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INVESTIGATING THE DENSITY PARAMETER IN A DISK MADE OF CADMIUM/ MAGNESIUM/BERYL

ISTRAŽIVANJE PARAMETRA GUSTINE DISKA IZRAĐENOG OD KADMIJUMA/ MAGNEZIJUMA/BERILIJUMA

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Keywords
- stresses
- disk
- density
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- angular speed

Abstract

This article deals with the study of density parameter in a disk made of cadmium, magnesium and beryl material, by using generalised strain measure. The mathematical model is based on stress-strain relation and equilibrium equation. Analytical solutions are presented for the disk made of cadmium, magnesium and beryl material. Effects of different pertinent parameters (i.e., density and angular speed) are considered for the disk made of cadmium, magnesium and beryl material. The behaviour of stress distribution and angular speed are investigated. From the obtained results, it is noticed that cadmium material disk requires a higher value of angular speed to yield at the internal surface in comparison to the disk made of magnesium/beryl material.

Ključne reči
- naponi
- disk
- gustina
- tečenje
- ugaona brzina

Izvod

U ovom radu proučava se parametar gustine diska izrađenog od materijala kadmijum, magnezijum i berilijuma, i to primenom generalisane mere deformacije. Matematički model se zasniva vezi napon-deformacija i jednačini ravnoteže. Analitička rešenja su predstavljena za disk od materijala kadmijum, magnezijum i berilijum. Uticaji različitih relevantnih parametara (na pr. gustina i ugaona brzina) se razmatraju za disk izrađen od kadmijuma, magnezijuma i berilijuma. Istraženo je ponašanje raspodele napona i ugaone brzine. Prema dobijenim rezultatima, uočava se da disk od kadmijuma zahteva veće vrednosti ugaone brzine za pojavu tečenja na unutrašnjoj površini, u poređenju sa diskom od materijala magnezijum/berilijum.

INTRODUCTION

Rotating disks play a very important role in many applications in mechanical, aerospace industries, and chemical processing such as in compressors, flywheels, turbo generators, high speed gear engines, compressors, sink fits, steam turbine pumps and computer disks, etc. Materials have been widely used in engineering and human health applications. Solutions for disks can be found in a large number of textbooks /1, 2/. Ghose /3/ analysed the thermal effect on the transverse vibration of a spinning disk with variable thickness parameter. Thakur et al. /4-10/ have investigated stress distribution in a disk with different parameters by using generalised strain measure. The objective of this research paper is to investigate the density parameter in a disk made of cadmium, magnesium, and beryl material by using transition theory and generalised strain measure.

BASIC GOVERNING EQUATION

Let us consider a thin disk with central bore of internal/ external radii as r_i and r_o and made of cadmium/magnesium/ beryl material as shown in Fig. 1. The disk is rotating about its axis with angular velocity ω at the internal surface. Let, the density of the disk be taken as, /6/:

$$\rho = \rho_0 (r / r_0)^{-m} , \tag{1}$$

where: ρ_0 is density at $r = b$; and m is density parameter.

The stress-strain relation of the elastic isotropic material is given by Thakur et al., /9/:

$$\tau_{rr} = \frac{1}{n}\frac{c_{11}c_{33}-c_{13}^2}{c_{33}}[2-\eta^n\{1+(1+T)^n\}]-2\frac{c_{66}}{n}[1-\eta^n],$$

$$\tau_{\theta\theta} = \frac{1}{n}\frac{c_{11}c_{33}-c_{13}^2}{c_{33}}[2-\eta^n\{1+(1+T)^n)\}]-2\frac{c_{66}}{n}[1-\eta^n(1+T)^n)],$$

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = \tau_{zz} = 0 \,. \tag{2}$$

The equation of equilibrium is given:

$$\frac{d}{dr}(r\tau_{rr}) - \tau_{\theta\theta} + \rho\omega^2 r^2 = 0 \,. \tag{3}$$

Transition points: using Eq.(2) into Eq.(3), we get:

$$\eta^{n+1}(1+T)^{n-1}\frac{dT}{d\eta} = \frac{\rho\omega^2 r^2 c_{33}}{c_{11}c_{33}-c_{13}^2} + \eta^n \frac{2c_{66}c_{33}}{n(c_{11}c_{33}-c_{13}^2)} \times$$

$$\times[1+nT-(1+T)^n - T\{1+(1+T)^n\}] , \tag{4}$$

where: $r\eta' = \eta T$ (T is function of η, and η is a function of r). $T \to \pm\infty$ (elastic to plastic state), and $T \to$ -1 (plastic to creep state), /4-10/.

