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THERMAL SAFETY MEASURES FOR CREEP DEFORMATION OF PRESSURIZED SPHERICAL SHELL FABRICATED FROM ISOTROPIC MATERIAL

TERMIČKE MERE SIGURNOSTI KOD DEFORMACIJE PUZANJA NA SFERNOJ LJUSCI OD IZOTROPNOG MATERIJALA POD PRITISKOM

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- shell
- thermal
- functionally graded material
- pressure

Abstract

This study presents a closed-form analytical solution for the thick spherical shell's thermal creep stress components. The thick-walled spherical shell under consideration comprises functionally graded isotropic material loaded axisymmetrically. It is no longer necessary to assume creep-strain laws when using Seth's transition theory which is based on the concept of generalised strain measure. The obtained result demonstrates that stresses are significantly impacted by the non-homogeneity features of the FGMs structure. The research concludes that a highly functionally graded thick spherical shell is on the safer side of design. This is because of the reason that less non-homogeneous spherical shell is having high circumferential stress as compared to the highly non-homogeneous spherical shell.

Ključne reči
- puzanje
- ljuska
- termički
- funkcionalni kompozitni materijal
- pritisak

Izvod

U radu je prikazano analitičko rešenje zatvorenog oblika za termičke napone puzanja u komponentama debelozide sferne ljuske. Razmatrana debelozida sferna ljuska je sačinjena od izotropnog funkcionalnog kompozitnog materijala, asimetrično opterećena. Sa primenom teorije prelaznih napona Seta, koja se bazira na konceptu generalisane mere deformacija, ne postoji potreba za pretpostavkom zakona deformacija puzanja. Dobijeni rezultat pokazuje da na napone u velikoj meri utiču karakteristike nehomogenosti FKM konstrukcije. Zaključuje se da sferna ljuska, od funkcionalnog kompozitnog materijala dobrih osobina, ima povoljnije osobine sigurnosti. Ovo je zbog toga što sferna ljuska smanjene nehomogenosti ima veći obimski napon u poređenju sa sfernom ljuskom povećane nehomogenosti.

INTRODUCTION

From the past few decades, considerable attention has been given to the study of the mechanics of highly deformed shells. This is due to the emergence of high technological applications in mechanical, automobile, biomedical, civil, architecture, aeronautical, and marine engineering. Within structural design, the spherical shell's structural strength and stiffness are highly valuable since they can bear the load. Therefore, it would be more cost-effective to use a spherical pressure vessel rather than a large pressure vessel to store a large volume of pressured liquid or gas. Because they are utilised in nuclear reactors, petrochemical facilities, and oil refineries, spherical pressure containers are the ideal option for high storage fluids. One of the hardest things to do is design materials for high temperature applications. Functionally graded materials (FGMs) allow the spatial variation of material properties to fully use the material everywhere. FGMs are designed with dynamic properties that include changing mechanical properties, chemical, thermal, magnetic, and electrical properties. Few recent researchers, i.e., Nie et al. /12/, Ghannad et al. /8/, and Gharooni et al. /9/ worked on the problems of elasticity for functionally graded pressure vessels. Creep plays an important part in design of structure and, therefore, to make the best use of shells and maximize their performance, creep stress and strain analysis of thick-walled shells is a focus of ongoing research. The creep theory was first suggested by Bailey /1/ for an idealised homogeneous material loaded uniaxially, accounting for the minimum creep rate strain, the transient creep strain, and the initial elastic strain. The analytical method of creep design within a nonlinear range was described by Freudenthal et al. /5/, while Penny /14/ looked at the creep of spherical shells with discontinuities. In 2008, Betton /2/ conducted research on the steady-state creep solution for engineering materials in a spherical vessel under internal pressure. Sharma et al. /17/ used the finite difference approach to determine the strain rates and thermal creep stresses in a functionally graded stainless steel composite cylinder. Nejad et al. /11/ investigated how stresses change with temperature and over time in spherical vessels composed of functionally graded materials. All of these authors used ad hoc, semi-empirical principles, and the infinitesimal strain theory to analyse the issues. A transition theory of elastic-plastic and creep deformation on a sound analytical base was developed by Seth /16/ and applied by a number of authors, including Bhatnagar et al. /3/, who examined an internally pressurized homogeneous, orthotropic rotating cylinder subjected to a steady state creep condition. Under internal pressure, Hulsulkar /10/ examined creep stresses in composite spherical shells. Gupta

/6/ examined the creep behaviour of a thick, isotropic spherical shell at steady state temperature and internal pressure and concluded that incompressible material shells needed high pressure to yield as compared to shells made of compressible material. Under steady-state temperature and internal pressure, Pankaj /13/ studied creep stresses for a thick isotropic spherical shell using finitesimal deformation. In a 2013 study, Sharma et al. /18/ examined the stresses and strains caused by thermal creep in a thick-walled, non-homogenous cylinder under both internal and external pressure. Under external loading a shell cannot change from an elastic state to creep state without passing through an intermediate state called a transition state. Borah /4/ explained at a transition state complete breakdown of the macroscopic structure causes the degeneracy of the shell. Spin, vorticity, rotation, and other nonlinear effects result from the shell's constituent particles rearranging themselves. This indicates that nonlinear factors are crucial at transitions and that ignoring them may lead to an inaccurate representation of the underlying physical reality. A thorough examination of the studies conducted on shells reveals a gap in the analysis of creep stresses caused by nonlinearity in the strain measure. As functionally graded isotropic materials used to create thick-walled spherical shells becoming more and more popular in engineering applications. Therefore, the objective of this research paper is to analyse creep stresses and strain rates under the influence of non-homogeneity parameter using the concept of finite deformation without considering Norton's law. The problem is solved by taking the generalised principal strain measures and the asymptotic solution obtained at transition points of the nonlinear differential equation defining the deformed state of a spherical shell.

This paper is an extension of Nejad et al. /11/ which includes the effect of compressibility without using Norton's law. Considering the non-homogeneity as the compressibility of the material in the shell,

$$\mathbb{C}=\mathbb{C}_0\left(\frac{r}{b}\right)^{-k}, \tag{1}$$

where: $a \le r \le b$; $k \le 0$ is non-homogeneity parameter; and $\mathbb{C}_0$ is a material constant.

