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EFFECT OF MAGNETIC FIELD ON THE ONSET OF CONVECTION IN A ROTATING JEFFREY NANOFLUID USING THE BRINKMAN MODEL FOR FREE-FREE, RIGID-RIGID AND RIGID-FREE BOUNDARY CONDITIONS

UTICAJ MAGNETNOG POLJA NA POJAVU KONVEKCIJE U ROTIRAJUĆEM JEFFREY NANOFLUIDU PRIMENOM BRINKMAN MODELA ZA GRANIČNE USLOVE: SLOBODNO-SLOBODNO, KRUTO-KRUTO I KRUTO-SLOBODNO

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Abstract

This paper investigates the impact of rotation and magnetic field on a Jeffrey nanofluid flow in a porous medium heated from below. We use the Brinkman model for the porous medium. In the Jeffrey nanofluid, the impacts of thermophoresis and Brownian motion are considered. Three boundary conditions - free-free, rigid-rigid, and rigid-free are investigated for stationary convection. The effects of the Darcy Brinkman number, porosity, Jeffrey parameter, Lewis number, nanoparticle Rayleigh number, modified diffusivity ratio, Chandrasekhar number, and Taylor number for all the above-mentioned boundary conditions are investigated analytically and graphically. The outcomes of the magnetic field are examined with consideration to how it can change the flow and heat transfer through the porous medium. In addition, here the system is considered to gain a better understanding of the connection between magnetic field effects and rotation on thermal instability.

Izvod

U radu se istražuje uticaj rotacije i magnetnog polja na protok Jeffrey nanofluida u poroznoj sredini, koja se zagreva odozdo. Koristimo Brinkman model za poroznu sredinu. U Jeffrey nanofluidu razmatramo uticaje termoforeze i Braunovog kretanja. Posmatramo tri granična uslova za stacionarnu konvekciju: slobodno-slobodno, kruto-kruto i kruto-slobodno. Za sve gore navedene granične uslove su analitički i grafički protumačeni uticaji Darsi Brinkman broja, poroznosti, Jeffrey parametra, Luisovog broja, Rejlejevog broja nanočestice, modifikovanog odnosa difuzivnosti, Čandrasekarovog broja i Tejlorovog broja. Istražene su posledice magnetnog polja, razmatranjem kako ono može da izmeni protok i prostiranje toplote u poroznoj sredini. Štaviše, u ovom sistemu se postiže bolje razumevanje povezanosti uticaja magnetnog polja i rotacije na termičku nestabilnost.

INTRODUCTION

When a fluid is heated from below, it generally causes thermal instability because the lighter liquid at the bottom rises to the surface while the heavier, colder liquid from the upper layer falls. Rayleigh /**Error! Reference source not found.**8/ studied the Bénard problem mathematically for the first time. The thermal instability of a Newtonian fluid is studied by Chandrashekar /4/ under several hydrodynamic and hydromagnetic assumptions. Nanofluids are mixtures of basic fluids such as water, ethylene glycol and other coolants, oil and other lubricants, bio-fluids and polymer solutions, etc. The term ‘nanofluid’ was first utilised by Choi /5/. Buongiorno /2/ provided an extensive overview of convective transport in nanofluids. Nanofluids have unique properties that make them potentially useful in many applications in heat transfer, including microelectronics, pharmaceutical processes, fuel cells and hybrid-powered engines, domestic refrigerators, chillers, nuclear reactors, heat exchangers in grinding, machining, in space, defence and ships and in boiler flue gas temperature reduction. Tzou /33-34/, Rana and Gautam /15/, and Chand and Rana /3/ studied the thermal instability problems of nanofluid. The nanofluid has a higher convective heat transfer coefficient and better thermal conductivity than the base fluid. There are various types of non-Newtonian nanofluids; one type that has caught the interest of various researchers is the Jeffrey fluid model. Non-Newtonian fluids are employed in many scientific and engineering fields, including the chemical and biological industries, food processing, textiles and geophysics. Jeffrey fluid is a fluid with high shear viscosity and linear viscoelasticity properties. Jeffrey's fluid model is less time derivative rather than convective derivative. The study of flow through porous layers has various applications in petroleum reservoirs, Earth’s molten cores, fluid filters, heat exchangers and human lungs, etc. Porous media improve heat conductivity by increasing the contact area between liquid, solid and nanofluids. Wooding /35/ investigates the Rayleigh instability in a thermal boundary layer flow via a porous medium. A thorough examination of convection in a porous media is given by Nield and Bejan /11/. Thermal instability in a porous medium takes place in numerous fields. It has numerous uses in the fields of geophysics, food processing, modelling oil reservoirs, thermal insulation development and nuclear reactors. Thermal instability in a horizontal nanofluid layer in a porous medium is investigated by Nield and Kuznetsov /12-13/ and then by the Darcy model. Kuznetsov and Nield /10/ studied the same by Brinkman model. Yadav et al. /36-38/ provide much more investigation on the start of nanofluid convection due to an applied magnetic field. Kumar et al. /8-9/ examined the effect of magnetic field in Jeffrey nanofluid for distinct boundary conditions and conclude that the parameter of magnetic field shows a stabilising effect. The current investigation is motivated by the growing number of fields in which nanofluids are being used. The impact of suspended particles on thermal convection in a Darcy-Brinkman porous medium using Rivlin-Ericksen fluid is investigated by Rana and Thakur /17/. The magnetic field plays an important role in the Rayleigh-Bénard convection in a layer of nanofluid and finds applications in biomedical engineering such as power plant cooling systems, MRI, plethora of engineering power plant cooling systems as well as computers. Zin et al. /39/ and Raju and Ojjela /14/ studied a magnetic field effect on the flow of Jeffrey nanofluid under various aspects. Nowadays there are numerous interests in the study of rotating fluids. Because of rotation, the momentum equation includes a Coriolis force term, which generates one non-dimensional rotation parameter called the Taylor number. Aggarwal /1/ looked at how rotation affects thermosolutal Rivlin-Ericksen fluid's convection that is permeable with suspended particles in a porous media whereas magnetic convection within a layer of nanofluid is studied by Gupta et al. /7/. Govender /6/ and Rana /16/ looked at the thermal instability of a rotating vertical porous layer that is saturated with a nanofluid. Sharma et al. /19-31/ and Sheu /32/ examine the various problems on thermal instability as well as thermosolutal convection in nanofluid with porous medium.

