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EFFECT OF MAGNETIC FIELD ON THERMAL INSTABILITY IN ROTATING JEFFREY NANOFLUID SATURATED BY A POROUS MEDIUM: FREE-FREE, RIGID-RIGID AND RIGID-FREE BOUNDARY CONDITIONS

UTICAJ MAGNETNOG POLJA NA TERMIČKU NESTABILNOST ROTIRAJUĆEG JEFFREY NANOFLUIDA ZASIĆENOG POROZNOM SREDINOM, GRANIČNIH ULOVA: SLOBODNO-SLOBODNO, KRUTO-KRUTO I KRUTO-SLOBODNO

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- convection
- nanofluid
- Jeffrey model
- Rayleigh number
- porous media

Abstract

The effect of a revolving Jeffrey nanofluid in a porous medium exposed to a magnetic field is examined in this paper. Three distinct boundary conditions are used to examine the system: free-free, rigid-rigid, and rigid-free. The study examines how the magnetic field impacts the behaviour of the spinning nanofluid inside the porous media using analytical methods and mathematical models. The results broaden our understanding of the connection between rotation, the magnetic field and the special rheological properties of the Jeffrey nanofluid in porous media. The impacts of the Rayleigh number, Lewis number, modified diffusivity ratio, Jeffrey parameter, Chandrashekar number, Taylor number, and porosity of the nanoparticles are investigated through the application of both mathematical and graphical approaches.

Ključne reči
- konvekcija
- nanofluid
- Jeffrey model
- Rejlejev broj
- porozna sredina

Izvod

U ovom radu se proučava uticaj na rotirajući Jeffrey nanofluid u poroznoj sredini, koji je izložen magnetnom polju. Primenjena su tri specifična granična uslova za istraživanje sistema: slobodno-slobodno, kruto-kruto, i kruto-slobodno. Primenom analitičkih metoda i matematičkih modela, izučava se uticaj magnetnog polja na ponašanje rotirajućeg nanofluida unutar porozne sredine. Dobijeni rezultati proširuju naše razumevanje povezanosti rotacije, magnetnog polja i posebnih reoloških osobina Jeffrey nanofluida u poroznoj sredini. Istražuju se uticaji Rejlejevog broja, Luisovog broja, modifikovanog odnosa difuzivnosti, Jeffrey parametra, Čandrašekarovog broja, Tejlorovog broja, i poroznost nanočestica, uz primenu matematičkog i grafičkog pristupa.

INTRODUCTION

Non-Newtonian fluids are employed in many different contexts, from everyday goods to manufacturing processes. Understanding their rheological behaviour is crucial in a variety of industries, including food processing, medicines, cosmetics, petroleum engineering and materials research. In many situations, scientists and engineers utilize rheological models to predict and describe the behaviour of non-Newtonian fluids. The Jeffrey fluid model is one type of non-Newtonian fluid model that enhances the conventional Newtonian model with elasticity effects. The elasticity of the fluid and a material parameter that symbolizes the shear rate determine the shear stress in a Jeffrey fluid. Spinning Jeffrey nanofluids are employed in a wider view of technological and commercial applications, where control over fluid behaviour, heat transfer and magnetic fields are essential.

There are various applications for magnetic materials, including heat exchangers, cooling systems, materials processing, magnetic drug targeting, biomedical applications, magnetorheological devices, cooling systems, and electrokinetic devices. These applications highlight the versatility of rotating Jeffrey nanofluids in a range of disciplines where their unique thermal and rheological properties, along with the influence of magnetic fields, can be applied for specific goals. Research in this area is expanding as engineers and scientists explore novel applications of nanofluids to enable advancements in technology.

Newtonian fluids include engine oil, soap solutions, sauces, foam, paints, lubricants and biological fluids like blood and synovial fluid. The modelling of non-Newtonian fluids has produced a number of constitutive relations due to the importance of non-Newtonian fluids in contemporary technology and industry. The Jeffrey non-Newtonian fluid model is one of these constitutive relations. A linear model called the Jeffrey fluid model substitutes time derivatives for convective derivatives. Jeffrey /5/ investigated the stability of a fluid layer that had been heated from below. He came up with a numerical solution to a few issues with the stability of a layer in a compressible fluid as temperature rises. Chandrasekhar /3/ has provided a thorough literature assessment on thermal instability in a Newtonian fluid. The Jeffrey fluid model has been researched by numerous researchers /1, 4, 6, 12-23/ and as a result, it is today regarded as the best fluid model to represent the behaviour of physiological and industrial fluids.

The convective flow in a porous material was researched by Lapwood /8/. The Rayleigh's instability of a thermal boundary layer in a flow through a porous media was explored by Wooding /27/. They discovered that the layer is stable under certain conditions, including a critical positive Rayleigh number for the system and a limited wave number for the critical neutral disturbance. Nield and Bejan /11/ worked on the problem of thermal convection in a porous medium.

The Buongiorno /2/ model-based investigation of hydrodynamic thermal convection issues in porous and non-porous media saturated by a nanofluid layer has attracted the attention of numerous researchers over the past ten years /1, 4, 6, 12-23, 26, 28/. Nanofluid is used in a wide range of industries, including the car industry, energy conservation and nuclear reactors, etc. Nanoparticle suspensions are widely used in medical applications, such as cancer treatment. Numerous engineering applications, including geothermal energy recovery, crude oil extraction, groundwater pollution and thermal energy storage. Different authors /1, 4, 6, 9-10, 12-23, 26-28/ investigated the natural convection of a nanofluid using Buongiorno's model and they found that nanofluids are effective coolants because of their improved thermal conductivities.

