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EFFECT OF FORCED VIBRATIONS OF BRAKE PADS ON THE THERMAL PERFORMANCE OF THE DISC: A NUMERICAL PERSPECTIVE

UTICAJ FORSIRANIH VIBRACIJA KOČIONIH OBLOGA NA TOPLOTNE KARAKTERISTIKE DISKA: NUMERIČKA PERSPEKTIVA

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Keywords
- brake disc
- brake pad
- vibration
- friction
- heat transfer
- simulation
- COMSOL Multiphysics®

Ključne reči
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- obloga kočnice
- vibracije
- trenje
- prenos toplote
- simulacija
- COMSOL Multiphysics®

Abstract

In this work we numerically investigate the thermal behaviour of automobile brake discs when the brake pads are subjected to horizontal vibrations. To determine the influence of these vibrations on the performance of the disc, the partial differential equation (PDE) of conductive heat transfer in the disc is solved numerically using the finite element method. The outer surfaces of the disc and pad are exposed to convective and radiant heat flows. We determine the influence of the amplitude and frequency of vibrations of the plate on the isotherms' distribution, maximum temperature, average temperature, and dissipation energy. As per the results, this study shows that adding a vibration force can reduce the heat dissipated in the disc and, therefore, improve the performance of the automobile braking system.

Izvod

U ovom radu numerički istražujemo termičko ponašanje automobilskih kočionih diskova, kada su kočione obloge opterećene horizontalnim vibracijama. Radi određivanja uticaja ovih vibracija na karakteristike diska, numerički se rešava parcijalna diferencijalna jednačina (PDJ) provođenja toplote u disku, primenom metode konačnih elemenata. Spoljne površine diska i obloge su izložene prenosu toplote konvekcijom i zračenjem. Određujemo uticaj amplitude i frekvencije vibracija ploče na raspodelu izotermi, maksimalne temperature, prosečne temperature i disipaciju energije. S obzirom na rezultate, u radu se pokazuje da se odvođenje toplote kroz disk može smanjiti zadavanjem sile vibracija, a samim tim, poboljšati performanse kočionog sistema automobila.

INTRODUCTION

An automobile is made up of a set of complex elements. Among them is the engine, undoubtedly the key element of car mechanics, because it allows the creation of rotational movement necessary for good performance. To ensure the safety of passengers, a braking mechanism is designed to slow down the vehicle, even stop it, or keep it stationary depending on different situations. Active safety of a vehicle is mainly influenced by properties of the installed braking system /1/.

A simple press on the pedal triggers the brakes to operate. To slow down vehicle movement, the disc, which is attached to the wheel, is pinched between two brake pads. This action brings the disc-pad couple into contact with friction. Once activated, this transforms the kinetic energy accumulated by the vehicle into heat. The braking system uses the principle of friction between two contacting surfaces in relative motion. The effectiveness of braking depends on several parameters such as the weight of the vehicle, the speed, the force exerted, and thermomechanical properties of friction materials (discs and pads). In extreme situations, the temperature of friction elements can reach 800 °C. Therefore, when braking, the majority of the heat energy is absorbed by the disc /2/. More specifically, the contact pressure made by the pads on the disc is the origin of mechanical and thermal fatigue. The latter is a direct consequence of thermal gradients and hot spots on the disc, caused by friction. Thermal gradients established on the disc contribute to its deformation and appearance of cracks, reducing braking performance. All these phenomena depend on the maximum temperature reached during braking, /3, 4/.

Thus, the energy transformation of braking produces heating of the pads and disc, and it is well known that heat can be transferred by three modalities: convection, conduction, and radiation.

One of the major concerns of researchers is to find ways to evacuate as much of the energy dissipated as heat to the external environment to reduce the hot spot temperature of the disc.

Kunt /5/ studied the variation of temperature of the brake disc by varying the mass of the vehicle and its speed. Reducing the mass by 300 kg reduced the temperature by 3.33 % at a speed of 25 m/s, and 6.03 % at a speed of 35 m/s.

Benramdane et al. /6/ presented the characterisation and analysis of heat transfer by conduction in a brake disc for different geometric configurations, heated to uniform braking temperature, and subjected to cooling. They found that the temperature distribution depends on several parameters: the number of holes, diameter of holes, and their distribution.

Bashir et al. /7/ addressed the modelling of frictional heating in a car disc brake using Comsol Multiphysics® software. The prediction of friction heat, temperature distribution, and thickness variation of brake disc was performed using finite element analysis technique. The model simulates the dynamics and heating by friction between the pad and disc. The frictional heat and temperature distribution have been calculated, as well as the stresses and variation in thickness of the disc. In addition, Sarkar et al. /8/ studied the ventilated disc brake thermal transfer rate by varying the shape of the material and vane. The materials of aluminium metal matrix composite (AMMC), asbestos, and gray cast iron (GCI) were used at different speeds to discover better performance.

