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ON THE ONSET OF STATIONARY CONVECTION ON JEFFREY NANOFLUID LAYER SATURATED WITH A POROUS MEDIUM: BRINKMAN MODEL

O POJAVI STACIONARNE KONVEKCIJE U SLOJU JEFFREY NANOFLUIDA KOJI JE ZASIĆEN POROZNOM SREDINOM: BRINKMAN MODEL

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Abstract

In this paper, we investigate the onset of convection in a Jeffrey nanofluid layer saturated with the porous medium using Darcy-Brinkmann model. Normal mode analysis and Galerkin type weighted residual method (GWRM) are used to analyse conservation equations. Effects of Brownian motion and thermophoresis are taken into account in the Jeffrey nanofluid model. The Buongiorno model deployed for nanoparticles incorporates the influences of thermophoresis and Brownian motion. Three cases of free-free, rigid-rigid and rigid-free boundaries are considered. For stationary convection, the effects of Darcy number, Jeffrey parameter, Lewis number, nanoparticle Rayleigh number, porosity and modified diffusivity ratio for all the above mentioned boundary conditions are investigated analytically and graphically. The numerical computed values of stationary thermal Rayleigh number are presented graphically for three distinct combinations of boundary conditions. The study is of great significance in many different areas such as automotive, pharmaceutical, geophysics, soil sciences, food processing, oceanography, limnology, etc., and excellent coincidence is found regarding the present paper and earlier published work.

Ključne reči
- toplotna nestabilnost
- nanočestice
- nanofluidi
- Braunovo kretanje

Izvod

U ovom radu istražujemo pojavu konvekcije u sloju Jeffrey nanofluida koji je zasićen poroznom sredinom, i to primenom Darcy-Brinkman modela. Za analize jednačina ravnoteže koriste se analiza u normalnom modu i analiza težinskim ostatkom tipa Galerkin (GWRM). Uticaji Braunovog kretanja i termoforeze se razmatraju u modelu Jeffrey nanofluida. Uvedeni model Buongiorno za nanočestice sadrži uticaje termoforeze i Braunovog kretanja. Razmotrena su tri slučaja slobodno-slobodno, kruto-kruto i kruto-slobodno graničnih uslova. Pri stacionarnoj konvekciji istraženi su uticaji Darcijevog broja, Jeffrey parametra, Luisovog broja, Rejlejevog broja za nanočestice, poroznosti i modifikovanog odnosa difuznosti za sve gore navedene granične uslove, i to analitički i grafički. Numerički sračunate vrednosti stacionarnog termičkog Rejlejevog broja su predstavljene grafički za tri kombinacije graničnih uslova. Ova istraživanja su od velikog značaja u mnogim oblastima kao što su automobilska industrija, farmaceutika, geofizika, nauka o tlu, procesiranje hrane, okeanografija, limnologija, itd., a uočava se izvanredno poklapanje rezultata sa onima u ranijim objavljenim radovima.

INTRODUCTION

A liquid that contains suspended submicroscopic solid particles, commonly referred to as nanoparticles, is referred to as a 'nanofluid'. The term was first used by Choi /20/. As stated by Masuda et al. /15/, the distinguishing characteristic of nanofluids is thermal conductivity enhancement.

According to Buongiorno and Hu /23/, this phenomenon raises the prospect of employing nanofluids in sophisticated nuclear systems. Buongiorno conducted a thorough analysis of convective transport in nanofluids and claims that an acceptable explanation for the unexpected rise in thermal conductivity and viscosity has not yet been discovered. He concentrated on the additional heat transfer improvement seen in convective conditions. Buongiorno points out that a number of writers have proposed that the dispersion of the suspended nanoparticles may be the cause of the enhanced convective heat transfer, but he contends that this impact is insufficient to account for the observed boost. Buongiorno comes to the conclusion that the presence of nanoparticles has no effect on turbulence, hence it is unable to account for the observed boost. The increase of heat transmission has also been attributed to particle rotation, but Buongiorno determines that this impact is insufficient to account for the result. Buongiorno developed a novel model based on the mechanics of the nanoparticle/base-fluid relative velocity after ruling out dispersion, turbulence and particle rotation as key factors for heat transfer amplification.

