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IMPLEMENTATION OF LAGRANGE INTERPOLATION FOR OPTIMISATION OF COMPUTATIONAL RESOURCE USED IN AE TOMOGRAPHY

IMPLEMENTACIJA LAGRANŽOVE INTERPOLACIJE ZA OPTIMIZACIJU RAČUNARSKIH RESURSA KORIŠĆENIH U AE TOMOGRAFIJI

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Keywords
- acoustic emission tomography (AET)
- elastic wave velocity
- Lagrange interpolation
- computational resource
- non-destructive testing

Abstract

Lagrange interpolation is applied to the source localisation to optimise the computational resources used in AET. Further, the potential of the optimisation is validated by numerical tests in which the original distribution is assumed to represent a damaged concrete plate. In the results of numerical tests, the percentage velocity error between the identified distributions and the original distribution is 2.59 % obtained by the implementation of Lagrange interpolation. This error is lower than the 8.81 % error of the conventional method. Thus, it is noted that the implementation of the interpolation contributes to the accurate identification in comparison with the conventional method. Moreover, if the result of the conventional AET is approximated as original distributions, the larger number of the candidates used in the localisation would be required. Therefore, it is confirmed that AET can be conducted with the optimisation of computational resources if the Lagrange interpolation is implemented.

Ključne reči
- akustična emisiona tomografija (AET)
- brzina elastičnih talasa
- Lagranžova interpolacija
- računarski resurs
- ispitivanje bez razaranja

Izvod

U ovom istraživanju, Lagranžova interpolacija se primenjuje na lokalizaciju izvora kako bi se optimizovali računski resursi korišćeni u AET. Dalje, potencijal ove optimizacije je potvrđen numeričkim testovima u kojima se pretpostavlja da originalna raspodela predstavlja oštećenu betonsku ploču. U rezultatima numeričkih testova, procenat greške brzine između identifikovanih raspodela i originalne raspodele je 2,59 % koji je dobijen primenom Lagranžove interpolacije. Ova greška je manja od greške konvencionalne metode, koja iznosi 8,81 %. Dakle, primena interpolacije doprinosi tačnijoj identifikaciji u poređenju sa konvencionalnom metodom. Štaviše, ako su rezultati konvencionalnog AET aproksimirani kao originalne raspodele, bio bi potreban veći broj kandidata korišćenih u lokalizaciji. Stoga, potvrđeno se da AET može biti sprovedena sa optimizacijom računarskih resursa ako se primeni Lagranžova interpolacija.

INTRODUCTION

Elastic wave tomography (EWT) is a non-destructive test method used to identify elastic wave velocity distributions and has been applied to visualizations of heterogeneities generated in materials /1, 2/. In applications of EWT, Chai et al. /3/ visualised internal heterogeneities of concrete structures using identified low velocity distributions. Generally, the velocity of defect areas is low in comparison with soundness area and defects are identified as low velocity distributions. Hence, it is noted that EWT has a potential applying to structural health monitoring of concrete structures. In addition, although measurements of elastic waves require installing sensors to the structures, the sensors are portable and can be installed to field tests: for instance, Sassa /1/ has attempted installing sensors to boreholes to identify the velocity distribution of grounds. Thus, it is expected that EWT can be applied to field tests and identify the velocity distributions of existing structures.

In an algorithm of EWT, the differences between measured and computed travel times from elastic wave sources to sensors, are minimised to identify elastic wave velocity distributions. A measured travel time is obtained by a difference between a pulse originated time at the sources and an arrival time at a sensor. Hence, in order to conduct EWT, the location of the sources and pulse originated times are required in the measurement. Although the large number of measured travel times is required for the identification of the velocity distributions since the tomography technique is conducted on the basis of an inverse problem, the large number of sources should be manually deployed to an analysis area. One method to generate these sources is the use of transducers. However, to use a large number of transducers, a specialised measurement apparatus with many channels is required. Moreover, if only one transducer is used and reinstalled at each source location, measurements must be conducted multiple times, as the transducer must be manually installed and removed. These challenges are in other methods used generating the sources as well as the use of the transducer.

Another tomography technique is acoustic emission tomography (AET) /4/ and AET has attempted to be applied to structural health monitoring /5, 6/. Though both tomography techniques have potential to identify the velocity distributions, AET does not require measurement of source locations and pulse originated times since a source localisation is implemented to obtain localised sources and computed pulse originated times in an algorithm of AET. If AET are applied to structural health monitoring, the sources can be easily generated by randomly hitting a hammer and/or other tools in an analysis area. Therefore, it is expected that the large number of sources is easily measured to identify the practical distributions.

