SAFETY ANALYSIS IN AN ELASTOPLASTIC ORTHOTROPIC ROTATING CYLINDER UNDER STEADY STATE TEMPERATURE

ANALIZA SIGURNOSTI ELASTOPLASTIČNOG ORTOTROPNOG ROTIRAJUĆEG CILINDRA SA STACIONARNOM TEMPERATUROM

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Adresa autora / Author's address: Department of Mathematics, ICFAI University, Himachal Pradesh, India P. Gulial **1** 0009-0009-0266-2298; P. Thakur 1 0000-0001-8119-2697, *email: **pankaj_thakur15@yahoo.co.in**

Abstract

This research paper deals with the study of thermoelastoplastic stress deformation in a rotating cylinder made of orthotropic material. The transition theory is applied to determining the presiding equation. An analytical solution for the rotating cylinder made of orthotropic material is presented. The behaviour of temperature, angular speed, and stress distribution are explored. The circumferential stress is maximal at the inner portion of the cylinder made of barite or copper material. With the introduction of thermal conditions, angular speed increases in the internal surface of barite material and decreases in that of copper material.

INTRODUCTION

Rotating cylinders have vast applications in engineering, such as ship propulsion systems, high-lift components such as aerofoils, and wind-driven devices. It is becoming increasingly less desirable to restrict design to the traditional elastic regime only due to the growing shortage of materials worldwide and their increased costs. Thick-walled cylindrical cross sections are commonly employed as pressure vessels to store industrial gases or transport high-pressure fluids. The elastoplastic deformation in a rotating cylinder was investigated by Hodge et al. /1/ by neglecting elastic and plastic strain hardening. Afterward, Bhatnagar et al. /2/ solved the problem for orthotropic cylinders. Zhao et al. /3/ explored techniques for finding stress deformation in thick-walled cylinders with distinct material parameters. It is found that for deformation, the value of strain energy depends on the initial or final geometry and also satisfies the boundary conditions. Additionally, Thakur /4/ examined the distribution of elasticplastic thermal stresses in a rotating cylinder-formed isotropic material. Furthermore, Thakur et al. /5, 6/ have investigated thermo-elastoplastic and creep analysis in a spherical shell and cylinder with axial load and pressure of orthotropic material. Moreover, the elastoplastic analysis of a cylinder with a shaft under loading and unloading conditions was investigated by Prokudin /7/. Moreover, Temesgen et al. /8/ examined thermal stress deformation in a functionally graded thick-walled rotating cylinder with uniform pressure. The

U ovom radu se istražuju deformacije usled termo-elastoplastičnih napona u rotirajućem cilindru od ortotropnog materijala. Primenjena je teorija prelaznih napona za određivanje odgovarajuće jednačine. Predstavljeno je analitičko rešenje za rotirajući cilindar od ortotropnog materijala. Istraženo je ponašanje temperature, ugaone brzine, kao i raspodela napona. Cirkularni napon ima maksimalnu vrednost u unutrašnjoj oblasti cilindra izrađenog od barita ili bakra. Uvođenjem termičkih uslova, ugaona brzina se povećava na unutrašnjoj površini za materijal barit, a opada kada je materijal bakar.

purpose of this research article is to investigate safety analysis in an elastoplastic orthotropic rotating cylinder under steady-state temperature using a generalised strain measure.

BASICS EQUATION

Let us consider a thick-walled cylinder made of orthotropic (e.g., barite) and isotropic (e.g., copper) material, having inner and outer radii *a* and *b*, which is rotating about its own axis with angular speed. Let steady-state temperature be applied to the inner surface at $r = a$.