The strain measure in generalised form /15/ is defined as

$$e_{ii}=\int_0^{e_{ij}^A}\left[1-2e_{ii}^A\right]^{\frac{n}{2}-1}de_{ii}^A=\frac{1}{n}\left[1-(1-2e_{ii}^A)^{\frac{n}{2}}\right], \tag{2}$$

where: n is nonlinear measure; and e_{ii}^A be finite component of principal strain measure.

OBJECTIVE

The objective of this paper is to evaluate creep stresses and the permanent strains at the steady state condition, resulting from loading of the shell under internal and external pressure with temperature. To explain the transition from elastic to creep, it is observed that the solid first deforms elastically. If the loading is sustained, plastic flow might set which leads to creep state. So, there exists an intermediate state in between the elastic and creep state that is known as transition state. Thus, a differential system defining the creep state should reach a critical value in the transition state. First, we need to recognise the transition state as an asymptotic one which eliminates the need to assume yield conditions, jump conditions, creep strain laws, etc.

MATHEMATICAL FORMULATION

Consider a thick-walled pressurized functionally graded spherical shell of internal and external radii a and b, respectively, as seen in Fig. 1. The shell's non-homogeneity is caused by variations in compressibility $\mathbb{C}$.

In spherical polar coordinates, displacements are given as

$$u_1 = r(1-\kappa),\ u_2 = 0,\ \text{and}\ u_3 = 0, \tag{3}$$

where: κ is a function of r only.

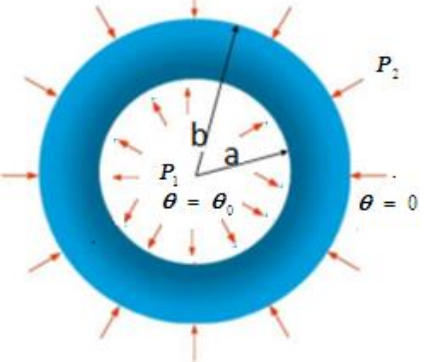


Figure 1. A pressurized functionally graded thick-walled spherical shell with internal pressureP_1 and external pressure P_2 at the boundary.

The finite components of strain are given as follows

$$e_{rr}=\frac{1}{n}\left[1-(r\kappa'+\kappa)^n\right],\quad e_{\phi\phi}=\frac{1}{n}\left[1-\kappa^n\right]=e_{zz},$$
$$e_{r\phi}=e_{\phi z}=e_{zr}=0, \tag{4}$$

where: $\kappa'=\dfrac{d\kappa}{dr}$.

Sokolinokoff's /19/ thermal stress-strain relation for isotropic materials is as follows:

$$T_{ij}=\lambda\delta_{ij}I_1+2\mu e_{ij}-\xi\theta\delta_{ij},\ (i, j = 1, 2, 3), \tag{5}$$

where: $I_1 = e_{kk}$; and T_{ij}, e_{ij} are stress and strain tensors, respectively; λ, μ are Lame's constants; δ_{ij} is the Kronecker delta; $\theta=\left(\dfrac{\theta_0 a}{b-a}\right)\left(\dfrac{b}{r}-1\right)$; and $\xi=\dfrac{3\alpha}{\mathbb{C}}$.

The equation for equilibrium is

$$\frac{d}{dr}(T_{rr})+\frac{(2T_{rr}-2T_{\phi\phi})}{r}=0. \tag{6}$$

Substituting Eq.(5) with the use of Eq.(4) in Eq.(6) yields a nonlinear differential equation in κ as

$$(P+1)^{n-1}P\kappa\frac{dP}{d\kappa}+(P+1)^nP-\frac{(4\mathbb{C}-3)\mathbb{C}'r}{(3-2\mathbb{C})\mathbb{C}\kappa^n n}[1-\kappa^n(P+1)^n]+$$
$$+2(1-\mathbb{C})P-\frac{r[1-\kappa^n]}{\kappa^n n}\frac{[8\mathbb{C}\mathbb{C}'-4\mathbb{C}^2\mathbb{C}-6\mathbb{C}']}{(3-2\mathbb{C}')\mathbb{C}}+\alpha\frac{(3-2\mathbb{C})r}{\kappa^n\mathbb{C}}\times$$
$$\times(\mathbb{C}\theta'-\theta\mathbb{C}')-\frac{2\mathbb{C}'}{n}[1-(P+1)^n]=0, \tag{7}$$

where: $P = r\kappa'/\kappa$; and $\mathbb{C} = 2\mu/(\lambda + 2\mu)$.

Equation (7) indicates that the possible transition points are $P \to -1$ and $P \to \pm\infty$.

Boundary conditions are as follows:

$$[T_{rr}]_{r=a}=-p_1,\quad [T_{rr}]_{r=b}=-p_2. \tag{8}$$

METHOD OF APPROACH

We observe that the material can transit from elastic to creep condition under internal or external loading. Because only principal stresses are examined, the transition can occur either when the principal stresses become critical or when the principal stress differential becomes critical. Several studies (Bhatnagar et al. /3/, Hulsulkar /10/, Gupta et al. /7/, Borah /4/, Sharma /18/) have demonstrated that transitioning across the principal stress differential results in the creep condition at critical point $P \to -1$.