Source localisation used in AET selects the sources from candidates located in an analytical model. The candidates are installed with regular intervals and if the interval of candidates is decreased, an accuracy of a localised source is raised. Although the accuracy of the localisation should be raised to improve the accuracy of identified velocity distributions, computational resources used in AET are raised because the number of candidates is increased in an analytical model. If an interval of candidates installed in an analytical area which is a square is halved, the number of candidates quadratically increases. Moreover, if the interval is halved in the cube, the number of candidates increases cubically. Thus, if the increment of candidates can be prevented, computational resources can be optimised.

Lagrange interpolation has a potential to localise sources at locations between the candidates. In the previous study /7/, Lagrange interpolation is applied to the source localisation and the sources are localised between the candidates. Thus, if Lagrange interpolation is applied to AET, it is expected that computational resources used in AET are optimised. Although Lagrange interpolation has been applied to AET, AET did not identify accurate velocity distributions. In this study, Lagrange interpolation is applied to AET using improved source localisation. Further, in order to provide reasons why an optimisation using the interpolation should be applied to AET using improved source localisation, the AET algorithm and the optimisation process are detailed. Furthermore, the performance of optimisation is validated by numerical tests which approximate concrete plates with a defect.

IDENTIFICATION OF ELASTIC WAVE VELOCITY DISTRIBUTIONS

Tomography techniques

Analytical models of tomography techniques consist of several cells. Further, homogeneous velocities are assumed in each cell and differences in velocities approximate the heterogenous velocity distributions. While complex distributions are identified if a size of the cell is minimised, the required measured data is raised. An example of a 2D analytical model is shown in Fig. 1. In Figure 1, the slowness which is the reciprocal of velocity, is set to each cell and computed travel times are obtained by use of the slowness. The computed travel time T_{ij} from source i to sensor j is defined as

$$T_{ij} = \sum_k S_k l_{ijk} , \quad (1)$$

where: S_k is slowness in the cell k; l_{ijk} is length of ray-path from source i to sensor j in cell k. It should be noted that Ray-Tracing /8/ is used to obtain l_{ijk} as Ray-Tracing considers the refractions and diffractions of elastic waves. Moreover, Ray-Tracing has been applied to EWT and EWT using Ray-Tracing improved the identified velocity distributions of the concrete specimen /9/. In order to identify the velocity distributions, error of T_{ij} is required and the error ΔT_{ij} is defined as

$$\Delta T_{ij} = T_{0ij} - T_{ij} , \quad (2)$$

where: T_{0ij} is measured travel time from source i to sensor j. In tomography techniques, S_k is computed by a minimisation of ΔT_{ij} on the basis of inverse problems and computed S_k is approximate to the original distributions. Thus, it is noted that S_k is the variable of tomography techniques. In addition, if the number of S_k is raised to identify the complex velocity distributions, the required number of measured travel times is raised owing to increments of the variables.

In order to obtain measured travel times, locations of the sources and pulse originated times of elastic waves from sources are required. The sources are manually installed to obtain measured travel times if EWT is conducted. The sources should be widely located in a model because ray-paths should be propagated to all of the cells in an analytical model to compute S_k by Eq.(2). The number of travel times should be sufficiently larger than the number of slowness since the number of travel times corresponds to the number of Eq.(2) used in an inverse problem. Thus, EWT has potential to be widely conducted for structural health monitoring if the sources are easily installed. On the other hand, the source localisation is implemented in AET to compute locations of sources and the pulse originated times, and it is noted that installing the sources is easier than in EWT analysis. Therefore, it is expected that AET is conducted for the practical identification of the distribution.

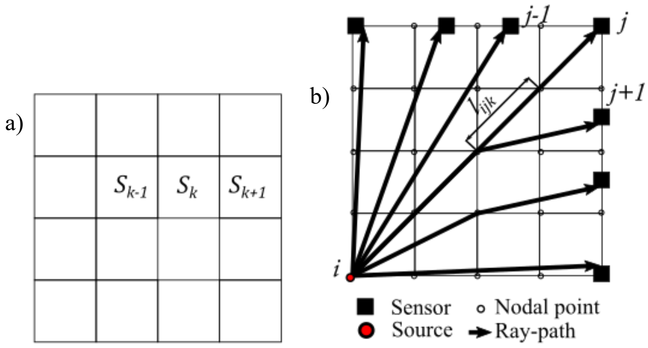

Figure 1. Illustration of tomography techniques: a) analytical model used in tomography techniques; b) approximated ray-paths.