In cylindrical coordinates, the displacement components are given by Seth, /9/:

$$
u_1 = r(1 - \xi)
$$
, $u_2 = 0$, $u_3 = dz$. (1)
Seth /10/ has given generalized components of strain as:

$$
\varepsilon_{rr} = \frac{1}{m} \Big[1 - (r\xi' + \xi)^m \Big], \quad \varepsilon_{\theta\theta} = \frac{1}{m} \Big[1 - \xi^m \Big],
$$

$$
\varepsilon_{zz} = \frac{1}{m} \Big[1 - (1 - d)^m \Big], \quad \varepsilon_{r\theta} = \varepsilon_{\theta z} = \varepsilon_{zr} = 0 \,, \tag{2}
$$

where: $r\xi' = \xi P$.

where:
$$
r\xi' = \xi P
$$
.
\nThe stress-strain relation is given by Thakur et al. /4/ $\tau_{rr} = \frac{c_{11}}{m} \Big[1 - (r\xi' + \xi)^m \Big] + \frac{c_{12}}{m} \Big[1 - \xi^m \Big] + \frac{c_{13}}{m} \Big[1 - (1 - d)^m \Big] - \alpha_1 \Theta$
\n $\tau_{\theta\theta} = \frac{c_{21}}{m} \Big[1 - (r\xi' + \xi)^m \Big] + \frac{c_{22}}{m} \Big[1 - \xi^m \Big] + \frac{c_{23}}{m} \Big[1 - (1 - d)^m \Big] - \alpha_2 \Theta$
\n $\tau_{zz} = \frac{c_{31}}{m} \Big[1 - (r\xi' + \xi)^m \Big] + \frac{c_{32}}{m} \Big[1 - \xi^m \Big] + \frac{c_{33}}{m} \Big[1 - (1 - d)^m \Big] - \alpha_3 \Theta$
\n $\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = 0$. (3)

Equilibrium equation is:

$$
\frac{d}{dr}(r\tau_{rr}) - \tau_{\theta\theta} + \rho\omega^2 r^2 = 0.
$$
\n(4)

1

$$
\begin{aligned}\n\text{transition points: inserting Eq.}(3) \text{ into Eq.}(4), \text{ we get:} \\
\xi P(1+P)^{m-1} \frac{dP}{d\xi} + P(P+1)^m + \frac{c_{12}}{c_{11}} P - \frac{1}{mc_{11}\xi^m} \times \\
&\times \left[(c_{11} - c_{21}) \left\{ 1 - \xi^m (P+1)^m \right\} + (c_{12} - c_{22}) (1 - \xi^m) + \right. \\
&\left. + (c_{13} - c_{23}) \left[1 - (1 - d)^m \right] \right] - \frac{\rho \omega^2 r^2}{c_{11}\xi^m} = 0. \quad (5)\n\end{aligned}
$$

The turning points ξ in Eq.(5) are $P \to \pm \infty$ (elastic to plastic state) and $P \rightarrow -1$ (plastic to creep state) by Seth /9, 10/. The boundary conditions are taken as:

 $\tau_{rr} = 0$ and $\Theta = \Theta_0$ at $r = a$,

$$
\tau_{rr} = 0 \text{ and } \Theta = 0 \text{ at } r = b ,
$$

$$
\Theta = \frac{\Theta_0 \ln(r/b)}{\ln(a/b)}.
$$
 (6)

Also,

and

$$
\int_{a}^{b} r \tau_{zz} dr = 0. \tag{7}
$$

SOLUTION

Let the transition function be taken as Thakur et al. /4- $2r^2$

6, 11-21/ and Seth /9, 10/:
\n
$$
\zeta = 1 - \frac{m}{(c_{11} + c_{12} + c_{13})} \left[\tau_{rr} - \frac{m\rho\omega^2 r^2}{2} \right] \equiv \frac{1}{(c_{11} + c_{12} + c_{13})} \times \sigma_1
$$
\n
$$
\times \left[c_{11} \xi^m (P+1)^m + c_{12} \xi^m + c_{13} (1-d)^m \frac{m\rho\omega^2 r^2}{2} \right].
$$
\n(8)

By taking logarithmic differentiations and the transition point Eq.(8), the transition function is given:

$$
R = Ar^{-k_1},\tag{9}
$$

where: $k_1 = (c_{11} - c_{21})/c_{11}$.