Dhir /9/ investigated the disc longevity and observed the overall rise in temperature by disc rotor geometry modification such as holes and airfoil vents. Ahmed et al. /10/ designed the disc brake with radial grooves. They adopted ANSYS® software for transient structural and thermal analysis and examined temperature changes throughout the discs with generated heat flux. They have also compared the result with a disc brake without grooves on the disc surface and found a 7 °C temperature difference. Pan and Cai /11/ focused their study on ventilated disc brakes. They estimated the temperature field by means of the thermodynamic coupling method and sequential coupling method. In this context, an approach that contemplates conjugate heat transfer problems /12, 13/ emerges as indispensable. Türkan et al. /14/ studied numerically the effect of vehicle speed on the variation of brake disc temperature using the three-dimensional Comsol® finite element programme by honing the steel (EI-696) and ceramic-metal (FMC-11) as materials for disc and pads, respectively.

Khatir et al. /15/ characterised and analysed the effect of thermal conduction of the brake disc for two geometric configurations and three distinct materials (standard steel, aluminium alloy, and gray cast iron), heated to uniform braking temperature and subject to cooling.

During braking, the disc undergoes two sudden variations in temperature and heat flow, from the start of braking, until moment $t = 0.5$ s there is a sudden and rapid rise in temperature and heat flow. After this moment and until the end of braking the temperature and heat flow decrease rapidly.

Sarkar et al. /8/ investigated and analysed the temperature distribution of the disc to identify critical temperature during operation using FEA analysis. Static thermal analysis has been carried out on the disc to evaluate and compare their performance, and the temperature distribution was analysed considering cooling parameters (convection and radiation). Comparative study is carried out between different materials, i.e., AMMC, asbestos and GCI.

Sugunarani and Santhosh /16/ analysed the temperature distribution of the brake disc during operation using Comsol Multiphysics® to calculate and predict the temperature distribution on the brake disc and identify the critical temperature. The results obtained from the analysis indicate that different materials exhibit different temperature distribution.

Ouyang et al. /17/ studied car disc brake squeal by transient analysis and detailed the combination of heat conduction analysis, contact analysis, and transient analysis of disc brake squeal. The contact pressure at the disc-pad interface is calculated and the information is used to define the friction induced heat flux. Finally a transient analysis is carried out, considering the influence on squeal generation of the contact pressure distribution affected by brake pad surface roughness and thermal deformation. A notable difference is noted between the dynamic responses obtained with the thermal effect and those without the thermal effect.

Park et al. /18/ used the finite element based commercial programme SAMCEF® to simulate the friction energy generated by sliding contact between disc and pad during braking. Sensitivity analysis was carried out in order to find the sensitive parameters of brake judder which were predicted after the phenomenon was confirmed.

Petinrin and Oji /19/ compared two brake pad materials (asbestos and aramid) by modelling the heat generation and dissipation in a disc brake during and after braking action. From their findings it was established that aramid as brake pad material was a good substitute for asbestos which was reportedly releasing toxic dust into the environment.

However, in this present work, the mathematical model for fully coupled thermoelastic instability problem of a disc brake system is developed from kinetic and potential energies of moving vehicles and analysed with gray cast iron brake disc and aramid brake pads for a car under braking actions on gradient surfaces: horizontal, descendent, and ascendant, as potential energy contribution has not been given attention in previous studies.

It is pertinent to state here that the wear action due to frictional heat generation between disc and pads is assumed to be infinitesimal and disregarded in this analysis.

Qi and Day /20/ discuss that using a designed experimental approach, the factors affecting interface temperature, including the number of braking applications, sliding speed, braking load, and type of friction material were studied. It was found that the number of braking applications had the strongest effect on friction interface temperature. The real contact area between disc and pad, i.e., pad regions where the bulk of kinetic energy is dissipated via friction, has a significant effect on braking interface temperature. For understanding the effect of real contact area on local interface temperatures and friction coefficient, finite element analysis (FEA) is conducted, and it is found that the maximal temperature at the friction interface does not increase linearly with decreasing contact area ratio. This finding is potentially significant in optimising the design and formulation of friction materials for stable friction and wear performance.

Zaid et al. /21/ investigated the disc brake rotor by finite element analysis. They conducted a study on ventilated disc brake rotor of a normal passenger vehicle with full load of capacity. The study more likely concerns the heat and temperature distribution on disc brake rotor. In this study, finite element analysis approach is conducted in order to identify temperature distributions and behaviours of disc brake rotor in transient response. ABAQUS/CAE® has been used as finite elements software to perform the thermal analysis on transient response. Thus, this study provides better understanding on the thermal characteristic of disc brake rotor and assists the automotive industry in developing optimum and effective disc brake rotors.

Singh and Shergill /22/ reported the thermal analysis of disc brake using Comsol®. Finite element analysis techniques are used to predict the temperature distribution and identify the critical temperature of brake disc. Considering all three modes of heat transfer (conduction, convection, and radiation) for three different materials of rotor disc were used (cast iron, aluminium, and ceramics). It was found that cast iron can be used for brake disc which will give moderate cooling at low temperature as compared to others. Ceramics have good cooling characteristics but are costly; they can be used in racing cars where high temperature is produced.