TR can be defined as

$$TR = T_{rr} - T_{\phi\phi} = \frac{2\mu\kappa^n}{n}[1-(P+1)^n] \,. \tag{9}$$

Applying logarithmic differentiation to Eq.(9) with respect to r and substituting in Eq.(7), one gets

$$\frac{d}{dr}(\log TR) = \frac{2\mathbb{C}'}{3-2\mathbb{C}} + \frac{nP}{r} + \Big\{ n(P+1)^n P - \frac{(4\mathbb{C}\mathbb{C}'-3\mathbb{C}')[1-\kappa^n(P+1)^n]}{\kappa^n(3\mathbb{C}-2\mathbb{C}^2)} + 2nP(1-\mathbb{C}) - \left(\frac{1}{\kappa^n}-1\right) \times \frac{(8\mathbb{C}\mathbb{C}'-4\mathbb{C}^2\mathbb{C}'-6\mathbb{C}')}{3\mathbb{C}-2\mathbb{C}^2} + \frac{n\alpha(3-2\mathbb{C})(\mathbb{C}\theta'-\theta\mathbb{C}')}{r\kappa^n\mathbb{C}} - 2\mathbb{C}[1-(P+1)^n] \Big\} / r[1-(P+1)^n] \,. \tag{10}$$

Taking asymptotic value of κ as $P \to -1$ and integrating the above equation, one gets

$$TR = A(3-2\mathbb{C})r^{-3n} \exp f \,, \tag{11}$$

where: A is constant of integration; $f = 2(1-n)(\mathbb{C}_0/k)(b/r)^k - F_1(r) - F_2(r) + F_3(r)$,

$$F_1(r) = \int \frac{1}{\kappa^n}\left(\frac{4\mathbb{C}\mathbb{C}'-3\mathbb{C}'}{3\mathbb{C}-2\mathbb{C}^2}\right)dr \,,$$

$$F_2(r) = \int \left(\frac{1}{\kappa^n}-1\right)\left(\frac{8\mathbb{C}\mathbb{C}'-4\mathbb{C}^2\mathbb{C}'-6\mathbb{C}'}{3\mathbb{C}-2\mathbb{C}^2}\right)dr \,,$$

and

$$F_3(r) = \int \frac{n\alpha(3-2\mathbb{C})(\mathbb{C}\theta'-\theta\mathbb{C}')}{r\kappa^n\mathbb{C}}dr \,. \tag{12}$$

Using the asymptotic value of κ as $P \to -1$ ($\kappa = D/r$, D being a constant), Eq.(12) becomes

$$F_1(r) = \int \frac{r^n}{D^n}\left(\frac{4\mathbb{C}\mathbb{C}'-3\mathbb{C}'}{3\mathbb{C}-2\mathbb{C}^2}\right)dr \,,$$

$$F_2(r) = \int \left(\frac{r^n}{D^n}-1\right)\left(\frac{8\mathbb{C}\mathbb{C}'-4\mathbb{C}^2\mathbb{C}'-6\mathbb{C}'}{3\mathbb{C}-2\mathbb{C}^2}\right)dr \,,$$

and

$$F_3(r) = \int \frac{n\alpha r''(3-2\mathbb{C})(\mathbb{C}\theta'-\theta\mathbb{C}')}{rD''\mathbb{C}}dr \,.$$

Equations (9) and (11) yield

$$T_{rr} - T_{\phi\phi} = ArF \,, \tag{13}$$

where: $F = (3-2\mathbb{C})r^{-3n-1}\exp f$.

Substituting the value of $T_{rr} - T_{\phi\phi}$ from Eq.(13) in Eq.(6) and integrating, we get

$$T_{rr} = B - 2A\int F dr \,, \tag{14}$$

where: B is a constant of integration.

The constants A and B are obtained by using boundary conditions Eq.(8) as

$$A = \frac{p_2 - p_1}{2\int_a^b F dr}, \quad B = \frac{p_2 - p_1}{\int_a^b F dr}\left[\int F dr\right]_{r=b} - p_2 \,.$$

As the non-homogeneity in the shell is due to variable compressibility $\mathbb{C}$ in the radial direction according to power law function given by Eq.(1), thermal creep stresses in a non-homogeneous shell under internal and external pressure are obtained as

$$T_{rr} = -p_2 + A_2\int_r^b F_1 dr, \quad T_{\phi\phi} = -p_2 + A_2\left[\int_r^b F_1 dr - rF_1\right], \tag{15}$$

where: $A_2 = \dfrac{p_2 - p_1}{\int_a^b F_1 dr}$,

$$f_1 = \frac{2(1-n)}{k}\mathbb{C}_0\left(\frac{r}{b}\right)^{-k} - \int \frac{kr^n b^{-1}\left(-4\mathbb{C}_0^2 \frac{r}{b}^{-2k-1} + 3\mathbb{C}_0 \frac{r}{b}^{-k-1}\right)}{D^n\left[3\mathbb{C}_0\left(\frac{r}{b}\right)^{-k} - 2\mathbb{C}_0^2\left(\frac{r}{b}\right)^{-2k}\right]}dr - \int\left(\frac{r^n}{D^n}-1\right)kr^n b^{-1} \times \frac{\left[-8\mathbb{C}_0^2\left(\frac{r}{b}\right)^{-2k-1} + 4\mathbb{C}_0^3\left(\frac{r}{b}\right)^{-3k-1} + 6\mathbb{C}_0\left(\frac{r}{b}\right)^{-k-1}\right]}{3\mathbb{C}_0\left(\frac{r}{b}\right)^{-k} - 2\mathbb{C}_0^2\left(\frac{r}{b}\right)^{-2k}}dr + \int \frac{nr^n\alpha\theta_0\left[3-2\mathbb{C}_0\left(\frac{r}{b}\right)^{-k}\right]\left[\mathbb{C}_0\left(\frac{r}{b}\right)^{-k}\frac{a}{b-a}\left(-\frac{b}{r^2}\right) + k\frac{a}{b-a}\left(\frac{b}{r}-1\right)\mathbb{C}_0\left(\frac{r}{b}\right)^{-k-1}\right]}{rD^n\mathbb{C}_0\left(\frac{r}{b}\right)^{-k}}dr,$$

$$F_1 = r^{-3n-1}\left[3-2\mathbb{C}_0\left(\frac{r}{b}\right)^{-k}\right]\exp f_1 \,.$$

Equation (15) evaluates thermal creep stresses for a thick-walled spherical shell under internal and external pressure with varying compressibility.

