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EFFECT OF MAGNETIC FIELD ON THERMAL INSTABILITY IN A POROUS MEDIUM LAYER SATURATED BY A JEFFREY NANOFLUID USING BRINKMAN MODEL: FREE-FREE, RIGID-RIGID, RIGID-FREE BOUNDARY CONDITIONS

UTICAJ MAGNETNOG POLJA NA TOPLOTNU NESTABILNOST KOD POROZNE SREDINE ZASIĆENE JEFFREY NANOFLUIDOM PRIMENOM BRINKMAN MODELA: GRANIČNIH USLOVA SLOBODNO-SLOBODNO, KRUTO-KRUTO, KRUTO-SLOBODNO

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- Jeffrey nanofluid
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- Brownian motion
- Brinkman model

Abstract

In this paper, we have used the Darcy-Brinkman model to study the influence of a magnetic field on a Jeffrey nanofluid layer saturated with a porous medium. The influence of thermophoresis and Brownian motion is incorporated into the Buongiorno model used for nanoparticles. Normal mode analysis and Galerkin method are used to analyse conservation equations. We have considered three different boundary conditions: free-free, rigid-rigid, and rigid-free. For stationary convection, the effects of Darcy-Brinkman number, Jeffrey parameter, nanoparticle Rayleigh number, Lewis number, porosity, modified diffusivity ratio and Chandrasekhar number for all the above-mentioned boundary conditions are investigated analytically and graphically.

Ključne reči
- Jeffrey nanofluid
- toplotna nestabilnost
- Braunovo kretanje
- Brinkman model

Izvod

U ovom radu smo primenili Darsi-Brinkman model za proučavanje uticaja magnetnog polja na Jeffrey nanofluidni sloj koji je zasićen poroznom sredinom. Uticaji termoforeze i Braunovog kretanja su uvedeni u Buonđornov model koji se primenjuje za nanočestice. Za analizu jednačina održanja, upotrebljeni su analiza u normalnom modu i metoda Galerkin. Razmotrili smo tri različita granična uslova: slobodno-slobodno, kruto-kruto i kruto-slobodno. U uslovima stacionarne konvekcije, za gore navedene granične uslove, analitički i grafički su proučeni uticaji Darsi Brinkman broja, Jeffrey parametra, Rejlejevog broja za nanočestice, Luisovog broja, poroznosti, modifikovanog odnosa difuzivnosti i Čandrasekarovog broja.

INTRODUCTION

A nanofluid is a fluid that contains particles the size of nanometers called nanoparticles. The term ‘nanofluid’ was initially utilized by Choi /3/. The important feature of the nanofluid is the enhancement of heat transmission which was reported by Masuda et al. /6/. Convection of nanofluids was analysed by Buongiorno /1/ and has generated a lot of interest in recent years. Nield and Kuznetsov /10/ found the onset of convection in a nanofluid layer. Nano-sized particles of metal oxides are used in several fields related to chemical engineering, medicine, and electronics. The convection in a layer saturated using two nanofluids was studied by Yadav et al. /27/. Non-Newtonian fluids are used in various fields of science and engineering like textiles, food processing, geophysics, chemical and biological industries. Jeffrey fluid is a non-Newtonian fluid with high shear viscosity and linear viscoelasticity properties. Jeffrey’s fluid model is less time derivative rather than convective derivative. The onset of stationary convection on Jeffrey nanofluid layer saturated with a porous medium was investigated by Sharma et al. /14/. Thermal convective instability in a Jeffrey nanofluid saturating with a porous medium: rigid-rigid and rigid-free boundary conditions was studied by Sharma et al. /23/. They found a numerical solution to various problems involving the stability of fluid layers as temperature decreases upwards. Nield and Kuznetsov /8-9/ and Nield /5/ have investigated the thermal instability in a porous layer saturated with a nanofluid. Tzou /24/ studied the instability of the nanofluid layer through experiments. Sheu /13/ has examined the thermal instability in a layer of porous medium saturated with a viscous nanofluid. The beginning of convection of thermal instability of a porous medium layer saturating a Jeffrey nanofluid was identified by Rana and Gautam /12/. The study of flow through porous layers has various applications in petroleum reservoirs, Earth’s molten cores, fluid filters, heat exchanger, human lungs, etc. Porous media improve heat conductivity by increasing the contact area between liquid, solid, and nanofluids. The Rayleigh instability of a thermal boundary layer flow through a medium that is porous was studied by Wooding /26/. A detailed study of convection in a porous media was given by Nield and Bejan /7/. Starting with the fundamental Darcy model, the investigation into porous media progressed to the Darcy-Brinkman model. Rana et al. /11/ have investigated the impact of suspended particles on thermal convection in Rivlin-Ericksen nanofluid layer saturating a Darcy Brinkman model. Sand, soil, sandstone are some examples of porous medium.

Several decades ago, it had been figured out how the magnetic field affects the beginning of convection. The influence of magnetic field on the Rayleigh Bénard convection in nanofluids has its important role in chemical engineering, biochemical engineering, industry, and many physical phenomena concerning geophysics and astrophysics. Magneto convection in a nanofluid layer was studied by Gupta et al. /25/. Many researchers /15-22/ have involved various types of fluid in their research work. Bhatia and Steiner /2/ and Chandrasekhar /4/ studied the thermal instability in a visco-elastic fluid layer in hydromagnetics. Because of its numerous applications in chemistry, physics, engineering science, and other fields, studying the magnetic field effects on fluids has become an important active area of research in recent years. In this paper, we have studied the impact of the magnetic field on thermal instability in Jeffrey nanofluid with a porous medium. To the best of the authors' knowledge, no research has been published yet on this topic.