MATHEMATICAL MODEL

Consider a porous layer of material enclosed between two planes $z^* = 0$ and $z^* = H$. The fluid layer receives heat from below and moves upwards with a gravitational force $g = (0, 0, -g)$. Let us take porosity ε, magnetic field $h = (0, 0, 1)$, permeability K, angular velocity $\Omega = (0, 0, \Omega)$ and hydrostatic pressure p. The temperature and volumetric fraction at the lower wall be T_h^* and ϕ_0^*, while at the upper wall are T_c^* and ϕ_1^*, respectively.

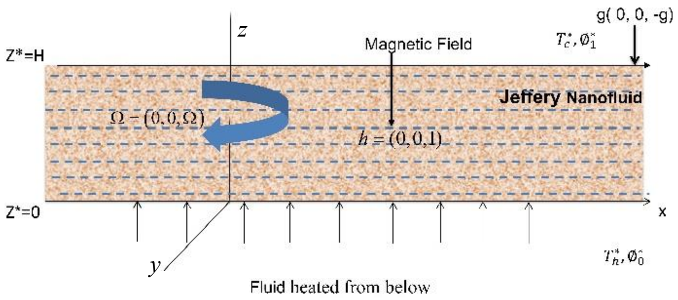

Figure 1. Physical configuration.

GOVERNING EQUATIONS

The equations for mass, momentum, thermal energy and nanoparticles are respectively given by Buongiorno /2/, Sheu /32/, and Rana and Gautam /15/,

$$\nabla^* . \mathbf{v}_D^* = 0 , \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\frac{\partial \mathbf{v}_D^*}{\partial t} = -\nabla^* p^* + \tilde{\mu}\nabla^{*2}\mathbf{v}_D^* - \frac{\mu}{K(1+\lambda)}\mathbf{v}_D^* + \frac{\mu_e}{4\pi}(h^*\nabla^*)h^* + \\ + \frac{\rho_f}{\varepsilon}(\mathbf{v}_D^* \times \Omega) + \left[\phi^*\rho_p + (1-\phi^*)\{\rho_f(1-\beta(T^*-T_c^*))\}\right]g , \tag{2}$$

$$(\rho c)_m \frac{\partial T^*}{\partial t^*} + (\rho c)_f \mathbf{v}_D^* . \nabla^* T^* = k_m \nabla^{*2} T^* + \varepsilon(\rho c)_p \times \\ \times\left[D_B \nabla^*\phi^* . \nabla^* T^* + (D_T / T_c^*)\nabla^* T^* . \nabla^* T^*\right], \tag{3}$$

$$\frac{\partial \phi^*}{\partial t^*} + \frac{1}{\varepsilon}\mathbf{v}_D^* . \nabla^*\phi^* = D_B \nabla^{*2}\phi^* + (D_T / T_c^*)\nabla^{*2} T^* . \tag{4}$$

The Maxwell equation is given as

$$\frac{\partial h^*}{\partial t^*} + (\mathbf{v}_D^* \nabla^*)h^* = (h^*\nabla^*)\,\mathbf{v}_D^* + \eta\nabla^{*2}h^* , \tag{5}$$

$$\nabla^* \cdot h^* = 0 . \tag{6}$$

We write $\mathbf{v}_D^* = (u^*, v^*, w^*)$.

We have presented the effective viscosity of porous medium by $\tilde{\mu}$, the effective heat capacity of porous medium by $(\rho c)_m$, k_m is effective thermal conductivity of the porous medium, $(\rho c)_p$ is effective heat capacity of the nanoparticle, $(\rho c)_f$ is heat capacity of fluid, and λ is the Jeffrey parameter. Here, ρ_f, μ, β, η, and μ_e are density, viscosity, volumetric expansion coefficient of the fluid, fluid electrical resistivity and magnetic permeability, respectively, while ρ_p is the density of particles. The coefficients that appear in Eqs. (3) and (4) are the Brownian diffusion coefficient D_B and the thermophoretic diffusion coefficient D_T.

According to Nield and Kuznetsov /12-13/, the boundary conditions are

$$w^* = 0,\ \ \frac{\partial w^*}{\partial z^*} + \lambda_1 H \frac{\partial^2 w^*}{\partial z^{*2}} = 0,\ \ \phi^* = \phi_0^*,\ \ T^* = T_h^* \ \text{ at } z^* = 0, \tag{7}$$

$$w^* = 0,\ \ \frac{\partial w^*}{\partial z^*} - \lambda_2 H \frac{\partial^2 w^*}{\partial z^{*2}} = 0,\ \ \phi^* = \phi_1^*,\ \ T^* = T_c^* \ \text{ at } z^* = H. \tag{8}$$

Dimensionless variables are defined as follows

$$(x,y,z) = \frac{(x^*, y^*, z^*)}{H},\ \ t = \frac{t^*\alpha_m}{\sigma H^2},\ \ (u,v,w) = \frac{(u^*, v^*, w^*)H}{\alpha_m}, \\ p = \frac{p^* K}{\mu\alpha_m},\ \ T = \frac{T^* - T_c^*}{T_h^* - T_c^*},\ \ \phi = \frac{\phi^* - \phi_0^*}{\phi_1^* - \phi_0^*},\ \ h = \frac{h^*}{h_0}, \tag{9}$$

where: $\sigma = (\rho c)_m/(\rho c)_f$; $\alpha_m = k_m/(\rho c)_f$.