From Eqs. (8) and (9), the radial stress is given:
\n
$$
\tau_r = \frac{c_{11} + c_{12} + c_{13}}{m} \left[1 - Ar^{-k_1} \right] - k_1 \alpha_1 \Theta - \frac{\rho \omega^2 r^2}{2} .
$$
 (10)

Using Eq. (6) into Eq. (10) , we get: q.(6) into Eq.(10), we get:
 $2k_1\alpha_1\Theta_0 + \rho\omega^2(a^2-b^2)$

$$
A = \frac{2k_1\alpha_1\omega_0 + \rho\omega(a-b)}{\rho\omega^2(a^2b^{-k_1}-b^2a^{-k_1}) + 2k_1\alpha_1\omega_0b^{-k_1}}
$$

$$
\frac{c_{11} + c_{12} + c_{13}}{m} = \frac{\rho\omega^2(a^2b^{-k_1}-b^2a^{-k_1}) + 2k_1\alpha_1\omega_0b^{-k_1}}{2(b^{-k_1}-a^{-k_1})}.
$$
 (11)

Substituting Eq.(11) into Eq.(10), we get:
\n
$$
\tau_r = \frac{\rho \omega^2 b^2 \left[R_0^2 (1 - R^{-k_1}) + R^2 (R_0^{-k_1} - 1) + R^{-k_1} - R_0^{-k_1} \right]}{2(1 - R_0^{-k_1})} + \beta_0 k_1 \left[\frac{1 - R^{-k_1}}{1 - R_0^{-k_1}} - \frac{\ln R}{\ln R_0} \right].
$$
\n(12)

Inserting Eq.(12) into Eq.(4), we get:
\n
$$
\tau_{\theta} = \tau_r + \frac{\rho \omega^2 b^2 k_1 R^{-k_1} (R_0^2 - 1) + 2k_1^2 \beta_0 R^{-k_1}}{2(1 - R_0^{-k_1})} - \frac{\beta_0 k_1}{\ln R_0}.
$$
\n(13)

Using Eq.(7) and the third equation of Eq.(3), we get: ing Eq.(7) and the third equation of Eq.(3), we get:
 $2\left[(\tau_r + \tau_\theta) - \frac{\rho \omega^2 b^2}{2} (1 + R_0^2) \right] + \frac{\beta_0}{\ln R_0} \left[\left(1 + \frac{\alpha_2}{\alpha_1} \right) k_2 - \frac{\alpha_3}{\alpha_1} \right]$ Using Eq.(7) and the third equation of Eq
 $\begin{bmatrix} zz = k_2 \left[(\tau_r + \tau_\theta) - \frac{\rho \omega^2 b^2}{2} (1 + R_0^2) \right] + \frac{\beta_0}{\ln R_0} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Using Eq.(7) and the third equation of Eq.(3), we get:
 $\tau_{zz} = k_2 \left[(\tau_r + \tau_\theta) - \frac{\rho \omega^2 b^2}{2} (1 + R_0^2) \right] + \frac{\beta_0}{\ln R_0} \left[\left(1 + \frac{\alpha_2}{\alpha_1} \right) k_2 - \frac{\alpha_3}{\alpha_1} \right] \times$ g Eq.(7) and the third equation of Eq.(3), we get:
 $\left[\frac{\rho \omega^2 b^2}{(z_+ + z_0) - \frac{\rho \omega^2 b^2}{(1 + R_0^2)}}\right] + \frac{\beta_0}{(1 + \frac{\alpha_2}{(1 + R_0^2)})} \left[\frac{\alpha_3}{(1 + \frac{\alpha_2}{(1 + R_0^2)})}\right]$ Using Eq.(7) and the third equation of Eq.(3), we get:
= $k_2 \left[(\tau_r + \tau_\theta) - \frac{\rho \omega^2 b^2}{2} (1 + R_0^2) \right] + \frac{\beta_0}{\ln R_0} \left[\left(1 + \frac{\alpha_2}{\alpha_1} \right) k_2 - \frac{\alpha_3}{\alpha_1} \right] \times$ and

$$
\times \left\{ \ln R - \frac{2}{(1 - R_0^2)} \left(\frac{R_0^2}{4} - \frac{1}{4} - \frac{R_0^2}{2} \ln R_0 \right) \right\},\tag{14}
$$

where: $R_0 = a/b$; $R = r/b$; $\beta_0 = \alpha_1 \Theta_0$, and $k_2 = c_{32}/(c_{12} + c_{22})$.

