC		SCSS		CCSS	
N	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
2	17.6582	67.6233	18.0086	56.9132	15.5601	81.6502	11.8324	65.3515	15.5601	81.6502
3	17.6577	66.6626	18.0034	55.2062	15.5306	56.5204	11.8303	50.2746	15.5306	56.5204
4	17.6577	66.6601	18.0018	55.1050	9.56743	15.9738	11.8301	50.1585	15.5306	55.8248
5	17.6577	66.6601	18.0018	55.1050	9.56743	15.9738	11.8301	50.1585	15.5306	55.8248

Table 6. Comparison of time period obtained for orthotropic parallelogram plate, from /8/, corresponding to m.

	$\alpha = 0.4$												
		$\beta = 0.0$		$\beta = 0.2$		$\beta = 0.4$		$\beta = 0.6$		$\beta = 0.8$		$\beta = 1.0$	
	m	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2
CCCC	0.0	0.11136	0.42672	0.10495	0.40332	0.09864	0.38095	0.09258	0.36000	0.08685	0.34055	0.08150	0.32264
		0.16035	**0.61629**	**0.15103**	**0.58211**	**0.14187**	**0.54947**	**0.13308**	**0.51890**	**0.12478**	**0.49053**	**0.11704**	**0.46442**
	0.2	0.11134	0.42663	0.10493	0.40322	0.09862	0.38086	0.09255	0.35990	0.08683	0.34046	0.08148	0.32252
		0.16309	**0.62562**	**0.15367**	**0.59097**	**0.14441**	**0.55792**	**0.13551**	**0.52688**	**0.12710**	**0.49816**	**0.11926**	**0.47168**
	0.4	0.11132	0.42654	0.10491	0.40313	0.09860	0.38076	0.09253	0.35978	0.08680	0.34033	0.08146	0.32242
		0.16578	**0.63479**	**0.15626**	**0.59970**	**0.14690**	**0.56621**	**0.13789**	**0.53479**	**0.12938**	**0.50570**	**0.12144**	**0.47884**
	0.6	0.11130	0.42644	0.10489	0.40304	0.09858	0.38067	0.09251	0.35968	0.08679	0.34024	0.08144	0.32233
		0.16842	**0.64384**	**0.15881**	**0.60837**	**0.14935**	**0.57444**	**0.14023**	**0.54259**	**0.13161**	**0.51309**	**0.12357**	**0.48588**
	0.8	0.11128	0.42635	0.10487	0.40291	0.09855	0.38054	0.09249	0.35959	0.08676	0.34014	0.08142	0.32220
		0.17102	**0.65276**	**0.16132**	**0.61685**	**0.15175**	**0.58248**	**0.14253**	**0.55025**	**0.13381**	**0.52034**	**0.12566**	**0.49279**
CCCF	0.0	0.24978	0.65468	0.24155	0.57067	0.23313	0.50024	0.22461	0.44227	0.21602	0.39477	0.20743	0.35579
		0.18087	**0.58993**	**0.17306**	**0.56084**	**0.16515**	**0.53244**	**0.15729**	**0.50517**	**0.14961**	**0.47935**	**0.14221**	**0.45516**
	0.2	0.24966	0.65392	0.24142	0.57007	0.23300	0.49977	0.22446	0.44190	0.21587	0.39446	0.20727	0.35553
		0.18375	**0.59876**	**0.17587**	**0.56935**	**0.16788**	**0.54054**	**0.15993**	**0.51293**	**0.15215**	**0.48676**	**0.14465**	**0.46219**
	0.4	0.24954	0.65317	0.24130	0.56945	0.23286	0.49926	0.22432	0.44149	0.21572	0.39415	0.20712	0.35528
		0.18659	**0.60749**	**0.17863**	**0.57771**	**0.17055**	**0.54855**	**0.16251**	**0.52056**	**0.15465**	**0.49408**	**0.14706**	**0.46917**
	0.6	0.24942	0.65242	0.24117	0.56885	0.23273	0.49879	0.22418	0.44111	0.21557	0.39386	0.20696	0.3503
		0.18938	**0.61607**	**0.18135**	**0.58591**	**0.17318**	**0.55644**	**0.16506**	**0.52810**	**0.15711**	**0.50124**	**0.14942**	**0.47605**
	0.8	0.24931	0.65169	0.24104	0.56825	0.23259	0.49832	0.22403	0.44074	0.21542	0.39355	0.20680	0.35478
		0.19213	**0.62452**	**0.18403**	**0.59405**	**0.17579**	**0.56417**	**0.16757**	**0.53552**	**0.15952**	**0.50834**	**0.15175**	**0.48283**
CFCF	0.0	0.24996	0.54623	0.23719	0.50407	0.22440	0.46527	0.21185	0.43043	0.19973	0.39958	0.18818	0.37237
		0.35632	**0.77579**	**0.33782**	**0.71735**	**0.31934**	**0.66332**	**0.30127**	**0.61462**	**0.28383**	**0.57124**	**0.26727**	**0.53285**
	0.2	0.24988	0.54617	0.23710	0.50401	0.22432	0.46521	0.21177	0.43037	0.19964	0.39952	0.18809	0.37228
		0.36163	**0.78754**	**0.34291**	**0.72835**	**0.32421**	**0.67356**	**0.30590**	**0.62420**	**0.28826**	**0.58016**	**0.27148**	**0.54123**
	0.4	0.24979	0.54607	0.23701	0.50391	0.22423	0.46511	0.21167	0.43030	0.19955	0.39942	0.18801	0.37222
		0.36684	**0.79916**	**0.34790**	**0.73916**	**0.32899**	**0.68367**	**0.31047**	**0.63360**	**0.29261**	**0.58899**	**0.27563**	**0.54953**
	0.6	0.24971	0.54601	0.23693	0.50385	0.22414	0.46505	0.21159	0.43021	0.19947	0.39936	0.18792	0.37215
		0.37200	**0.81060**	**0.35286**	**0.74984**	**0.33370**	**0.69360**	**0.31498**	**0.64290**	**0.29691**	**0.59766**	**0.27972**	**0.55760**
	0.8	0.24962	0.54592	0.23684	0.50376	0.22405	0.46496	0.21150	0.43015	0.19938	0.39930	0.18783	0.37209
		0.37705	**0.82184**	**0.35770**	**0.76039**	**0.33838**	**0.70347**	**0.31941**	**0.65201**	**0.30113**	**0.60627**	**0.28374**	**0.56565**
CSCF	0.0	0.25777	0.54010	0.24761	0.49157	0.23703	0.44749	0.22622	0.40872	0.21532	0.37520	0.20450	0.34642
		0.36524	**0.75543**	**0.35019**	**0.69140**	**0.33461**	**0.63272**	**0.31875**	**0.58060**	**0.30289**	**0.53511**	**0.28729**	**0.49568**
	0.2	0.25766	0.53992	0.24750	0.49141	0.23692	0.44733	0.22609	0.40857	0.21520	0.37508	0.20438	0.34633
		0.37061	**0.76699**	**0.35541**	**0.70202**	**0.33964**	**0.64252**	**0.32358**	**0.58971**	**0.30755**	**0.54353**	**0.29174**	**0.50354**
	0.4	0.25755	0.53973	0.24739	0.49122	0.23680	0.44718	0.22597	0.40844	0.21507	0.37495	0.20425	0.34624
		0.37589	**0.77830**	**0.36053**	**0.71251**	**0.34460**	**0.65226**	**0.32836**	**0.59863**	**0.31213**	**0.55182**	**0.29614**	**0.51130**
	0.6	0.25745	0.53954	0.24728	0.49106	0.23668	0.44702	0.22585	0.40831	0.21495	0.37482	0.20413	0.34611