Deressa and Ambie /23/ analysed different approaches, methods and simplifications in the numerical modelling of temperature fields of a rotating disc and stationary brake pads. One of the most important requirements when developing such models, in addition to the accurate determination of maximal temperature and flash temperature occurring in the actual contact zones of the pads with the disc, is the variability of friction coefficient and the intensity of wear rate. In the developed model, we are interested in shortening the calculation time.

Yevtushenko and Grzes /24/ have examined the effect of different temporal friction power profiles on temperature distribution on brake disc and brake lining. Analysis of heat generation between disc and lining has been made by means of finite elements method by using COMSOL Multiphysics software, /22, 25/.

Kohli et al. /26/ studied the effect of brake pad material on brake efficiency, replacing currently used materials with others with improved properties. A static analysis was taken into account to validate the ductility of the material. Sudden braking can cause significant mechanical and thermal stress which could damage the brake disc. After validation, it was concluded that carbon-ceramic disc brakes perform better than others in structural and thermal aspects for the same boundary and loading conditions.

The present work investigates the heating reduction of the brake disc by adding an exciting force to the brake pads during braking. The main goal is to determine the influence of this vibrational force on the thermal behaviour of the disc, taking into account both convection and radiation.

PROBLEM FORMULATION

The geometry under investigation is presented in Fig. 1, while geometric and physical characteristics of the disc and pad are given in Table 1.

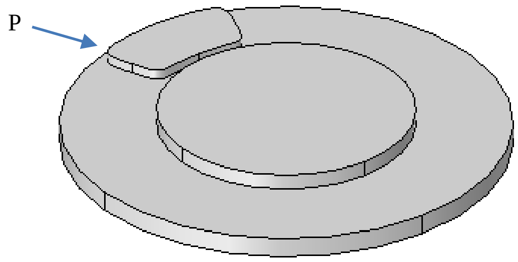

Figure 1. Disc geometry and brake pad.

Table 1. Geometric and physical characteristics.

Characteristics	Disc	Pad
External radius (mm)	140	138
Internal radius (mm)	80	83
Thickness (mm)	13	6.5
Density (kg/m^3)	7870	2000
Thermal conductivity (W/mK)	82	8.7
Heat capacity (J/kgK)	449	935
Emissivity	0.28	0.8

GOVERNING EQUATIONS

The temperature distribution in the disc is governed and determined by the conduction heat transfer equation:

$$\rho Cp\frac{\partial T}{\partial t}=k\nabla^2 T+q \ . \tag{1}$$

To be solved together with initial and boundary conditions, namely:

$$T(0,r,\theta,z)=T_{air}=27\ °\mathrm{C} \ . \tag{2}$$

On the external disc surface:

$$Q_0=h(T_{air}-T)+\sigma\varepsilon_d(T_{air}^4-T^4) \ . \tag{3}$$

On the external pad surface:

$$Q_0=h(T_{air}-T)+\sigma\varepsilon_p(T_{air}^4-T^4) \ . \tag{4}$$

On the disc-pad perfect contact:

$$T_d=T_p, \quad k_d\frac{\partial T}{\partial z}=k_p\frac{\partial T}{\partial z} \ . \tag{5}$$

On the symmetry surface:

$$\frac{\partial T}{\partial z}=0 \ . \tag{6}$$

On the (assumed perfect) contact zone, the heat power generated is defined by:

$$q(r,t)=-f_f V_d(r,t)=-\frac{1}{8A}m\frac{r}{r_m}a(V_0+at) \ . \tag{7}$$

The heat dissipation from disc and brake pad surfaces to the ambient air is described by convection and radiation as follows:

$$q_{diss}=-h(T-T_{air})-\varepsilon\sigma(T^4-T_{air}^4) \ . \tag{8}$$

During braking, the brake pad is animated by a harmonic movement in the plane of the friction track whose speed is given by:

$$V_p(t)=2\pi fb\sin(2\pi ft) \ . \tag{9}$$

NUMERICAL SOLUTION

The system of governing Eqs.(1)-(9) is solved numerically by finite element method. After several tests, the obtained results are organised in Table 2, choosing the extra fine mesh (M5), as shown in Fig. 2, to ensure the optimal condition verifying the maximization precision with a minimization of analysis time.

Table 2. Meshing of specimens.

Mesh	Type	Nombre of elements	T_{max} (°C)
M1	Coarse	652 E 3D, 674 E 2D, 152 E 1D	160.16
M2	Normal	1212 E 3D, 1206 E 2D, 204 E 1D	162.07
M3	Fine	1816 E 3D, 1762 E 2D, 252 E 1D	163.12
M4	Finer	3048 E 3D, 2896 E 2D, 312 E 1D	164.53
M5	Extra fine	5828 E 3D, 5388 E 2D, 424 E 1D	165.42
M6	Extremely fine	16580 E 3D, 14854 E 2D, 664 1D	165.42

The fixed values of some characteristics and parameters are presented in Table 3.