Many researchers /1, 4, 6, 9-24, 26, 28/ have researched thermal convection in a viscoelastic nanofluid layer saturating a porous media and discovered that viscoelastic nanofluids have applications in a variety of automotive sectors and biomedical engineering. Understanding the effects of rotation and magnetic field is essential for studying thermal instability in a fluid layer heated from below. Numerous industries, such as fluid machinery, geophysics, mechanical engineering, and the petroleum industry, can benefit from this information. Researchers have investigated the effects of rotation and magnetoconvection on thermal instability using a variety of nano viscoelastic fluids. They have found that stationary convection is encouraged by rotation. Considering the many applications previously addressed, the main objective of this work is to examine the effects of rotation and magnetic field on the onset of thermal instability in Jeffrey nanofluid in a porous medium. To the best of the writer's knowledge, no inquiry has been carried out regarding this.

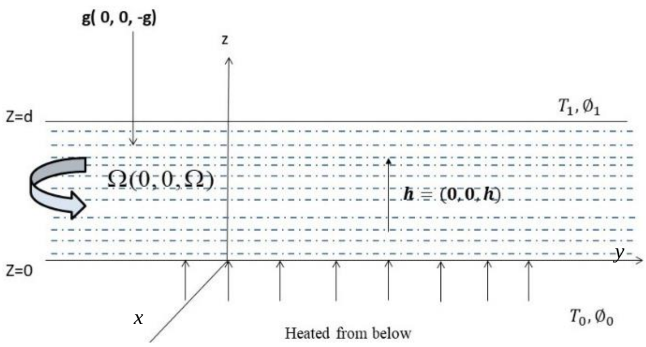

Figure 1: Physical Configuration

MATHEMATICAL FORMULATION

Here we consider a rotating horizontal layer of Jeffrey nanofluid of thickness d, in the presence of magnetic field $h = h(0, 0, 1)$ between planes $z = 0$ and $z = d$ (as shown in Fig. 1). The fluid layer is heated from the bottom and working to top with gravity $g = g(0, 0, -g)$. The temperature T and volumetric fraction ϕ of the nanoparticle, at the upper boundary T_1 and ϕ_1, and at lower boundary T_0 and ϕ_0 in respect, with $T_0 > T_1$ and $\phi_0 > \phi_1$.

GOVERNING EQUATIONS

The governing equations of Jeffrey nanofluid under the influence of magnetic field and rotation in a porous medium, as formulated by Buongiorno /2/ and Chandrasekhar /3/, are

$$\nabla.\mathbf{q_D} = 0 \,, \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\left(\frac{\partial \mathbf{q_D}}{\partial t} + \frac{(\mathbf{q_D}.\nabla)\mathbf{q_D}}{\varepsilon}\right) = -\nabla p - \frac{\mu}{k_1(1+\lambda_3)}\mathbf{q_D} + \frac{2\rho_f}{\varepsilon}(\mathbf{q_D} \times \Omega) +$$
$$+\frac{\mu_e}{4\pi}(h.\nabla)h + [\phi\rho_P + (1-\phi)\rho_f\{1-\alpha(T-T_1)\}]\mathbf{g} \,, \tag{2}$$

$$\left[\frac{\partial}{\partial t} + \frac{\mathbf{q_D}.\nabla}{\varepsilon}\right]\phi = D_B\nabla^2\phi + \frac{D_T}{T_1}\nabla^2 T \,, \tag{3}$$

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f\, \mathbf{q_D}.\nabla T = k_m \nabla^2 T + \varepsilon(\rho c)_p \times$$
$$\times\left[D_B \nabla\phi.\nabla T + \frac{D_T}{T_1}\nabla T.\nabla T\right]. \tag{4}$$

The Maxwell equations are given below:

$$\frac{\partial h}{\partial t} + (\mathbf{q_D}.\nabla)h = (h\nabla)\mathbf{q_D} + \eta\nabla^2 h \,, \tag{5}$$

$$\nabla h = 0 \,. \tag{6}$$

Here, $\lambda_3 = \lambda_1/\lambda_2$, ρ_f, ρ_p, $(\rho c)_f$, and η represent Jeffrey parameters (λ_3, a fraction of the stress relaxation-time parameter λ_1 and strain relaxation parameter λ_2), density of fluid, fluid pressure, heat capacity of fluid, and resistivity of the fluid, respectively.

Equations (1)-(6) reduce in nondimensional form to:

$$\nabla.\mathbf{q_D} = 0 \,, \tag{7}$$

$$\left(\frac{1}{V_a}\frac{\partial}{\partial t} + \frac{1}{1+\lambda_3}\right)w = -\nabla p - R_m\hat{e}_z + R_a T\hat{e}_z - R_n\phi\hat{e}_z +$$
$$+\sqrt{T_a}(\mathbf{q_D} \times \hat{e}_z) + Q\frac{Pr_1}{Pr_2}(h\nabla)h \,, \tag{8}$$

$$\frac{1}{\sigma}\frac{\partial\phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q_D}.\nabla\phi = \frac{1}{Le}\nabla^2\phi + \frac{N_A}{Le}\nabla^2 T \,, \tag{9}$$

$$\frac{\partial T}{\partial t} + \mathbf{q_D}.\nabla T = \nabla^2 T + \frac{N_B}{Le}\nabla\phi.\nabla T + \frac{N_A N_B}{Le}\nabla T.\nabla T \,, \tag{10}$$

$$\frac{\partial h}{\partial t} + \sigma(\mathbf{q_D}.\nabla)h = \sigma(h\nabla)\mathbf{q_D} + \sigma\frac{Pr_1}{Pr_2}\nabla^2 h \,, \tag{11}$$