Equations (1)-(8) become

$$\nabla . \mathbf{v} = 0 , \tag{10}$$

$$\frac{1}{\sigma \mathrm{Va}} . \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \mathrm{Da}\nabla^2\mathbf{v} - \frac{\mathbf{v}}{1+\lambda} + \mathrm{Q}\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}(h\nabla)h + \\ + \sqrt{\mathrm{Ta}}\,(v \times \hat{e}_z) - \mathrm{Rm}\,\hat{e}_z - \mathrm{Rn}\phi\hat{e}_z + \mathrm{Ra}\,T\hat{e}_z , \tag{11}$$

$$\frac{\partial T}{\partial t} + \mathbf{v} . \nabla T = \nabla^2 T + \frac{N_B}{\mathrm{Le}}\nabla\phi . \nabla T + \frac{N_A N_B}{\mathrm{Le}}\nabla T . \nabla T , \tag{12}$$

$$\frac{1}{\sigma}\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{v} . \nabla\phi = \frac{1}{\mathrm{Le}}\nabla^2\phi + \frac{N_A}{\mathrm{Le}}\nabla^2 T , \tag{13}$$

$$\frac{\partial h}{\partial t}+\sigma(\mathbf{v}\nabla)h=\sigma(h\nabla)\mathbf{v}+\sigma\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}\nabla^2 h\,, \tag{14}$$

$$\nabla\cdot h=0\,. \tag{15}$$

The dimensionless boundary conditions are

$$w=0,\ \frac{\partial w}{\partial z}+\lambda_1\frac{\partial^2 w}{\partial z^2}=0,\ \phi=0,\ T=1 \ \text{ at } z=0, \tag{16}$$

$$w=0,\ \frac{\partial w}{\partial z}-\lambda_2\frac{\partial^2 w}{\partial z^2}=0,\ \phi=1,\ T=0 \ \text{ at } z=1. \tag{17}$$

Here, Da $= K/H^2$ is Darcy number; Da $= \tilde{\mu}K/\mu H^2$ is Darcy-Brinkman number; $\mathrm{Pr}_1 = \mu/\rho\alpha_m$ is Prandtl number; $\mathrm{Pr}_2 = \mu/\rho\eta$ is magnetic Prandtl number; Le $= \alpha_m/D_B$ is Lewis number; Va $= \varepsilon\,\mathrm{Pr}/\mathrm{Da}$ is Vadasz number; Q $= \mu_e h_0^2 K/4\pi\eta\mu$ is Chandershekar number; Ta $= (2\Omega H^2\rho/\mu)^2$ is the Taylor number; Ra $= \rho g\beta KH(T_h^* - T_c^*)/\mu\alpha_m$ is thermal Rayleigh-Darcy number; Rm $= [\rho_p\phi_1^* + \rho(1-\phi_1^*)]gKH/\mu\alpha_m$ is basic density Rayleigh number; $N_A = D_T(T_h^* - T_c^*)/D_B T_c^*(\phi_1^* - \phi_0^*)$ is modified diffusivity rate; Rn $= (\rho_p - \phi_0^*)gKH/\mu\alpha_m$ is concentration Rayleigh number; and $N_B = (\rho c)_p(\phi_1^* - \phi_0^*)/(\rho c)_m$ is modified particle-density increment, in respect.

BASIC SOLUTIONS

Following Nield and Kuznetsov /12-13/. The time independent fundamental states for nanofluids are expressed as

$$\mathbf{v}=\mathbf{0},\ T=T_b(z),\ \phi=\phi_b(z),\ p=p_b(z),\ h=(0,0,1)\,. \tag{18}$$

Using Eq.(18) in Eqs.(10)-(13), those equations reduce to

$$-\frac{dp_z}{dz}+\mathrm{Q}\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}\left(\frac{\partial h}{\partial z}\right)\hat{e}_z-\mathrm{Rm}\,\hat{e}_z-\mathrm{Rn}\,\phi_b\hat{e}_z+\mathrm{Ra}\,T_b\hat{e}_z=0\,, \tag{19}$$

$$\frac{d^2T_b}{dz^2}+\frac{N_B}{\mathrm{Le}}\frac{d\phi}{dz}\frac{dT_b}{dz}+\frac{N_AN_B}{\mathrm{Le}}\left(\frac{dT_b}{dz}\right)^2=0\,, \tag{20}$$

$$\frac{d^2\phi_b}{dz^2}+N_A\frac{d^2T_b}{dz^2}=0\,. \tag{21}$$

Using boundary conditions Eqs.(16) and (17), the solution of Eq.(21) is

$$\phi_b=-N_AT_b+(1-N_A)z+N_A\,. \tag{22}$$

Putting the value of ϕ_b in Eq.(20), we get

$$\frac{d^2T_b}{dz^2}+\frac{(1-N_A)N_B}{\mathrm{Le}}\frac{dT_b}{dz}=0\,. \tag{23}$$

Neglecting the higher power term, Eq.(23) becomes

$$T_b=\frac{-e^{-(1-N_A)N_B/\mathrm{Le}}\left[1-e^{-(1-N_A)N_B/\mathrm{Le}(1-z)}\right]}{1-e^{-(1-N_A)N_B/\mathrm{Le}}}\,. \tag{24}$$

The approximated solution for Eqs.(22) and (24), gives

$$T_b=1-z,\ \phi_b=z\,. \tag{25}$$

This result coincides with the result of Sharma et al. /19-27/ and Kumar et al. /8-9/.