Source localisation used in AET

The source localisation selects the source from candidates of sources located in analytical models. A conceptual diagramme of the source localisation is shown in Fig. 2. In Fig. 2, the illustrated dots imply the candidates and the candidates are selected by uses of approximated ray-paths from sensors to the candidate. It should be noted that the approximated ray-paths are computed by Ray-Tracing. If the ray-path is approximated, the computed travel time from sensor to the candidate is obtained by the use of Eq.(1). Hence, pulse originated times P_{ij} can be computed as subtraction of arrival times of computed travel times and are defined as

$$P_{ij} = A_j - T_{ij}\,, \tag{3}$$

where: A_{ij} is arrival time of an elastic wave at sensor j. Equation (3) is noted that the number of P_{ij} in each candidate is the same as the number of installed sensors. Thus, variances of pulse originated times in each candidate are obtained and the variance σ_i^2 at the candidate i is defined as

$$\sigma_i^2 = \frac{1}{n}\sum_j^n (\bar{P}_i - P_{ij})^2\,, \tag{4}$$

where: n is number of sensors; $\bar{P}_i$ is the average of pulse originated times at candidate i. If the localised sources at the candidate approximate the location of the original source, σ_i^2 approaches 0 because the lengths of ray paths are more accurate than others. Therefore, the source which is the minimum σ_i^2 in all of candidates, is selected. Further, if the localised source is accurate, it is expected that $\bar{P}_i$ is approximated measured pulse originated time.

In Eq.(2), T_{0ij} and T_{ij} could be computed as subtraction of A_j of $\bar{P}_i$ and the use of the length of ray-path from localised source i to sensor j. It is noted that the accuracy of the localised sources contributes to accurate identification. The accuracy of the localised source can be improved if the number of candidates in a model are increased to shrink the interval of candidates. However, computational resources used in AET are raised because the candidates should be memorised in Random Access Memory (RAM) to conduct the source localisation. Therefore, an optimisation method should be applied to the source localisation.

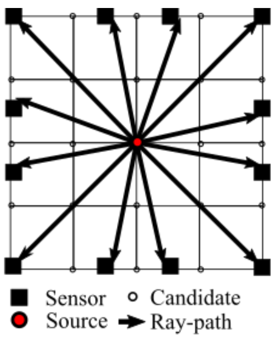

Figure 2. Illustration of source localisation based on ray-tracing.

Optimisation of computational resources on the basis of Lagrange interpolation

In order to conduct the optimisation, the suppression for incrementing the candidates should be proposed. Lagrange interpolation has potential to suppress the increment of the candidates if the sources are localised between candidates. The outline of source localisation using Lagrange interpolation is shown in Fig. 3. Moreover, Lagrange interpolation implemented in the source localisation is defined as

$$L(u) = \sum_i^m \sigma_i^2 \left(\prod_{\substack{l\\ i\neq 1}}^m \frac{u-u_l}{u_i-u_l} \right), \tag{5}$$

where: m is number of selected candidates; u is coordinate of the source; u_i and u_l are coordinates of the candidate i, l; and σ_i^2 is the variance of the pulse originated time obtained at candidate i. In Eq.(3), the output of $L(u)$ implies the variance of the pulse originated times and $L(u)$ is a quadratic function if m is equal to 3. It should be noted that the candidate localised as the sources by Ray-Tracing and others on either side of the localised candidate shown in Fig. 3a, are applied to Eq.(3). According to the algorithm of the source localisation based on Ray-Tracing, the candidate in which the minimum variance is computed is selected. Thus, it is noted that u used in the computation of minimum $L(u)$ implies the sources location. Moreover, the coordinate of the source is obtained by the following Eq.(6),

$$\frac{\partial L}{\partial u} = 0\,. \tag{6}$$

Owing to the implementation of the interpolation function defined as the quadratic function, the output of $L(u)$ has the vertex. Thus, the original source u_0 is obtained by the solution of Eq.(6). Moreover, Fig. 3b implies the example of computing the coordinate of u_0 in the horizontal axis. If $L(u)$ is applied to candidates in the vertical axis, it is expected that the coordinate of u_0 in the vertical axis, is obtained.