ELASTO-PLASTIC DEFORMATION

The circumferential stress is given by Thakur et al. /7/,

$$\tau_{\theta\theta} = k_1 r^{-\frac{2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}} . \qquad (5)$$

Substituting Eq.(4) into Eq.(2) and using Eq.(1), we get:

$$\tau_{rr} = \frac{k_2}{r} + k_1 \frac{r^{-\frac{2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}}}{1-\frac{2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}} - \frac{\rho_0\omega^2 r^{2-m}}{(3-m)r_0^{-m}} . \qquad (6)$$

Substituting Eq.(5) and Eq.(6) into

$$\varepsilon_{\theta\theta} = \frac{1}{2}[1-\eta^2] = \frac{1}{E}\left[\tau_{\theta\theta} - \frac{c_{11}c_{33}-c_{13}^2-2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}\tau_{rr}\right],$$

is given by /7/, and we get

$$\eta = \sqrt{1-\frac{2(c_{11}c_{33}-c_{13}^2-2c_{66}c_{33})}{E(c_{11}c_{33}-c_{13}^2)}\left[\frac{\rho_0\omega^2 r^{2-m}}{(3-m)r_0^{-m}} - \frac{k_2}{r}\right]} . \qquad (7)$$

Substituting Eq.(7) into $u = r - r\eta$, /7/, we get:

$$u = r - r\sqrt{1-\frac{2(c_{11}c_{33}-c_{13}^2-2c_{66}c_{33})}{E(c_{11}c_{33}-c_{13}^2)}\left[\frac{\rho_0\omega^2 r^{2-m}}{(3-m)r_0^{-m}} - \frac{k_2}{r}\right]} , \qquad (8)$$

$E = 2c_{66}\left(2-\frac{2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}\right)$ is Young's modulus. Using $\tau_{rr} = 0$ at $r = r_0$, and $u = 0$ at $r = r_i$ into Eqs. (6) and (8) we get:

$$k_2 = \frac{\rho_0\omega^2 r_i^{3-m}}{(3-m)r_0^{-m}}, \text{ and } k_1 = \frac{\rho_0\omega^2(r_0^{3-m}-r_i^{3-m})\left(1-\frac{2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}\right)}{(3-m)r_0^{1-m} r_0^{-\frac{2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}}} .$$

Substituting k_1 and k_2 into Eqs. (5), (6), and (8), we get

$$\tau_{\theta\theta} = \frac{\rho_0\omega^2(c_{11}c_{33}-c_{13}^2-2c_{66}c_{33})(r_0^{3-m}-r_i^{3-m})}{(3-m)(c_{11}c_{33}-c_{13}^2)r_0^{1-m}}\left(\frac{r}{r_0}\right)^{-\frac{2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}} \qquad (9)$$

$$\tau_{rr} = \frac{\rho_0\omega^2(c_{11}c_{33}-c_{13}^2-2c_{66}c_{33})}{(3-m)r_0^{-m} r(c_{11}c_{33}-c_{13}^2)} \times \left[(r_0^3-r_i^3)\left(\frac{r}{r_0}\right)^{-\frac{2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}} + r_i^{3-m} - r^{3-m}\right], \text{ and} \qquad (10)$$

$$u = r - r\sqrt{1-\frac{2\rho_0\omega^2(c_{11}c_{33}-c_{13}^2-2c_{66}c_{33})(r^{3-m}-r_i^{3-m})}{(3-m)r_0^{1-m}Er(c_{11}c_{33}-c_{13}^2)}} . \qquad (11)$$

From Eqs.(9)-(10), we get:

$$\tau_{rr} - \tau_{\theta\theta} = \frac{\rho_0\omega^2 b^2}{(3-m)R}\left[(1-R_0^{3-m})R^{\frac{c_{11}c_{33}-c_{13}^2-2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}} \times \frac{2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2} + R_0^{3-m} - R^{3-m}\right], \qquad (12)$$

where: $R = r/r_0$ and $R_0 = r_i/r_0$.