Introducing the following non-dimensional components as

$$R_0 = \frac{a}{b}, \quad R = \frac{r}{b}, \quad \sigma_{rr} = \frac{T_{rr}}{Y}, \quad \sigma_{\phi\phi} = \frac{T_{\phi\phi}}{Y}, \quad \sigma_{zz} = \frac{T_{zz}}{Y}, \quad P_1 = \frac{p_1}{Y}, \quad P_2 = \frac{p_2}{Y}.$$

The Eq.(15) in non-dimensional form can be written as

$$\sigma_{rr} = A_3\int_R^1 F_2 dR - P_2, \quad \sigma_{\phi\phi} = A_3\left(\int_R^1 F_2 dr - bRF_2\right) - P_2 = \sigma_{zz}, \tag{16}$$

where: $A_3 = \dfrac{P_2 - P_1}{\int_{R_0}^1 F_2 dR}$, $F_2 = (Rb)^{-3n-1}(3-2\mathbb{C}_0(R)^{-k})\exp f_2$,

$$f_2 = \frac{2(1-n)}{k}\mathbb{C}_0(R)^{-k} - \int \frac{k(bR)^n[-4\mathbb{C}_0^2(bR)^{-2k-1} + 3\mathbb{C}_0(bR)^{-k-1}]}{D^n[3\mathbb{C}_0(R)^{-k} - 2\mathbb{C}_0^2(R)^{-2k}]}dR -$$

$$-\int\left(\frac{(bR)^n}{D^n}-1\right)\frac{k(bR)^n[-8\mathbb{C}_0^2(R)^{-2k-1}+4\mathbb{C}_0^3(R)^{-3k-1}+6\mathbb{C}_0(R)^{-k-1}]}{3\mathbb{C}_0(R)^{-k}-2\mathbb{C}_0^2(R)^{-2k}}dR+$$

$$+\int\frac{n(bR)^n\theta_1[3-2\mathbb{C}_0(R)^{-k}]\left[\mathbb{C}_0(R)^{-k}\left(-\frac{R_0R^{-2}}{(1-R_0)b}\right)+k\frac{R_0(1-R)}{(1-R_0)Rb}\mathbb{C}_0(R)^{-k-1}\right]bdR}{RbD^n\mathbb{C}_0(R)^{-k}}.$$

STRAIN RATES

When the creep sets in, the strain should be replaced by strain rates. The thermal stress-strain relation Eq.(5) can be written as

$$\dot{e}_{ij}=\frac{1+\nu}{Y}T_{ij}-\frac{\nu}{Y}\delta_{ij}\Theta+\frac{3\alpha\theta}{\mathbb{C}}, \quad (17)$$

where: $\dot{e}_{ij}$ is strain rate tensor with respect to flow parameter t; and $\Theta = T_{rr} + T_{\phi\phi} + T_{zz}$; and $\nu = (1 - C)/(2 - C)$ is Poisson's ratio.

Equation (4) on differentiating with respect to t, gives

$$\dot{e}_{\phi\phi}=-\kappa^{n-1}\dot{\kappa}, \quad (18)$$

for Swainger measure (n = 1)

$$\dot{\varepsilon}_{\phi\phi}=-\dot{\kappa}. \quad (19)$$

where: $\dot{\varepsilon}_{\phi\phi}$ is Swainger's strain measure.

The asymptotic value of κ from Eq.(9), as $P \to -1$, is

$$\kappa=\left(\frac{n}{2\mu}\right)^{\frac{1}{n}}(T_{rr}-T_{\phi\phi})^{\frac{1}{n}}. \quad (20)$$

Using Eqs. (18), (19), and (20) in Eq.(17), we get

$$\dot{\varepsilon}_{\phi\phi}=\left[\frac{n}{2\mu}(T_{rr}-T_{\phi\phi})\right]^{\frac{1}{n}-1}\left[\frac{1+\nu}{Y}T_{ij}-\frac{\nu}{Y}\delta_{ij}\Theta+\frac{3\alpha\theta}{\mathbb{C}}\right], \text{ i.e.,}$$

$$\dot{\varepsilon}_{rr}=\left[\frac{3-2\mathbb{C}}{N(2-\mathbb{C})}\right]^{N-1}(\sigma_{rr}-\sigma_{\phi\phi})^{N-1}\left[\sigma_{rr}-\left(\frac{1-\mathbb{C}}{2-\mathbb{C}}\right)(\sigma_{\phi\phi}+\sigma_{zz})+\mathbb{C}\right], \quad (21)$$

$$\dot{\varepsilon}_{\phi\phi}=\left[\frac{3-2\mathbb{C}}{N(2-\mathbb{C})}\right]^{N-1}(\sigma_{rr}-\sigma_{\phi\phi})^{N-1}\left[\sigma_{\phi\phi}-\left(\frac{1-\mathbb{C}}{2-\mathbb{C}}\right)(\sigma_{rr}+\sigma_{zz})+\frac{3\alpha\theta}{\mathbb{C}}\right], \quad (22)$$

and

$$\dot{\varepsilon}_{zz}=\left[\frac{3-2\mathbb{C}}{N(2-\mathbb{C})}\right]^{N-1}(\sigma_{rr}-\sigma_{\phi\phi})^{N-1}\left[\sigma_{zz}-\left(\frac{1-\mathbb{C}}{2-\mathbb{C}}\right)(\sigma_{rr}+\sigma_{\phi\phi})+\frac{3\alpha\theta}{\mathbb{C}}\right]. \quad (23)$$

These are the constitutive equations for determining the creep strains for $N = 1/n$.

NUMERICAL DISCUSSION

The material property of the spherical shell fabricated of functionally graded material (FGM) is defined as: compressibility coefficient $\mathbb{C}_0 = 0.5$. The material considered is mild steel. The internal and external radii of the shell are taken as a = 1 and b = 2, respectively. The parameters of compressibility are considered as k = -5, -3, -1. To observe the influence of various parameters, i.e., temperature $\theta_1 = \alpha\theta_0$, strain measure N, and pressure P_1 and P_2 on thick-walled spherical shell fabricated of (FGM), Table 1 and graphs between radii ratio and creep stresses for various pressure and temperature combinations show effects of various parameters, such as temperature, strain measurement, and pressure, on thick-walled spherical shells fabricated of FGM. When the pressure vessel is exposed to assumed pressure, the material comprising the vessel is subjected to pressure load, and hence, stressed from all directions. The principal stresses resulting from this pressure are functions of the radius of the element under consideration due to the spherical shape of the pressure vessel, as well as the applied pressure. Creep strain distribution is also shown graphically.