		0.38114	**0.78948**	**0.36559**	**0.72288**	**0.34947**	**0.66175**	**0.33307**	**0.60749**	**0.31664**	**0.55999**	**0.30048**	**0.51890**
	0.8	0.25735	0.53935	0.24716	0.49088	0.23657	0.44686	0.22573	0.40816	0.21482	0.37470	0.20400	0.34602
		0.38629	**0.8005**	**0.37058**	**0.73306**	**0.35428**	**0.67117**	**0.33769**	**0.61619**	**0.32110**	**0.56810**	**0.30474**	**0.52641**
SFSF	0.0	0.20138	1.11710	0.18698	1.06710	0.17313	1.01960	0.16018	0.97503	0.14829	0.93331	0.13750	0.89441
		0.28765	**1.60250**	**0.26692**	**1.53230**	**0.24702**	**1.46560**	**0.22844**	**1.40270**	**0.21140**	**1.34380**	**0.19595**	**1.28860**
	0.2	0.20133	1.11700	0.18693	1.06700	0.17308	1.01950	0.16013	0.97490	0.14824	0.93321	0.13745	0.89432
		0.29229	**1.62820**	**0.27132**	**1.55740**	**0.25116**	**1.48990**	**0.23233**	**1.42630**	**0.21506**	**1.36660**	**0.19938**	**1.31070**
	0.4	0.20128	1.11690	0.18687	1.06690	0.17303	1.01940	0.16008	0.97481	0.14819	0.93312	0.13740	0.89423
		0.29688	**1.65360**	**0.27564**	**1.58200**	**0.25524**	**1.51380**	**0.23617**	**1.44950**	**0.21866**	**1.38910**	**0.20277**	**1.33250**
	0.6	0.20122	1.11680	0.18682	1.06680	0.17297	1.01930	0.16003	0.97471	0.14814	0.93302	0.13735	0.89413
		0.30138	**1.67860**	**0.27990**	**1.60640**	**0.25924**	**1.53740**	**0.23992**	**1.47230**	**0.22219**	**1.41120**	**0.20609**	**1.35400**
	0.8	0.20117	1.11670	0.18676	1.06670	0.17292	1.01920	0.15998	0.97462	0.14809	0.93293	0.13730	0.89401
		0.30581	**1.70320**	**0.28409**	**1.63020**	**0.26319**	**1.56060**	**0.24364**	**1.49480**	**0.22567**	**1.43300**	**0.20935**	**1.37510**