Table 3. Parameters for the numerical solution.

Parameter	Value
Vehicle mass m (kg)	1800
Wheel radius R (mm)	250
Initial velocity of the vehicle V_0 (m/s)	25
Deceleration of the vehicle a (m/s^2)	10
Friction coefficient μ (-)	0.3
Air temperature T_{air} (°C)	27
Braking start time (s)	2
Braking end time (s)	4

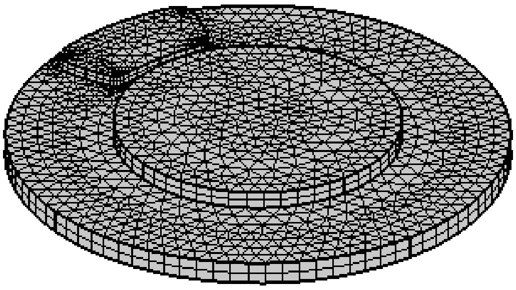

Figure 2. Computational domain mesh.

RESULTS AND DISCUSSION

Figures 3 and 4 display the distribution of isotherms for different times (before, during and after braking) and for different amplitudes of oscillatory movement of the pad, while the frequency is assumed to equal 10 Hz. During the braking operation, we see that the rim fixing bowl on the disc is almost not thermally influenced by the braking operation. The only area affected by this operation is the friction track.

In the presence and absence of vibrations, the temperature on the friction track increases with increasing time until reaching its maximal value in the middle of braking (t = 3 s). From this moment, it begins to decrease until it reaches room temperature.

The maximal temperature value decreases in the presence of plate vibration. This reduction evolves proportionally with the amplitude of vibration.

