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EFFECT OF NON-HOMOGENEITY ON THE STRESSES AND TEMPERATURE DISTRIBUTION IN A THICK-WALLED CIRCULAR CYLINDER

UTICAJ NEHOMOGENOSTI NA RASPODELE NAPONA I TEMPERATURE KOD DEBELOZIDOG KRUŽNOG CILINDRA

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Keywords
- stress
- pressure
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- yielding

Abstract

This article deals with the study of non-homogeneity on the stresses and temperature distribution in a thick-walled circular cylinder by using generalised strain measure. Mathematical modelling is based on stress-strain relation, non-linear differential equation and equilibrium equation. The effects of temperature and pressure are considered for the non-homogeneous thick-walled circular cylinder. The behaviour of stress distribution, pressure, and temperature are investigated. From the obtained results, it is noticed that hoop stress is maximal at outer surface for non-homogeneous circular cylinder and non-homogeneity increases the tangential stress at outer surface significantly. The thermal effects reduce the tangential stress at the external surface of a non-homogeneous circular cylinder extensively. Results are discussed numerically and graphically.

Ključne reči
- napon
- pritisak
- temperatura
- cilindar
- tečenje

Izvod

U ovom radu izučavamo nehomogenost i njen uticaj na raspodelu napona i temperature kod debelozidog kružnog cilindra, primenom generalisane mere deformacija. Matematičko modeliranje se zasniva na relaciji napon-deformacija, nelinearnoj diferencijalnoj jednačini i na jednačini ravnoteže. Razmatraju se uticaji temperature i pritiska kod nehomogenog debelozidog kružnog cilindra. Istražuje se ponašanje raspodele napona, pritiska i temperature. Prema dobijenim rezultatima, primećuje se da je obimski napon maksimalan na spoljnjoj površini za nehomogeni kružni cilindar, a sa uvećanjem nehomogenosti dolazi do značajnog porasta tangencijalnog napona na spoljnjoj površini. Toplotni uticaji u velikoj meri smanjuju tangencijalni napon na spoljnjoj površini nehomogenog kružnog cilindra. Rezultati su opisani i diskutovani numerički i grafički.

INTRODUCTION

The growing industrial demand for axisymmetric and spherical components or their elements has drawn the attention of designers and scientists to this particular field of activity. The progressive scarcity of materials worldwide, combined with their therefore higher costs, makes it increasingly unattractive to limit design to the usual elastic regime alone. The thick-walled cylinders subjected to temperatures and uniform pressures find several applications, e.g., in nuclear power plants, chemical industries, pressure vessels intended for storage of industrial gases or as the means for transportation of high pressurised fluids. Gao /1/ and Bonn et al. /2/ have investigated the elastic-plastic deformations in a thick-walled cylindrical tube under uniform pressure. The solution is based on finite strains, Hencky deformation theory and von Mises yield criterion. Orçan /3/ carried out stress and deformation in an elastic-perfectly plastic cylindrical rod with uniform internal heat generation by using Tresca's yield condition and associated flow rule. An analytical solution for stresses by considering the Bauschinger effect and Tresca yield criterion in thick-walled cylindrical vessel made of elastic linear-hardening material was given by Darijani et al. /4/. An elastic-plastic analytical model of thick-walled circular cylinder subjected to uniform pressure by using generalised strain measure and transition theory was given by Thakur /5/. Kamal et al. /6/ investigated stress and strain in thick-walled cylinders subjected to thermal gradient by using finite element method. The objective of this article is to investigate the effect of non-homogeneity on the stresses and temperature in a thick-walled circular cylinder by using the concept of generalised strain measure. Results have been presented graphically and are discussed.

MATHEMATICAL MODEL AND BASIC GOVERNING EQUATION

Let us consider a thick-walled cylinder with internal radius r_i and external radius r_0. The internal wall is subjected to temperature Θ_0 and pressure p_{int} (see Fig. 1). Let the non-homogeneity as the compressibility of the cylinder material be given by:

$$c = c_0 r^{-k} , \tag{1}$$

where: $r \in [r_i, r_0]$; c_0, and k (< 0) are constants. The displacement components in cylindrical polar co-ordinate are given by /7/:

$$u = r(1-\beta); \quad v = 0; \quad w = dz\,, \tag{2}$$

where: β is function of $r = \sqrt{(x^2 + y^2)}$ only; and d is a constant.