Bold values are from /8/

Table 7. Comparison of frequency obtained for rectangular plate and obtained in /17/ corresponding to *m*.

	$\alpha = 0.0$						
	m	$\beta = 0.2$		$\beta = 0.4$		$\beta = 0.6$	
		λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
CCCC	0.0	18.58210	71.26085	19.58386	75.32703	20.64751	79.78275
		17.69009	**70.95179**	**18.46363**	**74.00829**	**19.29844**	**77.30619**
	0.2	18.58478	71.95179	19.58693	75.33397	20.65099	79.79032
		17.41533	**69.73072**	**18.18466**	**72.70806**	**19.00451**	**75.92179**
	0.4	18.58745	71.27348	19.59001	75.34091	20.65447	79.79788
		17.16211	**68.57137**	**17.91795**	**71.47497**	**18.72359**	**74.61030**
	0.6	18.59012	71.27979	19.59308	75.34785	20.65795	79.80545
		16.91961	**67.46866**	**17.66263**	**70.30338**	**18.45475**	**73.36546**
	0.8	18.59280	71.28610	19.59615	75.35479	20.66143	79.81301
		16.68709	**66.41808**	**17.41791**	**69.18828**	**18.19716**	**72.18177**

Bold values are from /17/

Table 8. Comparison of frequency obtained for orthotropic parallelogram plate and obtained in /5, 16/ corresponding to β.

	$\alpha = 0.0$						
		$\theta = 0°$		$\theta = 45°$		$\theta = 75°$	
	β	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
CCCC	0.0	12.29991	47.29683	10.20204	38.96943	8.236021	31.23626
		12.29991(a)	47.29683(a)	10.20204(a)	38.96943(a)	8.236021(a)	31.23626(a)
		12.29991(b)	47.29683(b)	10.20204(b)	38.96943(b)	8.236021(b)	31.23625(b)
	0.2	12.95164	49.86952	10.72632	41.00294	8.605456	32.57915
		13.59406(a)	52.30063(a)	11.27253(a)	43.07660(a)	9.090629(a)	34.47687(a)
		13.30670(b)	51.26746(b)	11.01818(b)	42.13835(b)	8.833342(b)	33.43458(b)
	0.4	13.65895	52.74715	11.29492	43.27130	9.004677	34.05189
		14.99186(a)	57.75569(a)	12.42439(a)	47.53016(a)	9.995941(a)	37.91090(a)
		14.42099(b)	55.79978(b)	11.92000(b)	45.74141(b)	9.489102(b)	35.87987(b)
	0.6	14.41051	55.90142	11.89916	45.75380	9.429454	35.64523
		16.46323(a)	63.54685(a)	13.63427(a)	52.24230(a)	10.93839(a)	41.49061(a)
		15.61409(b)	60.80032(b)	12.88514(b)	{49.70862(b)	10.19108(b)	38.53703(b)
	0.8	15.19713	59.30373	12.53177	48.42933	9.875870	37.34955
		17.98677(a)	69.59111(a)	14.88549(a)	57.14975(a)	11.90823(a)	45.18163(a)
		16.86533(b)	66.18671(b)	13.89720(b)	53.97782(b)	10.92928(b)	41.37430(b)
	1.0	16.01172	62.92669	13.18710	51.27748	10.34049	39.15525
		19.54777(a)	75.82864(a)	16.16656(a)	62.20682(a)	12.89853(a)	48.95898(a)
		18.16060(b)	71.88982(b)	14.94474(b)	58.49637(b)	11.69597(b)	44.36403(b)

Values (a) are from /5/
Values (b) are from /16/

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