The mechanical property of the sphere such as compressibility is assumed to be varying through the radius. Figures 2-7 have been drawn for creep stresses with various temperature and pressure combinations for linear and nonlinear strain measure when internal pressure is more than that of external pressure. Radial and circumferential stresses are calculated at the internal and external surface of the spherical shell. Figures 2-3 are drawn with internal pressure P_1 = 1.0 and external pressure P_2 = 0.3. Distributions of creep stress components σ_{rr} and $\sigma_{\phi\phi}$ for values k = -5, -3, -1 are plotted in Fig. 2. It must be noted from Fig. 2a that the circumferential stress increases as k decreases and that the circumferential stress for different values of k approaches tensile from compressive. The absolute maximum of circumferential stress occurs at the outer edge. It means that the maximum shear stress which is $\tau_{\max} = (\sigma_{\phi\phi} - \sigma_{rr})/2$ for each value of k, will be very high on the outer surface of the vessel. It is observed from Fig. 2 that circumferential stresses are maximum at outer surface with linear measure. Also, these stresses are maximum at external surface for highly non-homogeneous (k = -1) spherical shell, as compared to less non-homogeneous spherical shell (k = -5) at room temperature. Circumferential stress shown in Fig. 2b remain tensile throughout, and decrease with increasing radius for k = -5, -3, -1 and reach the minimum value somewhere towards the inner radius followed by an increase with a further increase of the radius. It can be seen in Fig. 2b that with the introduction of thermal effects, the circumferential stresses for the highly non-homogeneous spherical shell are tensile, while these stresses approach tensile from compressive for the less non-homogeneous spherical shell. Also, with increase in thermal effects, these circumferential stresses decrease significantly which can be seen in Table 1.

With the change in measure from linear to nonlinear, the circumferential stresses decrease significantly as can be seen in Fig. 3. Also, it is noticed in Fig. 3a that circumferential stress approaches tensile from compressive without thermal effects. With the introduction of temperature, these circumferential stresses become tensile as can be seen from Fig. 3b. Also, it is noticed that with increase in temperature, circumferential stresses for the highly non-homogeneous spherical shell are maximum at internal surface, while these circumferential stresses are maximum at the external surface for less non-homogeneous spherical shell. As can be seen from Table 1, with increase in nonlinearity of the measure from N = 3 to N = 5, these circumferential stresses decrease significantly and are maximum at the external surface. Figures 4-5 show the effect of increased internal pressure on circumferential stresses. With increase in internal pressure, circumferential stresses increase, as can be seen in Figs. 4-5, and are tensile in nature. These stresses decrease with increase in temperature as can be seen in Figs. 4b and 5b. It has also been noticed that highly non-homogeneous spherical shell is having a high circumferential stress, as compared to the less non-homogeneous spherical shell at room temperature, while reverse is the case with temperature.

Table 1. Thermo-creep circumferential stresses in a thick-walled FGM spherical shell under internal and external pressure.

Pressure		$P_1 = 1$, $P_2 = 0.3$			$P_1 = 1.7$, $P_2 = 0.3$			$P_1 = 1.7$, $P_2 = 1$		
Non-homogeneity	θ_i	$R = r/b$			$R = r/b$			$R = r/b$		
		0.5	0.75	1	0.5	0.75	1	0.5	0.75	1
		$n = 1$ (linear measure)								
$k = -5$	0	-0.471	0.49446	0.85299	-0.64226	1.28892	2.00598	-1.1711	-0.2055	0.15299
	2	-0.410	0.36142	1.30361	-0.52051	1.02283	2.90723	-1.1103	-0.3386	0.60361
	4	-0.377	0.362	1.26968	-0.45401	1.024	2.83936	-1.077	-0.338	0.56968
$k = -3$	0	-0.520	0.41975	1.21368	-0.74031	1.13949	2.72737	-1.2202	-0.2803	0.51368
	2	-0.3955	0.35008	1.37279	-0.49108	1.00015	3.04558	-1.0955	-0.3499	0.67279
	4	-0.3503	0.35765	1.29894	-0.4006	1.0153	2.89788	-1.0503	-0.3423	0.59894
$k = -1$	0	-0.8289	0.0738	2.97937	-1.35783	0.44761	6.25874	-1.5289	-0.6262	2.27937
	2	0.12481	0.37633	0.77065	0.549627	1.05265	1.84131	-0.5752	-0.3237	0.07065
	4	-0.0314	0.37022	0.94951	0.237252	1.04044	2.19903	-0.7314	-0.3298	0.24951
		$n = 1/3$ (nonlinear measure)								
$k = -5$	0	-0.2757	0.4539	0.83788	-0.25136	1.20779	1.97576	-0.9757	-0.2461	0.13788
	2	-0.0055	0.42164	0.70664	0.289061	1.14329	1.71328	-0.7055	-0.2784	0.00664
	4	0.29129	0.36521	0.66655	0.882569	1.03041	1.6331	-0.4087	-0.3348	-0.0335
$k = -3$	0	-0.2579	0.43881	0.89813	-0.21575	1.17761	2.09625	-0.9579	-0.2612	0.19813
	2	0.01716	0.4027	0.77569	0.334326	1.1054	1.85138	-0.6828	-0.2973	0.07569
	4	0.37875	0.3558	0.63802	1.0575	1.0116	1.57603	-0.3212	-0.3442	-0.062
$k = -1$	0	-0.2198	0.42069	0.93719	-0.1397	1.14139	2.17439	-0.9198	-0.2793	0.23719
	2	0.52841	0.35948	0.48209	1.35682	1.01896	1.26419	-0.1716	-0.3405	-0.2179
	4	3.38017	0.02795	-0.2859	7.06035	0.35589	-0.2719	2.68017	-0.6721	-0.9859
		$n = 1/5$ (nonlinear measure)								
$k = -5$	0	-0.1475	0.4654	0.67179	0.005093	1.2308	1.64359	-0.8475	-0.2346	-0.0282
	2	-0.0397	0.45191	0.62181	0.220581	1.20382	1.54362	-0.7397	-0.2481	-0.0782
	4	0.07833	0.43633	0.57324	0.456658	1.17266	1.44648	-0.6217	-0.2637	-0.1268
$k = -3$	0	-0.1278	0.45024	0.72452	0.044433	1.20048	1.74904	-0.8278	-0.2498	0.02452
	2	-0.0157	0.43409	0.68043	0.268584	1.16818	1.66085	-0.7157	-0.2659	-0.0196
	4	0.10309	0.41709	0.63686	0.50618	1.13419	1.57371	-0.5969	-0.2829	-0.0631
$k = -1$	0	-0.1004	0.42866	0.78462	0.099175	1.15732	1.86923	-0.8004	-0.2713	0.08461
	2	0.13869	0.40645	0.64503	0.577377	1.1129	1.59006	-0.5613	-0.2936	-0.055
	4	0.53798	0.36463	0.45201	1.37595	1.02926	1.20401	-0.162	-0.3354	-0.248