According to Buongiorno, the base fluid velocity and a relative velocity (which he refers to as slip velocity) may be combined to form the nanoparticle absolute velocity. He thought about each of the following seven slide processes in turn: gravity settling, fluid drainage, inertia, Brownian diffusion, thermophoresis, and diffusiophoresis.

On the basis of the transport equations of Buongiorno /22/, the Bénard issue (the commencement of convection in a horizontal layer evenly heated from below) for a nanofluid was investigated by Tzou /1-2/ and Nield and Kuznetsov /12-14/. Nield and Kuznetsov investigated the Horton-Rogers-Lapwood issue, which is the equivalent flow problem in a porous medium. According to Kuznetsov and Nield /17-18/ the Brinkman model is added to that inquiry to further it. A Darcy number thus is added as a new parameter as a result.

A few researchers /3-11, 15-16, 21/ studied heat convection in a viscoelastic nanofluid layer saturating a porous medium and found that viscoelastic nanofluid has uses in a number of automotive industries and biomedical engineering. This study's main objective is to examine the effects of the Jeffery parameter and other variables in a porous layer saturated with a viscoelastic nanofluid heated from below, which is an extension of the paper studied by Rana and Gautam /11/, for free-free boundary conditions.

MATHEMATICAL MODEL

Consider a porous layer of material between two planes $z^* = 0$ and $z^* = H$. The fluid layer is heated from below and working in the upwards direction with a gravity force $g = (0,0,-g)$. The temperature and volumetric fraction at the lower wall be T_h^* and ϕ_0^* while at the upper wall it is T_c^* and ϕ_1^*, respectively. We consider a porous medium with porosity ε and permeability K.

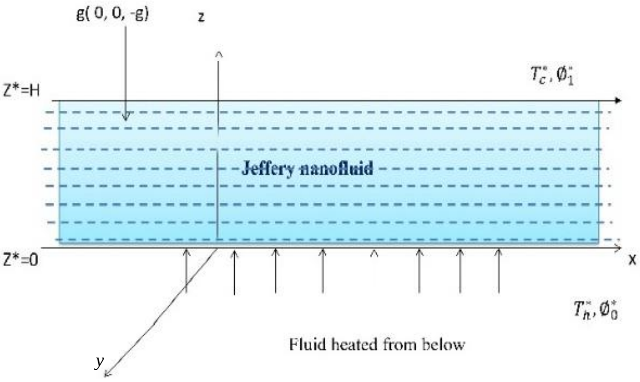

Figure 1. Physical sketch of the problem.

GOVERNING EQUATIONS

The conservation equation of mass, momentum, thermal energy and nanoparticles, respectively are:

$$\nabla^* . v_D^* = 0, \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\frac{\partial v_D^*}{\partial t^*} = -\nabla^* p^* + \tilde{\mu}\nabla^{*2} v_D^* - \frac{\mu}{K(1+\lambda)} v_D^* + \left[\phi^*\rho_p + (1-\phi^*)\{\rho_f(1-\beta(T-T_c^*))\}\right] g, \tag{2}$$

$$(\rho c)_m \frac{\partial T^*}{\partial t^*} + (\rho c)_f v_D^* . \nabla^* T^* = k_m \nabla^{*2} T^* + \varepsilon(\rho c)_p \times \left[D_B \nabla^*\phi^* . \nabla^* T^* + (D_T / T_c^*)\nabla^* T . \nabla^* T^*\right], \tag{3}$$

$$\frac{\partial \phi^*}{\partial t^*} + \frac{1}{\varepsilon} v_D^* . \nabla^* \phi^* = D_B \nabla^{*2}\phi^* + (D_T / T_c^*)\nabla^{*2} T^*. \tag{4}$$

We write $\mathbf{v}_D^* = (u^*, v^*, w^*)$.