MATHEMATICAL MODEL

Let us consider a layer of Jeffrey nanofluid contained between two planes $z^* = 0$ and $z^* = H$. The layer of fluid is heated from below and working upwards direction with a gravity force $g(0, 0, -g)$. The temperature and volumetric fraction at the lower wall are T_h^* and ϕ_0^* while at the upper wall are T_c^* and ϕ_1^*, respectively. We consider a porous medium with porosity ε, permeability K, and hydrostatic pressure p.

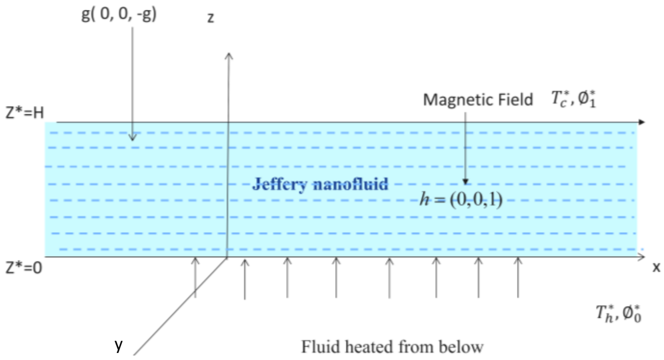

Figure 1. Physical sketch of the problem.

GOVERNING EQUATIONS

The conservation equations of mass, momentum, thermal energy and nanoparticles are given by Nield and Kuznetsov /9/ under Boussinesq approximation in a porous medium, respectively

$$\nabla^* \mathbf{v}_D^* = 0\,, \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\frac{\partial \mathbf{v}_D^*}{\partial t^*} = -\nabla^* p^* + \tilde{\mu}\nabla^{*2}\mathbf{v}_D^* - \frac{\mu}{K(1+\lambda)}\mathbf{v}_D^* + \frac{\mu_e}{4\pi}(h^*\nabla^*)h^* + \left[\phi^*\rho_p + (1-\phi^*)\{\rho_f(1-\beta(T-T_c^*))\}\right]g\,, \tag{2}$$

$$(\rho c)_m \frac{\partial T^*}{\partial t^*} + (\rho c)_f\, \mathbf{v}_D^*.\nabla^* T^* = k_m \nabla^{*2}T^* + \varepsilon(\rho c)_p \times \left[D_B\nabla^*\phi^*.\nabla^*T^* + (D_T/T_c^*)\nabla^*T.\nabla^*T^*\right], \tag{3}$$

$$\frac{\partial \phi^*}{\partial t^*} + \frac{1}{\varepsilon}\mathbf{v}_D^*.\nabla^*\phi^* = D_B\nabla^{*2}\phi^* + (D_T/T_c^*)\nabla^{*2}T^*\,, \tag{4}$$

The Maxwell equation is given as

$$\frac{\partial h^*}{\partial t^*} + (\mathbf{v}_D^*\nabla^*)h^* = (h^*\nabla^*)\mathbf{v}_D^* + \eta\nabla^{*2}h^*\,, \tag{5}$$

$$\nabla^*.h^* = 0\,, \tag{6}$$

we write $\mathbf{v}_D^* = (u^*, v^*, w^*)$.

Here, ρ_f, μ, β, η, and h are the density, viscosity, volumetric expansion coefficient of the fluid, fluid electrical resistivity, and magnetic field, respectively, while ρ_p is the density of particles. We have introduced effective viscosity $\tilde{\mu}$, effective heat capacity $(\rho c)_m$, k_m effective thermal conductivity of the porous medium, and λ is the Jeffrey parameter. The coefficients that appear in Eqs. (3) and (4) are the Brownian diffusion coefficient D_B and thermophoretic diffusion coefficient D_r. On the boundaries, we use the assumption that the volumetric fraction and temperature of the nanoparticles are both constant. According to Kuznetsov and Nield /5/, the boundary conditions are

$$w^* = 0,\ \frac{\partial w^*}{\partial z^*} + \lambda_1 H\frac{\partial^2 w^*}{\partial z^{*2}} = 0,\ T^* = T_h^*,\ \phi^* = \phi_0^* \ \text{ at } z^* = 0, \tag{7}$$

$$w^* = 0,\ \frac{\partial w^*}{\partial z^*} - \lambda_2 H\frac{\partial^2 w^*}{\partial z^{*2}} = 0,\ T^* = T_c^*,\ \phi^* = \phi_1^* \ \text{ at } z^* = H. \tag{8}$$

As follows, we present dimensionless variables. We define

$$(x,y,z) = (x^*, y^*, z^*)/H,\quad t = t^*\alpha_m/\sigma H^2,$$
$$(u,v,w) = (u^*, v^*, w^*)H/\alpha_m,\quad p = p^*K/\mu\alpha_m,$$
$$\phi = \frac{\phi^* - \phi_0^*}{\phi_1^* - \phi_0^*},\quad T = \frac{T^* - T_c^*}{T_h^* - T_c^*},\quad h = \frac{h^*}{h_0}\,, \tag{9}$$

where: $\alpha_m = \dfrac{k_m}{(\rho c)_f}$, $\sigma = \dfrac{(\rho c)_m}{(\rho c)_f}$.