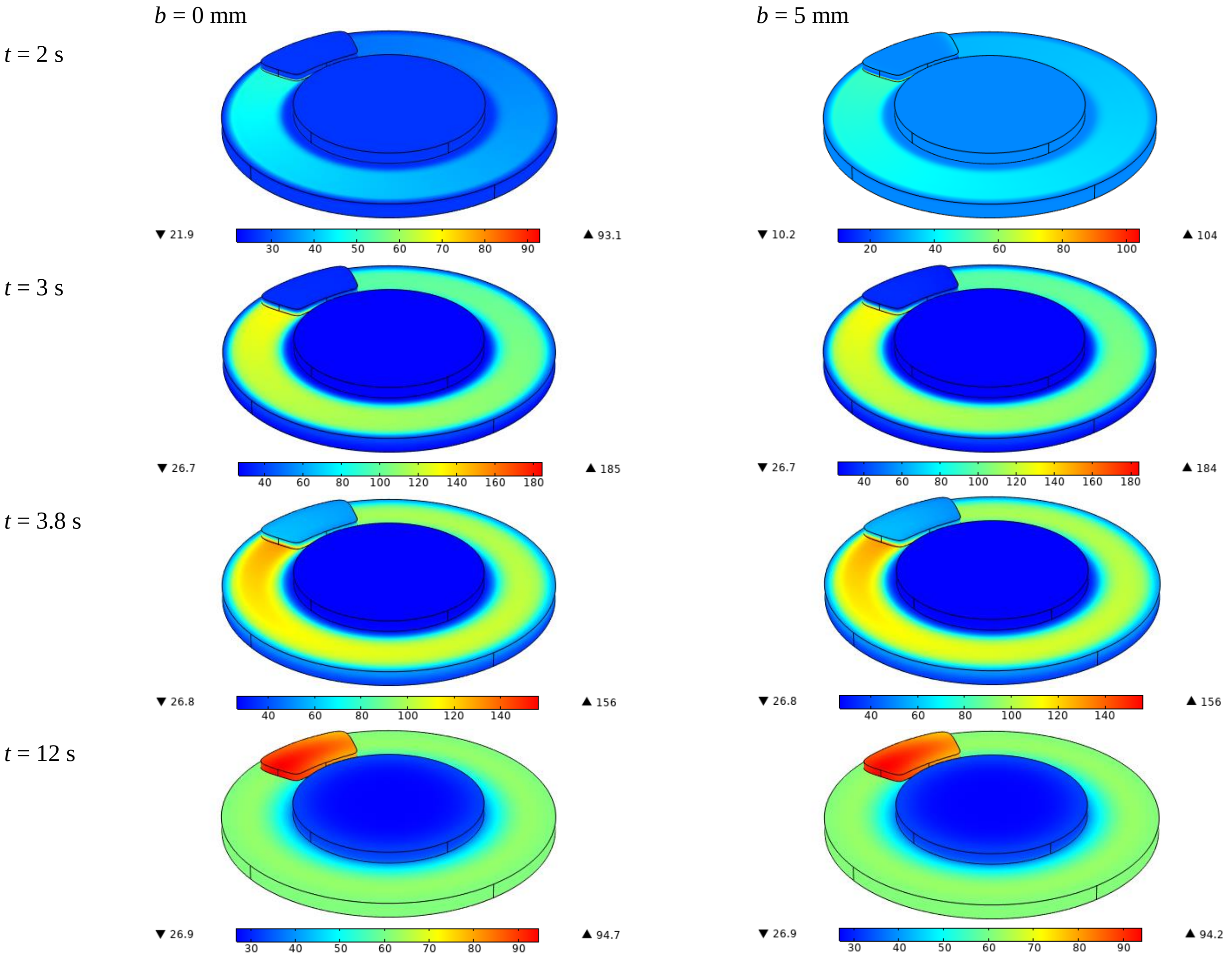

Figure 3. Isotherms for different times without vibration (b = 0) and with vibration (b = 5 mm).

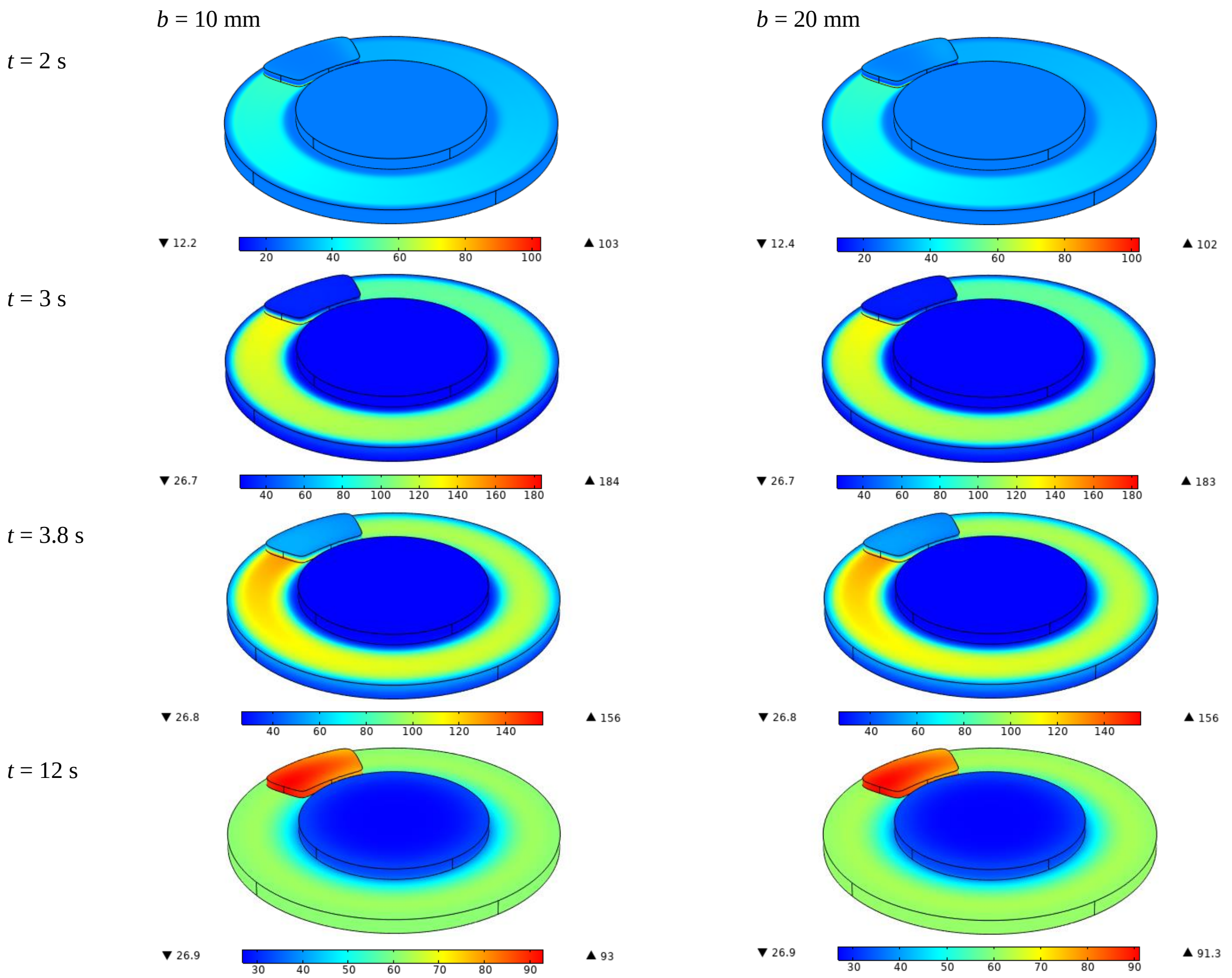

Figure 4. Isotherms at different times, with vibration (b = 10 mm; b = 20 mm).