Here, ρ_f, μ, and β are the density, viscosity, and volumetric expansion coefficient of the fluid, while ρ_p is the density of particles. We have introduced effective viscosity $\tilde{\mu}$, the effective heat capacity $(\rho c)_m$ and effective thermal conductivity of the porous medium k_m. The coefficients that appear in Eqs. (3) and (4) are the Brownian diffusion coefficient D_B and the thermophoretic diffusion coefficient D_T.

On the boundaries, we assume that the temperature and volumetric fraction of nanoparticles are both constant. The boundary conditions are therefore,

$$w^* = 0, \quad \frac{\partial w^*}{\partial z^*} + \lambda_1 H \frac{\partial^2 w^*}{\partial z^{*2}} = 0, \quad T^* = T_h^*, \quad \phi^* = \phi_0^* \text{ at } z^* = 0 \tag{5}$$

$$w^* = 0, \quad \frac{\partial w^*}{\partial z^*} - \lambda_2 H \frac{\partial^2 w^*}{\partial z^{*2}} = 0, \quad T^* = T_c^*, \quad \phi^* = \phi_1^* \text{ at } z^* = H \tag{6}$$

As follows, we present dimensionless variables. We define

$$(x,y,z) = (x^*, y^*, z^*)/H, \quad t = t^*\alpha_m / \sigma H^2,$$
$$(u,v,w) = (u^*, v^*, w^*)H / \alpha_m,$$
$$p = p^* K / \mu\alpha_m, \quad \phi = \frac{\phi^* - \phi_0^*}{\phi_1^* - \phi_0^*}, \quad T = \frac{T^* - T_c^*}{T_h^* - T_c^*}, \tag{7}$$

where:
$$\alpha_m = \frac{k_m}{(\rho c)_f}, \quad \sigma = \frac{(\rho c)_m}{(\rho c)_f}. \tag{8}$$

Equations (1)-(6) take the form

$$\nabla . \mathbf{v} = 0, \tag{9}$$

$$\frac{1}{\sigma V_a} . \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + D_a \nabla^2 \mathbf{v} - \frac{\mathbf{v}}{1+\lambda} - R_m \hat{e}_z - R_n \hat{e}_z + R_a \hat{e}_z, \tag{10}$$

$$\frac{\partial T}{\partial t} + \mathbf{v} . \nabla T = \nabla^2 T + \frac{N_B}{L_e}\nabla\phi . \nabla T + \frac{N_A N_B}{L_e}\nabla T . \nabla T, \tag{11}$$

$$\frac{1}{\sigma}\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{v} . \nabla\phi = \frac{1}{L_e}\nabla^2\phi + \frac{N_A}{L_e}\nabla^2 T, \tag{12}$$

$$w = 0, \quad \frac{\partial w}{\partial z} + \lambda_1 \frac{\partial^2 w}{\partial z^2} = 0, \quad T = 1, \quad \phi = 0 \quad \text{at} \quad z = 0, \tag{13}$$

$$w = 0, \quad \frac{\partial w}{\partial z} - \lambda_2 \frac{\partial^2 w}{\partial z^2} = 0, \quad T = 0, \quad \phi = 1 \quad \text{at} \quad z = 1. \tag{14}$$

