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SYMMETRY OF RECTANGULAR PLATE MADE OF ISOTROPIC MATERIAL

SIMETRIJA PRAVOUGAONE PLOČE OD IZOTROPNOG MATERIJALA

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- isotropic material
- plate
- stress
- tension
- compression

Abstract

The objective of this paper is to present the study of stress distribution in a rectangular plate made of isotropic material by using transition theory. The analysis also includes the neutral surface separating the tension and compression region. It has been observed that circumferential stresses are maximal at the neutral surface of the rectangular plate made of incompressible material as compared to the rectangular plate when it is made of compressible material. Numerical results are shown graphically.

Ključne reči
- izotropni materijal
- ploča
- napon
- zatezanje
- pritisak

Izvod

Cilj ovog rada je u predstavljanju istraživanja raspodele napona u pravougaonoj ploči od izotropnog materijala, primenom teorije prelaznih napona. U analizi se takođe uzima u obzir neutralna površina, koja razdvaja oblasti zatezanja i pritiska. Primećuje se da su obimski naponi maksimalni na neutralnoj površini pravougaone ploče izvedene od nestišljivog materijala u poređenju sa pravougaonom pločom, kada je ona izvedena od stišljivog materijala. Numerički rezultati su prikazani grafički u vidu dijagrama.

INTRODUCTION

The problems of elastic-plastic bending of rectangular plates have been studied by numerous researchers. The widespread use of plate structures in many fields of technology such as mechanics, civil engineering, aerospace, and marine is due to their intrinsic properties. Wojtaszak /1/ computed the maximal deflection, moments, and shear for a rectangular loaded plate with flanged edge. Gupta /2/ studied the problem of elastic-plastic transition in the flexion of rectangular sheets using the theory of transition. Matsuda et al. /3/ analysed the problem of elastoplastic bending of a rectangular plate using numerical integration and incremental variable elasticity. Jain et al. /4/ discussed the problem of elastic plastic bending of rectangular plates using Ilyushin's theory. Thakur et al. /5/ discussed the problem of the distribution of thermal constraints in a rectangular rubber/copper plate and glass materials using transition theory. In this paper, we discuss the stress distribution in a rectangular plate made of compressible/incompressible material by using Seth's transition theory.

GOVERNING EQUATION

We consider a rectangular plate referred to as an *x-y-z* system of rectangular coordinates and determine the position of the origin 0 of the *x-y-z* system at the corner of the middle plane of the plate. Let the rectangular plate be bent in the shape of a right circular cylinder and having two edges as generator, and the bending moment *M* per unit length is applied perpendicular to the plane. The displacement components are taken as: $u = x - f(r)$; $v = y - A\theta$; $w = \alpha z$; where A and α are constants; $f(r)$ is a function of r, and r is a function of x and y, respectively. The generalised strain components are given (Seth /6/):

$$e_{rr} = \frac{1}{2}\left[1 - f'^2\right],\quad e_{\theta\theta} = \frac{1}{2}\left[1 - \left(\frac{A}{r}\right)^2\right],$$
$$e_{zz} = \frac{1}{2}\left[1 - (1-\alpha)^2\right],\quad e_{r\theta} = e_{\theta z} = e_{zr} = 0, \tag{1}$$

where: $f' = df/dr$; and e_{rr}, $e_{\theta\theta}$, e_{zz} be strain components. The stress-strain relation is given by Seth /6, 7/:

$$T_{rr} = \frac{\mu}{C}\left\{1 - f'^2 + \left[1 - \left(\frac{A}{r}\right)^2\right](1-C) + 2(1-C)k - 2(1-C)\xi\Theta\right\},$$
$$T_{\theta\theta} = \frac{\mu}{C}\left\{\left[1 - \left(\frac{A}{r}\right)^2\right](1 - f'^2)(1-C) + (1-C)k - 2(1-C)\xi\Theta\right\},$$
$$T_{zz} = \frac{2\mu}{C}\left\{k + \frac{1-C}{2}(1 - f'^2) + \left[1 - \left(\frac{A}{r}\right)^2\right](1-C) - 2(1-C)\xi\Theta\right\},$$
$$T_{r\theta} = T_{\theta z} = T_{zr} = 0, \tag{2}$$

where: $c = \frac{2\mu}{\lambda + 2\mu} = \frac{1-2\nu}{1-\nu}$; and $k = \frac{1}{2}\left[1 - (1-\alpha)^2\right]$.