Equation (13) gives:
 $-\tau = -\frac{\rho \omega^2 b^2}{r} k_1 R^{-k_1} \left[R_0^2 - 1 \right]_{r,k} R_r \left[k_1 R^{-k_1} + 1 \right]$ (15)

Equation (13) gives:
\n
$$
\tau_{\theta} - \tau_r = -\frac{\rho \omega^2 b^2}{2} k_1 R_0^{-k_1} \left[\frac{R_0^2 - 1}{1 - R_0^{-k_1}} \right] + k_1 \beta_0 \left[\frac{k_1 R^{-k_1}}{(1 - R_0^{-k_1})} + \frac{1}{\ln R_0} \right] (15)
$$

From Eq.(15), $|\tau \omega \rangle = \tau_r r|$ has a maximum value at $R = R_0$,

refore, yielding will be given as:
 $\begin{bmatrix} \rho \omega^2 b^2 \mu R^{-k_1} \end{bmatrix} R_0^2 - 1 \end{bmatrix} + k_1 R_0 \begin{bmatrix} k_1 R_0^{-k_1} \mu R_1^{-k_2} \end{bmatrix} + \frac{1}{R_0} \begin{bmatrix} k_1 R_1^{-k_1} \mu R_1^{-k_2} \end$

therefore, yielding will be given as:
\n
$$
|\tau_{\theta} - \tau_r|_{R=R_0} = \left| \frac{\rho \omega^2 b^2}{2} k_1 R_0^{-k_1} \left[\frac{R_0^2 - 1}{1 - R_0^{-k_1}} \right] + k_1 \beta_0 \left[\frac{k_1 R_0^{-k_1}}{(1 - R_0^{-k_1})} + \frac{1}{\ln R_0} \right] \right| = Y
$$

and angular speed for the initial yielding surface becomes:
\n
$$
\Omega_{\text{initial yielding}}^2 = \frac{\rho \omega^2 b^2}{Y} = \left| \frac{2(1 - R_0^{-k_1})}{k_1 R_0^{-k_1} (R_0^2 - 1)} \right| \left[1 - k_1 \beta_1 \right] \left| \frac{k_1 R_0^{-k_1}}{(1 - R_0^{-k_1})} + \frac{1}{\ln R_0} \right| \left| (16) \right|
$$
\nwhere: $\beta_1 = \beta_1 / Y$

where: $\beta_1 = \beta_0/Y$.

The non-dimensional stresses for the initial yielding sur-

The non-dimensional success of the initial yielding
$$
\sinh \theta
$$
 and $\sinh \theta$ is the same as $\cosh \theta$ and $\cosh \theta$ is the same as $\cosh \theta$ and $\cosh \theta$ is the same as $\cosh \theta$ and $\cosh \theta$.
\n
$$
\sigma_{\theta} = \sigma_r + \frac{\sinh \theta}{2(1 - R_0^{-k_1})} + \beta_1 k_1 \left[\frac{k_1 R^{-k_1}}{1 - R_0^{-k_1}} - \frac{1}{\ln R_0} \right],
$$
\n
$$
\sigma_r = \frac{\Omega^2}{\sinh \theta} \left[R_0^2 (1 - R^{-k_1}) + R^2 (R_0^{-k_1} - 1) + R^{-k_1} - R_0^{-k_1} \right] + 2(1 - R_0^{-k_1}) + \beta_1 k_1 \left[\frac{1 - R^{-k_1}}{1 - R_0^{-k_1}} - \frac{\ln R}{\ln R_0} \right],
$$
\n
$$
\sigma_z = k_4 \left[(\sigma_r + \sigma_\theta) - \frac{\Omega^2}{2} (1 + R_0^2) + \frac{\beta_1}{\ln R_0} \left[(1 + \beta_2) k_4 - \beta_3 \right] \times \left\{ \ln R - \frac{2}{(1 - R_0^2)} \left(\frac{R_0^2}{4} - \frac{1}{4} - \frac{R_0^2}{2} \ln R_0 \right) \right\}, \quad (17)
$$