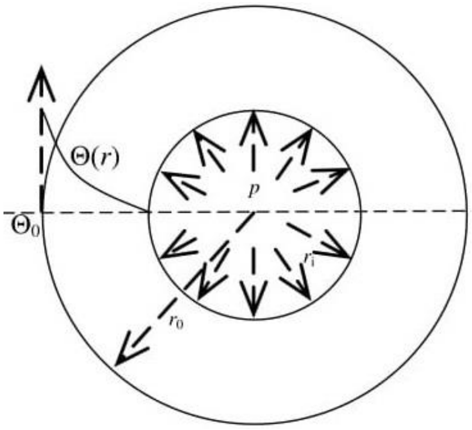


Figure 1. Geometry of thick-walled circular cylinder.

The generalised components of strain are given /7, 8/:

$$\varepsilon_r = \frac{1}{n}[1-(r\eta'+\eta)^n]\,, \quad \varepsilon_\theta = \frac{1}{n}[1-\eta^n]\,,$$

$$\varepsilon_z = \frac{1}{n}[1-(1-d)^n]\,, \quad \varepsilon_{r\theta} = \varepsilon_{\theta z} = \varepsilon_{zr} = 0\,, \tag{3}$$

where: $\eta' = d\eta/dr$.

Stress-strain relation: the stress-strain relation for thermal elastic isotropic material is given by /9/:

$$\tau_r = \frac{2\mu}{n}[1-(r\eta'+\eta)^n] + \frac{\lambda}{n}[3-\eta^n-(r\eta'+\eta)^n-(1-d)^n] - \xi\Theta,$$

$$\tau_\theta = \frac{2\mu}{n}[1-\eta^n] + \frac{\lambda}{n}[3-\eta^n-(r\eta'+\eta)^n-(1-d)^n] - \xi\Theta\,,$$

$$\tau_z = \frac{2\mu}{n}[1-(1-d)^n] + \frac{\lambda}{n}[3-\eta^n-(r\eta'+\eta)^n-(1-d)^n] - \xi\Theta\,,$$

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = 0\,. \tag{4}$$

Equation of equilibrium: $\dfrac{\partial\tau_r}{dr} + \dfrac{(\tau_r - \tau_\theta)}{r} = 0\,.$ (5)

Transition points: using Eq.(4) and Eq.(1) into Eq.(5), we get nonlinear differential equation in term of η as:

$$n\eta T(T+1)^{n-1}\frac{dT}{d\eta} = \left\{ r\left(\frac{\mu'}{\mu} - \frac{c'}{c}\right) \times \left[\frac{1}{\eta^n}\{(3-3c)-(1-c)(1-d)^n\} - (T+1)^n - (1-c)\right] - nT \times \left[(1-c)+(T+1)^n - \frac{nc\Theta_0}{2\mu\eta^n \ln(r_i/r_0)}\left\{\xi + \xi' r \ln\frac{r}{r_0}\right\}\right] + c[1-(T+1)^n] + \frac{rc\eta^n}{\eta^n - 2 + (1-d)^n}\right\}, \tag{6}$$

where: $c = 2\mu/(\lambda + 2\mu)$; and $r\eta' = \eta T$. The transition points in Eq.(6) are $T \to \pm\infty$ and $T \to -1$.

Boundary conditions:

$$\tau_r = -p_{int} \quad \text{and} \quad \Theta = \Theta_0 \quad \text{at} \quad r = r_i\,, \text{ and}$$
$$\tau_r = 0 \quad \text{and} \quad \Theta = 0 \quad \text{at} \quad r = r_0\,, \tag{7}$$

where: Θ_0 is constant given by: $\Theta_0 = \dfrac{\Theta_0 \ln(r/r_0)}{\log(r_i/r_0)}$. The resultant force transmitted by the wall in axial direction is equal to $l = \pi r_i^2 p_{int}$, i.e.:

$$2\pi\int_{r_i}^{r_0} r\tau_z dr = \pi r_i^2 p_{int}\,. \tag{8}$$

PROBLEM SOLUTION

At the transition point $T \to \pm\infty$, the transition function is taken through the principal stress as given by /5, 7-26/. Let transition function ζ be defined as:

$$\zeta = \tau_r - \frac{\lambda}{n}k + \alpha\Theta(3-2c)T \equiv \frac{2\mu}{cn}[c-\eta^n(1-c) + (T+1)^n] + \alpha\Theta(3-2c)\left(1-\frac{2\mu}{c}\right). \tag{9}$$