PERTURBATION SOLUTIONS

We apply perturbations to the basic solution. As we write,

$$\mathbf{v}=\mathbf{0}+\mathbf{v}',\ p=p_b+p',\ T=T_b+T',\ \phi=\phi_b+\phi',\ h=(0,0,1)+h'\,. \tag{26}$$

Using Eq.(26) in Eqs.(10)-(17) and linearising the terms by ignoring the product of prime quantities, these equations become

$$\nabla.\mathbf{v}'=0\,, \tag{27}$$

$$\frac{1}{\sigma\mathrm{Va}}\frac{\partial w'}{\partial t}=-\nabla p'+\mathrm{Da}\nabla^2\mathbf{v}'-\frac{\mathbf{v}'}{1+\lambda}+\mathrm{Q}\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}\left(\frac{\partial h'}{\partial z}\right)\hat{e}_z+$$
$$+\sqrt{\mathrm{Ta}}\,(\mathbf{v}'\times\hat{e}_z)+\mathrm{Ra}\,T'\hat{e}_z-\mathrm{Rn}\,\phi'\hat{e}_z\,, \tag{28}$$

$$\frac{\partial T'}{\partial t}-w'=\nabla^2T'+\frac{N_B}{\mathrm{Le}}\left(\frac{\partial T'}{\partial z}-\frac{\partial\phi'}{\partial z}\right)-\frac{2N_AN_B}{\mathrm{Le}}\frac{\partial T'}{\partial z}\,, \tag{29}$$

$$\frac{1}{\sigma}\frac{\partial\phi'}{\partial t}+\frac{1}{\varepsilon}w'=\frac{1}{\mathrm{Le}}\nabla^2\phi'+\frac{N_A}{\mathrm{Le}}\nabla^2T'\,, \tag{30}$$

$$\frac{\partial h'}{\partial t}=\sigma(0,0,1)\nabla w'+\sigma\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}\nabla^2h'\,, \tag{31}$$

$$\nabla\cdot h'=0\,. \tag{32}$$

The boundary conditions are

$$w'=0,\ \frac{\partial w'}{\partial z}+\lambda_1\frac{\partial^2 w'}{\partial z^2}=0,\ \phi'=0,\ T'=0 \ \text{ at } z=0, \tag{33}$$

$$w'=0,\ \frac{\partial w'}{\partial z}-\lambda_2\frac{\partial^2 w'}{\partial z^2}=0,\ \phi'=0,\ T'=0 \ \text{ at } z=1. \tag{34}$$

The six unknowns u', v', w', p', T' and ϕ' reduce to three by operating on Eq.(28) multiplied by $\hat{e}_z$.curl.curl and also using Eq.(27), we get

$$\left(\frac{1}{\sigma\mathrm{Va}}\frac{\partial}{\partial t}+\frac{1}{1+\lambda}\right)^2\nabla^2w'-\mathrm{Da}\left(\frac{1}{\sigma\mathrm{Va}}\frac{\partial}{\partial t}+\frac{1}{1+\lambda}\right)\nabla^4w'+$$
$$+\mathrm{Q}\left(\frac{1}{\sigma\mathrm{Va}}\frac{\partial}{\partial t}+\frac{1}{1+\lambda}\right)\frac{\partial^2w'}{\partial z^2}+\mathrm{Ta}\frac{\partial^2w'}{\partial z^2}-\left(\frac{1}{\sigma\mathrm{Va}}\frac{\partial}{\partial t}+\frac{1}{1+\lambda}\right)\times$$
$$\times\mathrm{Ra}\nabla_H^2T'+\left(\frac{1}{\sigma\mathrm{Va}}\frac{\partial}{\partial t}+\frac{1}{1+\lambda}\right)\mathrm{Rn}\nabla_H^2\phi'=0\,, \tag{35}$$

where: $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ and $\nabla_H^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ are the two-dimensional Laplace operators.

NORMAL MODE ANALYSIS

The disturbances are analysed by normal mode analysis, as follows

$$(w',T',\phi')=\left[W(z),\Theta(z),\Phi(z)\right]\exp(ilx+imy+st)\,, \tag{36}$$

where: l and m are the wave numbers along x and y directions, respectively, and s is the growth rate.

Substituting Eq.(36) in Eqs. (29), (30), (33), (34) and Eq.(35), we get

$$\left[\mathrm{Da}(D^2-a^2)^2-\left(\frac{s}{\sigma\mathrm{Va}}+\frac{1}{1+\lambda}\right)(D^2-a^2)-\mathrm{Q}D^2-\frac{\mathrm{Ta}\,D^2}{\left(\frac{s}{\sigma\mathrm{Va}}+\frac{1}{1+\lambda}\right)}\right]W-\mathrm{Ra}\,a^2\Theta+\mathrm{Rn}\,a^2\Phi=0\,, \tag{37}$$

$$W+\left(D^2+\frac{N_A}{\mathrm{Le}}D-\frac{2N_AN_B}{\mathrm{Le}}-a^2-s\right)\Theta-\frac{N_B}{\mathrm{Le}}D\Phi=0\,, \tag{38}$$

$$\frac{1}{\varepsilon}W-\frac{N_A}{\mathrm{Le}}(D^2-a^2)\Theta-\left(\frac{1}{\mathrm{Le}}(D^2-a^2)-\frac{s}{\sigma}\right)\Phi=0\,. \tag{39}$$

The boundary conditions are

$$W=0,\ DW+\lambda_1D^2W=0,\ \Theta=0,\ \Phi=0 \ \text{ at } z=0, \tag{40}$$

$$W=0,\ DW-\lambda_2D^2W=0,\ \Theta=0,\ \Phi=0 \ \text{ at } z=1, \tag{41}$$

where: $a=\sqrt{(l^2+m^2)}$ is dimensionless horizontal wave number; and $d/dz = D$.

According to Chandrasekhar /4/, boundary conditions are:

1) Free-free boundaries

$$W=D^2W=\Theta=\Phi=0 \ \text{ at } z=0,1. \tag{42}$$

2) Rigid-rigid boundaries

$$W = DW = \Theta = \Phi = 0 \quad \text{at } z = 0, 1. \qquad (43)$$

3) Rigid-free boundaries

$$W = DW = \Theta = \Phi = 0 \quad \text{at } z = 0, \qquad (44)$$

$$W = D^2W = \Theta = \Phi = 0 \quad \text{at } z = 1. \qquad (45)$$

LINEAR STABILITY ANALYSIS FOR FREE-FREE BOUNDARIES

The assumed solutions for W, Θ and Φ are of the form

$$W = W_0 \sin\pi z,\ \ \Theta = \Theta_0 \sin\pi z,\ \ \Phi = \Phi_0 \sin\pi z\,. \qquad (46)$$

