Abdelmalek Abdelmalek[1], *Mohammed Hadj Meliani*[2*], *Abdelaziz Lairedj*[1], *Farouk Bouchakour*[1]

STUDY OF THE VIBRATORY BEHAVIOUR OF SPHERICAL 3D LAMINATED COMPOSITE SHELL

ISTRAŽIVANJE PONAŠANJA VIBRACIJA SFERNE LJUSKE 3D LAMINIRANOG KOMPOZITA

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Adresa autora / Author's address:
[1] Mechanics, Modeling and Experimentation Laboratory L2ME, Faculty of Technology, Bechar University, Bechar, Algeria
A. Abdelmalek https://orcid.org/0009-0009-6255-5988 ;
A. Lairedj https://orcid.org/0000-0001-6639-9884 ;
[2] LPTPM, Faculty of Technology, Hassiba Benbouali University of Chlef, Chlef, Algeria M. Hadj Meliani https://orcid.org/0000-0003-1375-762X , *email: m.hadjmeliani@univ-chlef.dz

Keywords
- vibration
- analysis
- shell
- composite element
- laminate

Ključne reči
- vibracije
- analiza
- ljuska
- kompozitni element
- laminat

Abstract

As part of this study, a vibration analysis of composite hulls is carried out as a function of various parameters. The aim of this analysis is to identify the frequency of the composite element. To do this, a comparative approach is taken to the results, that are obtained under different structural and material conditions. As part of the analyses carried out on spherical shells with a single support, developed from transverse layers laminated in three dimensions, it is observed that natural frequencies predicted by the elements show variable results depending on the angle of orientation of the layers. The model selected for the simulation is chosen according to the specific criteria of study, and the results are validated by existing data in scientific literature.

Izvod

Vibraciona analiza kompozitnog trupa je izvedena, kao deo ovih istraživanja, u funkciji raznih parametara. Cilj ove analize je identifikacija frekvencije kompozitnog elementa. Da bi se to postiglo, primenjen je uporedni pristup prema rezultatima, dobijenim pod različitim uslovima konstrukcije i materijala. Kao deo analize izvedene za sferne ljuske sa jednim osloncem, formirane od poprečnih slojeva laminata u tri dimenzije, uočava se da prirodne frekvencije predviđene elementima pokazuju promenljive rezultate u zavisnosti od ugla orijentacije slojeva. Izabran je model za simulaciju prema specifičnom kriterijumu istraživanja, a rezultati su verifikovani shodno postojećim podacima u naučnoj literaturi,

INTRODUCTION

Composite materials are being used extensively in many engineering applications, including aerospace, marine, and civil infrastructure, because they have many favourable mechanical properties. These include high rigidity to weight and low density, Thai et al. /1/. During the course of its operational life, the laminate material studied by L.H. Wun et al. /2/ has been observed to undergo a reduction in elastic modulus and a corresponding degradation in strength as a result of temperature variation. In particular, plates, shell and beams, have recently become the focus of computational and analytical research. Patil et al /3/ provide a methodology to accurately predict process-induced shape distortions, additionally.

Recently Xue et al. /4/ studied the mechanical response and damage evolution under impact loading of added glass fibres of different angles to the surface layer of a fibre composite. The results show that increasing the thickness of surface glass fibres can effectively enhance the impact resistance of carbon fibre laminates. A single layer of glass fibres at 90° provides better protection than at 45°. The initial and most evident consequence of temperature is the decline in material characteristics of the structure, which gives rise to a transformation in the vibration characteristics of the composite structure, /5/. Shells and plates with simply supported edges, subjected to a bisinusoidal thermal load, are analysed. Different aspect and radius-to-thickness ratios are taken into account. The results demonstrate the efficacy of the shell element in the study of thermomechanical problems of composite structures /6/. Abdelmalek et al. /7/ employ the micromechanical approach to investigate the linear vibration behaviour of laminated composite materials in a thermal environment. Abed and Majeed /9/ analyse the effect of boundary conditions on the harmonic response of laminated plates. In their study, Pradhan and Murmu /10/ conduct a thermo-mechanical vibration analysis of both functionally graded beams and functionally graded sandwich structures. Upadhyay et al. /11/ suggest that the stress components resulting from a temperature change can be calculated using the classical laminated plate theory. Akbaş /12/ conducts a comprehensive study on dynamical displacements of a laminated composite cantilever beam under a sinusoidal dynamic load, building upon the extensive research of numerous experts who have investigated the static, dynamic, and stability analyses of laminated structures. The higher-order isoparametric finite element shows that shear deformation theory, which ignores thickness-stretching and slope discontinuities, leads to vibration responses that depend strongly on support conditions, thickness ratio, and aspect ratio. Lal et al. /13/ confirmed this using a modified non-polynomial displacement field based on Carrera's Unified Formulation. The numerical results for mean and standard deviation demonstrate that the characteristics of free vibration of a plate subjected to thermal loading are significantly influenced by a number of factors. As the stiffness factor exceeds a certain threshold, the frequency parameter undergoes a gradual shift, as demonstrated by Adams et al. /14/. Reissner /15/ and Mindlin /16/ develop a more accurate thick plate model by considering the effect of transverse shear deformation and rotary inertia. In the context of the first-order shear deformation theory (FSDT), it is assumed that a straight line originally normal to the middle surface will remain in a straight line. An extensive study by Forsat /18/ has shown that the current model is in perfect agreement with the analysis of bending and free vibration of laminated plates and shells, composites and sandwiches. Accordingly, it has been established that the proposed calculation method can be used for bending analysis of thick laminated plates and shells, taking into account other boundary conditions and for different fibre orientations. Future work could be considered to improve the current formulation and apply it to other types of materials.