$$\nabla h = 0 \,. \tag{12}$$

Here, we have used the nondimensional variables:

$$(u', v', w') = \left(\frac{u, v, w}{\kappa_m}\right)d \,;\; (x', y', z') = \left(\frac{x, y, z}{d}\right);\; T' = \frac{T - T_1}{T_0 - T_1} \,;$$
$$t' = \frac{t\kappa_m}{\sigma d^2} \,;\; \phi' = \frac{\phi - \phi_0}{\phi_1 - \phi_0} \,;\; p' = \frac{pk_1}{\mu\kappa_m} \,,$$

where: Prandtl number is $Pr_1 = \mu/\rho_f\kappa_m$; Taylor number is $T_a = (2\Omega\rho d^2/\varepsilon\mu)^2$; Darcy number is $D_a = k_1/d^2$; Vadasz number is

$V_a = \varepsilon P_r/D_a$; Rayleigh number is $R_a = \rho_f g\beta dk(T_0 - T_1)/\mu_f \kappa_m$; nanoparticle Rayleigh number $R_n = (\rho_p - \rho_f)(\phi_1 - \phi_0)gk_1 d/\mu\kappa_m$; modified particle density increment is $N_B = \varepsilon(\rho c)_p(\phi_1 - \phi_0)/(\rho c)_f$; Lewis number is $Le = \kappa_m/D_B$; modified diffusivity ratio $N_A = D_T(T_0 - T_1)/D_B T_1(\phi_1 - \phi_0)$; $\nabla^2 = (\partial/\partial x^2) + (\partial/\partial y^2) + (\partial/\partial z^2)$ is a Laplacian operator; basic density Rayleigh number $R_m = (\rho_p\phi_0 + \rho_f(1 - \phi_0))gk_1 d$; $\nabla_H^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$ is a horizontal Laplacian operator; magnetic Prandtl number $P_{r2} = \mu/\rho\eta$; and Chandrasekhar number $Q = \mu_e H^2 d^2/4\pi\eta\mu$.

The initial boundary conditions are:

$$w = 0,\ T = T_0,\ \phi = \phi_0 \ \text{at}\ z = 0$$

and

$$w = 0,\ T = T_1,\ \phi = \phi_1 \ \text{at}\ z = d \tag{13}$$

BASIC STATE SOLUTIONS

Following Nield and Kuznetsov /9-10/, Sheu /24-25/, Sharma et al. /15-23/, the basic state of nanofluid is assumed and does not depend on time and is described as:

$$\mathbf{q_D}(u,v,w) = 0 \Rightarrow u = v = w = 0,$$
$$p = p_b(z),\ T = T_b(z),\ \phi = \phi_b(z). \tag{14}$$

The basic variable is represented by subscript b.

Therefore, when the basic state defined in Eq.(14) is substituted into Eqs.(8)-(10), these equations reduce to:

$$0 = -\frac{d}{dz}p_b(z) - R_m\hat{e}_z + R_a T_b(z)\hat{e}_z - R_n\phi_b(z)\hat{e}_z + $$
$$+Q\frac{Pr_1}{Pr_2}\left(\frac{\partial h}{\partial z}\right)k + \sqrt{T_a}(\mathbf{q_D}\times\hat{e}_z), \tag{15}$$

$$\frac{d^2}{dz^2}\phi_b(z) + N_A\frac{d^2}{dz^2}T_b(z) = 0, \tag{16}$$

$$\frac{d^2}{dz^2}T_b(z) + \frac{N_A}{Le}\frac{d}{dz}\phi_b(z).\frac{d}{dz}T_b(z) + \frac{N_A N_B}{Le}\left(\frac{d}{dz}T_b(z)\right)^2 = 0. \tag{17}$$

Using boundary conditions Eq.(14) in Eq.(17), we get

$$\phi_b(z) = (1 - T_b)N_A + (1 - N_A)z. \tag{18}$$

Putting Eq.(18) in Eq.(17), we get

$$\frac{d^2}{dz^2}T_b(z) + \frac{N_A}{Le}(1 - N_A)\frac{d}{dz}T_b(z) + \frac{N_A N_B}{Le}\left(\frac{d}{dz}T_b(z)\right)^2 = 0.$$

Eradicating the higher degree term, we get

$$\frac{d^2}{dz^2}T_b(z) + \frac{N_A(1 - N_A)}{Le}\frac{d}{dz}T_b(z) = 0. \tag{19}$$

The solution of differential Eq.(15) with boundary condition Eq.(13) is

$$T_b(z) = \frac{e^{\frac{-N_B(1-N_A)z}{Le}}\left[1 - e^{\frac{-N_B(1-N_A)(1-z)}{Le}}\right]}{1 - e^{\frac{-N_B(1-N_A)}{Le}}}. \tag{20}$$

According to Buongiorno /2/ hypothesis, the approximation solution for Eqs.(19) and (20) are given as

$$T_b = 1 - z \quad \text{and} \quad \phi_b = z. \tag{21}$$

These results are similar with the result obtained by Buongiorno /2/, Nield and Kuznetsov /9-10/, Sheu /24-25/, and Sharma et al. /15-23/.