Equations (1)-(8) take the form

$$\nabla.\mathbf{v} = 0\,, \tag{10}$$

$$\frac{1}{\sigma V_a}.\frac{\partial \mathbf{v}}{\partial t} = -\nabla p + D_a\nabla^2\mathbf{v} - \frac{\mathbf{v}}{1+\lambda} + Q\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}(h\nabla)h - R_m\hat{e}_z - R_n\phi\hat{e}_z + R_aT\hat{e}_z\,, \tag{11}$$

$$\frac{\partial T}{\partial t} + \mathbf{v}.\nabla T = \nabla^2 T + \frac{N_B}{L_e}\nabla\phi.\nabla T + \frac{N_A N_B}{L_e}\nabla T.\nabla T\,, \tag{12}$$

$$\frac{1}{\sigma}\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{v}.\nabla\phi = \frac{1}{L_e}\nabla^2\phi + \frac{N_A}{L_e}\nabla^2 T\,, \tag{13}$$

$$\frac{\partial h}{\partial t} + \sigma(\mathbf{v}.\nabla)h = \sigma(h\nabla).\mathbf{v} + \sigma\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}\nabla^2 h\,, \tag{14}$$

$$\nabla h = 0\,, \tag{15}$$

$$w = 0,\ \frac{\partial w}{\partial z} + \lambda_1\frac{\partial^2 w}{\partial z^2} = 0,\ T = 1,\ \phi = 0 \ \text{ at } z = 0\,, \tag{16}$$

$$w = 0,\ \frac{\partial w}{\partial z} - \lambda_2\frac{\partial^2 w}{\partial z^2} = 0,\ T = 0,\ \phi = 1 \ \text{ at } z = 1\,. \tag{17}$$

Here, $\mathrm{Pr}_1 = \mu/\rho\alpha_m$, is the Prandtl number, $\mathrm{Pr}_2 = \mu/\rho\eta$ is the magnetic Prandtl number, $D_a = K/H^2$ is the Darcy number, $D_a = \tilde{\mu}K/\mu H^2$ is the Darcy-Brinkman number, $L_e = \alpha_m/D_B$ is the Lewis number, $V_a = \varepsilon\mathrm{Pr}/D_a$ is the Vadasz number, $Q = \mu_e h_0^2 K/4\pi\eta\mu$ is the Chandrasekhar number, $R_a = \rho g\beta KH(T_h^* - T_c^*)/\mu\alpha_m$ is thermal Darcy-Rayleigh number, $R_m = [\rho_p\phi_1^* + \rho(1-\phi_1^*)]gKH/\mu\alpha_m$ is basic density Rayleigh number, $R_n = (\rho_p - \rho)(\phi_1^* - \phi_0^*)gKH/\mu\alpha_m$ is the concentration Rayleigh number, $N_A = D_T(T_h^* - T_c^*)/D_B T_c^*(\phi_1^* - \phi_0^*)$ is the modified diffusivity rate, and $N_B = (\rho c)_p(\phi_1^* - \phi_0^*)/(\rho c)_m$ is the modified particle-density increment, respectively.

BASIC SOLUTIONS

The time independent fundamental states for nanofluids are expressed as Sheu /13/ and Rana et al. /11/

$$\mathbf{v}=0,\ T=T_b(z),\ \phi=\phi_b(z),\ p=p_b(z),\ h=(0,0,1)\,. \tag{18}$$

Using Eq.(18) in Eqs.(10)-(13), those equations reduce to

$$-\frac{dp_z}{dz}+Q\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}\left(\frac{\partial h}{\partial z}\right)\hat{e}_z-R_m\hat{e}_z-R_n\phi_b\hat{e}_z+R_aT_b\hat{e}_z=0\,, \tag{19}$$

$$\frac{d^2T_b}{dz^2}+\frac{N_B}{L_e}\frac{d\phi}{dz}.\frac{dT_b}{dz}+\frac{N_AN_B}{L_e}\left(\frac{dT_b}{dz}\right)^2=0\,, \tag{20}$$

$$\frac{d^2\phi_b}{dz^2}+N_A\frac{d^2T_b}{dz^2}=0\,. \tag{21}$$

Using boundary conditions Eqs. (16) and (17), the solution of Eq.(21) is

$$\phi_b=-N_AT_b+(1-N_A)z+N_A\,. \tag{22}$$

Substituting the value of ϕ_b in Eq.(21), we get

$$\frac{d^2T_b}{dz^2}+\frac{(1-N_A)N_B}{L_e}.\frac{dT_b}{dz}=0\,. \tag{23}$$

Neglecting the higher power term, solution of Eq.(23) is given by

$$T_b=\frac{-e^{-(1-N_A)N_B/L_e}\left[1-e^{-(1-N_A)N_B(1-z)/L_e}\right]}{1-e^{-(1-N_A)N_B/L_e}}\,. \tag{24}$$