Substituting Eq.(46) in Eqs.(37)-(39) and integrating each term individually within limits $z = 0$ to $z = 1$, we get

$$\begin{bmatrix} \mathrm{Da}\,J^2\left(\frac{s}{\sigma\mathrm{Va}}+\frac{1}{1+\lambda}\right)J+\pi^2\mathrm{Q}+\frac{\mathrm{Ta}\,\pi^2}{\left(\frac{s}{\sigma\mathrm{Va}}+\frac{1}{1+\lambda}\right)} & -\mathrm{Ra}\,a^2 & \mathrm{Rn}\,a^2 \\ 1 & -(J+s) & 0 \\ \frac{1}{\varepsilon} & \frac{N_A}{\mathrm{Le}}J & \left(\frac{J}{\mathrm{Le}}+\frac{s}{\sigma}\right) \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (47)$$

The eigenvalue of the system of linear Eq. (47) is given as

$$\mathrm{Ra} = \frac{1}{a^2}\left\{ \mathrm{Da}\,J^2\left(\frac{s}{\sigma\mathrm{Va}}+\frac{1}{1+\lambda}\right)J+\mathrm{Q}\pi^2+\frac{\mathrm{Ta}\,\pi^2}{\left(\frac{s}{\sigma\mathrm{Va}}+\frac{1}{1+\lambda}\right)}(J+s)\right\} - \frac{\left(\frac{N_A J}{\mathrm{Le}}+\frac{J+s}{\varepsilon}\right)}{\left(\frac{J}{\mathrm{Le}}+\frac{s}{\sigma}\right)}\mathrm{Rn}\,. \qquad (48)$$

Stationary convection for free-free boundaries

For stationary convection $s = 0$ in Eq.(48), we obtain

$$\mathrm{Ra}^S = \frac{\mathrm{Da}(\pi^2+a^2)^3}{a^2} + \frac{(\pi^2+a^2)^2}{a^2(1+\lambda)} + \frac{\pi^2(\pi^2+a^2)\mathrm{Q}}{a^2} + \frac{\pi^2(\pi^2+a^2)(1+\lambda)\mathrm{Ta}}{a^2} - \left(\frac{\mathrm{Le}}{\varepsilon}+N_A\right)\mathrm{Rn}\,. \qquad (49)$$

When Da = 0, the critical wave number is determined by minimising thermal Rayleigh-Darcy number Ra with respect to a. Thus the critical wave number must satisfy

$$\left(\frac{\partial \mathrm{Ra}}{\partial a^2}\right)_{a=a_c} = 0\,.$$

Equation (49) gives

$$a_c = \pi\,. \qquad (50)$$

This result is identical with the original work of Nield and Kuznetsov /12/.

LINEAR STABILITY ANALYSIS FOR RIGID-RIGID BOUNDARIES

The assumed solutions for W, Θ, and Φ are of the form

$$W = W_0(z^2-2z^3+z^4),\ \ \Theta = \Theta_0(z-z^2),\ \ \Phi = \Phi_0(z-z^2)\,. \qquad (51)$$

Substituting Eq. (51) in Eqs. (37)-(39) and integrating each term individually within limits $z=0$ to $z=1$, we get

$$\begin{bmatrix} 2\mathrm{Da}(504+24a^2+a^4)+(12+a^2)\left(\frac{s}{\sigma\mathrm{Va}}+\frac{1}{1+\lambda}\right)+12\mathrm{Q}+\frac{12\mathrm{Ta}}{\left(\frac{s}{\sigma\mathrm{Va}}+\frac{1}{1+\lambda}\right)} & -9\mathrm{Ra}\,a^2 & 9\mathrm{Rn}\,a^2 \\ 3 & -14(10+a^2+s) & 0 \\ \frac{3}{\varepsilon} & 14\frac{N_A}{\mathrm{Le}}(10+a^2) & \frac{14(10+a^2)}{\mathrm{Le}}+\frac{14s}{\sigma} \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (52)$$

The eigenvalue of the system of linear Eq.(52) is given as

$$\mathrm{Ra} = \frac{28}{27a^2}\left[\mathrm{Da}(504+24a^2+a^4)+12+a^2\left(\frac{s}{\sigma\mathrm{Va}}+\frac{1}{1+\lambda}\right)+12\mathrm{Q}+\frac{12\mathrm{Ta}}{\left(\frac{s}{\sigma\mathrm{Va}}+\frac{1}{1+\lambda}\right)}\right](10+a^2+s) - \frac{N_A(10+a^2)+\frac{\mathrm{Le}(10+a^2+s)}{\varepsilon}}{10+a^2+\frac{s\,\mathrm{Le}}{\sigma}}\,. \qquad (53)$$

Stationary convection for rigid-rigid boundaries

For stationary convection $s = 0$ in Eq.(53), we obtain

$$\mathrm{Ra}^S = \frac{28}{27a^2}\left[\mathrm{Da}(504+24a^2+a^4)+12+a^2\left(\frac{1}{1+\lambda}\right)+12\mathrm{Q}+12(1+\lambda)\mathrm{Ta}\right](10+a^2) - \left(N_A+\frac{\mathrm{Le}}{\varepsilon}\right)\mathrm{Rn}\,. \qquad (54)$$

When Da = 0, the critical wave number at the onset of instability is determined by minimising thermal Rayleigh-Darcy number Ra with respect to a. Thus the critical wave number must satisfy

$$\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c} = 0\,.$$

Equation (54) gives

$$a_c = 3.31\,. \qquad (55)$$

This result is identical with the original work of Nield and Kuznetsov /12/.