PERTURBATION SOLUTIONS

We introduce a small perturbation on the basic state and investigate the stability of the system:

$$q_D(u,v,w) = 0 + q_D^*(u,v,w),\quad p = p_b + p^*$$

$$\phi = \phi_b + \phi^*,\quad h = (0,0,1) + h^*,\quad T = T_b + T^*. \tag{22}$$

Using Eq.(22) in Eqs.(7)-(12), linearizing the resulting equations by neglecting nonlinear terms, the following non-dimensional perturbed equations are

$$\nabla.\mathbf{q_D^*} = 0, \tag{23}$$

$$\left(\frac{1}{Va}\frac{\partial}{\partial t} + \frac{1}{1+\lambda_3}\right)q_D^* = -\nabla p^* + R_a T^*\hat{e}_z - R_n\phi^*\hat{e}_z +$$
$$+\sqrt{T_a}(q_D^*\times\hat{e}_z) + Q\frac{Pr_1}{Pr_2}\hat{e}_z\frac{\partial h^*}{\partial z}, \tag{24}$$

$$\frac{1}{\sigma}\frac{\partial\phi^*}{\partial t} + \frac{q_D^*}{\varepsilon} = \frac{1}{Le}\nabla^2\phi^* + \frac{N_A}{Le}\nabla^2 T^*, \tag{25}$$

$$\frac{\partial T^*}{\partial t} - q_D^* = \nabla^2 T^* + \frac{N_B}{Le}(\nabla T^* - \nabla\phi^*) - \frac{2N_A N_B}{Le}\nabla T^*, \tag{26}$$

$$\frac{\partial h^*}{\partial t} = \sigma(h\nabla)q_D^* + \sigma\frac{Pr_1}{Pr_2}\nabla^2 h^*, \tag{27}$$

$$\nabla h^* = 0, \tag{28}$$

and the boundary conditions are

$$w^* = T^* = \phi^* = 0 \quad \text{at} \quad z = 0 \quad z = 1. \tag{29}$$

Operating Eq.(24) with $\hat{e}_z$.curl.curl, we get

$$\left(\frac{1}{Va}\frac{\partial}{\partial t} + \frac{1}{1+\lambda_3}\right)^2\nabla^2 w^* + T_a\frac{\partial^2 w^*}{\partial z^2} + Q\left(\frac{1}{Va}\frac{\partial}{\partial t} + \frac{1}{1+\lambda_3}\right)\frac{\partial^2 w^*}{\partial z^2} -$$
$$-\left(\frac{1}{Va}\frac{\partial}{\partial t} + \frac{1}{1+\lambda_3}\right)R_a\nabla_H^2 T^* + \left(\frac{1}{Va}\frac{\partial}{\partial t} + \frac{1}{1+\lambda_3}\right)R_n\nabla_H^2\phi^* = 0 \tag{30}$$

NORMAL MODE ANALYSIS

The disturbances are analysed by normal mode analysis as follows:

$$[w^*, T^*, \phi^*] = [W(z), \Theta(z), \Phi(z)]e^{(ilx + imy + \omega t)}, \tag{31}$$

where: ω is the growth rate; lx and ly are wave numbers along x and y, respectively.

Using Eq.(31) in Eq.(30), Eqs. (25) and (26), we get

$$\left[\left(\frac{\omega}{Va} + \frac{1}{1+\lambda_3}\right)(D^2 - a^2) + QD^2 + \frac{T_a D^2}{\left(\frac{\omega}{Va} + \frac{1}{1+\lambda_3}\right)}\right]W +$$
$$+a^2 R_a\Theta - a^2 R_n\Phi = 0, \tag{32}$$

$$\frac{W}{\varepsilon} - \frac{N_A}{Le}(D^2 - a^2)\Theta - \left\{\frac{1}{Le}(D^2 - a^2) - \frac{\omega}{\sigma}\right\}\Phi = 0, \tag{33}$$

$$W + \left\{\frac{N_B}{Le}D + (D^2 - a^2) - \frac{2N_A N_B}{Le}D - \omega\right\}\Theta - \frac{N_B}{Le}D\Phi = 0, \tag{34}$$

Here, $D = d/dz$, $a^2 = l^2 + m^2$, and boundary conditions in normal mode are:

$$W = 0,\ \Theta = 0,\ \Phi = 0,\ D^2 W = 0 \ \text{at} \ z = 0,$$
$$W = 0,\ \Theta = 0,\ \Phi = 0,\ D^2 W = 0 \ \text{at} \ z = 1. \tag{35}$$

LINEAR STABILITY ANALYSIS FOR FREE-FREE BOUNDARIES

We suppose the solution to W, Θ, and Φ are of the form

$$W = W_0(1 - z)^2 z^2,\quad \Theta = \Theta_0 z(1 - z),\quad \Phi = \Phi_0 z(1 - z). \tag{36}$$

Putting Eq.(36) into Eqs.(32)-(34), integrating each equation from $z = 0$ to $z = 1$ and performing some integrations by parts, we obtain the following matrix equation:

$$\begin{bmatrix} \left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)(Q\pi^2)+\frac{T_a\pi^2}{\left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)} & -a^2R_a & a^2R_n \\ \frac{1}{\varepsilon} & \frac{N_A}{Le}(\pi^2+a^2) & \frac{(\pi^2+a^2)}{Le}+\frac{\omega}{\sigma} \\ 3 & -\{(\pi^2+a^2)+\omega\} & 0 \end{bmatrix}\times$$

$$\times\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (37)$$

Non-trivial solution of Eq.(37) is

$$\begin{vmatrix} \left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)(Q\pi^2)+\frac{T_a\pi^2}{\left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)} & -a^2R_a & a^2R_n \\ \frac{1}{\varepsilon} & \frac{N_A}{Le}(\pi^2+a^2) & \frac{(\pi^2+a^2)}{Le}+\frac{\omega}{\sigma} \\ 3 & -\{(\pi^2+a^2)+\omega\} & 0 \end{vmatrix}=0 \quad (38)$$

$$R_a=\frac{\left(\frac{\omega}{Va}+\frac{1}{1+\lambda_3}\right)(\pi^2+a^2)(\pi^2+a^2+\omega)}{a^2}+\frac{Q\pi^2(\pi^2+a^2+\omega)}{a^2}+$$
$$+\frac{T_a\pi^2(\pi^2+a^2+\omega)}{a^2\left(\frac{\omega}{Va}+\frac{1}{1+\lambda_3}\right)}-\frac{\varepsilon N_A(\pi^2+a^2)L_e(\pi^2+a^2+\omega)}{(\pi^2+a^2)\sigma+\omega L_e}\frac{\sigma}{\varepsilon}R_n. \quad (39)$$