According to Buongiorno /1/, the approximated solution for Eqs. (22) and (24) gives

$$T_b=1-z,\quad \phi_b=z\,. \tag{25}$$

PERTURBATION SOLUTIONS

We now superimpose perturbations on the basic solution. We write,

$$\mathbf{v}=0+\mathbf{v}',\quad p=p_b+p',\quad T=T_b+T'\,,$$
$$\phi=\phi_b+\phi',\quad h=(0,0,1)+h'\,. \tag{26}$$

Using Eq.(26) in Eqs.(10)-(17) and linearising the terms by ignoring the product of prime quantities, the following equations are obtained:

$$\nabla.\mathbf{v}'=0\,, \tag{27}$$

$$\frac{1}{\sigma V_a}\frac{\partial\mathbf{v}'}{\partial t}=-\nabla p'+D_a\nabla^2\mathbf{v}'-\frac{\mathbf{v}'}{1+\lambda}+Q\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}\left(\frac{\partial h'}{\partial z}\right)\hat{e}_z+$$
$$+R_aT'\hat{e}_z-R_n\phi'\hat{e}_z\,, \tag{28}$$

$$\frac{\partial T'}{\partial t}-w'=\nabla^2T'+\frac{N_B}{L_e}\left(\frac{\partial T'}{\partial z}-\frac{\partial\phi'}{\partial z}\right)-\frac{2N_AN_B}{L_e}\frac{\partial T'}{\partial z}\,, \tag{29}$$

$$\frac{1}{\sigma}\frac{\partial\phi'}{\partial t}+\frac{1}{\varepsilon}w'=\frac{1}{L_e}\nabla^2\phi'+\frac{N_A}{L_e}\nabla^2T'\,, \tag{30}$$

$$\frac{\partial h'}{\partial t}=\sigma(0,0,1)\nabla w'+\sigma\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}\nabla^2h'\,, \tag{31}$$

$$\nabla.h'=0\,, \tag{32}$$

$$w'=0,\ \frac{\partial w'}{\partial z}+\lambda_1\frac{\partial^2w'}{\partial z^2}=0,\ T'=0,\ \phi'=0\quad \text{at}\quad z=0\,, \tag{33}$$

$$w'=0,\ \frac{\partial w'}{\partial z}-\lambda_2\frac{\partial^2w'}{\partial z^2}=0,\ T'=0,\ \phi'=0\quad \text{at}\quad z=1\,. \tag{34}$$

The six unknowns u', v', w', p', T' and ϕ' can be reduced to three by operating on Eq.(28) multiplied by $\hat{e}_z$*.curl.curl* and also using Eq.(27), we get

$$\frac{1}{\sigma V_a}\frac{\partial}{\partial t}\nabla^2w'-D_a\nabla^4w'+\frac{\nabla^2w'}{1+\lambda}+Q\frac{\partial^2w'}{\partial z^2}=R_a\nabla_H^2T'-R_n\nabla_H^2\phi' \tag{35}$$

where: $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ and $\nabla_H^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplace operator.

NORMAL MODE ANALYSIS

The disturbances are analysed by normal mode analysis are as follows

$$(w',T',\phi')=\left[W(z),\Theta(z),\Phi(z)\right]\exp(ilx+imy+st)\,, \tag{36}$$

where: s is the growth rate; and l and m are wave numbers along x and y directions, respectively.

Substituting Eq.(36) in Eqs.(28)-(30), (33)-(34), and (35), we get

$$\left[D_a(D^2-a^2)^2-\left(\frac{1}{1+\lambda}+\frac{s}{\sigma V_a}\right)(D^2-a^2)-QD^2\right]W-$$
$$-R_aa^2\Theta+R_na^2\Phi=0\,, \tag{37}$$

$$W+\left(D^2+\frac{N_A}{L_e}D-\frac{2N_AN_B}{L_e}-a^2-s\right)\Theta-\frac{N_B}{L_e}D\Phi=0\,, \tag{38}$$

$$\frac{1}{\varepsilon}W-\frac{N_A}{L_e}(D^2-a^2)\Theta-\left(\frac{1}{L_e}(D^2-a^2)-\frac{s}{\sigma}\right)\Phi=0\,, \tag{39}$$

$$W=0,\ DW+\lambda_1D^2W=0,\ \Theta=0,\ \Phi=0\quad \text{at}\quad z=0\,, \tag{40}$$

$$W=0,\ DW-\lambda_2D^2W=0,\ \Theta=0,\ \Phi=0\quad \text{at}\quad z=1\,, \tag{41}$$

where: $D = d/dz$; and $a^2 = l^2 + m^2$ is the dimensionless wave number.