where: $\sigma \theta = \tau \theta \theta / Y$, $\sigma_r = \tau_{rr}/Y$, $\sigma_z = \tau_{zz}/Y$ and $\beta_3 = \alpha_3/\alpha_1$.

SUBSEQUENT YIELDING SURFACE

For fully plastic surfaces, $c_{13} = c_{12} = c_{11}$, $c_{33} = c_{31} = c_{32}$ ty plastic surfaces, $c_{13} = c_{12} = c_{11}$, $c_{33} = c_{31} = c_{32}$
 $c_{21} = c_{22}$, Eqs.(16)-(17) become:
 $\Omega_{\text{fully plastic}}^2 k_5 R^{-k_3} (R_0^2 - 1) + R_k \left[\frac{k_5 R^{-k_3}}{R} - \frac{1}{R} \right]$

$$
\sigma_{\theta} = \sigma_r + \frac{\Omega_{\text{fully plastic}}^2}{2(1 - R_0^{-k_3})} + \beta_1 k_3 \left[\frac{k_5 R^{-k_3}}{1 - R_0^{-k_3}} - \frac{1}{\ln R_0} \right]
$$

\n
$$
\sigma_r = \frac{\Omega_{\text{fully plastic}}^2 \left[R_0^2 (1 - R^{-k_3}) + R^2 (R_0^{-k_3} - 1) + R^{-k_3} - R_0^{-k_3} \right]}{2(1 - R_0^{-k_3})} + \beta_1 k_3 \left[\frac{k_5 R^{-k_3}}{1 - R_0^{-k_3}} - \frac{1}{\ln R_0} \right]
$$

\n
$$
\sigma_r = \frac{\Omega_{\text{fully plastic}}^2 \left[R_0^2 (1 - R^{-k_3}) + R^2 (R_0^{-k_3} - 1) + R^{-k_3} - R_0^{-k_3} \right]}{2(1 - R_0^{-k_3})} + \beta_1 k_3 \left[\frac{1 - R^{-k_3}}{1 - R_0^{-k_3}} - \frac{\ln R}{\ln R_0} \right],
$$

\n
$$
\sigma_z = k_4 \left[(\sigma_r + \sigma_\theta) - \frac{\Omega_{\text{initial yielding}}^2}{2} (1 + R_0^2) \right] + \frac{\beta_1}{\ln R_0} \times \left[(1 + \beta_2) k_4 - \beta_3 \right] \left\{ \ln R - \frac{2}{(1 - R_0^2)} \left(\frac{R_0^2}{4} - \frac{1}{4} - \frac{R_0^2}{2} \ln R_0 \right) \right\},
$$

and

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$$
\Omega_{\text{fully plastic}} = \left| \frac{2(1 - R_0^{-k_3})}{k_3 R_0^{-k_3} (R_0^2 - 1)} \right| \left[1 - k_3 \beta_1 \left| \frac{k_5 R_0^{-k_3}}{(1 - R_0^{-k_3})} + \frac{1}{\ln R_0} \right| \right] (18)
$$

where: $k_3 = (c_{11} - c_{22})/c_{11}$; and $k_4 = c_{22}/(c_{12} + c_{22})$.