LINEAR STABILITY ANALYSIS FOR RIGID-FREE BOUNDARIES

The assumed solutions for W, Θ, and Φ are of the form

$$W = W_0(3z^2 - 5z^3 + 2z^4),\ \ \Theta = \Theta_0(z - z^2),\ \ \Phi = \Phi_0(z - z^2)\,. \tag{56}$$

Substituting Eq.(56) in Eqs.(37)-(39) and integrating each term individually within limits $z = 0$ to $z = 1$, we get

$$\begin{bmatrix} 2\,\mathrm{Da}(4536 + +432a^2 + 19a^4) + 216 + 19a^2\left(\dfrac{s}{\sigma \mathrm{Va}} + \dfrac{1}{1+\lambda}\right) + 216\mathrm{Q} + \dfrac{216\mathrm{Ta}}{\left(\dfrac{s}{\sigma \mathrm{Va}} + \dfrac{1}{1+\lambda}\right)} & -39\,\mathrm{Ra}\,a^2 & 39\,\mathrm{Rn}\,a^2 \\ 13 & -14(10 + a^2 + s) & 0 \\ \dfrac{13}{\varepsilon} & 14\dfrac{N_A}{\mathrm{Le}}(10 + a^2) & \dfrac{14(10+a^2)}{\mathrm{Le}} + \dfrac{14s}{\sigma} \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{57}$$

The eigenvalue of the system of linear Eq.(57) is given by

$$\mathrm{Ra} = \frac{28}{507a^2}\left[\mathrm{Da}(4536 + 432a^2 + 19a^4) + 216 + 19a^2\left(\frac{s}{\sigma \mathrm{Va}} + \frac{1}{1+\lambda}\right) + 12\mathrm{Q} + \frac{12\mathrm{Ta}}{\dfrac{s}{\sigma \mathrm{Va}} + \dfrac{1}{1+\lambda}}\right](10 + a^2 + s) - \frac{N_A(10+a^2) + \dfrac{\mathrm{Le}(10 + a^2 + s)}{\varepsilon}}{10 + a^2 + \dfrac{s\,\mathrm{Le}}{\sigma}}\,. \tag{58}$$

Stationary convection for rigid-free boundaries

For stationary convection $s = 0$ in Eq.(58), we obtain

$$\mathrm{Ra} = \frac{28}{507a^2}\left[\mathrm{Da}(4536 + 432a^2 + 19a^4) + 216 + 19a^2\left(\frac{1}{1+\lambda}\right) + 216\mathrm{Q} + 216(1+\lambda)\mathrm{Ta}\right](10 + a^2) - \left(N_A + \frac{\mathrm{Le}}{\varepsilon}\right)\mathrm{Rn}\,. \tag{59}$$

When Da = 0, the critical wave number at the onset of instability is acquired by minimising thermal Rayleigh-Darcy number Ra with respect to wave number a. Thus the critical wave number must satisfy

$$\left(\frac{\partial \mathrm{Ra}}{\partial a^2}\right)_{a=a_c} = 0\,.$$

Equation (59) gives, $\quad a_c = 3.27\,.$ (60)

This result is identical with the original work of Nield and Kuznetsov /12/.

RESULTS

In this research paper, we investigate the effect of magnetic field on the onset of convection in a rotating Jeffrey nanofluid using the Brinkman model for free-free, rigid-rigid and rigid-free boundary conditions. The impact of different parameters like: Darcy-Brinkman number, Jeffrey parameter, modified diffusivity ratio, Lewis number, porosity parameter, concentration Rayleigh number and Chandrasekhar number on stationary convection have been analysed analytically and plotted graphically for free-free, rigid-rigid and rigid-free boundaries.

To look into the effects of Darcy-Brinkman number Da, Lewis number Le, nanoparticle Rayleigh number Rn, porosity ε, modified diffusivity ratio N_A, Chandrasekhar number Q and Taylor number Ta. We examine the behaviours of $\partial\mathrm{Ra}^S/\partial\mathrm{Da} > 0$, $\partial\mathrm{Ra}^S/\partial N_A > 0$, $\partial\mathrm{Ra}^S/\partial\mathrm{Le} > 0$, $\partial\mathrm{Ra}^S/\partial\varepsilon < 0$, $\partial\mathrm{Ra}^S/\partial\mathrm{Q} > 0$, $\partial\mathrm{Ra}^S/\partial\mathrm{Ta} > 0$, analytically from Eq.(49). According to these inequalities, the following parameters like Darcy Brinkman number, Jeffrey parameter, modified diffusivity ratio, Lewis number, Chandrasekhar number and Taylor number have stabilising effects, and on the other hand, the following parameters as porosity and nanoparticle Rayleigh number have destabilising effects.

Equation (49) gives,

$$\frac{\partial \mathrm{Ra}^S}{\partial \lambda} = -\frac{1}{(1+\lambda)^2}\frac{(\pi^2 + a^2)^2}{a^2} + \frac{\pi^2(\pi^2 + a^2)\mathrm{Ta}}{a^2}\,,$$

$$\frac{\partial \mathrm{Ra}^S}{\partial \lambda} > 0\,, \quad \text{if} \quad \mathrm{Ta} > \frac{1}{(1+\lambda)^2}\frac{(\pi^2 + a^2)}{\pi^2}\,,$$

therefore the Jeffrey parameter λ has stabilising effect. But in the absence of rotation, the Jeffrey parameter λ has a destabilising effect. This result is equivalent to Rana /16/.

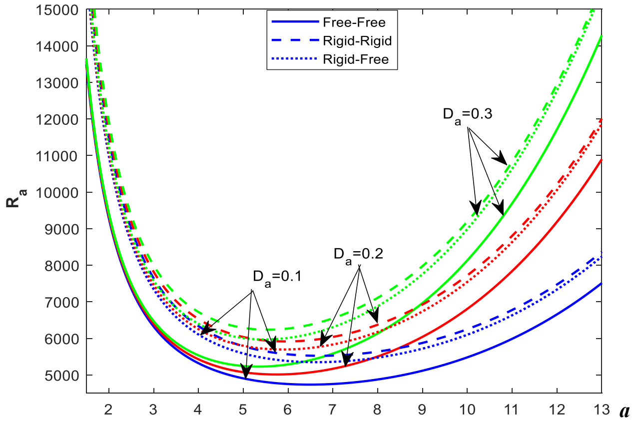

Figure 2. Impact of the Darcy-Brinkman number on the thermal Rayleigh-Darcy number.