NON-OSCILLATORY CONVECTION (FREE-FREE)

For the case of steady state we put $\omega = 0$ in Eq.(39), we get

$$R_{a_s}=\frac{1}{1+\lambda_3}\frac{(a^2+\pi^2)^2}{a^2}+\frac{Q\pi^2(\pi^2+a^2)}{a^2}+\frac{T_a\pi^2(1+\lambda_3)(\pi^2+a^2)}{a^2}-$$
$$-\left(N_A+\frac{L_e}{\varepsilon}\right)R_n. \quad (40)$$

Equation (40) represents the dispersion relation for the effect of the Jeffrey parameter, Lewis number, nanoparticle's Rayleigh number, modified diffusivity ratio, medium porosity, magnetic field, and Taylor number.

Now we find the critical wave number by minimizing R_a with respect to a^2, thus the critical wave number must satisfy

$$\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c}=0. \quad (41)$$

In Eq.(41), neglect the parameter of magnetic field and Taylor number, then the critical wave number is $a_c = \pi$.

This result is validated with the original result of Nield et al. /7, 9-11/.

RIGID-RIGID BOUNDARIES

We confine our analysis to the one-term Galerkin approximation. Appropriate trial functions satisfying the boundary conditions are now

$$W=0,\ \Theta=0,\ \Phi=0,\ DW=0 \text{ at } z=0,$$
$$W=0,\ \Theta=0,\ \Phi=0,\ DW=0 \text{ at } z=1. \quad (42)$$

LINEAR STABILITY ANALYSIS FOR RIGID-RIGID BOUNDARIES

We suppose the solution to W, Θ, and Φ is of the form: $W = W_0(1-z)^2z^2$, $\Theta = \Theta_0 z(1-z)$, $\Phi = \Phi_0 z(1-z)$. (43)

Substituting solution Eq.(43) into Eqs.(32)-(34), integrating the equations from $z = 0$ to $z = 1$, we obtain the following

$$\begin{bmatrix} \left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)\left(\frac{2}{105}+\frac{a^2}{630}\right)+Q\left(\frac{2}{105}\right)+\frac{T_a}{\left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)}\left(\frac{2}{105}\right) & -\frac{a^2R_a}{140} & \frac{a^2R_n}{140} \\ \frac{1}{140\varepsilon} & \frac{N_A}{Le}\left(\frac{1}{3}+\frac{a^2}{30}\right) & \frac{1}{L_e}\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30\sigma} \\ 3 & -\left\{\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30}\right\} & 0 \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (44)$$

Non-trivial solution of Eq.(44) is

$$\begin{vmatrix} \left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)\left(\frac{2}{105}+\frac{a^2}{630}\right)+Q\left(\frac{2}{105}\right)+\frac{T_a}{\left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)}\left(\frac{2}{105}\right) & -\frac{a^2R_a}{140} & \frac{a^2R_n}{140} \\ \frac{1}{140\varepsilon} & \frac{N_A}{Le}\left(\frac{1}{3}+\frac{a^2}{30}\right) & \frac{1}{L_e}\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30\sigma} \\ 3 & -\left\{\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30}\right\} & 0 \end{vmatrix}=0. \quad (45)$$

$$R_a=\frac{28}{27a^2}\left[\left(\frac{1}{1+\lambda_3}\right)(12+a^2)+12Q+\frac{12T_a}{\left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)}\right]\{(10+a^2)+\omega\}-\frac{\left[\frac{(10+a^2)+\omega}{\varepsilon}+\frac{N_A}{L_e}(10+a^2)\right]}{\frac{(10+a^2)}{L_e}+\frac{\omega}{\sigma}}R_n. \quad (46)$$

NON-OSCILLATORY CONVECTION (RIGID-RIGID)

For the case of steady-state, we put $\omega = 0$ in Eq.(33) and obtain

$$R_a + \left(\frac{Le}{\varepsilon} + N_A\right)R_n =$$
$$= \frac{28}{27a^2}\left[\left(\frac{1}{1+\lambda_3}\right)(12+a^2) + 12Q + 12T_a(1+\lambda_3)\right](10+a^2) . \quad (47)$$

Equation (47) represents the dispersion relation with different parameters as Jeffrey parameter, Lewis number, nanoparticle's Rayleigh number, modified diffusivity ratio, medium porosity, magnetic field, and Taylor number.

The critical wave number at the onset of instability is obtained by minimising R_a with respect to a^2, thus the critical wave number must satisfy,

$$\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c} = 0 . \quad (48)$$

In Eq.(48), neglecting the parameter of magnetic field and Taylor number, the critical wave number is

$$a_c = 3.31 . \quad (49)$$

This result is validated with the original result of Nield et al. /7, 9-11/.