PHYSICAL MODEL PARAMETERS AND MODELLING

In order to obtain a thorough understanding of through-thickness properties, it is necessary to conduct an analysis on a shell composite model. In this section we focus on the key considerations for building a composite model by selecting the appropriate shell elements 99 and 281, as illustrated in Fig. 1 a and b.

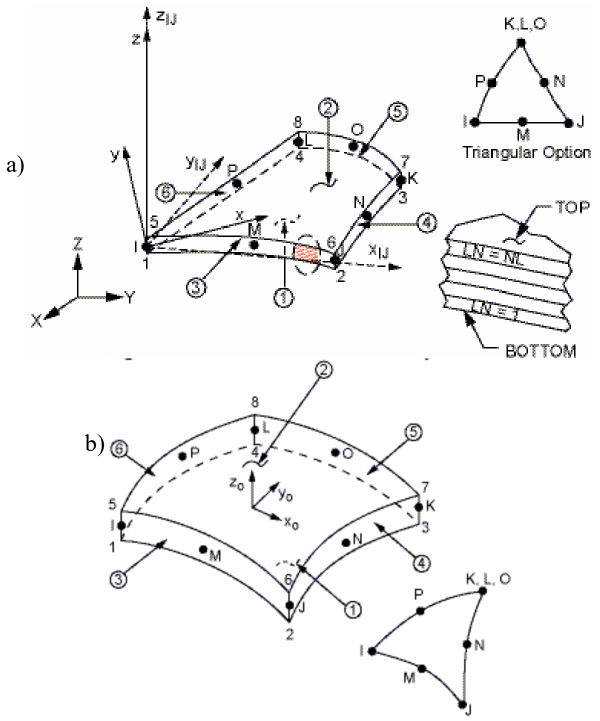

Figure 1. Composites models: a) shell 99; b) shell 281.

NUMERICAL APPLICATION

Three-dimensional shell elements

Consider the model of an orthotropic shell in Fig. 2. The spherical coordinates of shell geometry are radius R, length a, thickness $h = 0.5$ mm, and curvature θ.

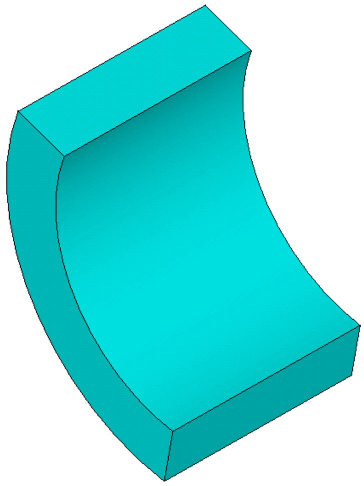

Figure. 2 Model of 3D shell.

The material properties shown in Table 1 are incorporated into ANSYS®; with lamination scheme /0°/90°/90°/0°/, simply supported boundary conditions.

Table 1. Material parameters.

Material	Property	Unit
Density	1	mg mm^{-3}
Young's modulus directions		
Plane 1–2	25×10^6	N/m^2
Plane 2–3	1×10^6	N/m^2
Plane 3–1	0.2×10^6	N/m^2
Shear modulus		
Plane 1–2	5×10^5	N/m^2
Plane 2–3	2×10^5	N/m^2
Plane 3–1	5×10^5	N/m^2
Poisson's ratio		
Plane 1–2	0.25	
Plane 2–3	0.3	
Plane 3–1	0.25	

Laminated shell are assumed to be of the same thickness, mass density, and made of the same linearly elastic composite material; the mesh consists of 12 × 12 = 144 elements on each side with 169 nodes (Fig. 3). The elements of the grid are quadrilateral elements with 8 nodes; the number of elements is given in order to obtain enough precision in the results.