The equation of equilibrium is:

$$\frac{d\tau_{rr}}{dr} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} = 0. \tag{3}$$

Inserting Eq.(2) into Eq.(3), we get:

$$\frac{df}{dp} = \frac{2f(fp)^2}{2(1-c)A^2 + cr^2\left[(a/r)^2 - (fp/r)^2\right] - 2(p-1)(fp)^2} \tag{4}$$

where: $rf' = fp$. The transition points from Eq.(4) are $p \to 0$ and $p \to \pm\infty$. Over the plane ends, the resultant force normal to the plane $z = const.$ must vanish, i.e.,

$$\int_a^b rT_{zz} dr = 0\,,$$

whereas on the straight edge $\theta = \pm z$, we have

$$\int_a^b T_{\theta\theta} dr = 0 \quad \text{and} \quad M = -\int_a^b rT_{\theta\theta} dr\,, \tag{5}$$

where: M is the bending moment.

PROBLEM SOLUTION

Elastic to plastic state (see /5-21/), let the transition function ξ be defined as:

$$\xi = 1 + (1-c)(1+2k) - \frac{c}{\mu} T_{rr} \equiv f'^2 + (1-c)\left(\frac{A}{r}\right)^2. \tag{6}$$

Taking logarithmic differentiation from Eq.(6) and using Eq.(4), we get:

$$\frac{d(\log\xi)}{dr} = \frac{(c/r)\left[(A/r)^2 - (fp/r)^2\right]}{(fp/r)^2 + (1-c)(A/r)^2}. \tag{7}$$

The transition point $p \to \pm\infty$ (corresponds to the tension region) and $p \to 0$ (corresponds to compression region), we obtain the transition values of ξ from Eq.(7) and after integration with respect to r as:

$$\xi = A_1 r^{-c} \ \text{for}\ p \to \pm\infty \quad \text{and} \quad \xi = A_2 r^{c/(1-c)} \ \text{for}\ p \to 0. \tag{8}$$

From Eq.(8) and Eq.(6), we get stress for the region of tension/compression as:

$$T_{rr} = \frac{\mu}{c}\left[1 + (1-c)(1+2k) - A_1 r^{-c}\right], \tag{9}$$

$$T_{rr} = \frac{\mu}{c}\left[1 + (1-c)(1+2k) - A_2 r^{c/(1-c)}\right]. \tag{10}$$

The second equation of Eq.(1) shows that $r = A$ is the unstretched longitudinal fibre. Let us assume that T_{rr}^* be the radial stress of T_{rr} of neutral axis at $r = A$. Eq.(9) and Eq. (10) become:

tension:

$$T_{rr} = \left(\frac{A}{r}\right)^c T_{rr}^* + \frac{\mu(2-c)}{c}\left[1 - \left(\frac{A}{r}\right)^c\right] \ \forall \ (\text{i.e., } a \le r \le A), \tag{11}$$

compression:

$$T_{rr} = \left(\frac{r}{A}\right)^{\frac{c}{1-c}} T_{rr}^* + \frac{\mu(2-c)}{c}\left[1 - \left(\frac{r}{A}\right)^{\frac{c}{1-c}}\right] \ \forall \ (\text{i.e., } A \le r \le b) \tag{12}$$

It follows from the results for simple shear under the condition of finite deformation /7/ that in the transition $\mu \to k$, the latter being the yield limit in shear. Substituting $T_{rr} = 0$ at $r = a$ and $r = b$ into Eq.(11) and Eq.(12), respectively, and taking $\mu \to k$ (neglecting higher order terms), /7/, we get:

$$T_{rr}^* = \frac{k(2-c)}{c}\left[1 - \left(\frac{a}{A}\right)^c\right] \ \forall \ a \le r \le A\,,$$

$$T_{rr}^* = \frac{k(2-c)}{c}\left[1 - \left(\frac{A}{b}\right)^{c/(1-c)}\right] \ \forall \ A \le r \le b\,. \tag{13}$$

Neutral surface: the radial stresses T_{rr} must be continuous across the neutral axis at $r = A$. Eq.(13) becomes:

$$A = a^{(1-c)/(2-c)} b^{1/(2-c)} \ \forall \ 0 \le c \le 1. \tag{14}$$

Tension region: substituting Eq.(13) into Eq.(11) and into Eq.(12), we get the radial stresses for the regions of tension and compression as:

$$T_{rr} = \frac{k(2-c)}{c}\left[1 - \left(\frac{a}{r}\right)^c\right] \ \forall \ a \le r \le A\,,$$

$$T_{rr} = \frac{k(2-c)}{c}\left[1 - \left(\frac{r}{b}\right)^{c/(1-c)}\right] \ \forall \ a \le r \le A\,. \tag{15}$$

Compression region: substituting Eq.(15) into Eq.(3), we get the circumferential stress in the regions of tension and compression as:

$$T_{\theta\theta} = \frac{k(2-c)}{c}\left[(c-1)\left(\frac{a}{r}\right)^c + 1\right] \ \forall \ a \le r \le A\,,$$

$$T_{\theta\theta} = \frac{k(2-c)}{c}\left[1 - \frac{1}{1-c}\left(\frac{r}{b}\right)^{c/(1-c)}\right] \ \forall \ a \le r \le A\,. \tag{16}$$