RIGID-FREE BOUNDARIES

We confine our analysis to the one-term Galerkin approximation. Appropriate trial functions satisfying the boundary conditions are now

$$W = 0, \quad \Theta = 0, \quad \Phi = 0, \quad DW = 0 \quad \text{at} \quad z = 0 ,$$
$$W = 0, \quad \Theta = 0, \quad \Phi = 0, \quad DW = 0 \quad \text{at} \quad z = 1 . \quad (50)$$

LINEAR STABILITY ANALYSIS FOR RIGID-FREE BOUNDARIES

We suppose the solution to W, Θ, and Φ is of the form

$$W = W_0 z^2(1-z)(3-2z), \ \Theta = \Theta_0 z(1-z), \ \Phi = \Phi_0 z(1-z). \quad (51)$$

Substituting solution Eq.(51) into Eqs.(32)-(34), and by integrating each equation from $z = 0$ to $z = 1$, we obtain the following matrix equation:

$$\begin{bmatrix} \left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)\left(\frac{12}{35}+\frac{19a^2}{630}\right)+Q\frac{12}{35}+\frac{T_a}{\left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)}\left(\frac{12}{35}\right) & -\frac{13}{420}a^2R_a & \frac{13}{420}a^2R_n \\ \frac{13}{420\varepsilon} & \frac{N_A}{Le}\left(\frac{1}{3}+\frac{a^2}{30}\right) & \frac{1}{Le}\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30\sigma} \\ \frac{13}{420} & -\left\{\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30}\right\} & 0 \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} . \quad (52)$$

Non-trivial solution of Eq.(52) is

$$\begin{vmatrix} \left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)\left(\frac{12}{35}+\frac{19a^2}{630}\right)+Q\frac{12}{35}+\frac{T_a}{\left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)}\left(\frac{12}{35}\right) & -\frac{13}{420}a^2R_a & \frac{13}{420}a^2R_n \\ \frac{13}{420\varepsilon} & \frac{N_A}{Le}\left(\frac{1}{3}+\frac{a^2}{30}\right) & \frac{1}{Le}\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30\sigma} \\ \frac{13}{420} & -\left\{\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30}\right\} & 0 \end{vmatrix} = 0 , \quad (53)$$

$$R_a = \frac{28}{507a^2}\left[\left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)(216+19a^2)+Q(216)+\frac{T_a(216)}{\left(\frac{\omega}{V_a}+\frac{1}{1+\lambda_3}\right)}\right]\left\{(10+a^2)+\omega\right\} - \frac{\left[\frac{(10+a^2)+\omega}{\varepsilon}+\frac{N_A}{L_e}(10+a^2)\right]}{\frac{(10+a^2)}{L_e}+\frac{\omega}{\sigma}}R_n . \quad (54)$$

NON-OSCILLATORY CONVECTION (RIGID-FREE)

For the case of steady-state, we put $\omega = 0$ in Eq.(55) and obtain

$$R_a + \left(\frac{Le}{\varepsilon}+N_A\right)R_n = \frac{28}{507a^2}\left[(216+19a^2)\left(\frac{1}{1+\lambda_3}\right)+216Q+216T_a(1+\lambda_3)\right](10+a^2) . \quad (55)$$

Equation (56) represents the dispersion relation with different parameters like Jeffrey parameter, Lewis number, nanoparticle's Rayleigh number, modified diffusivity ratio, medium porosity, magnetic field, and Taylor number.

The critical wave number is at the onset of instability, is obtained by minimizing R_a with respect to a^2, thus the critical wave number must satisfy

$$\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c} = 0 . \quad (56)$$

Equation (56) neglects the parameter of magnetic field and Taylor number, then the critical wave number is

$$a_c = 3.27 . \quad (57)$$

This result is validated with the original result of Nield et al. /7, 9-11/.

RESULTS AND DISCUSSIONS

In this paper we have analysed the effects of Jeffrey parameter, Lewis number, nanoparticle's Rayleigh number, modified diffusivity ratio, magnetic field, Talyor number and medium porosity on the onset of stationary convection by considering Jeffrey nanofluids in the presence of free-free, rigid-rigid and rigid-free boundary conditions. We have analysed the effects analytically and presented them graphically.

Figure 2 shows the graph of Ra_s w.r.t. wave number a for different values of $\lambda_3 = 0.3$, 0.5, 0.9 and by fixing other parameters $N_A = 10$, $Le = 1000$, $\varepsilon = 0.6$, $R_n = -1$, $T_a = 300$, $Q = 100$. It is clear from Fig. 2 that with increase in the value of λ_3, Ra_s goes on increasing and hence shows the stabilising effect on stationary convection. It is also clear from Fig. 2 that rigid-rigid boundary condition has a more stabilising impact on stationary convection as compared to rigid-free boundary conditions. Thus, λ_3 delays the onset of convection.

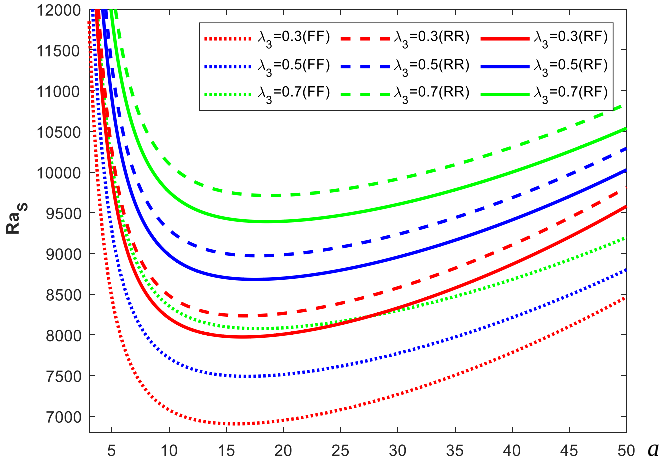

Figure 2. Ra_s vs. a for distinct values of λ_3.