Taking the logarithmic differentiation of Eq.(9) with respect to r and using Eq.(6) and taking the asymptotic value $T \to \pm\infty$, we get:

$$\frac{d(\ln\zeta)}{dr} = -\frac{c}{r}\,. \tag{10}$$

Integrating Eq.(10) with respect to r, we get:

$$\zeta = A\exp F(r)\,, \tag{11}$$

where: $$F(r) = -\int\frac{c}{r}dr\,. \tag{12}$$

Equations (9) and (11), become:

$$\tau_r = A\exp F(r) + \frac{\lambda}{n}[3-(1-d)^n] - \alpha\Theta(3-2c)\,. \tag{13}$$

Using Eq.(7) into Eq.(14), we get:

$$A = \frac{\alpha\Theta_0[3-2c(r_i)] - p_{int}}{\exp F(r_i) - \exp F(r_0)}\,,$$

and $$\frac{\lambda}{n}[3-(1-d)^n] = A\exp F(r_0)\,. \tag{14}$$

Substituting Eq.(14) into Eq.(13) and Eq.(3), we get:

$$\tau_r = A\exp[F(r) - \exp F(r_0)] - \alpha\Theta(3-2c)\,, \tag{15}$$

and $$\varepsilon_z = \frac{\exp F(r_0)}{\lambda}\left\{\frac{p_{int} - \alpha\Theta_0[3-2c(r_i)]}{\exp F(r_i) - \exp F(r_0)}\right\} - \frac{2}{n}\,. \tag{16}$$

Substituting Eq.(15) into Eq.(5), we get:

$$\tau_\theta = A\left\{[(1-c)\exp F(r) - \exp F(r_0)] - \frac{\alpha\Theta_0}{\ln(r_i/r_0)} \times [(3-2c)[1+\ln(r/r_0)] - 2rc'\ln(r/r_0)]\right\}. \tag{17}$$

The third equation of Eq.(4) gives:

$$\tau_z = \left(\frac{1-c}{2-c}\right)(\tau_r + \tau_\theta) + \frac{c\lambda(3-2c)}{(1-c)(2-c)}\left[\varepsilon_z - \alpha\frac{\Theta_0\ln(r/r_0)}{\log(r_i/r_0)}\right]. \tag{18}$$

Using Eq.(16) and Eq.(8) into Eq.(18), we get:

$$l = 2\pi\left\{\left[\frac{1-c(r_i)}{2-c(r_i)}\right]pa^2 + \int_a^b \frac{c'r^2\tau_r}{(2-c)^2}dr - \lambda\int_a^b \frac{\alpha\Theta cr(3-2c)}{(1-c)(2-c)}dr + \left\{\frac{\{p_{int} - \alpha\Theta_0[3-2c(r_i)]\}\exp F(r_0)}{\exp F(r_i) - \exp F(r_0)} - \frac{2\lambda}{n}\right\}\right\}. \tag{19}$$