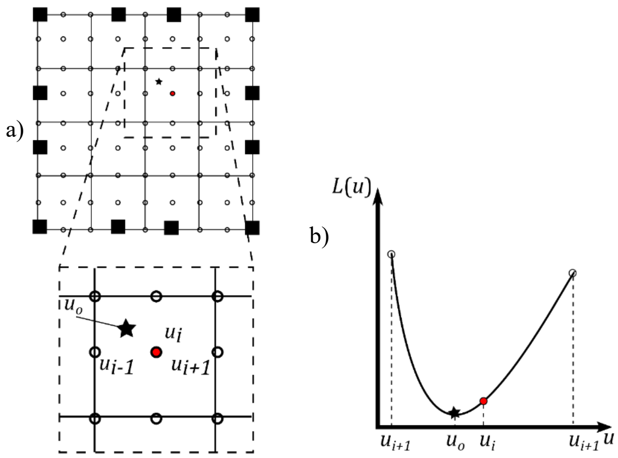

Figure 3. Illustration of source localisation using Lagrange interpolation: a) example of source located between candidates; b) Lagrange interpolation using the coordinate in the horizontal axis.

In the previous study /7/, $L(u)$ is applied to the source localisation and AET is conducted with localised sources. However, the identified velocity distribution has not clearly approximated the original distribution in the numerical test. In the numerical test, the candidates used in the source localisation are located in the nodal points of the cells. In the used model in the previous study, if the interval of the candidates is minimised, the number of cells is raised. Thus, it implies that the required number of travel times increases because the variables of the tomography techniques set in the cells are raised. Therefore, the velocity distribution which was of low accuracy was possibly identified by the use of the insufficient number of travel times.

The accuracy of the source localisation using $L(u)$ depends on the interval as the vertex of $L(u)$ approaches u_0 with minimising the interval of the candidates. Hence, the interval of candidates should be considered as accuracy of the source localisation and the suppression of computational resources. The minimisation of the interval should be considered to not increase the cells. In this study, the candidates are located inside of cells and between nodal points as well as nodal points. Kobayashi et al. /6/ have conducted AET with these improved candidates. Thus, the results note that AET can be conducted with these improved candidates. Furthermore, the minimisation of the interval is not related to the increment of the cells, and it is expected that the accurate distribution is identified by AET using $L(u)$ with the practical interval if these improved candidates are applied.

INITIAL CONDITIONS OF THE NUMERICAL TEST

In the numerical test, AET is conducted to approximate the original velocity distribution shown in Fig. 4. Moreover, the analytical model is shown in Fig. 5, and the initial conditions are listed in Table 1. The original distribution is referred to a damaged concrete plate. Further, the soundness area is shown in red cells. The used velocity of soundness concrete refers to application of EWT in the concrete structure /10/ and the used velocity is 4000 m/s. The damaged area is shown in green cells, and the velocity of damaged concrete is assumed 3000 m/s because the velocity is lower than the soundness.

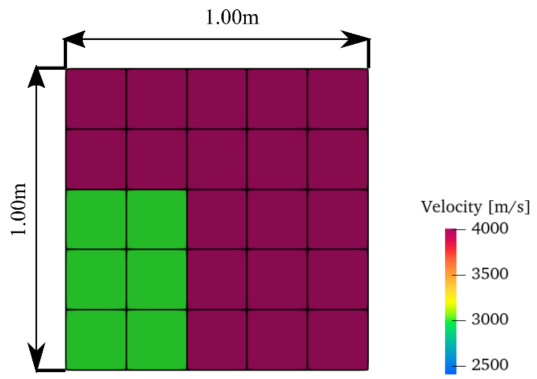

Figure 4. Original velocity distribution.

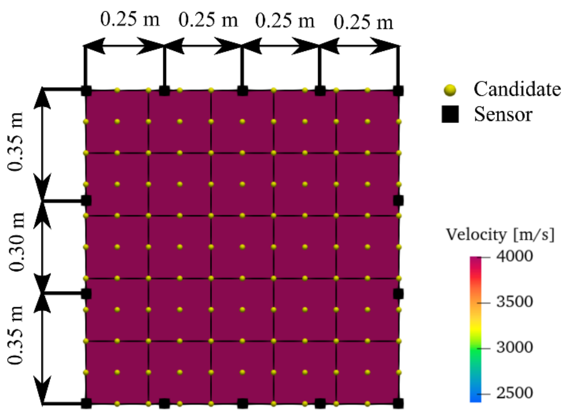

Figure 5. Analytical model used in numerical tests.