According to Chandrasekhar /4/, the boundary conditions should be as follows:

1) free-free boundaries

$$W=D^2W=\Theta=\Phi=0\quad \text{at}\quad z=0,1\,, \tag{42}$$

2) rigid-rigid boundaries

$$W=DW=\Theta=\Phi=0\quad \text{at}\quad z=0,1\,, \tag{43}$$

3) rigid-free boundaries

$$W=DW=\Theta=\Phi=0\quad \text{at}\quad z=0\,, \tag{44}$$

$$W=D^2W=\Theta=\Phi=0\quad \text{at}\quad z=1\,. \tag{45}$$

The assumed solutions for W, Θ, and Φ, for all boundary conditions are taken as follows:

1) for free-free boundaries

$$W=W_0\sin\pi z,\ \Theta=\Theta_0\sin\pi z,\ \Phi=\Phi_0\sin\pi z\,, \tag{46}$$

2) for rigid-rigid boundaries

$$W=W_0(z^2-2z^3+z^4),\ \Theta=\Theta_0(z-z^2),\ \Phi=\Phi_0(z-z^2), \tag{47}$$

3) for rigid-free boundaries

$$W=W_0(3z^2-5z^3+2z^4),\ \Theta=\Theta_0(z-z^2),\ \Phi=\Phi_0(z-z^2). \tag{48}$$

LINEAR STABILITY ANALYSIS FOR FREE-FREE BOUNDARIES

Substituting Eq.(46) in Eqs.(37)-(39) and integrating each term individually within limits $z = 0$ to $z = 1$, we get

$$\begin{bmatrix} D_a J\left(J+\frac{s}{\sigma V_a}\right)+\frac{J}{1+\lambda}+\pi^2 Q & -R_a a^2 & R_n a^2 \\ 1 & -(J+s) & 0 \\ \frac{1}{\varepsilon} & \frac{N_A}{L_e}J & \left(\frac{J}{L_e}+\frac{s}{\sigma}\right) \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{49}$$

where: $J = \pi^2 + a^2$.

The eigenvalue to the system of linear Eq.(49) is given as

$$R_a=\frac{1}{a^2}\left\{D_a J\left(J+\frac{s}{\sigma V_a}\right)+\left(\frac{J}{1+\lambda}+\pi^2 Q\right)(J+s)-\frac{R_n a^2\left(\frac{N_A J}{L_e}+\frac{J+s}{\varepsilon}\right)}{\left(\frac{J}{L_e}+\frac{s}{\sigma}\right)}\right\}. \tag{50}$$

Stationary convection for free-free boundaries

For stationary convection $s = 0$ in Eq.(50), we obtain

$$R_a^S=\frac{D_a(\pi^2+a^2)^3}{a^2}+\frac{(\pi^2+a^2)^2}{a^2(1+\lambda)}+\frac{\pi^2(\pi^2+a^2)Q}{a^2}-\left(\frac{L_e}{\varepsilon}+N_A\right)R_n . \tag{51}$$

For the case when $D_a = 0$, the critical wave number is obtained by minimising thermal Darcy-Rayleigh number R_a with respect to a^2, thus the critical wave number must satisfy $\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c}=0$.

Equation (51) gives

$$a_c=\pi . \tag{52}$$

On the other hand when D_a is large compared with unity, the critical wave number is obtained by minimising thermal Rayleigh-Darcy number R_a with respect to a^2. Thus, the critical wave number must satisfy $\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c}=0$.

Equation (51) gives

$$a_c=\frac{\pi}{\sqrt{2}}, \tag{53}$$

which coincides with Kuznetsov and Nield, /5/.

In order to investigate the effects of the Darcy-Brinkman number, Jeffrey parameter, Lewis number, nanoparticle Rayleigh number, porosity, modified diffusivity ratio, and Chandrasekhar number, we examine the behaviour of $\frac{\partial R_a^S}{\partial D_a}>0$, $\frac{\partial R_a^S}{\partial \lambda}<0$, $\frac{\partial R_a^S}{\partial N_A}>0$, $\frac{\partial R_a^S}{\partial L_e}>0$, $\frac{\partial R_a^S}{\partial \varepsilon}<0$, $\frac{\partial R_a^S}{\partial R_n}<0$, $\frac{\partial R_a^S}{\partial Q}>0$ analytically from Eq.(51). According to these inequalities, the following parameters have stabilising effects: Darcy Brinkman number, modified diffusivity ratio, Lewis number, Chandrasekhar number, and Taylor number, and on the other hand, the following parameters have destabilising effects: porosity, Jeffrey parameter, and nanoparticle Rayleigh number.

LINEAR STABILITY ANALYSIS FOR RIGID-RIGID BOUNDARIES

Substituting Eq.(47) in Eqs.(37)-(39) and integrating each term individually within limits $z = 0$ to $z = 1$, after applying Galerkin's first approximation, we get

$$\begin{bmatrix} 2D_a(504+24a^2+a^4)+(12+a^2)\left(\frac{1}{1+\lambda}+\frac{s}{\sigma V_a}\right)+12Q & -9R_a a^2 & 9R_n a^2 \\ 3 & -14(10+a^2+s) & 0 \\ \frac{3}{\varepsilon} & 14\frac{N_A}{L_e}(10+a^2) & \frac{14(10+a^2)}{L_e}+\frac{14s}{\sigma} \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{54}$$

The eigenvalue to the system of linear Eq.(54) is given as

$$R_a=\frac{28}{27a^2}\left[D_a(504+24a^2+a^4)+12+a^2\left(\frac{1}{1+\lambda}+\frac{s}{\sigma V_a}\right)+12Q\right](10+a^2+s)-\frac{N_A(10+a^2)+\frac{L_e(10+a^2+s)}{\varepsilon}}{10+a^2+\frac{sL_e}{\sigma}} . \tag{55}$$

Stationary Convection for rigid-rigid boundaries

For stationary convection $s = 0$ in Eq.(55), we obtain

$$R_a^S=\frac{28}{27a^2}\left[D_a(504+24a^2+a^4)+12+a^2\left(\frac{1}{1+\lambda}\right)+12Q\right]\times(10+a^2)-\left(N_A+\frac{L_e}{\varepsilon}\right)R_n . \tag{56}$$

For the case when $D_a = 0$, the critical wave number is obtained by minimising thermal Darcy-Rayleigh number R_a with respect to a^2, thus, the critical wave number must satisfy

$$\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c}=0 .$$

Equation (56) gives

$$a_c=3.31 . \tag{57}$$

This result is identical with Kuznetsov and Nield /5/.