Substituting Eq.(2) into Eqs.(15)-(18), we get the transitional stresses as:

$$\tau_r = [\alpha\Theta_0(3-2c_0r_i^{-k}) - p_{int}]\frac{\exp\left(\frac{c_0r^{-k}}{k}\right)-\exp\left(\frac{c_0r_0^{-k}}{k}\right)}{\exp\left(\frac{c_0r_i^{-k}}{k}\right)-\exp\left(\frac{c_0r_0^{-k}}{k}\right)} - \frac{\alpha\Theta_0\ln(r/r_0)(3-2r^{-k}c_0)}{\log(r_i/r_0)}, \tag{20}$$

$$\tau_\theta = \tau_r - [\alpha\Theta_0(3-2c_0r_i^{-k}) - p_{int}]\frac{c_0r^{-k}\exp\left(\frac{c_0r^{-k}}{k}\right)}{\exp\left(\frac{c_0r_i^{-k}}{k}\right)-\exp\left(\frac{c_0r_0^{-k}}{k}\right)} - \frac{\alpha\Theta_0}{\ln(r_i/r_0)}[2r^{-k}kc_0\ln(r/r_0)+(3-2r^{-k}c_0)], \tag{21}$$

$$\tau_z = (\tau_\theta+\tau_r)\left(\frac{1-c_0r^{-k}}{2-c_0r^{-k}}\right) + \frac{\lambda c_0r^{-k}(3-2c_0r^{-k})}{(1-c_0r^{-k})(2-c_0r^{-k})}\left(\varepsilon_z - \alpha\frac{\Theta_0\ln(r/r_0)}{\log(r_i/r_0)}\right), \tag{22}$$

where
$$\varepsilon_z = \frac{p_{int}-\alpha\Theta_0(3-2c_0r_i^{-k})}{\exp\left(\frac{c_0r_i^{-k}}{k}\right)-\exp\left(\frac{c_0r_0^{-k}}{k}\right)}\frac{\exp\left(\frac{c_0r_0^{-k}}{k}\right)}{\lambda} - \frac{2}{n}. \tag{23}$$

Equations (20)-(21) become:

$$\tau_\theta - \tau_r = \frac{-c_0r^{-k}\{\alpha\Theta_0(3-2c_0r_i^{-k})-p_{int}\}}{\exp\left(\frac{c_0r_i^{-k}}{k}\right)-\exp\left(\frac{c_0r_0^{-k}}{k}\right)}\exp\left(\frac{c_0r^{-k}}{k}\right) - 2\alpha c_0kr^{-k}\frac{\Theta_0\ln(r/r_0)}{\log(r_i/r_0)} - \frac{\alpha\Theta_0}{\ln(r_i/r_0)}(3-2c_0r^{-k})\,. \tag{24}$$

Initial yielding: it has been seen that from Eq.(24), that $|\tau_\theta-\tau_r|$ is maximum at $r = (r_0^ke^2)^{1/k}$, $c_0 = -kr_0^ke^2$, and $k < 0$. Therefore yielding of a non-homogeneous cylinder takes place at $r = (r_0^ke^2)^{1/k} = r_1$ (say), depending upon values of $c_0 = -kr_0^ke^2$ and k, Eq.(24) becomes:

$$|\tau_\theta-\tau_r|_{r=r_1} = \left|\frac{-c_0r_1^{-k}\{\alpha\Theta_0(3-2c_0r_i^{-k})-p_{int}\}}{\exp\left(\frac{c_0r_i^{-k}}{k}\right)-\exp\left(\frac{c_0r_0^{-k}}{k}\right)}\exp\left(\frac{c_0r_1^{-k}}{k}\right) - 2\alpha c_0kr_1^{-k}\frac{\Theta_0\ln(r_1/r_0)}{\log(r_i/r_0)} - \frac{\alpha\Theta_0}{\ln(r_i/r_0)}(3-2c_0r_1^{-k})\right| \cong Y, \tag{25}$$

where: Y is yielding stress; and e (say value = 2.178) be the exponential.

Investigation pressure: from Eq.(25), the pressure required for initial yielding is given:

$$P_i = \frac{p_{int}}{Y} = \frac{1}{|A_1|} - \Theta_1\frac{|A_2|}{|A_1|}, \tag{26}$$

where: $A_1 = \dfrac{k}{e[\exp(-R_0^{-k}e^2)-\exp(-e^2)]}$; $R_0 = r_i/r_0$; $A_2 = \dfrac{3-2k}{\ln R_0} - \dfrac{k(3+2kR_0^{-k}e^2)}{e[\exp(-R_0^{-k}e^2)-\exp(-e^2)]}$; $R_0 = r/r_0$; and $\Theta_1 = \alpha\Theta_0/Y$.