From Eq.(7), we have:

$$T_{zz} = \left(\frac{1-C}{2-C}\right)(T_{rr} + T_{\theta\theta}) + \frac{2\mu(3-2C)}{(2-C)} e_{zz}\,. \tag{17}$$

Moment of couples: substituting Eq.(14) and Eq.(16) into boundary condition Eq.(5), the moment of couple per unit width at the ends is given:

$$M = -\int_a^b r\tau_{\theta\theta} dr = -\left[\int_a^A rT_{\theta\theta} dr + \int_A^b rT_{\theta\theta} dr\right] =$$

$$= \frac{k}{c}\Big\{(C-1)a\left[b(2-C)^{(1-C)/C} k^{(1-C)/C} - a\right] +$$

$$+ \frac{(b^2 - a^2)}{2} - b^{(C+1)/(1-C)}\left[b^{2-C} - a(2-c)^{1/C} k^{1/C}\right]\Big\}. \tag{18}$$

Equations (15)-(16) and Eq.(18) are the same as in Thakur et al. /5/, with neglecting the thermal condition.

RESULT AND DISCUSSION

For calculating the stress distribution in region of tension, neutral, and compression, based on the above analysis, the following values have been taken: $C = 0.00$ (incompressible material); 0.45; and 0.65 (compressible material), in respect.

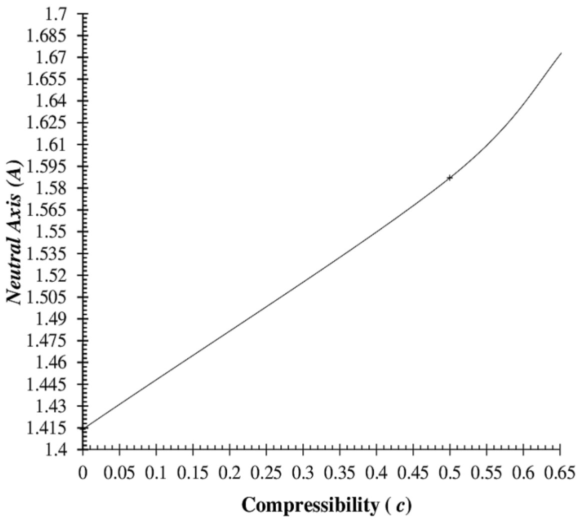

Figure 1. Graph of neutral axis A vs. compressibility c.

Curves are drawn between compressibility (C = 0; 0.45; 0.65) and the neutral axis (A) (see Fig. 1). It is observed that with increasing compressibility of materials, the value of the neutral axis may also be increased and the value of the neutral axis on the surface of tension must concentrate on compression.

In Fig. 2, curves are drawn between stress versus radii ratio (*r/a*). It is observed that the circumferential stress is maximal at the neutral surface of the rectangular plate made of incompressible material, as compared to the rectangular plate made of compressible material (i.e., $C = 0.45$ and 0.65). Rectangular plate made of incompressible material is more comfortable than that of the compressible material.

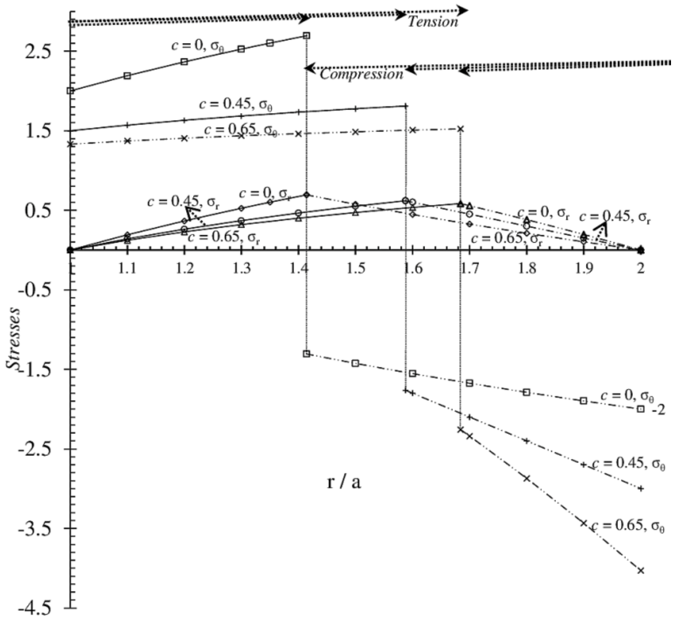

Figure 2. Stress vs. radii ratio.

CONCLUSIONS

The main findings are given as follows:

- rectangular plate made of incompressible material is more comfortable than that of compressible material,
- the result is the same as given by Thakur et al. /5/.

