Pankaj Thakur

CREEP DEFORMATION IN A THICK-WALLED SPHERICAL SHELL HAVING STEADY STATE TEMPERATURE

DEFORMACIJA PUZANJA U DEBELOZIDNOJ SFERNOJ LJUSCI SA STACIONARNOM TEMPERATUROM

Originalni naučni rad / Original scientific paper
Rad primljen / Paper received: 10.10.2022
https://doi.org/10.69644/ivk-2024-03-0301

Adresa autora / Author's address:
Department of Mathematics, Faculty of Science and Technology, ICFAI University, Himachal Pradesh, India
P. Thakur 0000-0001-8119-2697,
email: pankaj_thakur15@yahoo.co.in

Keywords

- isotropic structure
- pressure
- temperature
- yielding
- shell

Ključne reči

- izotropna struktura
- pritisak
- temperatura
- tečenje
- ljuska

Abstract

This article deals with the study of creep deformation in a thick-walled spherical shell under steady state temperature by using transition theory and generalised strain measure. Mathematical modelling is based on stress-strain relation and equilibrium equation. Analytical solutions are presented in a thick-walled spherical shell made of saturated clay, steel and rubber materials. The shell made of rubber material requires higher pressure to yield in comparison to the shell made of saturated clay/steel material without thermal effects.

Izvod

U ovom radu se proučava deformacija puzanja u debelozidnoj sfernoj ljusci sa stacionarnom temperaturom, primenom teorije prelaznih napona i generalisanih mera deformacija. Matematičko modeliranje se bazira na relaciji napon-deformacija i na jednačini ravnoteže. Analitička rešenja su predstavljena kod debelozidne sferne ljuske izrađene od zasićenog zemljišta, čelika i gume. Za ljusku izrađenu od gume je potreban veći pritisak za pojavu tečenja, u poređenju sa ljuskom od zasićenog zemljišta/čelika bez termičkih uticaja.

INTRODUCTION

Analysis of spherical shell structures in aerospace, chemical, civil, and mechanical industries such as in high-speed centrifugal separators, gas turbines for high-power aircraft engines, certain rotor systems, spinning satellite, and structures, are important for safety purpose and long life of shell structures. To increase the life of spherical shells, it is therefore very important for engineers to study the safety analysis in spherical shells under various environments. The spherical shell made of isotropic materials is presented in most standard textbooks /1-6/ with evaluated solutions for stresses and displacements in a thick spherical shell subjected to various load conditions. Seth /7/ has analysed stress distribution in shells/tubes under pressure by using the transition theory.

In this paper, we investigate the creep deformation in a thick-walled isotropic spherical shell under steady-state temperature and uniform pressure by using the concept of generalised strain measures and asymptotic solution through the principal stresses-difference. Results are presented graphically and are discussed.

GOVERNING EQUATION

We consider a spherical shell having internal and external radii a and b, respectively, subjected to internal pressure p_i and a steady state temperature Θ_0 applied at the internal surface of the shell.

Basic governing equation

Due to the spherical symmetry of the structure, the components of displacements in spherical co-ordinates (r, ϕ, z) are given by /8/:

$$u = r(1-\beta),\ \ v = 0,\ \ w = dz, \tag{1}$$

where: β is position function, depending on $r = \sqrt{(x^2 + y^2 + z^2)}$ only. The generalised strain measures are given by /8, 9/:

$$\varepsilon_{ii}^{M} = \frac{1}{n}\left[1-(1-2\hat{\varepsilon}_{ii})^{n/2}\right],\ \ (i = 1, 2, 3), \tag{2}$$

where: n = -2, -1, 0, 1, 2 give Green, Cauchy, Hencky, Swainger, and Almansi measures, respectively; and $\overset{A}{e_{ii}}$ be Almansi finite strain components. From Eq.(2), the generalised components of strain are obtained as:

$$\varepsilon_{rr} = \frac{1}{n}\left[1-(r\eta'+\eta)^{n}\right],\ \ \varepsilon_{\varphi\varphi} = \frac{1}{n}\left[1-\eta^{n}\right] = e_{zz},$$
$$\varepsilon_{r\varphi} = \varepsilon_{\varphi z} = \varepsilon_{zr} = 0, \tag{3}$$

where: n is measure; ε_{rr}, $\varepsilon_{\varphi\varphi}$, and ε_{zz} are strain components; and $\eta' = d\eta/dr$.