On the other hand when D_a is large compared with unity, the critical wave number obtained by minimising thermal Darcy-Rayleigh number R_a with respect to a^2. Thus the critical wave number must satisfy

$$\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c}=0\,.$$

Equation (56) gives $a_c = 3.12\,.$ (58)

This result is identical with Kuznetsov and Nield /5/.

LINEAR STABILITY ANALYSIS FOR RIGID-FREE BOUNDARIES

Substituting Eq.(49) in Eqs.(37)-(39) and integrating each term individually within limits $z = 0$ to $z = 1$, after applying Galerkin's first approximation, we get

$$\begin{bmatrix} 2D_a(4536+432a^2+19a^4)+(216+19a^2)\left(\frac{1}{1+\lambda}+\frac{s}{\sigma V_a}\right)+216Q & -39R_a a^2 & 39R_n a^2 \\ 13 & -14(10+a^2+s) & 0 \\ \frac{13}{\varepsilon} & 14\frac{N_A}{L_e}(10+a^2) & \frac{14(10+a^2)}{L_e}+\frac{14s}{\sigma} \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (59)$$

The eigenvalue to the system of linear Eq.(59) is given by

$$R_a=\frac{28}{507a^2}\left[D_a(4536+432a^2+19a^4)+216+19a^2\left(\frac{1}{1+\lambda}+\frac{s}{\sigma V_a}\right)\right](10+a^2+s)-\frac{N_A(10+a^2)+\frac{L_e(10+a^2+s)}{\varepsilon}}{10+a^2+\frac{sL_e}{\sigma}}\,. \quad (60)$$

Stationary convection for rigid-free boundaries

For stationary convection $s = 0$ in Eq.(60), we obtain

$$R_a=\frac{28}{507a^2}\left[D_a(4536+432a^2+19a^4)+216+19a^2\left(\frac{1}{1+\lambda}\right)+216Q\right](10+a^2)-\left(N_A+\frac{L_e}{\varepsilon}\right)R_n\,. \quad (61)$$

For the case when $D_a = 0$, the critical wave number is obtained by minimising thermal Darcy-Rayleigh number R_a with respect to wave number a^2, thus the critical wave number must satisfy $\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c}=0\,.$

Equation (61) gives $a_c = 3.27\,.$ (62)

On the other hand when D_a is large compared with unity, the critical wave number obtained by minimising thermal Darcy-Rayleigh number R_a with respect to wave number a^2. Thus the critical wave number must satisfy $\left(\frac{\partial Ra}{\partial a^2}\right)_{a=a_c}=0\,.$

Equation (61) gives $a_c = 2.67\,.$ (63)

This result is similar to the result of Kuznetsov and Nield /5/.

RESULTS AND DISCUSSION

In this research paper, we studied the impact of magnetic field on thermal instability in a porous medium layer saturated by a Jeffrey nanofluid using Brinkman nanofluid for free-free, rigid-rigid, rigid-free boundaries. The impact of different parameters like Darcy-Brinkman number, Jeffrey parameter, modified diffusivity ratio, Lewis number, porosity parameter, concentration Rayleigh number, and Chandrasekhar number on stationary convection have been analysed analytically and plotted graphically for free-free, rigid-rigid and rigid-free boundaries.

Figure 2 illustrates the graph of R_a with respect to wave number a for various values of D_a = 0.1, 0.2, 0.3. Fixing other parameters as λ = 0.2, N_A = 5, L_e = 1000, ε = 0.6, R_n = –1, Q = 100, it is obvious from Fig. 2 that as D_a goes on increasing with the rise in R_a. Thus, D_a has a stabilising effect on stationary convection. Also, we have analysed that D_a has a more stabilising effect in rigid-rigid boundaries. Thus, D_a delays the onset of convection of the system.

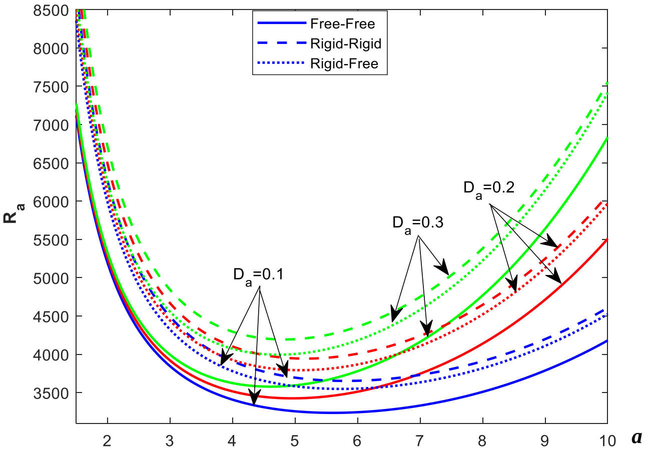

Figure 2. Variation of R_a with wave number a, for various values of Darcy Brinkman number.