Figures 6-7 show the effect of increasing external pressure to circumferential stresses. With increase in external pressure, circumferential stresses decrease significantly as can be seen in Figs. 6-7. Also, without thermal effects these circumferential stresses are approaching tensile from compressive for linear measure, as can be seen in Fig. 6a.

With the change in measure from linear to nonlinear and increase of temperature, stresses decrease significantly as can be seen in Figs. 7a and 7b.

Figures 8-11 are drawn for creep strain rates with various temperatures and pressure combinations for linear and nonlinear measure when internal pressure is higher than external pressure.

It is observed from Fig. 8 that circumferential strain rates are maximum at the external surface with linear measure. Also, these strain rates are maximum at external surface for highly non-homogeneous spherical shell, as compared to the less non-homogeneous spherical shell at room temperature. Also, it is noticed that circumferential strain rates for less non-homogeneous spherical shell are tensile, while these strain rates approach tensile from compressive for the highly non-homogeneous spherical shell. With the introduction of thermal effects, strain rates are maximum at the internal surface. Also, less non-homogeneous spherical shell is having maximum strain rate as compared to the highly non-homogeneous spherical shell. It is observed from Fig. 9 that with the change in measure from linear to nonlinear, strain rates decrease. As can be seen from Figs. 8b and 9b that with the introduction of thermal effects, strain rates increase significantly which further increase with increase in temperature.

At room temperature circumferential strain rates are maximum at the external surface, while with the introduction of temperature, strain rates are maximum at the internal surface. It is noticed from Figs. 10-11 that with increase in internal pressure, strain rates increase significantly. At room temperature, these circumferential strain rates are maximum at the external surface, while with the introduction of thermal effects, these strain rates are maximum at internal surface for linear and nonlinear measure. Also, it is noticed that at room temperature, circumferential strain rates are high for highly non-homogeneous thick-walled spherical shell, as compared to the less non-homogeneous spherical shell, while reverse is the case with the introduction of thermal effects. Also, circumferential stresses are maximum for the less non-homogeneous spherical shell, as compared to the highly non-homogeneous spherical shell. The internal pressure must exceed external pressure in order to avoid inward buckling.

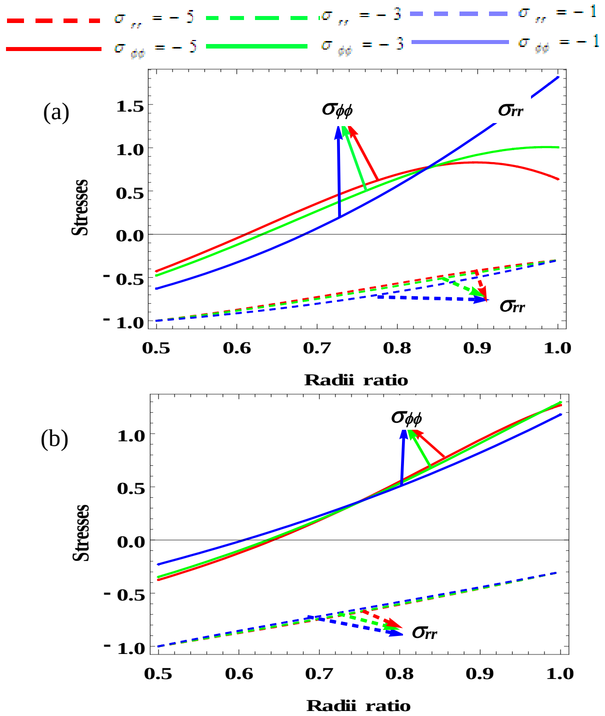


Figure 2. Thermal creep stresses (θ_1 = 0, 2 resp. a) and b)) for thick-walled FGM spherical shell under pressure P_1 = 1 and P_2 = 0.3, with linear measure n = 1.

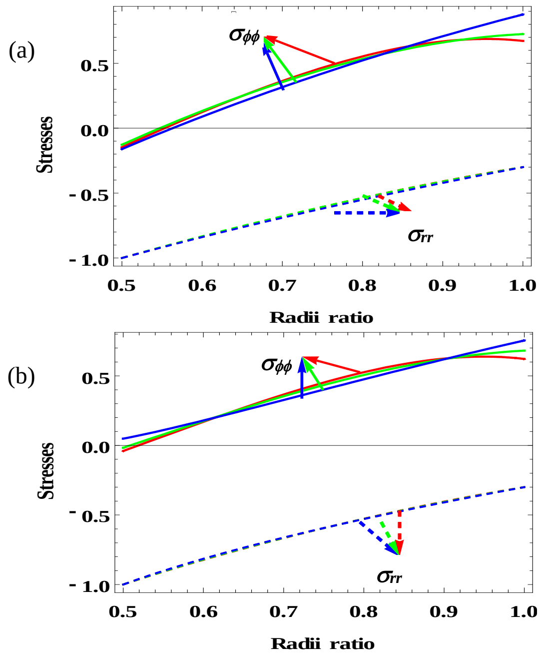


Figure 3. Thermal creep stresses (θ_1 = 0, 2 resp. a) and b)) for thick-walled FGM spherical shell under pressure P_1 = 1 and P_2 = 0.3, with nonlinear measure n = 1/5.