Stress-strain relation: for thermal elastic isotropic material, the stress-strain relationships are presented /1, 4/ as:

$$\tau_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij} - \xi\Theta\delta_{ij},\ \ (i, j, k = 1,2,3). \tag{4}$$

Further, Θ has to satisfy:

$$\nabla^2\Theta = 0. \tag{5}$$

The temperature satisfying Laplace Eq.(5) with boundary condition:

$$\Theta = \Theta_0\ \text{ at } r = a,\ \ \Theta = 0\ \text{ at } r = b, \tag{6}$$

where: Θ_0 is constant, and given by:

$$\Theta = \frac{\Theta_0 a}{(b-a)}\left[\frac{b}{r}-1\right]. \tag{7}$$

Using Eq.(3) into Eq.(4), we get:

$$\tau_{rr} = \frac{\lambda+2\mu}{n}\left[1-(r\eta'+\eta)^n\right]+\frac{2\lambda}{n^m}\left[1-\eta^n\right]-\xi\Theta,$$

$$\tau_{\varphi\varphi} = \tau_{zz} = \frac{\lambda}{n}\left[1-(r\eta'+\eta)^n\right]+\frac{2(\lambda+\mu)}{n^m}\left[1-\eta^n\right]-\xi\Theta,$$

$$\tau_{r\varphi} = \tau_{\varphi z} = \tau_{zr} = 0. \tag{8}$$

Equation of equilibrium: the equations of motion are all satisfied except:

$$\frac{d}{dr}(r\tau_{rr}) - \frac{2(\tau_{rr}-\tau_{\varphi\varphi})}{r} = 0. \tag{9}$$

Asymptotic solution at transition points: using Eqs.(6)-(8) into Eq.(9), we get nonlinear differential equation in η as:

$$\left\{\eta T(T+1)^{n-1}\frac{dT}{d\eta}+T(T+1)^n\right\}+2(1-C)T(1-\eta^n)^{m-1}+$$
$$+\frac{n^m C\xi\bar{\Theta}_0}{2n\mu r\beta^n}-\frac{2C}{n\beta^n}\left\{\left[1-\eta^n(T+1)^n\right]-(1-\eta^n)\right\}=0, \tag{10}$$

where: $r\eta' = \eta T$.

Critical or transition points: transition points of η in Eq.(10) are $T \to 0$ and $T \to \pm\infty$.

Boundary condition: the boundary conditions are

$$\tau_{rr} = p_i \text{ at } r = a \text{ and } \tau_{rr} = 0 \text{ at } r = b. \tag{11}$$

PROBLEM SOLUTION

For finding the creep stress distribution, the transition function Ψ is taken through the principal stresses' difference /7-25/ at the transition point $T \to -1$ as:

$$\Psi = \tau_{rr} - \tau_{\varphi\varphi} = \frac{2\mu}{n}\left[\left\{1-\eta^n(T+1)^n\right\}-(1-\eta^n)\right]. \tag{12}$$

Taking the logarithmic differentiation of Eq.(12) with respect to η and using Eq.(10), and after that by taking the asymptotic value at $T \to -1$, we get:

$$\frac{d(\log\Psi)}{d\eta} = \frac{n(3-2C)\eta^{n-1}}{\eta^n}-\frac{nC\xi\bar{\Theta}_0}{2\mu r\eta^{n+1}}+\frac{2C}{\eta}. \tag{13}$$