Figure 2 illustrates the variation of Ra with respect to wave number a for distinct values of Da = 0.1, 0.2, 0.3. Adjusting other parameter as $\lambda = 0.2$, $N_A = 5$, Le = 1000, $\varepsilon = 0.6$, Rn = -1, Q = 100, and Ta = 100. It shows that Da goes on increasing with the rise in the value of Ra. Thus, Da provides a stabilising impact on stationary convection. Also, we have analysed that Da has a more stabilising effect for rigid-rigid boundaries. Thus, Da delays the onset of convection.

Figure 3 illustrates the variation of Ra with respect to wave number a for distinct values of $\lambda = 0.2$, 0.5, 0.9. Adjusting other parameter as Da = 0.1, $N_A = 5$, Le = 1000, $\varepsilon =$

0.6, Rn = -1, Q = 100, and Ta = 100. It shows that Ra goes on increasing with the rise in the value λ. Thus, λ provides a stabilising impact on stationary convection. Also, we have analysed that λ has a more stabilising effect for rigid-rigid boundaries. Thus, λ delays the onset of convection.

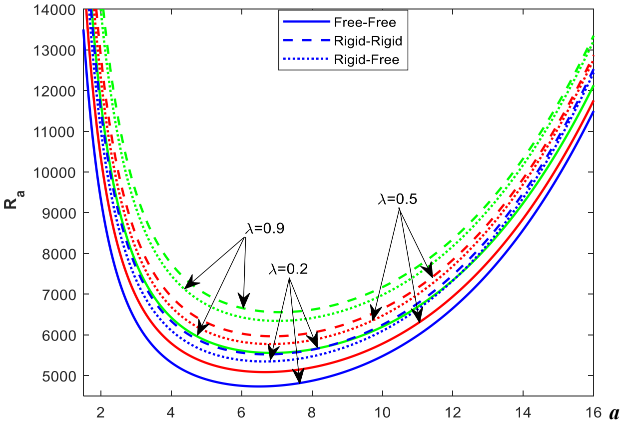

Figure 3. Impact of Jeffrey param. on thermal Rayleigh-Darcy number.

Figure 4 illustrates the variation of Ra with wave number a for distinct values of N_A = 1, 5, 10. Adjusting other parameter as Da = 0.1, λ = 0.2, Le = 1000, ε = 0.6, Rn = -1, Q = 100, and Ta = 100. It shows that Ra goes on increasing with the rise in N_A. Thus, N_A provides a stabilising impact on stationary convection. Also, we have analysed that N_A has more stabilising effect for rigid-rigid boundaries. Thus, N_A delays the onset of convection.

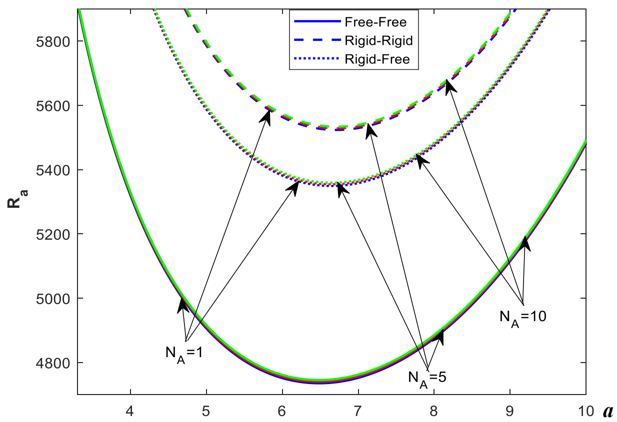

Figure 4. Impact of modified diffusivity ratio on thermal Rayleigh-Darcy number.

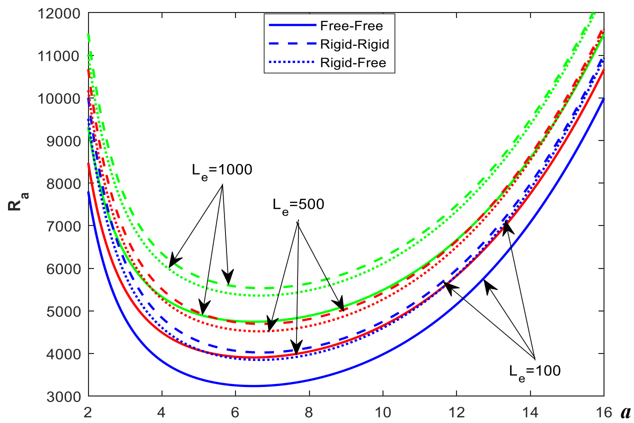

Figure 5. Impact of Lewis number on thermal Rayleigh-Darcy number.

Figure 5 illustrates the variation of Ra with wave number a for distinct values of Le = 100, 500, 1000. Adjusting other parameters as Da = 0.1, λ = 0.2, N_A = 5, ε = 0.6, Rn = -1, Q = 100 and Ta = 100. It shows that Ra goes on increasing with the rise in Le. Thus, Le provides a stabilising impact on stationary convection and it demonstrates that Le has a more stabilising effect for rigid-rigid boundaries. Thus, Le delays the onset of convection.

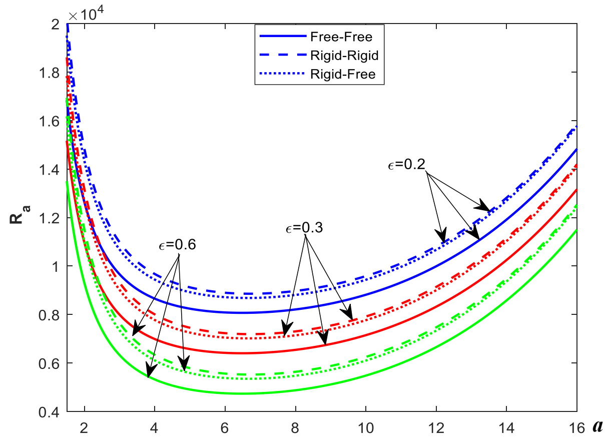

Figure 6. Impact of porosity param. on thermal Rayleigh-Darcy number.