An analytical model used in the numerical test is the square in which one side of the square is 1.00 m. The model consists of 25 cells. Thus, it is noted that 25 variables are applied to AET. 14 sensors are installed around the model. Hence, it is expected that the ray-paths are propagated in all of cells. The number of candidates is 121 and the interval is 0.1 m. The applied arrival times to numerical tests are obtained by Ray-Tracing conducted on the original distribution. Moreover, original sources applied to Ray-Tracing are randomly located in the original velocity distribution and the number of sources is 300. In the first step of AET, the homogenous velocity distribution which is 4000 m/s is set in the model. Thus, in the numerical test, the damaged area is iteratively identified from the homogenous velocity distribution.

In the conventional source localisation based on Ray-Tracing, the sources are selected from the candidates. If the sources between candidates are localised by the use of the interpolation function and the identification of the velocity distribution is improved rather than the result of the conventional method, it implies that AET can be conducted with the optimised number of candidates and it is expected that the optimisation of computational resources is verified.

Table 1. Initial conditions.

Velocity of soundness area (m/s)	4000
Velocity of damaged area (m/s)	3000
One side of the square (m)	1.00
Number of cells	25
Number of sensors	14
Number of candidates	121
Interval of candidates (m)	0.10
Number of sources	300

RESULTS AND DISCUSSION

AET is conducted using initial conditions listed in Table 1, and results of the identifications are shown in Fig. 6.

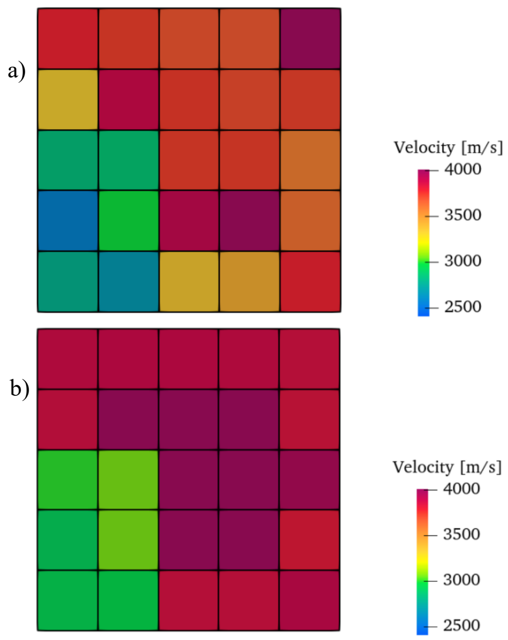

Figure 6. Identified elastic wave velocity distributions: a) result of conventional method; b) result of the implemented interpolation.

As shown in Fig. 6a, the identified distribution visualises damaged area in the bottom left. Although the low velocity visualised by the yellow cells is identified, these yellow cells belong to the soundness area in the original distribution. On the other hand, according to Fig. 6b, the visualised low velocity is improved by the implementation of the interpolation and it is confirmed that the identified distribution by the implementation more closely approximates the original distribution in comparison with the result of the conventional method if the interval of the candidates is fixed.

Results of localisation are shown in Fig. 7. It should be noted that the white dots are original sources, and black dots are localised sources. As shown in Fig. 7a, localised sources are not located between the candidates as localised sources are selected from the candidates. On the other hand, as shown in Fig. 7b, Lagrange interpolation localises sources between the candidates. It is noted that Lagrange interpolation has potential to improve the accuracy of the source localisation based on Ray-Tracing.