Fully-plastic state: for the full plasticity (i.e., $c_0 \to 0$), Eq.(25) becomes:

$$\left|\frac{p_{int}k}{(r_i^{-k}-r_0^{-k})e^2r_0^k} - 3\alpha\Theta_0\left[\frac{k}{(r_i^{-k}-r_0^{-k})e^2r_0^k} - \frac{1}{\log(r_0/r_i)}\right]\right| \cong Y$$

and the pressure required for fully-plastic state is given as:

$$P_f = \frac{p_{int}}{Y_1} = \left|\frac{e^2(R_0^{-k}-1)}{k}\right| + 3\Theta_1\left|\frac{k\ln(1/R_0)-e^2(R_0^{-k}-1)}{k\ln(1/R_0)}\right|. \tag{27}$$

Stresses for full plasticity are obtained by taking $c_0 \to 0$ in Eqs.(20)-(22), and we get:

$$\sigma_r = \frac{\tau_r}{Y} = 3\Theta_1\left[\frac{R^{-k}-1}{R_0^{-k}-1} - \frac{\ln R}{\ln R_0}\right] - P_f\left[\frac{R^{-k}-1}{R_0^{-k}-1}\right], \tag{28}$$

$$\sigma_\theta = \frac{\tau_\theta}{Y} = 3\Theta_1\left[\frac{(1-k)R^{-k}-1}{R_0^{-k}-1} - \frac{1+\ln R}{\ln R_0}\right] - P_f\left[\frac{(1-k)R^{-k}-1}{R_0^{-k}-1}\right]. \tag{29}$$

$$\sigma_z = \frac{\tau_z}{Y} = \frac{3kR^{-k}(P_f-3\Theta_1)}{2R_0^{-k}(1-R_0^{-k})} + \frac{\sigma_r+\sigma_\theta}{2}. \tag{30}$$

The axial force given is as:

$$L = \frac{l}{Yb^2} = \frac{3\pi k(P_f-3\Theta_1)(1-R_0^{2-k})}{(2-k)(1-R_0^{-k})} + P_f\pi R_0^2. \tag{31}$$

In Eqs.(28)-(31), by taking $k \to 0$ and $\Theta_1 \to 0$, the present results reduce to the previously published outcomes done by /18/ thus to make sure of the validity of the presented solutions.

NUMERICAL RESULTS AND DISCUSSION

To evaluate the effects of pressure and temperature on a circular cylinder made of non-homogeneous materials and non-homogeneity as the compressibility of the material in the cylinders given as $c = c_0r^{-k}$, the following numerical values are taken as: $k = -2, -2.5, -3$; $\Theta_1 = 0$ and 0.175, in respect. For a thick-walled circular cylinder made of non-homogeneous material (say $k < 0$) and the non-homogeneity increases radially, yielding will takes place at the radius r, where $r_i < r < r_0$ and different temperatures (say $\Theta_1 = 0$, 0.175), depending upon the values of k and c_0 (see Table 1).

From Table 1, it shows that the percentage increase in pressure required for the initial yielding state to become fully-plastic has been discussed numerically. It can also be seen from Table 1, that a thick-walled cylinder with radii ratio $b/a = 4$ requires higher percentage values in pressure (i.e., $P =$ 106.90 %, 95.28 %, 89.68 %), to become fully plastic as compared to the thick-walled cylinder with radii ratio b/a = 8 (i.e., P = 89.68 %, 86.14 %, 84.90 %), respectively. Further increasing the dimensionless temperature (i.e., Θ_1 = 0.175), the percentage ratio (i.e., P %) also increases, for the initial yielding state to become a fully plastic state, in respect. Further, the value of percentage ratio increases with increasing temperature and decreases with increasing radii ratio b/a (say b/a = 6, 8), and vice versa. Moreover, the non-homogeneous thick-walled cylinder having larger radii ratio requires lower percentage values in pressure to become fully plastic in comparison to a smaller radii ratio.

Table 1. Percentage in pressure.