Initial yielding surface: from Eq.(12) it is seen that $|\tau_{\theta\theta} - \tau_{rr}|$ is maximal at $R = R_0$, therefore yielding of the surface becomes:

$$\left|\tau_{\theta\theta} - \tau_{rr}\right|_{R=R_0} = \left|\frac{\rho_0\omega^2 b^2(1-R_0^3)}{(3-m)R_0} R_0^{\frac{c_{11}c_{33}-c_{13}^2-2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}} \frac{2c_{66}c_{33}}{c_{11}c_{33}-c_{13}^2}\right| = Y$$

and angular speed for initial yielding surface is given:

$$\Omega_i^2 = \left|\frac{(3-m)R_0}{(1-R_0^{3-m})CR_0^{1-C}}\right| , \qquad (13)$$

where: $C = 2c_{66}c_{33}/(c_{11}c_{33} - c_{13}^2)$ and $\omega_I = (\Omega_i/r_0)\sqrt{(Y/\rho_0)}$.

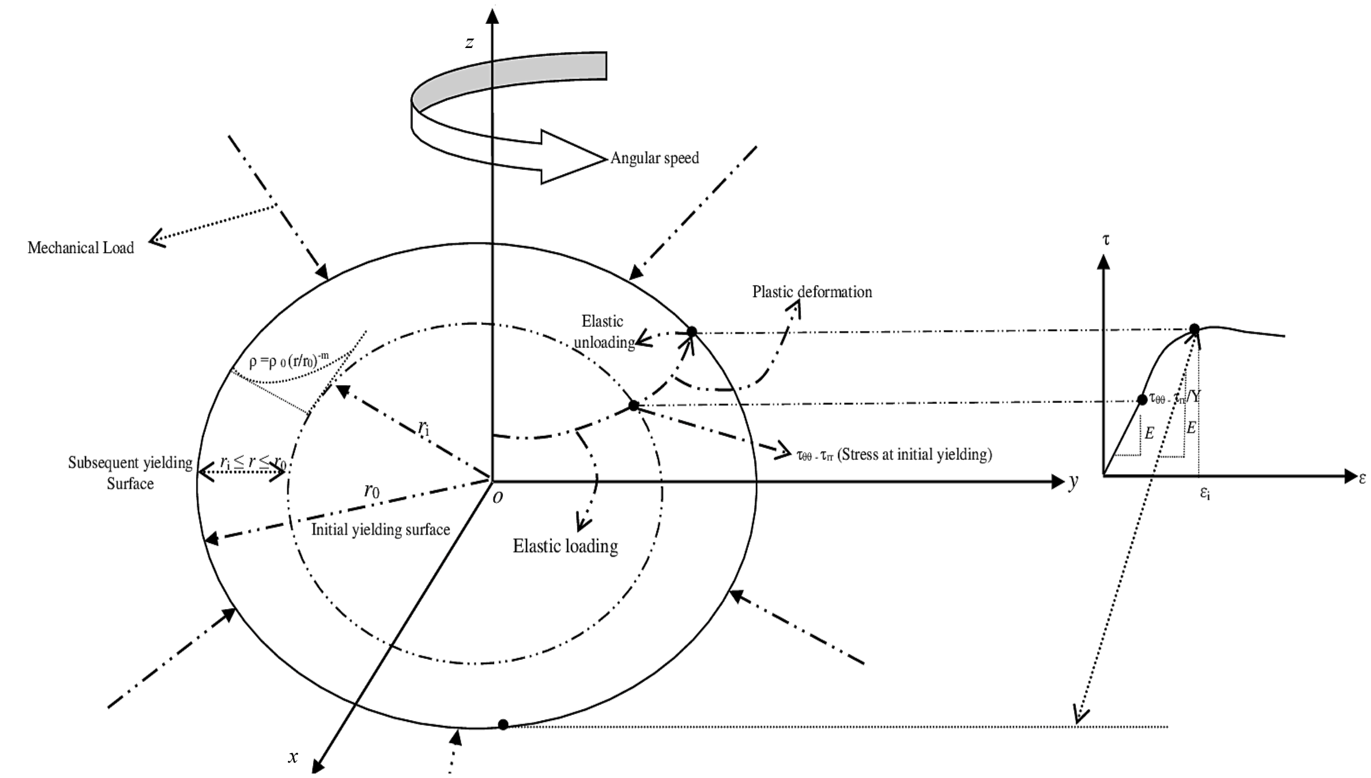

Figure 1. Geometry of the disk.

Subsequent yielding surface: from Eq.(12) that $|\tau_{\theta\theta} - \tau_{rr}|$ is maximal at $R = 1$ and $C \to 1/2$ given by /9 / for fully-plastic state, therefore yielding of the subsequent surface becomes:

$$\left|\tau_{\theta\theta} - \tau_{rr}\right|_{R=1} = \left|\frac{\rho\omega_f^2 b^2 (1 - R_0^{3-m})}{2(3-m)}\right| = Y ,$$

and angular speed for subsequent yielding surface becomes:

$$\Omega_f^2 = \left|\frac{2(3-m)}{1 - R_0^{3-m}}\right|, \quad \forall m \neq 3 \cdot \tag{14}$$

Stresses and displacement for initial yielding surface: From Eqs.(8)-(11) and Eq.(13), we get:

$$\sigma_\theta = \frac{(1-C)R^{-C} R_0^C}{C} ,$$