Integrating Eq.(13) with respect to r, we get:

$$\Psi = Ar^{-2C}\eta^{n(3-2C)}\exp f_1, \tag{14}$$

where: A is a constant of integration and $f_1 = \frac{C\xi\bar{\Theta}_0}{2\mu}\int\frac{dr}{r^2\eta^n}$. The asymptotic value of η as $T \to -1$ is D/r; D being a constant, therefore, Eq.(14) becomes:

$$\Psi = \tau_{rr}-\tau_{\varphi\varphi} \equiv Ar^{-2C}D^{n(3-2C)}r^{-n(3-2C)}\exp f_1. \tag{15}$$

Using Eq.(15) into Eq.(9), and integrating, we get:

$$\tau_{rr} = -2A\int r^{-2C-1}D^{n(3-2C)}r^{-n(3-2C)}\exp f_1 dr + B. \tag{16}$$

Using boundary conditions Eq.(11) into Eq.(16), we get:

$$B = 2A\int r^{-2C-1}D^{n(3-2C)}r^{-n(3-2C)}\exp f_1 dr,$$

$$A = \frac{-p_i}{2\int_a^b r^{-2C-1}D^{n(3-2C)}r^{-n(3-2C)}\exp f_1 dr}.$$

Now substituting the value of constants A and B into Eqs.(15)-(16), we get:

$$\tau_{rr} = -p_i\frac{\int_r^b r^{-3n+2C(n-1)-1}\exp f_1 dr}{\int_a^b r^{-3n+2C(n-1)-1}\exp f_1 dr},$$

$$\tau_{\varphi\varphi} = \tau_{zz} = \tau_{rr} + \frac{p_i r^{-3n+2C(n-1)-1}\exp f_1}{2\int_a^b r^{-3n+2C(n-1)-1}\exp f_1 dr}, \tag{17}$$

where: $f_1 = \frac{\alpha E(3-2C)\bar{\Theta}_0 r^{n-1}}{Y(n-1)D^n}$.

Initial yielding: it has been seen that $|\tau_{rr} - \tau_{\phi\phi}|$ are maximum at $r = a$, therefore yielding will take place at the internal surface of the shell and Eq.(17) becomes:

$$\left|\tau_{rr}-\tau_{\varphi\varphi}\right|_{r=a} = \frac{p_i a^{-3n+2C(n-1)-1}\exp f_2}{2\int_a^b r^{-3n+2C(n-1)-1}\exp f_2 dr} \equiv Y, \tag{18}$$

where: $f_2 = \frac{\alpha E(3-2C)\bar{\Theta}_0 a^{n-1}}{Y(n-1)D^n}$.

Equations (17)-(18) for incompressible material (i.e., $C \to 0$) become:

$$\tau_{rr} = -p_i\frac{\int_r^b r^{-3n-1}\exp f_1 dr}{\int_a^b r^{-3n-1}\exp f_1 dr},$$

$$\tau_{\varphi\varphi} = \tau_{zz} = \tau_{rr} + \frac{p_i r^{-3n-1}\exp f_1}{2\int_a^b r^{-3n-1}\exp f_1 dr},$$

$$\frac{Y}{p_i} = \frac{a^{-3n-1}\exp f_2}{2\int_a^b r^{-3n-1}\exp f_2 dr}, \tag{19}$$

where: $f_3 = \frac{3\alpha E\bar{\Theta}_0 a^{n-1}}{Y(n-1)D^n}$.