ISOTROPIC MATERIALS

For isotropic material, the elastic constants are $c_{12} = c_{21} =$ $c_{32} = c_{23} = c_{31} = c_{13} = c_{11} - 2c_{66}$, $\lambda = c_{12}$, $\mu = (c_{11} - c_{12})/2$, $c_{11} = c_{22} = c_{33}$, and $c = (c_{11} - c_{12})/c_{11}$. Eq.(18) becomes: 33, and $c = (c_{11} - c_{12})$
 $cR^{-c}(R_0^2)$ $\frac{1}{1 - R_0^{-c}}$ + $\beta_1 c \left[\frac{cR}{1 - R_0^{-c}} - \frac{1}{\ln R_0} \right]$ $-(c_{12})/c_{11}$. Eq.(18) becomes:
 (R_0^2-1) cR^{-c} 1 $\frac{C}{r} + \frac{\Omega_{\text{isotropic initial}}^2 cR^{-c} (R_0^2 - 1)}{2(1 - R_0^{-c})} + \beta_1 c \left[\frac{cR^{-c}}{1 - R_0^{-c}} - \frac{cR^{-c}}{\ln R} \right]$ $\frac{(c_{11} - c_{12})/c_{11}}{cR^{-c}(R_0^2 - 1)} + \beta_1 c \left[\frac{cR}{R_0^2 - 1} \right]$ $\frac{R}{R_0^{c}}$ (R_0^{-1}) $+ \beta_1 c \left[\frac{cR^{-c}}{1 - R_0^{-c}} - \frac{1}{\ln R} \right]$ $\sigma_{\theta} = \sigma_r + \frac{\Omega_{\text{isotropic initial}}^2 c R^{-c} (R_0^2 - 1)}{2(1 - R_0^{-c})} + \beta_1 c \left[\frac{c R^{-c}}{1 - R_0^{-c}} - \frac{1}{\ln \Omega} \right]$ c_{33} , and $c = (c_{11} - c_{12})/c_{11}$. Eq.(18) becomes:
 $\Omega_{\text{isotropic initial}}^2 c R^{-c} (R_0^2 - 1) + R_c \left[c R^{-c} - 1 \right]$ $= \sigma_r + \frac{\Omega^2}{\text{isotropic initial}} \frac{cR^{-c}(R_0^2 - 1)}{2(1 - R^{-c})} + \beta_1 c \left[\frac{cR^{-c}}{1 - R^{-c}} - \frac{1}{\ln R_0} \right],$ $\frac{1}{1-R_0^{-c}} \left(\frac{1}{R_0^{-c}} \right) + \beta_1 c \left[\frac{1}{1-R_0^{-c}} - \frac{1}{\ln R_0} \right],$ 2(1- R_0^{-C}) $\left[1-R_0^{-C} \ln \right]$
 $\left[R_0^2 (1 - R^{-C}) + R^2 (R_0^{-C} - 1) + R^{-C} - R_0^{-C} \ln \right]$ $\boldsymbol{0}$ $-R_0^{-c}$)
 $(1 - R^{-c}) + R^2 (R_0^{-c} - 1)$ $\frac{2^{(1)}+R^{-}(1)}{2(1-R_0^{-c})}$ $\begin{bmatrix} 1-R_0^{-c} & \ln R \end{bmatrix}$
 c^c ₂ + $R^2(R_0^{-c} - 1)$ + R^{-c} - R_0^{-c} $r =$ $\frac{1}{2(1 - R^{-c})}$ $2(1 - R_0^{-c})$ $\left[1 - R_0^{-c} \ln R_0^{2}(1 - R^{-c}) + R^2(R_0^{-c} - 1) + R^{-c} - R\right]$ $\sigma_r = \frac{2(1 - R)}{2}$ $\left[1 - R_0^{-c} \ln R_0\right]$ $\left[-c\right] + R^2 (R_0^{-c} - 1) + R^{-c} - R_0^{-c}$ − $-2(1 - R_0^{-c})$ $+ P_1^c$ $\left[1 - R_0^{-c} \ln R_0\right]$