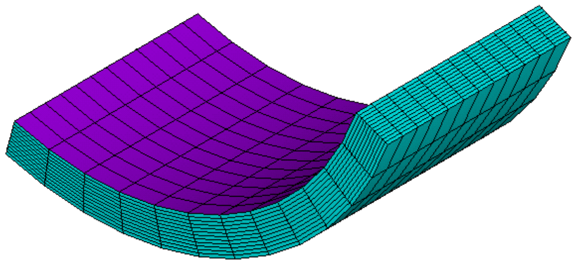

Figure. 3. Mesh 3D with a quadrilateral element Shell281.

The un-symmetric simply supported laminated spherical shells ($R_x = R_y = R$) are considered with different R/a ratios.

In the first part of this study, we validated the choice of simulation models for multilayer composite shells. This validation is carried out using some results available in the scientific literature.

Table 2.Non-dimensional fundamental frequency,
$\varpi = \omega a^2 \sqrt{\rho / E_2} / h$, 0°/90°/90°/0°.

R/a	a/h = 100			a/h = 10		
	present ϖ_1	present ϖ_2	ϖ^*	present ϖ_1	present ϖ_2	ϖ^*
1	128.35	132.45	126.35	17.26	19.08	16.15
2	68.31	77.56	67.38	14.62	15.54	13.44
3	47.42	55.50	46.01	13.31	14.87	12.79
4	37.09	42.06	35.24	13.02	14.33	12.55
5	30.08	31.47	28.83	12.96	14.07	12.44
10	17.38	20.95	16.71	12.48	13.83	12.28

ϖ^* /17/, ϖ_1 present element shell99, ϖ_2 present element shell281

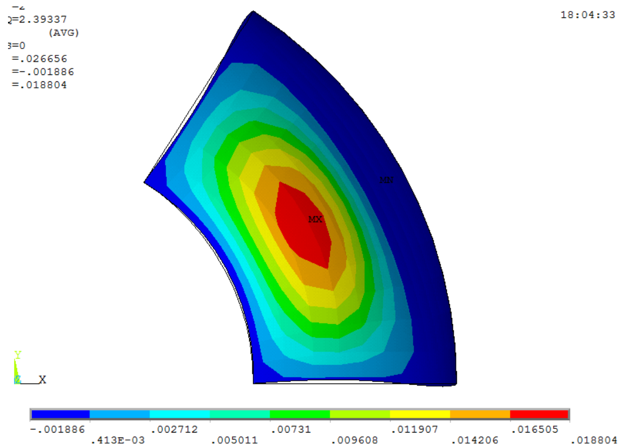

Figure 4. Frequencies of first mode, shell99 element.

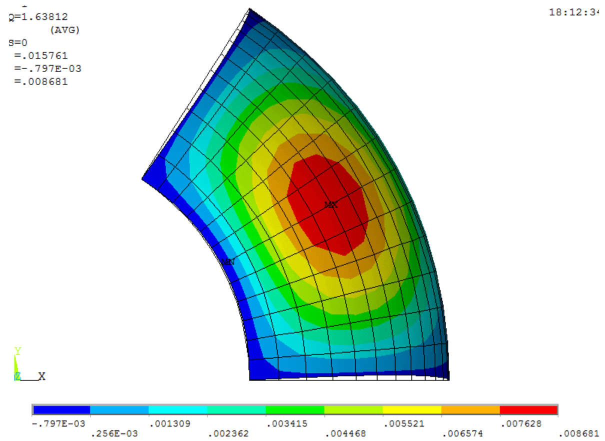

Figure 5. Frequencies of first mode, shell281 element.

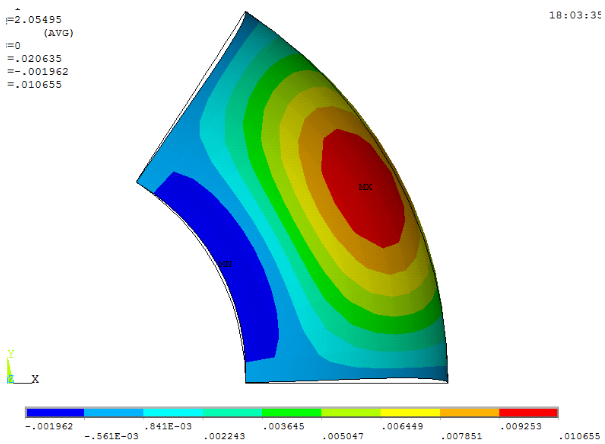

Figure 6. Frequencies of second mode, shell99 element.