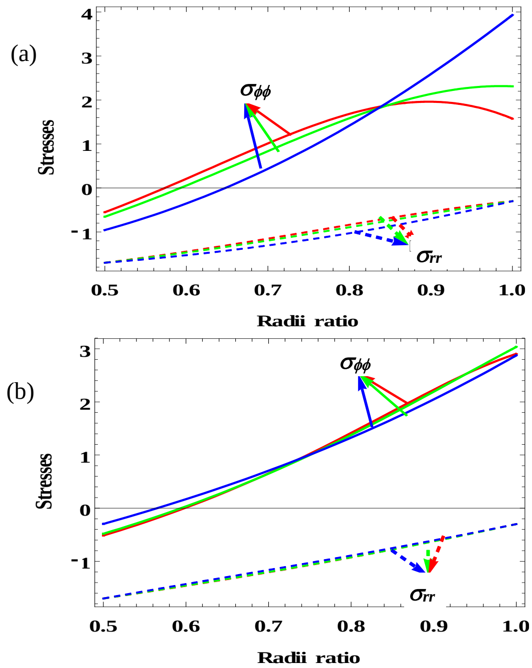


Figure 4. Thermal creep stresses (θ_1 = 0, 2 resp. a) and b)) for thick-walled FGM spherical shell under pressure P_1 = 1.7 and P_2 = 0.3, with linear measure n = 1.

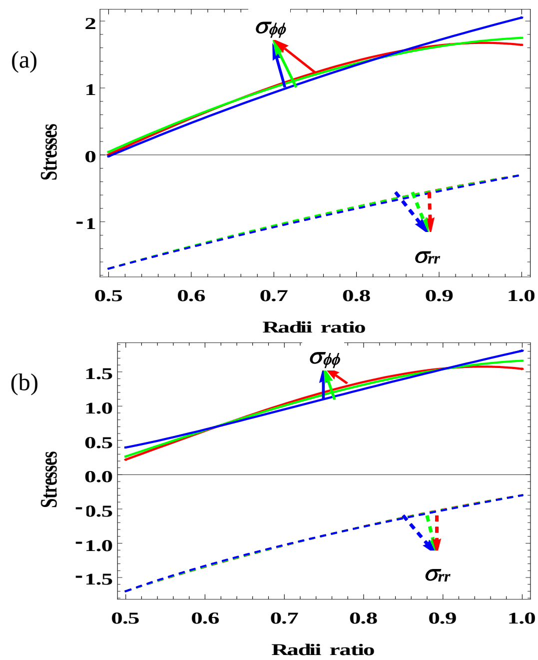
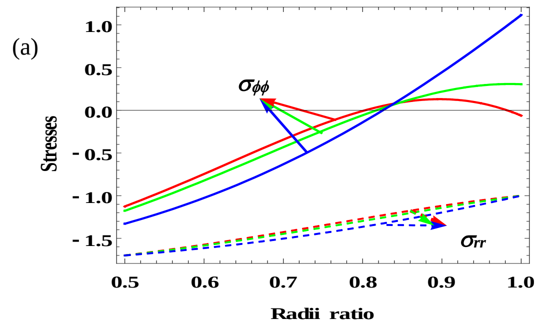


Figure 5. Thermal creep stresses (θ_1 = 0, 2 resp. a) and b)) for thick-walled spherical shell under pressure P_1 = 1.7 and P_2 = 0.3, with nonlinear measure n = 1/5.



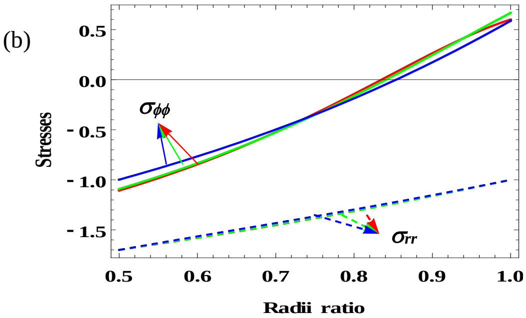


Figure 6. Thermal creep stresses ($\theta_1 = 0, 2$ resp. a) and b)) for thick-walled FGM spherical shell under pressure $P_1 = 1.7$ and $P_2 = 1$, with linear measure $n = 1$.

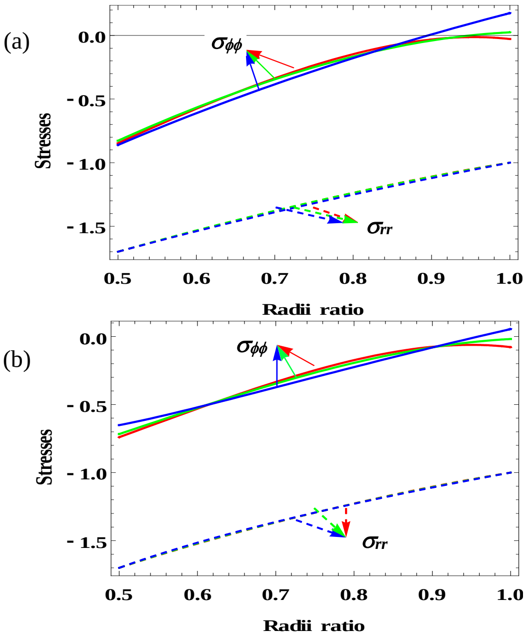


Figure 7. Thermal creep stresses ($\theta_1 = 0, 2$ resp. a) and b)) for thick-walled FGM spherical shell under pressure $P_1 = 1.7$ and $P_2 = 1$, with nonlinear measure $n = 1/5$.