Ω²

sotropic initial $\left[R_0^2(1 - R^{-c}) + R^2(R_0^{-c} - 1) + R^{-c} - R_0^{-c}\right]$ $=\frac{\Omega_{\text{isotropic initial}}^2 \left[R_0^2 (1 - R^{-c}) + R^2 (R_0^{-c} - 1) + R^{-c} - R_0^{-c} \right]}{2(1 - R_0^{-c})}$ $\sqrt{1-c}\left[\frac{1-R_0^{-c}}{1-R_0}\right]$ $1 - R^{-c}$ ln $1 - R_0^{-c}$ ln *c c* $\beta_1 c \left| \frac{1 - R^{-c}}{1 - R_0^{-c}} - \frac{\ln R}{\ln R_0} \right|$ − − $+\beta_1 c \left[\frac{1 - R^{-c}}{1 - R^{-c}} - \frac{\ln R}{1 - R} \right],$ $\left[1 - R_0^{-c} \quad \ln R_0\right]$, inital yielding 2 $\binom{2}{0}$ + $\frac{p_1}{\ln R_0}$ 1 $\mathcal{L}_z = \left(\frac{1-c}{2-c}\right) \left[(\sigma_r + \sigma_\theta) - \frac{\Omega^2}{2} (1 + R_0^2) \right] + \frac{1}{\ln \theta}$ $\frac{c}{c}$ $\left[(\sigma_r + \sigma_\theta) - \frac{\Omega^2}{2} (1 + R_0^2) \right] + \frac{\beta_1}{\ln R}$ $\sigma_z = \left(\frac{1-c}{2-c}\right) \left[(\sigma_r + \sigma_\theta) - \frac{\Omega^2}{\Omega^2} (1 + R_0^2) \right] + \frac{\beta_1}{\ln R_0} \times$ $\begin{bmatrix} 1-R_0^{-c} & \ln R_0 \end{bmatrix}$ $(1+\beta_2)$ $2 \left(1 + \beta_2\right)\left(\frac{1-c}{2-c}\right) - \beta_3\left[\ln R - \frac{2}{(1-R_0^2)}\left(\frac{R_0^2}{4} - \frac{1}{4} - \frac{R_0^2}{2}\ln R_0\right)\right]$ $\left[\frac{1-c}{2-c}\right]-\beta_3\right]\left\{\ln R-\frac{2}{(1-R_0^2)}\left(\frac{R_0^2}{4}-\frac{1}{4}-\frac{R_0^2}{2}\right)\right\}$ $\sigma_z = \left(\frac{1}{2-c}\right) \left[\frac{(\sigma_r + \sigma_\theta) - \frac{\sigma_{\text{max}} - \sigma_{\text{max}}}{2}(1 + R_0^{\sigma})\right] + \frac{1}{\ln R_0} \times$
 $\left[\left(1 + \beta_2\right) \left(\frac{1-c}{2-c}\right) - \beta_3\right] \left\{\ln R - \frac{2}{(1 - R_0^2)} \left(\frac{R_0^2}{4} - \frac{1}{4} - \frac{R_0^2}{2}\ln R_0\right)\right\},\right.$ $\times \left[\left(1+\beta_2\right) \left(\frac{1-c}{2-c}\right) - \beta_3 \right] \left\{ \ln R - \frac{2}{\left(1-R_0^2\right)} \left(\frac{R_0^2}{4} - \frac{1}{4} - \frac{R_0^2}{2} \ln R_0 \right) \right\},\right\}$, and Ω^2 isotropic initial $= \left| \frac{2(1 - R_0^{-c})}{cR_0^{-c}(R_0^2 - 1)} \right| \left[1 - c\beta_1 \left| \frac{cR_0^{-c}}{(1 - R_0^{-c})} + \frac{1}{\ln R_0} \right| \right]$ $\frac{2(1-R_0^{-c})}{(1-R_0^{-c})}$ $\left[\left[1-c\beta_1\right] \frac{cR_0^{-c}}{(1-R_0^{-c})} + \frac{1}{\ln R_0^{-c}}\right]$ (c) $(1-R_0^{\pi})$ $($ $+$ 4
 c_R $\frac{-R_0^{-c}}{c}$ $\left| 1 - c\beta_1 \frac{cR_0^{-c}}{(1 - R_0^{-c})} \right|$ $\frac{2(1-R_0^{-c})}{cR_0^{-c}(R_0^2-1)}\left[1-c\beta_1\frac{cR_0^{-c}}{(1-R_0^{-c})}+\frac{1}{\ln R}\right]$ $(1-R_0^{\sigma}) \times 4$ 4 - $(1-R_0^{-c})$ $\frac{(1 - R_0^{-c})}{(R_0^2 - 1)} \left\| 1 - c\beta_1 \frac{cR_0^{-c}}{(1 - R_0^{-c})} + \frac{cR_0^{-c}}{(1 - R_0^{-c})} \right\|$ $\begin{bmatrix} | & (1 - R_0^2) \ (1 - R_0^2) \ (1 - R_0^{-c}) \ (1$ $\Omega_{\text{isotropic initial}}^2 = \frac{2(1 - R_0^{-c})}{2P_0^{-c}(R_0^2 - 1)} \left[1 - c\beta_1 \left| \frac{cR_0^{-c}}{(1 - R_0^{-c})} + \frac{1}{\ln R_0} \right| \right] (19)$ $\frac{1}{(-1)^{n}}\left\|1-c\beta_1\left|\frac{cR_0^{-c}}{(1-R_0^{-c})}+\frac{1}{\ln R_0}\right|\right\}$ (19)