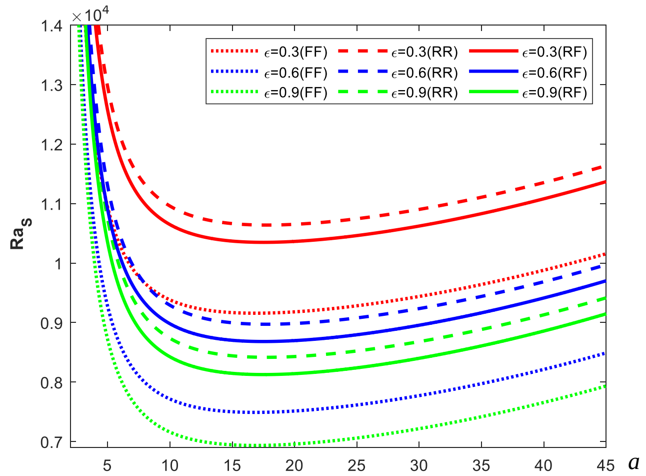

Figure 3. Ra_s vs. a for distinct values of ε.

Figure 3 shows the graph of Ra_s w.r.t. wave number a for different values of $\varepsilon = 0.3$, 0.6, 0.9 and by fixing the other parameters as $N_A = 10$, $Le = 1000$, $R_n = -1$, $\lambda_3 = 0.5$, $T_a = 300$, $Q = 100$. It is clear from Fig. 3 that within increase in the value of ε, Ra_s goes on decreasing and hence shows the destabilising effect on stationary convection. It is also clear from Fig. 3 that rigid-rigid boundary condition has a more destabilising impact on stationary convection as compared to rigid-free boundary conditions. Thus, ε also enhances the onset of convection.

Figure 4 shows the graph of Ra_s w.r.t. wave number a for different values of $Le = 500$, 1000, 1500 and by fixing the other parameters as $N_A = 10$, $\varepsilon = 0.6$, $R_n = -1$, $\lambda_3 = 0.5$, $T_a = 300$, $Q = 100$. It is clear from Fig. 4 that with increase in the value of Le, Ra_s goes on increasing and hence shows the stabilising effect on stationary convection. It is also clear from Fig. 4 that rigid-free boundary condition has a more stabilising impact on stationary convection as compared to rigid-rigid boundary conditions. Thus, Le delays the onset of convection.

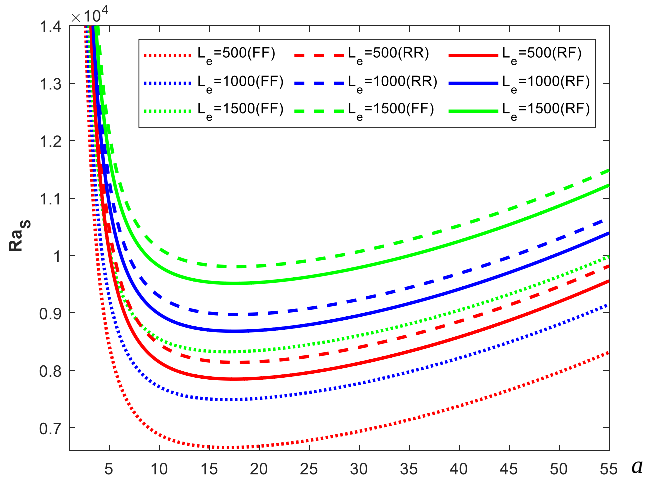

Figure 4. Ra_s vs. a for distinct values of Le.

Figure 5 shows the graph of Ra_s w.r.t. wave number a for different value of $N_A = 1$, 5, 10 and by fixing the other parameters as $\lambda_3 = 0.5$, $Le = 1000$, $\varepsilon = 0.6$, $R_n = -1$, $T_a = 300$, and $Q = 100$. It is clear from Fig. 5 that within increase in the value of N_A, Ra_s goes on increasing and hence shows the stabilising effect on stationary convection. It is also clear from Fig. 5 that rigid-rigid boundary condition has more stabilising impact on stationary convection as compared to rigid-free boundary conditions. Thus, N_A delays the onset of convection.

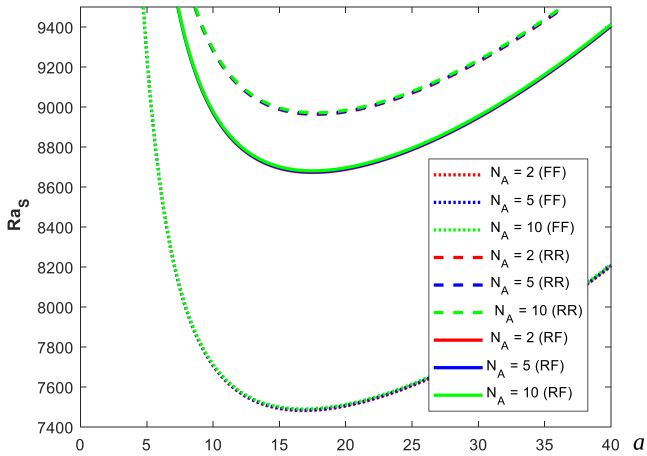

Figure 5. Ra_s vs. a for distinct values of N_A.