Neglecting thermal condition: taking $\Theta_0 = 0$ into Eq.(19), we get:

$$\tau_{rr} = -p_i\frac{(b/r)^{3n-2C(n-1)}-1}{(b/a)^{3n-2C(n-1)}-1},$$

$$\tau_{\varphi\varphi} = \tau_{zz} = -p_i\frac{\frac{1}{2}\{n(3-2C)-2(1-C\}(b/r)^{3n-2C(n-1)}-1}{(b/a)^{3n-2C(n-1)}-1}. \tag{20}$$

Equation (20) for incompressible material reduces to:

$$\tau_{rr} = -p_i\frac{(b/r)^{3n}-1}{(b/a)^{3n}-1},$$

$$\tau_{\varphi\varphi} = \tau_{zz} = -p_i\frac{\frac{1}{2}\{3n-2\}(b/r)^{3n}-1}{(b/a)^{3n}-1}. \tag{21}$$

Equations (20)-(21) are same as obtained by Gupta et al., /25/.

NUMERICAL ILLUSTRATION AND DISCUSSION

To investigate the effect of yielding pressure versus thickness ratios for the spherical shell made of incompressible/compressible (*say* rubber ν = 0.5; saturated clay ν = 0.42; and steel ν = 0.27) by /1/. For mild steel, the following values have been taken /17/: $\alpha = 7.5\times10^{-6}$ per °F; $E = 3\times10^7$ lb/in^2; ν = 0.5, 0.42, 0.27; $Y = 3\times10^4$ lb/in^2; Θ_0 = 0 °F, 40 °F, 80 °F; n = 3, 5; and a = 0.5 (inner radius); b = 1 (outer radius); $r \in (a, b)$, respectively.

Figure 1 is portrayed in order to demonstrate the behaviour of yielding pressure Y/p_i versus thickness ratios at different temperature and measure $n = 3$, $n = 5$, respectively. In the absence of thermal effect, it has been observed that yielding of thinner shells as well as thicker shell occurs at the same pressure.

With the introduction of thermal effect, the thinner shell yields at higher pressure in comparison to the thicker shell. This yielding pressure goes on increasing with increasing measure $n = 3$, 5, and temperature $\Theta = 40$ °F, 80 °F, respectively. Moreover, shell made of incompressible material (*say* rubber, $\nu = 0.5$) needs superior pressure to yield, in comparison to shell made of compressible materials (*say* saturated clay, $\nu = 0.42$; steel $\nu = 0.27$).