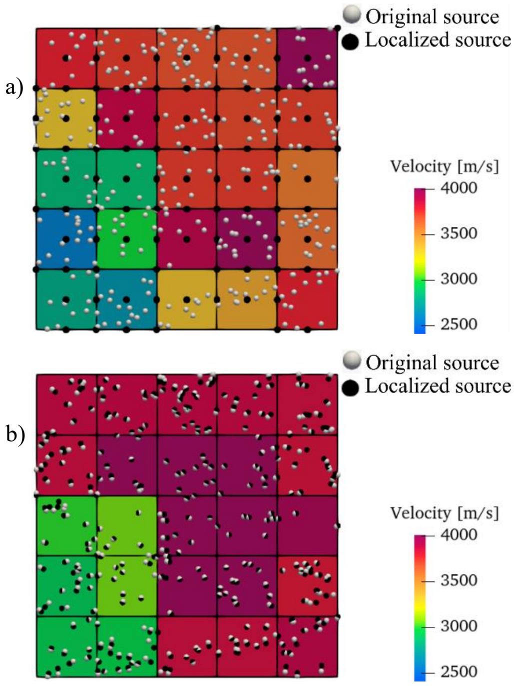

Figure 7. Original and localised sources: a) result of conventional method; b) result of implementing the interpolation.

Errors of AET are listed in Table 2. The errors of the identified velocities are represented by percentage velocity errors between the identified distributions and original distribution. Moreover, the computation of percentage velocity errors refers to the previous study on elastic wave tomography /2/. The average of the localisation errors is the average distance between localised sources and original sources. Further, the average of the localisation errors is computed by the use of 300 sources. According to Table 2, the percentage velocity errors of 2.59 % obtained by the implementation is lower than 8.81 % obtained by the conventional method, and it is noted that the contribution of Lagrange interpolation to the accurate identification is validated by the percentage velocity errors.

As shown in Table 2, the averages of localisation errors are 0.0488 m for the conventional method and 0.00788 m for the implementation of interpolation. Thus, it is confirmed that the accuracy of the localisation is improved by the implementation, and the improved source locations contribute to the accurate identification. In Table 2, the average errors of 0.00788 m obtained by the implementation of the interpolation is approximately one-tenth of the interval of candidates of 0.1 m. Moreover, in the previous study /7/, the average of localisation errors was approximately one-tenth of the interval of the candidates as well if the localisation implementing the interpolation is conducted on the original distribution. Therefore, it is confirmed that the accuracy of the localisation with the implementation of the interpolation is approximately one-tenth of the interval of the candidates. In the results of the implementation of the interpolation, the damaged area and the soundness area are clearly distinguished. Thus, the distribution identified by the implementation is more accurate in comparison with the results of the previous study. In addition, it is noted that the candidates should be placed at points between candidates and where there are internal cells as well as nodal points of cells.

Table 2. Errors of identifications and the localisations.

	Conventional method	Implementation of the interpolation
Percentage velocity errors (%)	8.81	2.59
Average of localisation errors (m)	0.0488	0.00788

Kobayashi et al. /11/ conducted the source localisation, and the results imply that the average localisation is the half of the interval of candidates. Thus, if conventional AET is conducted to identify the distribution approximated as the result of the implementation, the interval of the candidates should be 0.02 m. Hence, 2601 candidates are required if the model is formed by the square having one side 1.00 m and it is expected that the number of candidates are optimised from 2601 to 121 of the initial condition, shown in Table 1, is applied to the AET analysis.

CONCLUSIONS

In order to conduct the optimisation of the computational resources used in AET, the implementation of Lagrange interpolation is proposed. In addition, the performance of the optimisation is validated by the numerical test and the conclusions obtained by numerical test are listed as follows.

- The accuracy of the identified velocity distributions is validated by the percentage velocity errors between the identified distributions and the original distribution. In the results of numerical test, the error of 2.59 % obtained by the implementation of Lagrange interpolation is lower than 8.81 % obtained by conventional method, and it is confirmed that the implementation of the interpolation contributes to the accurate identification in comparison with the conventional method if the interval of the candidates is fixed.
- In the results of the source localisation, the average errors of 0.00788 m obtained by the implementation of the interpolation is approximately one-tenth of the interval 0.1 m. Therefore, it is noted that the accuracy of the localisation implementing the interpolation is approximately one-tenth of the interval of the candidates.

- If conventional AET is conducted to identify the distribution approximated as the result of the implementation in this numerical test, it is expected that the interval of the candidates would need to be 0.02 m. Although 2601 candidates are required if the model is formed by the square with a side of 1.00 m, AET implementing the interpolation could be conducted with 121 candidates to identify the accurate velocity distribution. Thus, it is confirmed that the implementation of Lagrange interpolation could be applied to the optimisation of computational resources as the increase in the number of candidates contributes to the increment of computational resources demand in AET.

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