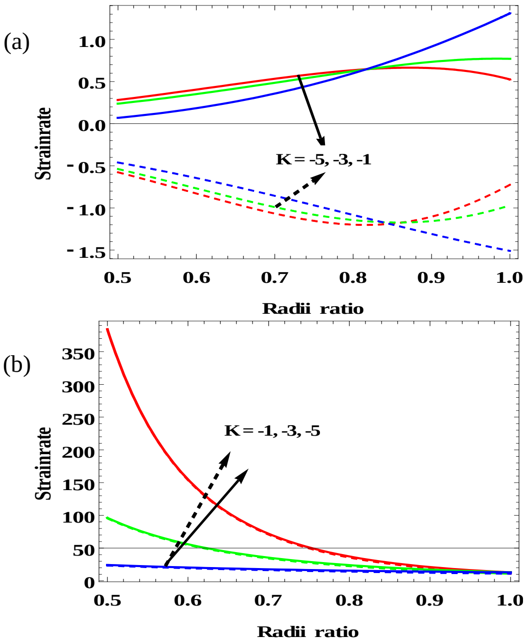


Figure 8. Creep strain rates ($\theta_1 = 0, 2$ resp. a) and b)) for thick-walled FGM spherical shell under pressure $P_1 = 1$ and $P_2 = 0.3$, with linear measure $n = 1$.

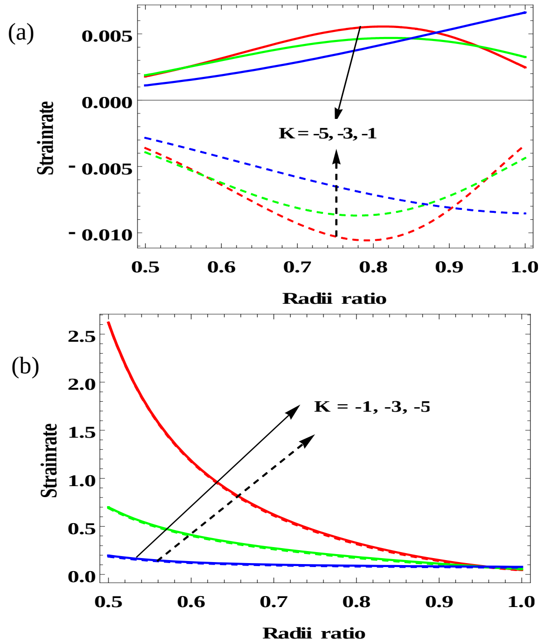


Figure 9. Creep strain rates ($\theta_1 = 0, 2$ resp. a) and b)) for thick-walled FGM spherical shell under pressure $P_1 = 1$ and $P_2 = 0.3$, with nonlinear measure $n = 1/5$.

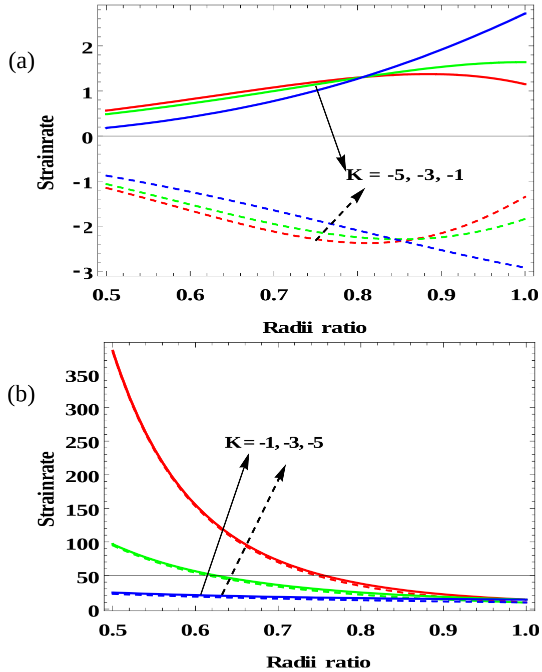
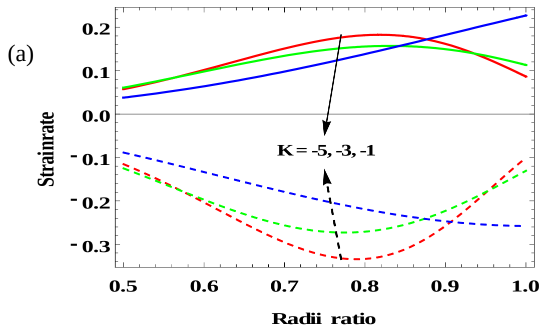


Figure 10. Creep strain rates ($\theta_1 = 0, 2$ resp. a) and b)) for thick-walled FGM spherical shell under pressure $P_1 = 1.70$ and $P_2 = 0.3$, with linear measure $n = 1$.



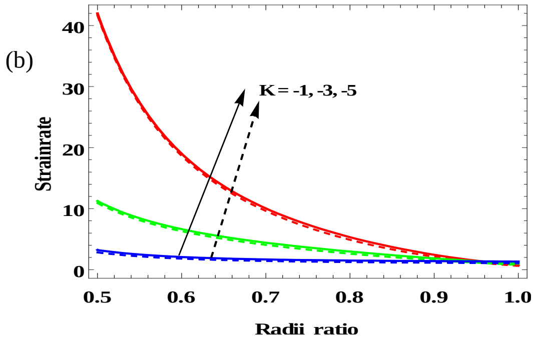


Figure 11. Creep strain rates (θ_1 = 0, 2 resp. a) and b)) for thick-walled FGM spherical shell under pressure P_1 = 1.70, and P_2 = 0.3, with nonlinear measure n = 1/5.

CONCLUSIONS

In this paper, thermal creep stresses and thermal creep strain rates are obtained for the thick-walled spherical shell made up of functionally graded material under internal and external pressure using the concept of transition theory and generalised principal strain measures. It has been concluded from the numerical discussion that by introducing a suitably chosen temperature gradient, the material in-homogeneity parameter has a significant influence on the mechanical behaviour of thick-walled spherical shells made up of functionally graded materials. Highly non-homogeneous thick-walled spherical shell with nonlinear strain measure is on the safer side of the design, as compared to less the non-homogeneous thick-walled spherical shell, because less non-homogeneous spherical shell is having high circumferential stresses, as compared to the highly non-homogeneous spherical shell, which leads to the idea of 'stress saving' thus minimizing the possibility of fracture of the spherical shell. This problem can help engineers to design a specific FGM sphere that can meet some special requirements.

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