Radii ratio (b/a)	constant $k < 0$	Temperature Θ_1	Compressibility c_0	Initial yielding surface r	Pressure P_i	P_f	Percentage in pressure for initial yielding to fully plastic state P (%)
4	-2	0	0.92	1.47	0.81	3.46	106.90
	-2.5		0.57	1.79	0.75	2.86	95.28
	-3		0.34	2.05	0.67	2.42	89.68
	-2	0.175	0.92	1.47	0.41	2.67	155.01
	-2.5		0.57	1.79	0.38	2.30	143.71
	-3		0.34	2.05	0.35	2.03	139.03
6	-2	0	0.41	2.21	0.95	3.59	94.03
	-2.5		0.21	2.69	0.82	2.92	88.17
	-3		0.10	3.08	0.71	2.45	85.83
	-2	0.175	0.41	2.21	0.73	3.06	104.24
	-2.5		0.21	2.69	0.66	2.59	98.36
	-3		0.11	3.08	0.58	2.25	95.99
8	-2	0	0.23	2.94	1.01	3.63	89.68
	-2.5		0.10	3.59	0.84	2.93	86.14
	-3		0.04	4.10	0.72	2.45	84.90
	-2	0.175	0.23	2.94	0.88	3.24	91.06
	-2.5		0.10	3.59	0.77	2.72	86.89
	-3		0.04	4.10	0.68	2.36	85.07

where: $P = [\sqrt{(P_f/P_i)} - 1]\times 100$ is the percentage (%) increase in pressure for initial yielding state to become fully plastic state and dimensionless temperature Θ_1 = 0.00, 0.0175.

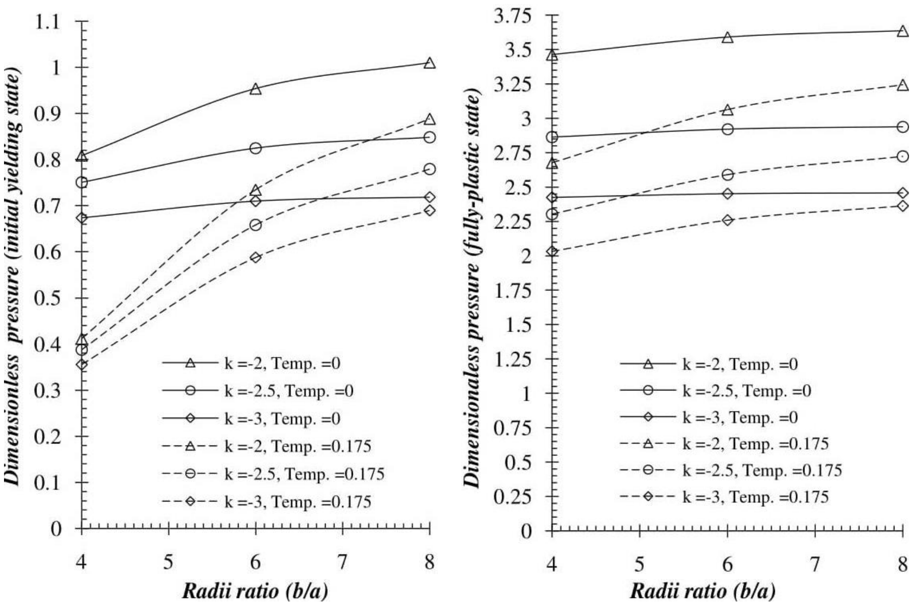


Figure 2. Graph between dimensionless pressures versus radii ratio with/without thermal effect: a) initial yielding sate; b) fully-plastic state with/without thermal effect.

In Fig. 2, curves are drawn between dimensionless pressure required for initial yielding/fully-plastic state versus radii ratio b/a = 4, 6, 8, and having temperature Θ_1 = 0 and 0.175, respectively.

A thick-walled circular cylinder made of non-homogeneous material with constant k = –2 requires higher dimensionless pressure to yield at the internal surface as compared to the thick-walled cylinder with constant k = –2.5, –3, and the value of pressure goes on increasing with the increase in radii ratio b/a = 4 and 6, for the initial/fully-plastic state. Further, the non-homogeneity of the thick-walled circular cylinder increases gradually with temperature (say Θ_1 = 0.175), the value of the dimensionless pressure decreases at the inner surface of the thick-walled cylinder for the initial/fully-plastic state.