$$\sigma_r = \frac{R_0^C}{CR(1 - R_0^{3-m})}[(1 - R_0^{3-m})R^{1-C} + R_0^{3-m} - R^{3-m}],$$

and

$$U = R - R\sqrt{1 - \frac{2(1-C)HR_0^C (R^{3-m} - R_0^{3-m})}{CR(1 - R_0^{3-m})}} , \tag{15}$$

where: $\sigma_\theta = \tau_{\theta\theta}/Y$; $\sigma_r = \tau_{rr}/Y$; $H = Y/E$ be non-dimensional components; and $C = 2c_{66}c_{33}/(c_{11}c_{33} - c_{13}^2)$.

Stresses and displacement for subsequent yielding surface: by taking $C \to 1/2$, /9/, for fully-plastic state, Eq.(15) become:

$$\sigma_\theta = \frac{\sqrt{R_0}}{2\sqrt{R}} ,$$

$$\sigma_r = \frac{2\sqrt{R_0}}{R(1 - R_0^{3-m})}[(1 - R_0^{3-m})\sqrt{R} + R_0^{3-m} - R^{3-m}] ,$$

and

$$U = R - R\sqrt{1 - \frac{2H(R^{3-m} - R_0^{3-m})}{R(1 - R_0^{3-m})}} . \tag{16}$$

Validation of results: Eqs.(15)-(16) show similar results after neglecting thermal gradient by /7/.

RESULTS AND DISCUSSION

To illustrate the analysis, we have to take the following values of elastic constants from the existing literature /4-10/. As a numerical example, elastic constants C_{ij} (10^{10} N/m^2) are given for transversely isotropic material such as cadmium (i.e., $C = 0.433$), magnesium (i.e., $C = 0.63$), and beryl (i.e., $C = 0.69$) given in /7, 10/, for the initial/subsequent yielding surface based on the following numerical values, taken as: $r_i = 1$, $r_o = 2$, and $m = 0$ and 2.7, respectively.