We can observe the temperature profile at point P, i.e., the hottest point on the disc-pad contact surface, located in the middle of the left border of the pad (Fig. 5). Figure 5 represents the evolution of temperature at this point as a function of time for different vibration amplitudes. During the braking period, the temperature increases with increasing time and reaches its maximum value around t = 3 s where the disc is exposed to strong heating. The temperature value at this instance decreases with increase in the amplitude of vibration, thus giving a reduction in the heating of the disc, as shown in the zoomed area.

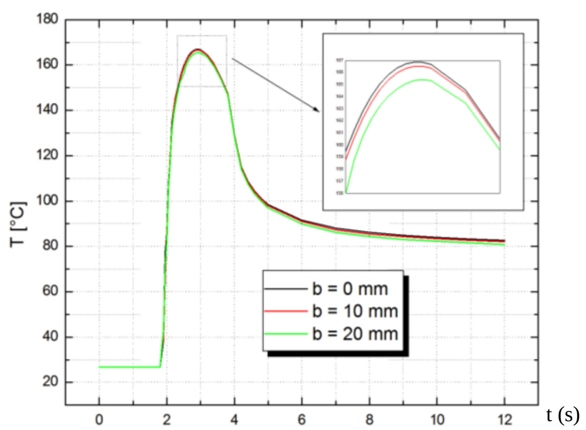

Figure 5. Temperature at point P vs. time for different amplitudes.

Figure 6 represents variation of temperature at the indicated point as a function of amplitude of vibrations of the pad at several times (before, during and after braking). In the absence and presence of vibrations, the temperature is proportional to time in interval (2 s, 3 s) and inversely proportional in the interval (3 s, 4 s) until reaching a minimum value corresponding to ambient temperature. At any time, the temperature at the point decreases by increasing the amplitude of vibrations. The histogram in Fig. 7 confirms these results.

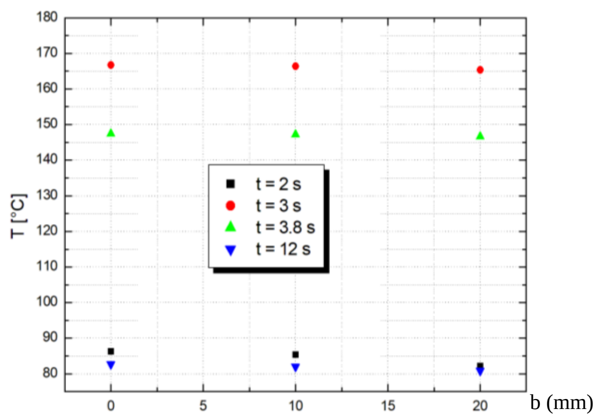

Figure 6. Temperature at point P vs. amplitude for different time steps.

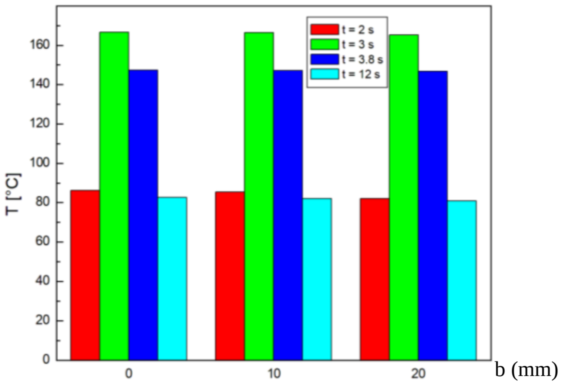

Figure 7. Temperature at point P vs. amplitude.

The variations of maximum temperature on the friction track as a function of time for different values of amplitude, and as a function of amplitude for different values of time are shown in Figs. 8 and 9. These variations show an extreme (max) in the vicinity of t = 3 s. Zooming in this area clearly shows this critical point which decreases with increase in vibration amplitude. From the end of braking, the disc begins to cool under the effect of heat transfer by convection until it reaches ambient temperature. The time required for cooling decreases as vibration amplitude increases, as shown in the zoomed area.

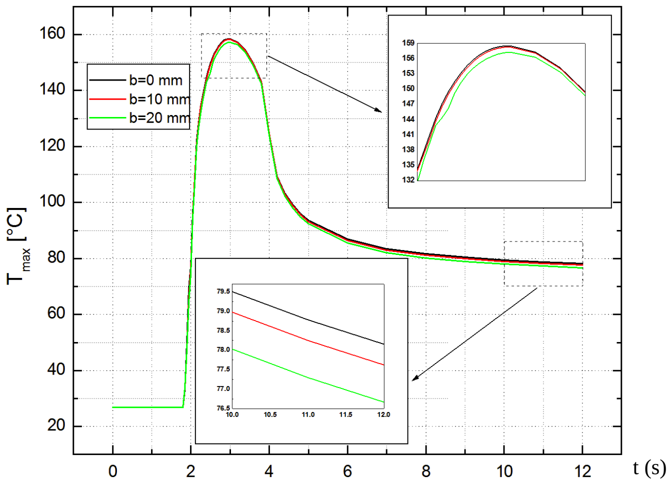

Figure 8. Variation of max. temperature vs time for different amplitudes.