 $\sqrt{2}$

The fully-plastic surface, Eq.(19), becomes:
\n
$$
\sigma_{\theta} = \sigma_r + \frac{\Omega_{\text{isotropic fully-plastic}}^2 (R_0^2 - 1)}{2 \ln R_0} - \frac{\beta_1}{\ln R_0},
$$
\n
$$
\sigma_r = \frac{\Omega_{\text{isotropic fully-plastic}}^2}{2} \left[(1 - R^2) + (R_0^2 + 1) \frac{\ln R}{\ln R_0} - \beta_1 \ln R \right],
$$
\n
$$
\sigma_z = \frac{1}{2} \left[(\sigma_r + \sigma_{\theta}) - \frac{\Omega^2}{\text{isotropic fully-plastic}} (1 + R_0^2) \right] + \frac{\beta_1}{\ln R_0} \left[\frac{1 + \beta_2}{2} - \beta_3 \right] \times \times \left\{ \ln R - \frac{2}{(1 - R_0^2)} \left[\frac{R_0^2}{4} - \frac{1}{4} - \frac{R_0^2}{2} \ln R_0 \right] \right\},
$$
\nand
$$
\Omega_{\text{isotropic fully-plastic}}^2 = \left| \frac{2 \ln R_0}{R_0^2 - 1} \right| \left[\ln R_0 \right] - \frac{\beta_1}{\ln R_0} \right].
$$
\n(20)

VALIDATION OF RESULTS

The present results from Eqs. $(19)-(20)$ are the same by neglecting thermal conditions compared to the results given by Thakur et al. /4/.

DISCUSSION

To emphasize the above, we have taken the following values of c_{ij} (units of 1010 N/m²): $c_{11} = 0.891$, $c_{12} = 0.46$, $c_{21} = 0.4614$, $c_{22} = 0.7842$, $c_{32} = 0.2676$ for orthotropic material $/6/$ (e.g., barite); $c = 0.5$ for isotropic material $/6/$ (e.g., copper). Figure 1 clearly manifests that a cylinder made of barite requires higher values of angular speed at the inner surface for the initial, fully plastic state as compared to copper. Figure 2 depicts that a cylinder made of barite needs

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a higher value of angular speed at the inner portion in comparison to a cylinder made of copper in its initial, fully plastic state. With the addition of the thermal effect, angular speed must increase at the inner portion of a cylinder made of orthotropic material and decrease for isotropic material.

Figure 3 depicts that circumferential stress is maximum at the inner portion of the cylinder made of barite or copper. Moreover, the rotating cylinder made of isotropic material must need more circumferential or axial stress in comparison to the cylinder made of orthotropic material, and the imposed boundary conditions eliminate the radial stress on the internal or external surface.

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STRUCTURAL INTEGRITY AND LIFE Vol. 24, No.3 (2024), pp. 309–313 *Abbreviations*

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