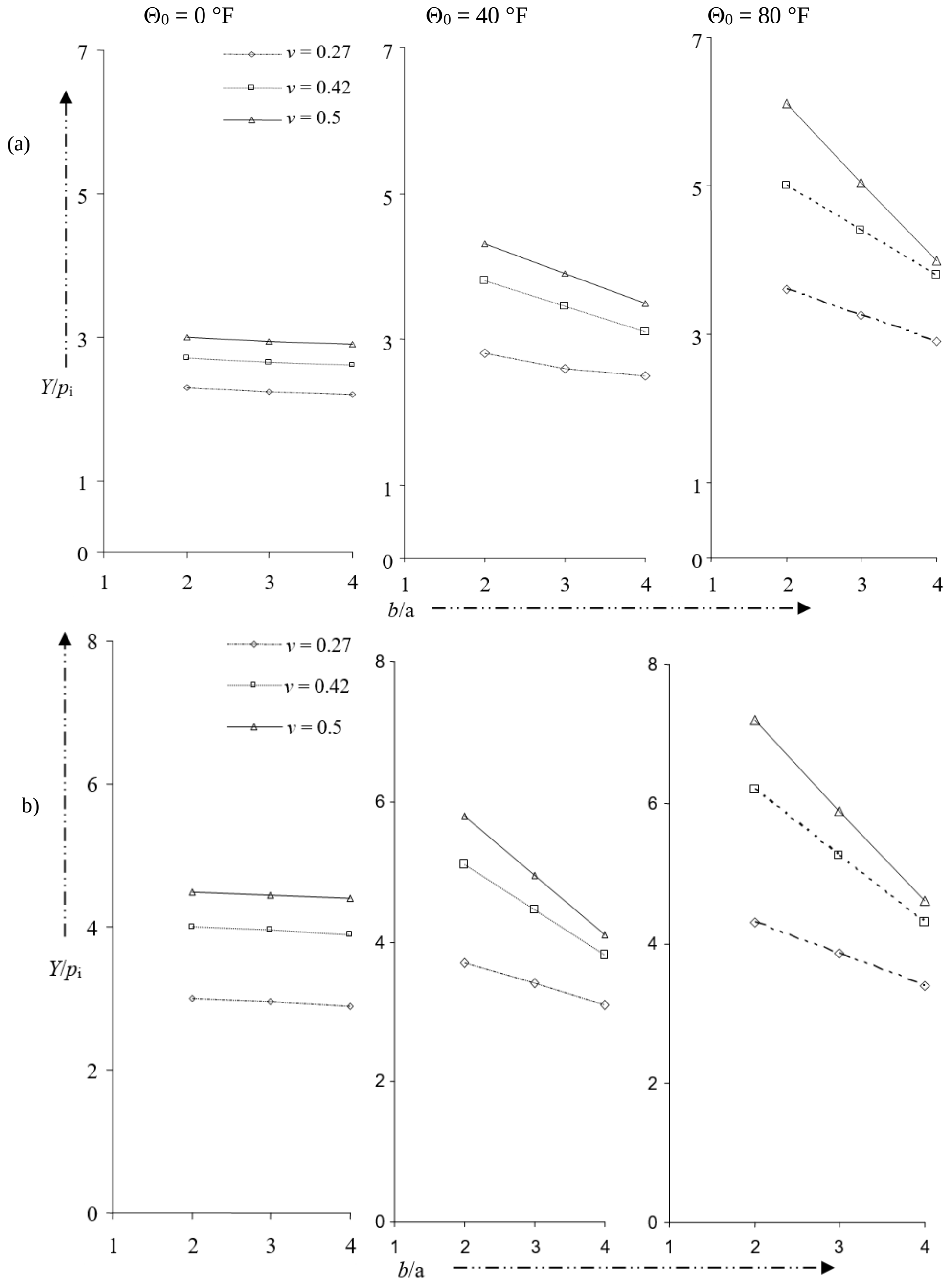

Figure 1. Yielding ratios Y/p_i vs. thickness ratios b/a at different temperatures and measures: (a) $n = 3$; (b) $n = 5$.

CONCLUSIONS

The main findings are given as follows:

- the shell made of rubber material requires higher pressure to yield in comparison to the shell made of compressible material without thermal effects,
- the result is same as given by Gupta et al. /25/.

ACKNOWLEDGEMENT

This research paper is specially dedicated to the Society for Structural Integrity and Life and IMS Institute in Serbia.