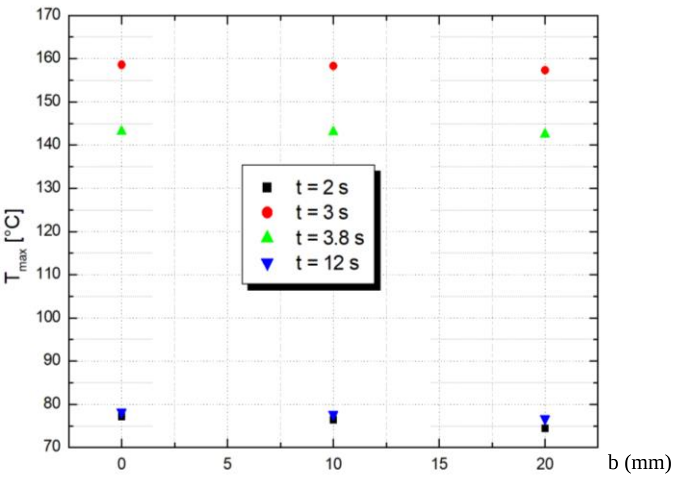

Figure 9. Variation of max. temperature vs. amplitude at different times.

The results are also confirmed by the plots of the average temperature on the friction track, as a function of time (Fig. 10) and amplitude (Fig. 11). Moreover, Fig. 12 represents the histogram of this temperature variation as a function of amplitude. These results confirm the previously discussed findings.

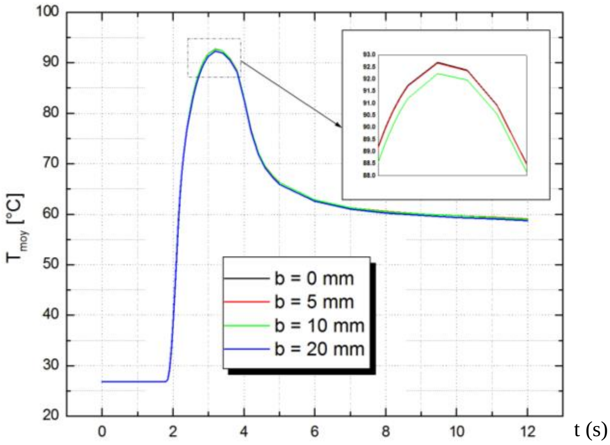

Figure 10. Average temperature vs. time.

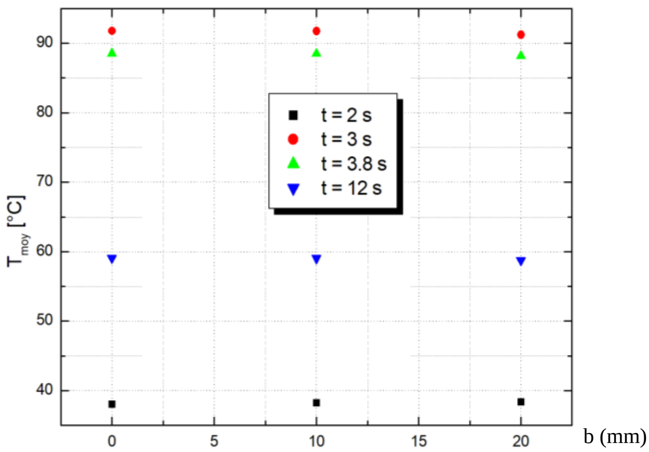

Figure 11. Average temperature vs. amplitude.

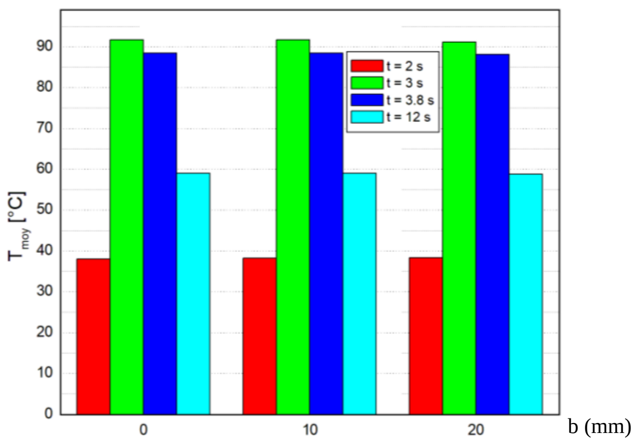

Figure 12. Average temperature vs. amplitude at different times.

The time evolution of the energy produced by braking and dissipated in the form of heat in the brake disc is presented in Figs. 13-15. The figures show that heat dissipation in the disc decreases with increasing values of vibration amplitude. The histogram in Fig. 15 illustrates the percentage of energy dissipated in the disc compared to the energy produced for different values of amplitude (with and without vibration).