Figure 3 is portrayed in order to demonstrate the behaviour of dimensionless stress distribution versus radii ratio $R = r/b$ in a non-homogeneous cylinder (i.e., $c = c_0 r^{-k}$) with temperature Θ_1 = 0 and 0.175, respectively. From Fig. 3, it is observed that hoop stress is maximum at outer surface for non-homogeneous circular cylinder and non-homogeneity increases the tangential stress at outer surface significantly. With the introduction of thermal effects, the values of the hoop stress decrease at the outer surface of the non-homogeneous circular cylinder. Moreover, thermal effects reduce the tangential stress at the external surface of a non-homogeneous circular cylinder extensively.

In Fig. 4, the geometry of the non-homogeneous circular cylinder and applied maximum/minimum radial/hoop stress at the inner and outer surface, is shown.

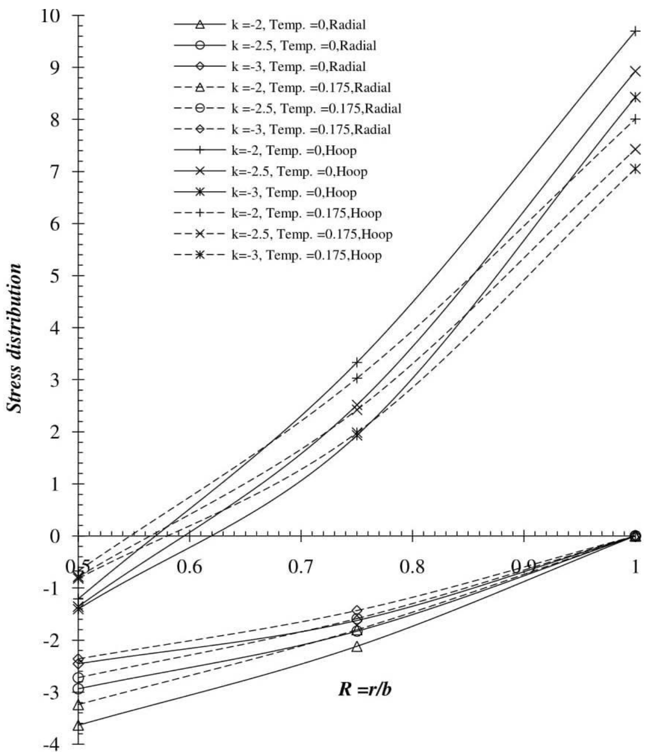
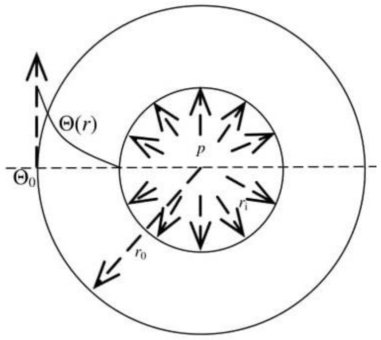
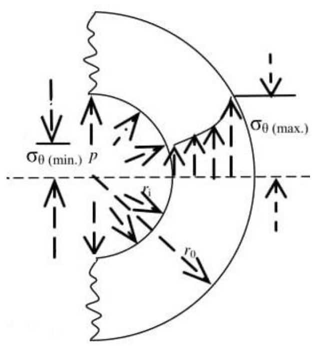
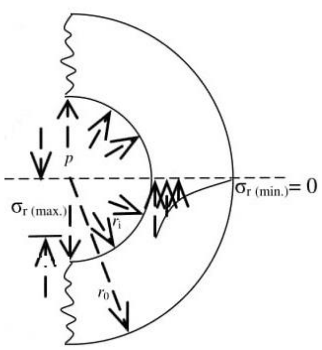


Figure 3. Graph between plastic dimensionless stress distribution vs. radii ratio $R = r/b$ in a non-homogeneous (i.e., $c = c_0 r^{-k}$) cylinder with different temperature.



thick cylinder





hoop stress distribution radial stress distribution

Figure 4. Stress distribution in thick-walled circular cylinder.

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