Figure 6 illustrates the variation of Ra with wave number a for distinct values of ε = 0.2, 0.3, 0.6. Adjusting other parameters as Da = 0.1, λ = 0.2, N_A 5, Le = 1000, Rn = -1, Q = 100, and Ta = 100. It shows that Ra goes on decreasing with the rise in ε. Thus, ε provides a destabilising impact on stationary convection and it demonstrates that ε has a more destabilising effect for free-free boundaries. Thus, ε enhances the onset of convection.

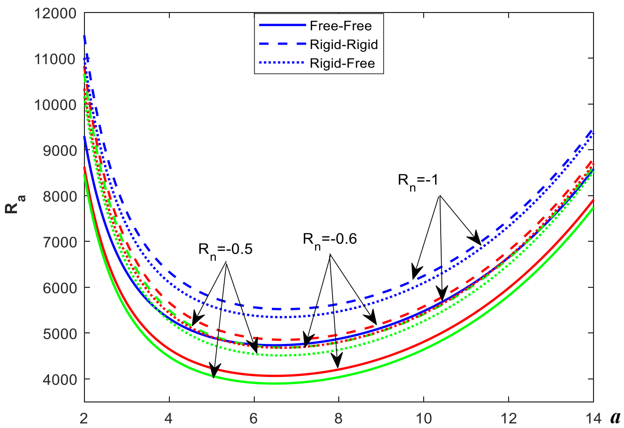

Figure 7. Impact of concentration Rayleigh number on thermal Rayleigh-Darcy number.

Figure 7 illustrates the variation of Ra with wave number a for distinct values of Rn = -1, -0.6, -0.5. Adjusting other parameters as Da = 0.2, λ = 0.2, N_A = 5, Le = 1000, ε = 0.6, Q = 100, and Ta = 100. It shows that Ra goes on decreasing with the rise in Rn. Thus, Rn provides a destabilising impact on stationary convection, and it demonstrates that Rn has a more destabilising effect for free-free boundaries. Thus, Rn enhances the onset of convection.

Figure 8 illustrates the variation of Ra with wave number a for distinct values of Q = 100, 200, 300. Adjusting other parameter as Da = 0.1, $\lambda = 0.2$, $N_A = 5$, Le = 1000, $\varepsilon = 0.6$, Rn = -1, and Ta = 100. It shows that Ra goes on increasing with rise in Q. Thus, Q provides a stabilising impact on stationary convection and demonstrates that Q has a more stabilising effect for rigid-rigid boundaries. Thus, Q delays the onset of convection.

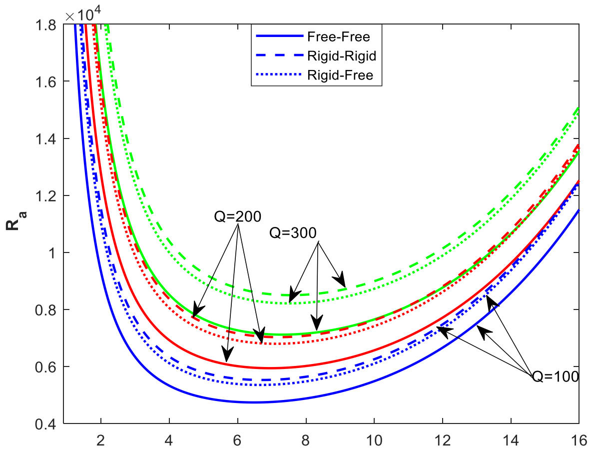

Figure 8. Impact of Chandrashekhar number on thermal Rayleigh-Darcy number.

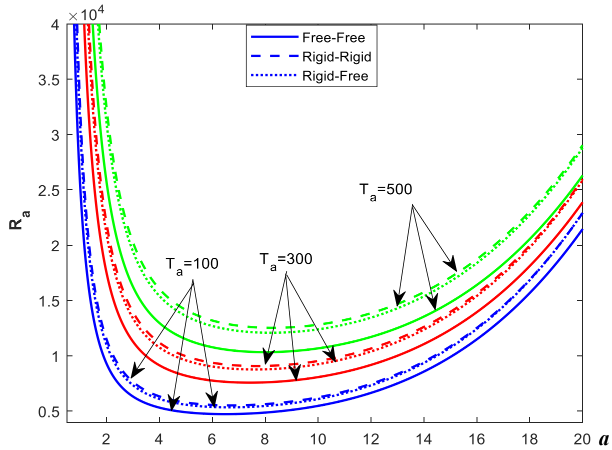

Figure 9. Impact of Taylor number on thermal Rayleigh-Darcy number.

Figure 9 illustrates the variation of Ra with wave number a for distinct values of Ta = 100, 300, 500. Adjusting other parameters as Da = 0.1, $\lambda = 0.2$, $N_A = 5$, Le = 1000, $\varepsilon = 0.6$, Rn = -1, and Q = 100. It shows that Ra goes on increasing with the rise in Ta. Thus, Ta provides a stabilising impact on stationary convection and demonstrates that Ta has more stabilising effect for rigid-rigid boundaries. Thus, Ta delays the onset of convection.

CONCLUSIONS

The following are the main outcomes we found in this article.

- Darcy Brinkman number Da, Lewis number Le, Chandrasekar number Q, Taylor number Ta, modified diffusivity ratio N_A and Jeffrey parameter λ have a stabilising influence on the system that delays the onset of convection.
- Concentration Rayleigh number Rn and porosity ε have destabilising impact on the system that enhances the onset of convection.
- In rigid-rigid boundaries, the system has a greater stabilising impact rather than free-free/rigid-free boundaries.
- Darcy Brinkman number Da, modified diffusivity ratio N_A, Lewis number Le, Chandrasekhar number Q, Taylor number Ta, and Jeffrey parameter λ have a more destabilising influence on stationary convection in the situation of free-free boundaries, as compared to rigid-rigid/rigid-free boundaries.

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