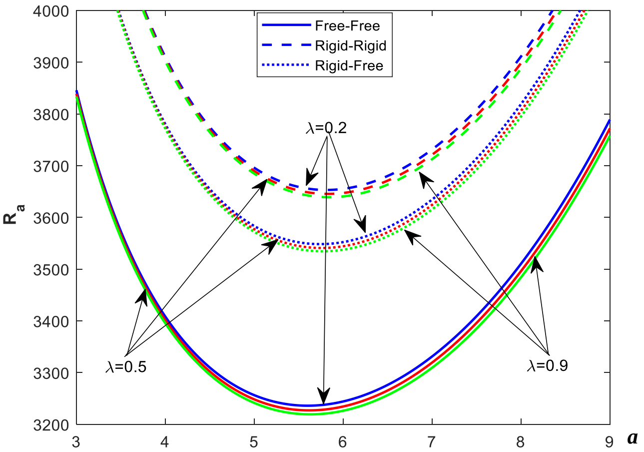

Figure 3. Variation of R_a with wave number a, for various values of Jeffrey parameter.

Figure 3 illustrates the graph of R_a with respect to wave number a for various values of $\lambda = 0.2, 0.5, 0.9$. Fixing other parameters as $D_a = 0.1$, $N_A = 5$, $L_e = 1000$, $\varepsilon = 0.6$, $R_n = -1$, $Q = 100$, it is obvious from Fig. 3 that R_a goes on decreasing with rise in λ. Thus, λ has a destabilising effect on stationary convection and it is also clear from the figure that it has a more destabilising effect in free-free boundaries. Thus, λ enhances the onset of convection of the system.

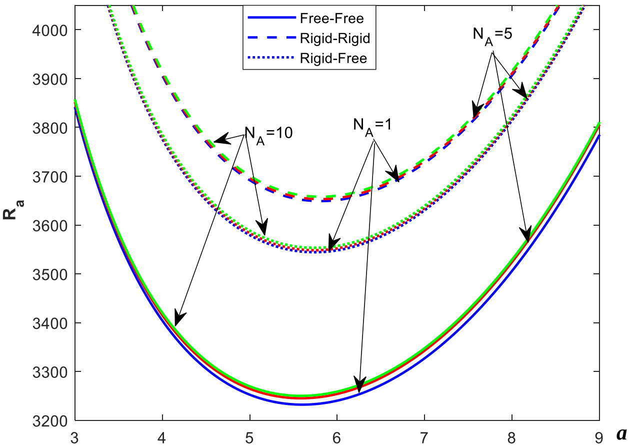

Figure 4. Variation of R_a with wave number a, for various values of modified diffusivity ratio.

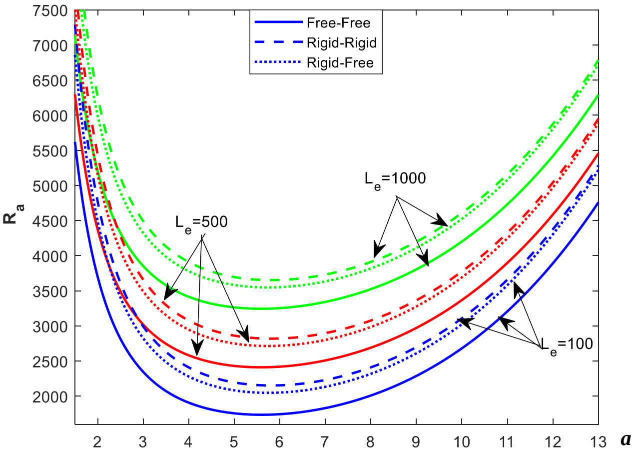

Figure 5. Variation of R_a with wave number a, for various values of Lewis number.

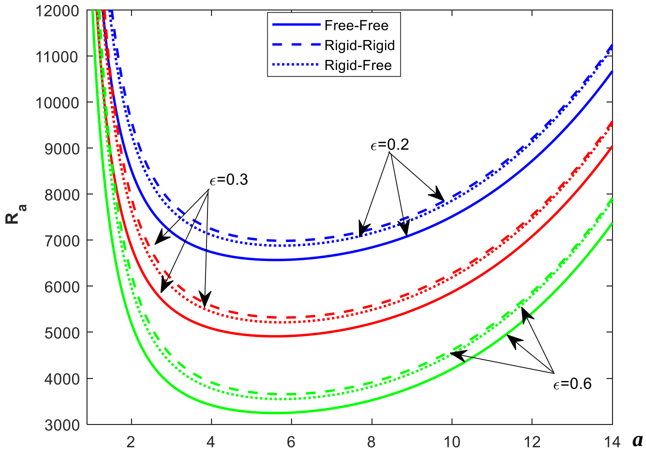

Figure 6. Variation of R_a with wave number a, for various values of porosity parameter.