Curves are drawn in Fig. 2, between angular speed Ω_i^2 versus radii ratios for initial yielding surface. It has been observed that cadmium material disk requires higher value of angular speed to yield at the internal surface in comparison to the disk made of magnesium/beryl material. With the introduction of density variation (say $m = 2.7$), angular speed also deceases at inner surface of the disk made of cadmium/ magnesium/beryl material.

In Fig. 3, curves are drawn between stress distributions versus radius ratio R. It is observed that the disk made of cadmium material requires maximum value of radial stress at the internal surface as compared to magnesium/beryl material. With the introduction of density parameter, quite effect has been seen for radial/circumferential stresses. Moreover, the rotating disk made of cadmium is on the safer side of the design as compared to magnesium/beryl.

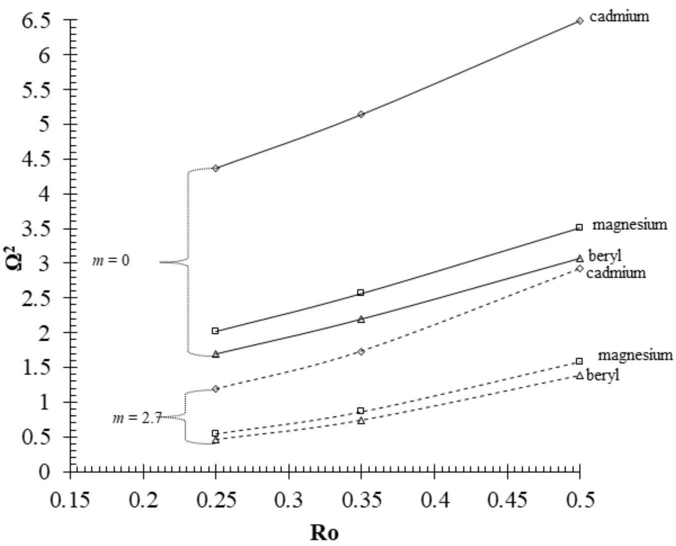

Figure 2. Graph between Ω^2 vs. R_0.

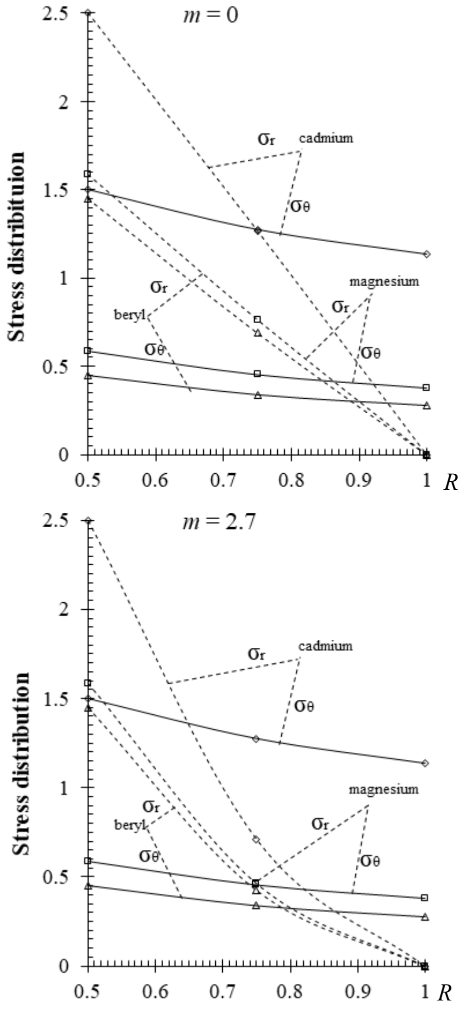

Figure 3. Stress distribution and displacement vs radius.

CONCLUSIONS

The main findings can be concluded as follows.

- Cadmium material disk requires higher value of angular speed to yield at the internal surface.

- The disk made of cadmium material requires maximum value of radial stress at the internal surface as compared to the magnesium/beryl material.
- The disk made of cadmium is more convenient than that of magnesium/beryl.

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