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REFERENCES

1. Wojtaszak, IA. (1937), *The calculation of maximum deflection, moments, and shear for uniformly loaded rectangular plate with clamped edges*, J Appl. Mech. 4(4): A173-A176. doi: 10.1115/1.4008810
2. Gupta, S.K. (1980), *Elastic-plastic bending of wide plates*, Ind. J Pure Appl. Math. 11(9): 1235-1241.
3. Matsuda, H., Sakiyama, T. (1988), *An analysis of elasto-plastic bending of rectangular plate*, Trans. Jap. Soc. Civ. Eng. 1988 (392): 141-149. doi: 10.2208/jscej.1988.392_141
4. Jain, R.K, Mazumdar, J. (1994), *Research note on the elastic-plastic bending of rectangular plates*, Int. J Plast. 10(7): 749-759. doi: 10.1016/0749-6419(94)90044-2
5. Thakur, P., Sethi, M., Gupta, N., Gupta, K. (2021), *Thermal effects in rectangular plate made of rubber, copper and glass materials*, J Rubber Res. 24: 147-155. doi: 10.1007/s42464-020-00080-6
6. Seth, B.R. (1962), *Transition theory of elastic-plastic deformation, creep and relaxation*, Nature, 195: 896-897. doi:10.1038/195896a0
7. Seth, B.R. (1970), *Transition conditions: The yield condition*, Int. J Non-Linear Mech. 5(2): 279-285. doi: 10.1016/0020-7462(70)90025-9
8. Thakur, P., Gupta N., Gupta K., Sethi, M. (2020), *Elastic-plastic transition in an orthotropic material disk*, Struct. Integr. Life, 20(2): 169-172.
9. Thakur, P., Chand, S., Sukhvinder, et al. (2020), *Density parameter in a transversely and isotropic disc material with rigid inclusion*, Struct. Integr. Life, 20(2): 159-164.
10. Sethi, M., Thakur P. (2020), *Elastoplastic deformation in an isotropic material disk with shaft subjected to load and variable density*, J Rub. Res. 23: 69-78. doi: 10.1007/s42464-020-00038-8
11. Thakur, P., Sethi, M. (2020), Elasto-plastic deformation in an orthotropic spherical shell subjected to a temperature gradient, Math. Mech. Solids, 25(1): 26-34. doi: 10.1177/1081286519857128
12. Thakur, P., Kumar N., Sukhvinder (2020), *Elasto-plastic density variation in a deformable disk*, Struct. Integr. Life, 20(1): 27-32.
13. Thakur, P., Kumar, N., Sethi, M. (2021), *Elastic-plastic stresses in a rotating disc of transversely isotropic material fitted with a shaft and subjected to thermal gradient*, Meccanica, 56: 1165-1175. doi: 10.1007/s11012-021-01318-2
14. Thakur, P., Sethi, M., Kumar, N., et al. (2021), *Thermal effects in a rotating disk made of rubber and magnesium materials and having variable density*, J Rubber Res. 24(3): 403-413. doi: 10.1007/s42464-021-00107-6
15. Thakur, P., Sethi, M., Gupta, K., Bhardwaj, R.K. (2021), *Thermal stress analysis in a hemispherical shell made of transversely isotropic materials under pressure and thermo-mechanical loads*, J Appl. Math. Mech. (ZAMM), 101(12). doi: 10.1002/zamm.202100208
16. Thakur, P., Sethi, M., Kumar, N., et al. (2021), *Analytical solution of hyperbolic deformable disk having variable density*, Mech. Solids, 56(6): 1039-1046. doi: 10.3103/S0025654421060194
17. Thakur, P., Sethi, M., Kumar, N., et al. (2022), *Stress analysis in an isotropic hyperbolic rotating disk fitted with rigid shaft*, Z Angew. Math. Phys. 73(1), Art. ID 23. doi: 10.1007/s00033-021-01663-y
18. Kumar, N., Thakur, P. (2021), *Thermal behaviour in a rotating disc made of transversely isotropic material with rigid shaft*, Struct. Integr. Life, 21(3): 217-223.
19. Kumar, A., Sood, S., Thakur, P., Kumar, N. (2022), *Safety analysis in an isotropic material disc fitted with a rigid shaft subjected to thermo-mechanical load by using transition theory*, Struct. Integr. Life, 22(2): 193-201.
20. Kumar, N., Thakur, P. (2022), *Behaviour of creep stress and strain rates in a solid disc made of isotropic material with thermal condition*, Struct. Integr. Life, 22(2): 202-209.
21. Chand, S., Sood, S., Thakur, P., Gupta, K. (2023), *Elastoplastic stress deformation in an annular disk made of isotropic material and subjected to uniform pressure*, Struct. Integr. Life, 23 (1): 61-64.