Figure 6 shows the graph of Ra_s w.r.t. wave number a for different values of $R_n = -1$, –0.5, –0.1, and by fixing the other parameters as $\lambda_3 = 0.5$, $N_A = 10$, $Le = 1000$, $\varepsilon = 0.6$, $T_a = 300$, and $Q = 100$. It is clear from Fig. 6 that within increase in the values of R_n, Ra_s goes on decreasing and hence shows the destabilising effect on stationary convection. It is also clear from Fig. 6 that rigid-rigid boundary condition has more destabilising impact on stationary convection as compared to rigid-free boundary conditions. Thus, R_n accelerates the onset of convection.

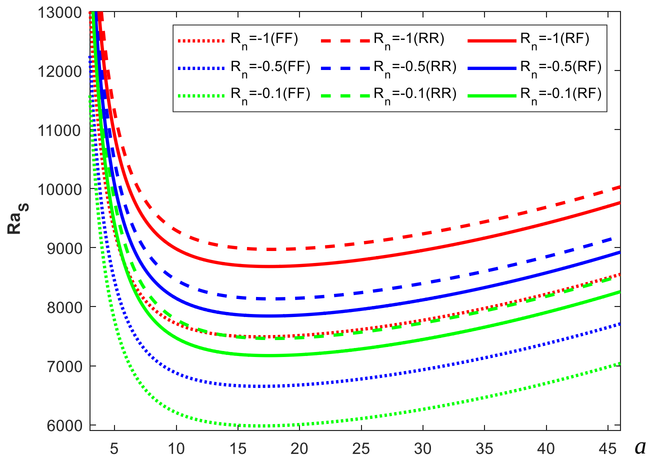

Figure 6. Ra_s vs. a for distinct values of R_n.

Figure 7 shows the graph of Ra_s w.r.t. wave number a for different values of Q = 50, 100, 150, and by fixing the other parameters as N_A = 10, ε = 0.6, R_n = –1, λ_3 = 0.5, T_a = 300, and Le = 1000. It is clear from Fig. 7 that with increase in the value of Q, Ra_s goes on increasing and hence shows the stabilising effect on stationary convection. It is also clear from Fig. 7 that rigid-rigid boundary condition has a more stabilising impact on stationary convection as compared to rigid-free boundary conditions. Thus, Q delays the onset of convection.

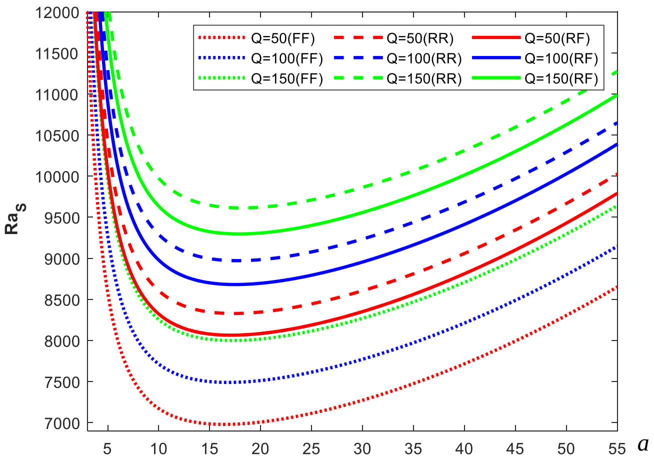

Figure 7. Ra_s vs. a for distinct values of Q.

Figure 8 shows the graph of Ra_s w.r.t. wave number a for different values of T_a = 200, 300, 400, and by fixing the other parameters as N_A = 10, ε = 0.6, R_n = –1, λ_3 = 0.5, T_a = 300, and Q = 100. It is clear from Fig. 8 that within increase in the values of T_a, Ra_s goes on increasing and hence shows the stabilising effect on stationary convection. It is also clear from Fig. 8 that rigid-rigid boundary condition has a more stabilising impact on stationary convection as compared to rigid-free boundary conditions. Thus, T_a delays the onset of convection.

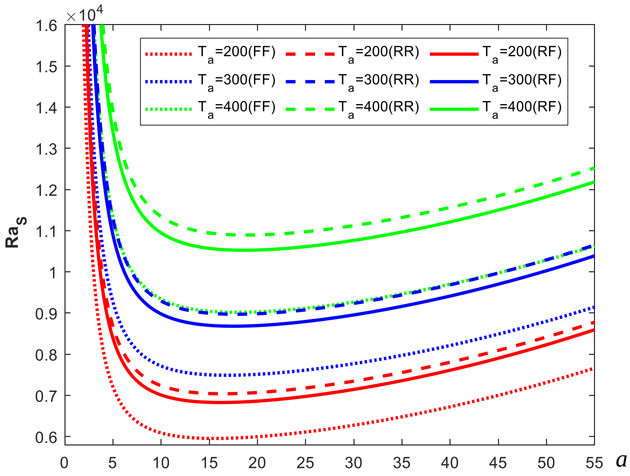

Figure 8. Ra_s vs. a for different values of T_a.

CONCLUSIONS

In this paper, we have analysed the stationary convection in the effect of magnetic field and rotation on thermal instability of Jeffrey nanofluid in a porous medium: free-free, rigid-rigid, and rigid-free boundary conditions. For this analysis we have utilised the GWR method.

We have drawn the following conclusions.

Nanoparticle's Rayleigh number R_n and medium porosity ε have destabilising effects on stationary convection.

Magnetic field Q, Taylor number T_a, Jeffrey parameter λ_3, Lewis number Le, and modified diffusivity ratio N_A, all have stabilising impact on stationary convection.

It is found that in case of rigid-rigid boundary condition, the system remains more stable rather than in the case of rigid-free boundary condition.

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