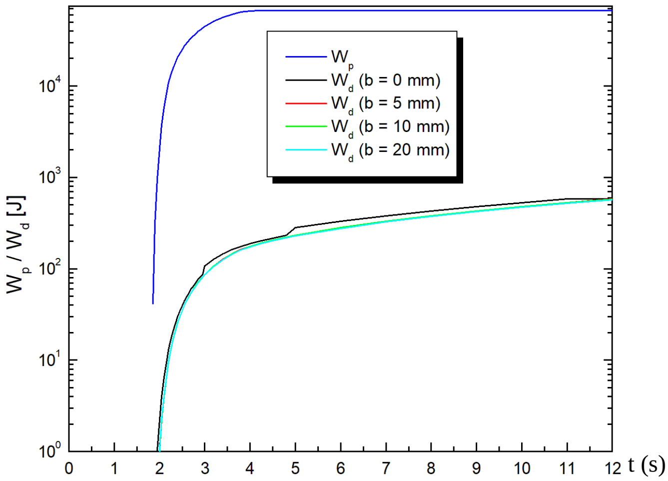

Figure 13. Comparison between produced and dissipated energies over time.

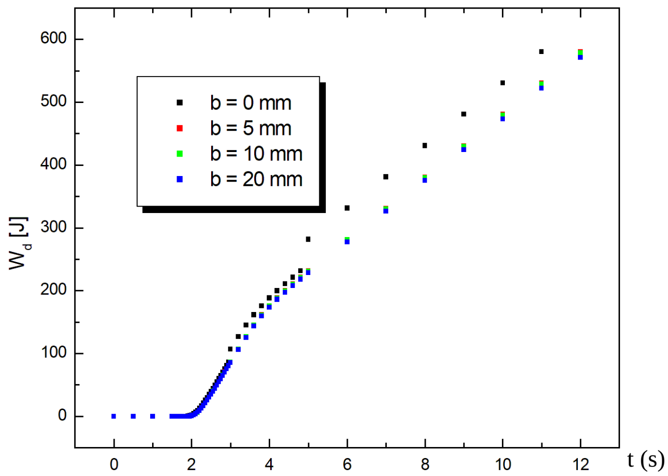

Figure 14. Dissipated energy vs. time.

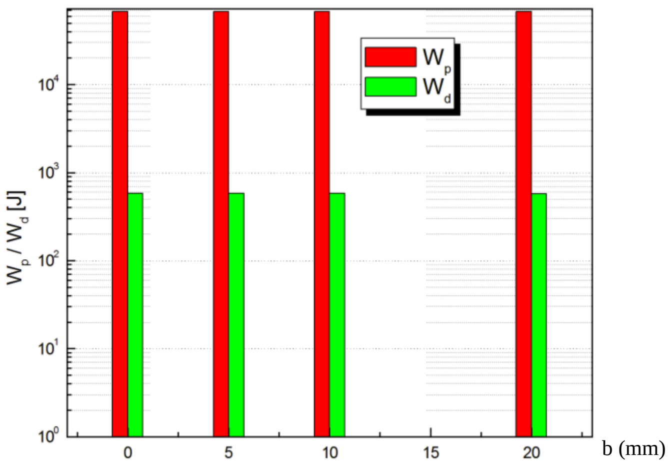

Figure 15. Histogram of produced and dissipated energy.

CONCLUSION

Improving the performance of an automobile braking system ensures the safety and comfort of passengers. In the present paper, we solve numerically the heat equation for brake discs when brake pads are subjected to horizontal vibrations. To preserve the rigidity of the disc and pad, we opted to add an exciting force to the brake pad in order to determine its impact on braking performance. Indeed, particular attention is paid to the influence of amplitude and frequency of vibrations of the plate on the distribution of isotherms, maximum temperature, average temperature, and dissipation energy. The main findings can be summarized as follows:
- maximum temperature decreases with increasing amplitude of horizontal vibrations;
- also the average temperature decreases with increasing amplitude;
- the energy dissipates as heat in the disc decreases with increasing amplitude of horizontal vibrations.

Thus, adding a vibration force can reduce heat dissipated in the disc, which can improve the performance of the automobile braking system.

List of symbols

A pad area (mm^2)
a vehicle deceleration (ms^{-2})
b vibration amplitude (m)
C_p specific heat (J/kgK)
f frequency
f_f friction force (N)
h convection coefficient (W/m^{2}K)
k thermal conductivity (W/mK)
m vehicle mass (kg)
Q power per volume unit (W/m^3)
Q_0 power per surface unit (W/m^2)
R radial coordinate (m)
r_m average radius (m)
T temperature (K)
T time (s)
V_0 initial vehicle speed (m/s)
V_p tangential disc speed (m/s)
Z vertical coordinate (m)

Greek symbols

Θ emissivity
q angular coordinate (m)
r Density
s Stefan-Boltzmann constant

Subscripts

air air
d disc
p pad

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