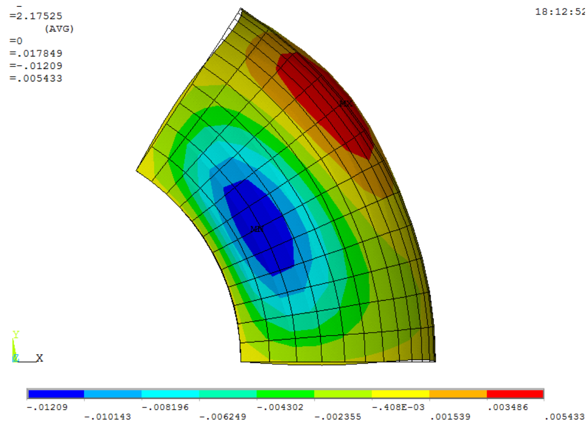

Figure 7. Frequencies of second mode, shell281 element.

The free vibration frequencies of laminated shells are presented in Table 2. In the conductions of spherical shells with simple support from laminated transverse layers (a/h = 100), the natural frequencies predicted by the present elements are in good agreement with those reported by Raja /17/ for shell90 and are not in agreement for shell281. Figures 4, 5, 6, and 7 illustrate the shapes of the first and second modes of a 0°/90°/90°/0° laminated shell with (R/a = 100), for the simple support of shell99 and shell281, respectively, for the 12×12 mesh.

Influence of fibre orientation angle on frequencies

In this section, we study the influence of the angle of orientation on the normalised frequency with a simply applied shell (Fig. 9). The first normalised frequencies derived from the present method are listed in Table 3. The results of the present method are compared with the results of Akavci /8/.

Table 3. Effect of angle of inclination on the non-dimensional frequency parameter ϖ, E_1/E_2 = 40.

angle	present ϖ	ϖ^*
$/0°/90°/_3$	11.48	11.51
$/60/-60°/_3$	11.43	-
$/45/-45/_3$	11.45	-
$/30/-30/_3$	11.43	-

ϖ^* /8/, ϖ present element shell99.

It is clear that present results exhibit high reliability to all solutions and are very stable in case of a wide angle of inclination $0°/90°/_3$, and the same results are obtained for 30° and 60°. Furthermore, it is imperative to refrain from creating a plate with inclined layers at an angle of $45°/-45°/_3$, (Fig. 8).

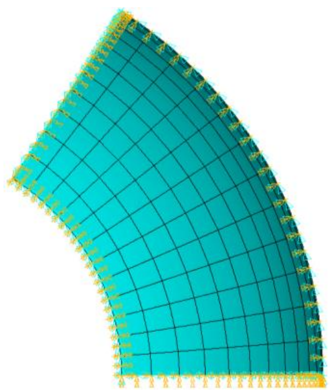

Figure 8. simply applied shell.

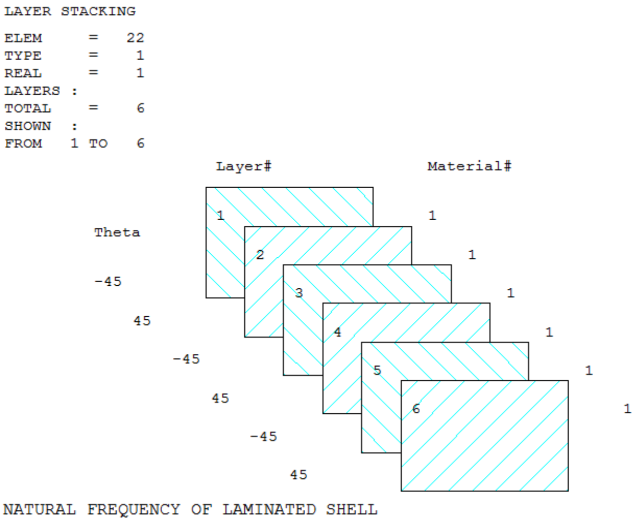

Figure 9. Six layer stack 45°/-45°/3 of shell.

CONCLUSION

This work constitutes a contribution to the study of free vibrations of cross-laminated shells in composite materials, characterised by anisotropic elastic mechanical behaviour. The simulation is conducted utilising the ANSYS® calculation code in three dimensions, employing a calculation procedure that adheres to the steps delineated by the code. The obtained results are subsequently validated through comparison with those reported in other scientific literature. The following conclusions can be drawn from the analysis of these results:

- the results from these finite element models are in good agreement with those obtained by the shell stratified theory;
- the geomantic ratio exerts a significant influence on the vibratory behaviour of laminated composite shells, especially with regard to the angle of orientation.

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