REFERENCES

1. Sokolnikoff, I.S., Mathematical Theory of Elasticity, 2nd Ed., McGraw-Hill Inc., New York, 1953.
2. Fung, Y.C., Foundations of Solid Mechanics, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1965.
3. Kraus, H., Creep Analysis, John Wiley & Sons, Inc., New York, 1980. ISBN 0471062553
4. Parkus, H., Thermoelasticity, Springer-Verlag, Wien, 1976. doi: 10.1007/978-3-7091-8447-9
5. Timoshenko, S., Woinowsky-Krieger, S., Theory of Plates and Shells (2nd Ed.), McGraw-Hill, New York, 1959. ISBN 0-07-064779-8
6. Chakrabarty, J., Theory of Plasticity, McGraw-Hill, New York, 1987.
7. Seth, B.R. (1963), *Elastic-plastic transition in shells and tubes under pressure*, J Appl. Math. Mech (ZAMM), 43(7-8): 345-351. doi: 10.1002/zamm.19630430706
8. Seth, B.R. (1962), *Transition theory of elastic-plastic deformation, creep and relaxation*, Nature, 195: 896-897. doi: 10.10 38/195896a0
9. Seth, B.R. (1966), *Measure-concept in mechanics*, Int. J Non-Linear Mech. 1(1): 35-40. doi: 10.1016/0020-7462(66)90016-3
10. Thakur, P, Sethi, M. (2020), *Creep deformation and stress analysis in a transversely material disk subjected to rigid shaft*, Math. Mech. Solids, 25(1): 17-25. doi: 10.1177/108128651985 7109
11. Thakur, P., Gupta, N., Gupta, K., Sethi, M. (2020), *Elastic-plastic transition in an orthotropic material disk*, Struct. Integr. Life, 20(2): 169-172.
12. Thakur, P., Chand, S., Sukhvinder et al. (2020), *Density parameter in a transversely and isotropic disc material with rigid inclusion*, Struct. Integr. Life, 20(2): 159-164.
13. Sethi, M. Thakur P. (2020), *Elastoplastic deformation in an isotropic material disk with shaft subjected to load and variable density*, J Rub. Res. 23: 69-78. doi: 10.1007/s42464-020-00038-8
14. Thakur, P., Sethi M. (2020), *Elastoplastic deformation in an orthotropic spherical shell subjected to temperature gradient*, Math. Mech. Solids, 25(1): 26-34. doi: 10.1177/108128651985 7128
15. Thakur, P., Kumar N., Sukhvinder (2020), *Elasto-plastic density variation in a deformable disk*, Struct. Integr. Life, 20(1): 27-32.
16. Thakur, P., Kumar, N., Sethi, M. (2021), *Elastic-plastic stresses in a rotating disc of transversely isotropic material fitted with a shaft and subjected to thermal gradient*, Meccanica, 56: 1165-1175. doi: 10.1007/s11012-021-01318-2
17. Thakur, P., Sethi, M., Kumar, N., et al. (2021), *Thermal effects in a rotating disk made of rubber and magnesium materials and having variable density*, J Rubber Res. 24(3): 403-413. doi: 10. 1007/s42464-021-00107-6
18. Thakur, P., Sethi, M., Gupta, N., Gupta, K. (2021), *Thermal effects in rectangular plate made of rubber, copper and glass materials*, J Rubber Res. 24(1): 147-155. doi: 10.1007/s42464-020-00080-6
19. Thakur, P., Sethi, M., Gupta, K., Bhardwaj, R.K. (2021), *Thermal stress analysis in a hemispherical shell made of transversely isotropic materials under pressure and thermo-mechanical loads*, J Appl. Math. Mech. (ZAMM), 101(12). doi: 10.1002/zamm.20 2100208
20. Thakur, P., Sethi, M., Kumar, N., et al. (2021), *Analytical solution of hyperbolic deformable disk having variable density*, Mech. Solids, 56(6): 1039-1046. doi: 10.3103/S002565442106 0194
21. Thakur, P., Sethi, M., Kumar, N., et al. (2022), *Stress analysis in an isotropic hyperbolic rotating disk fitted with rigid shaft*, Z Angew. Math. Phys. 73(1), Art. ID 23. doi: 10.1007/s00033-02 1-01663-y
22. Kumar, N., Thakur, P. (2021), *Thermal behaviour in a rotating disc made of transversely isotropic material with rigid shaft*, Struct. Integr. Life, 21(3): 217-223.
23. Kumar, A., Sood, S., Thakur, P., Kumar, N. (2022), *Safety analysis in an isotropic material disc fitted with a rigid shaft subjected to thermo-mechanical load by using transition theory*, Struct. Integr. Life, 22(2): 193-201.
24. Kumar, N., Thakur, P. (2022), *Behaviour of creep stress and strain rates in a solid disc made of isotropic material with thermal condition*, Struct. Integr. Life, 22(2): 202-209.
25. Gupta, S.K., Bhardwaj, P.C., Rana, V.D. (1988), *Thermo-creep transition of a thick isotropic spherical shell under internal pressure*, Ind. J Pure Appl. Math. 19(12): 1239-1248.