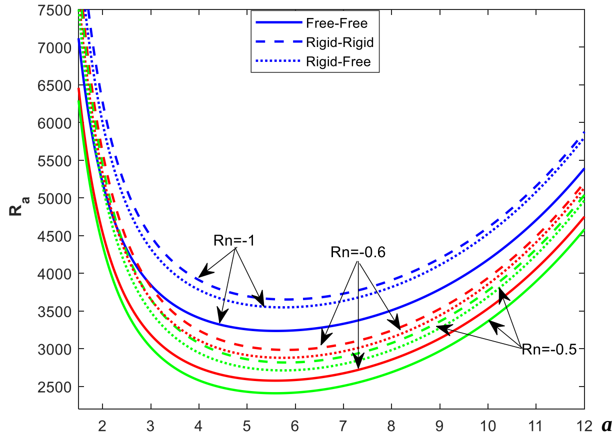

Figure 7. Variation of R_a with wave number a, for various values of concentration Rayleigh number.

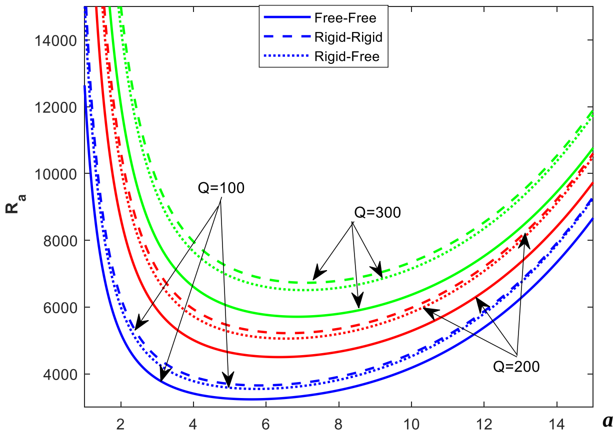

Figure 8. Variation of R_a with wave number a, for various values of Chandershekhar number.

Figure 4 illustrates the graph of R_a with respect to wave number a for various values of $N_A = 1, 5, 10$. Fixing other parameters as $D_a = 0.1$, $\lambda = 0.2$, $L_e = 1000$, $\varepsilon = 0.6$, $R_n = -1$, $Q = 100$, it is obvious from Fig. 4 that R_a goes on increasing with rise in the value of N_A. Thus, N_A has a stabilising effect, and it is also obvious from the figure that it has more stabilising effect in rigid-rigid boundaries. Thus, N_A delays the onset of convection of the system.

Figure 5 illustrates the graph of R_a with respect to wave number a for various values of $L_e = 100, 500, 1000$. Fixing other parameters as $D_a = 0.1$, $\lambda = 0.2$, $N_A = 5$, $\varepsilon = 0.6$, $R_n = -1$, $Q = 100$, it is obvious from the figure that as R_a goes on increasing with rise in L_e. Thus, it has a stabilising effect on stationary convection and Fig. 5 demonstrates that L_e has a more stabilising effect in rigid-rigid boundaries. Thus, L_e delays the onset of convection of the system.

Figure 6 illustrates the graph of R_a with respect to wave number a for various values of $\varepsilon = 0.2, 0.3, 0.6$. Fixing other parameters as $D_a = 0.1$, $\lambda = 0.2$, $N_A = 5$, $L_e = 1000$, $R_n = -1$, $Q = 100$, it is obvious from the figure that as R_a goes on decreasing with the rise in the values of ε. Thus, ε shows a destabilising effect and it is also clear from Fig. 6 that it has a more destabilising effect in free-free boundaries. Thus, ε enhances the onset of convection of the system.

Figure 7 illustrates the graph of R_a with respect to wave number a for various values of $R_n = -1, -0.6, -0.5$. Fixing other parameters as $D_a = 0.2$, $\lambda = 0.2$, $N_A = 5$, $L_e = 1000$, $\varepsilon =$

0.6, Q = 100, it is obvious from the figure that as R_a goes on decreasing with the rise in the value of R_n. Thus, R_n has a destabilising effect, and it is also clear from Fig. 7 that R_n has a more destabilising effect in free-free boundaries. Thus, R_n enhances the onset of convection of the system.

Figure 8 illustrates the graph of R_a with respect to wave number a for various values of Q = 100, 200, 300. Fixing other parameters as D_a = 0.1, λ = 0.2, N_A = 5, L_e = 1000, ε = 0.6, R_n = –1, it is obvious from the figure that R_a goes on increasing with the rise in the value of Q. Thus, Q has a stabilising effect, and it is also clear from Fig. 7 that Q has a more stabilising effect in rigid-rigid boundaries. Thus, Q delays the onset of convection of the system.

CONCLUSIONS

In this article, we use linear stability analysis to make the following key conclusions:

(i) Darcy Brinkman number, modified diffusivity ratio, Lewis number, and Chandershekhar number, have stabilising influence on the system.

(ii) Jeffrey parameter, porosity parameter, and concentration Rayleigh number enhance the start of convection on the system.

(iii) In case of rigid-rigid boundaries, the system has greater stabilising impact rather than free-free/rigid-free boundaries.

(iv) It was also found that parameters like Darcy Brinkman number, modified diffusivity ratio, Lewis number, and Chandershekhar number have a more destabilising effect on stationary convection in the situation of free-free boundaries, as compared to rigid-rigid/rigid-free boundaries.

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