Here, $P_r = \frac{\mu}{\rho_f \alpha_m}$ is the Prandtl number; $D_a = \frac{K}{H^2}$ is Darcy number; $L_e = \frac{\alpha_m}{D_B}$ is the Lewis number; $V_a = \frac{\varepsilon P_r}{D_a}$ is the Vadasz number; $R_a = \frac{\rho g \beta K H (T_h^* - T_c^*)}{\mu\alpha_m}$ is the thermal Rayleigh-Darcy number; $R_m = \frac{\left[\rho_p \phi_1^* + \rho(1-\phi_1^*)\right] gKH}{\mu\alpha_m}$ is the basic density Rayleigh number, $R_n = \frac{(\rho_p - \rho)(\phi_1^* - \phi_0^*) gKH}{\mu\alpha_m}$ is concentration Rayleigh number, $N_A = \frac{D_T (T_h^* - T_c^*)}{D_B T_c^* (\phi_1^* - \phi_0^*)}$ is the modified diffusivity rate, and $N_B = \frac{(\rho c)_p}{(\rho c)_m}(\phi_1^* - \phi_0^*)$ is modified particle-density increment, respectively.

BASIC SOLUTIONS

The time independent fundamental states for nanofluids are expressed as

$$\mathbf{v}=0,\ T=T_b(z),\ \phi=\phi_b(z),\ p=p_b(z)\,. \quad (15)$$

Using Eq.(15) in Eqs. (9), (10), (11) and (12), these equations reduce to

$$-\frac{dp_z}{dz}-R_m\hat{e}_z-R_n\phi_b\hat{e}_z+R_aT_b\hat{e}_z=0\,, \quad (16)$$

$$\frac{d^2T_b}{dz^2}+\frac{N_B}{L_e}\frac{d\phi}{dz}.\frac{dT_b}{dz}+\frac{N_AN_B}{L_e}\left(\frac{dT_b}{dz}\right)^2=0\,, \quad (17)$$

$$\frac{d^2\phi_b}{dz^2}+N_A\frac{d^2T_b}{dz^2}=0\,. \quad (18)$$

Using boundary conditions Eqs. (13) and (14), the solution of Eq.(18) is

$$\phi_b=-N_AT_b+(1-N_A)z+N_A\,. \quad (19)$$

Substituting the value of ϕ_b in Eq.(17), we get

$$\frac{d^2T_b}{dz^2}+\frac{(1-N_A)N_B}{L_e}.\frac{dT_b}{dz}=0\,. \quad (20)$$

Neglecting the higher power term, solution of Eq.(20) is given by

$$T_b=\frac{-e^{-(1-N_A)N_B/L_e}\left[1-e^{-(1-N_A)N_B/L_e(1-z)}\right]}{1-e^{-(1-N_A)N_B/L_e}}\,. \quad (21)$$

According to Buongiorno, the approximated solution for Eqs. (19) and (21), gives

$$T_b=1-z,\ \phi_b=z\,. \quad (22)$$

PERTURBATION SOLUTION

We now superimpose perturbations on the basic solution. We write,

$$\mathbf{v}=0+\mathbf{v}',\ p=p_b+p',\ T=T_b+T',\ \phi=\phi_b+\phi'\,. \quad (23)$$

Using Eq.(23) in Eqs.(9)-(14) and linearizing the terms by neglecting the product of prime quantities, the following equations are obtained:

$$\nabla.\mathbf{v}'=0\,, \quad (24)$$

$$\frac{1}{\sigma Va}\frac{\partial\mathbf{v}'}{\partial t}=-\nabla p'+Da\nabla^2\mathbf{v}'-\frac{\mathbf{v}'}{1+\lambda}+RaT'\hat{\mathbf{e}}_\mathbf{z}-Rn\phi'\hat{\mathbf{e}}_\mathbf{z}, \quad (25)$$

$$\frac{\partial T'}{\partial t}-w'=\nabla^2T'+\frac{N_B}{L_e}\left(\frac{\partial T'}{\partial z}-\frac{\partial\phi'}{\partial z}\right)-\frac{2N_AN_B}{L_e}\frac{\partial T'}{\partial z}, \quad (26)$$

$$\frac{1}{\sigma}\frac{\partial\phi'}{\partial t}+\frac{1}{\varepsilon}=\frac{1}{L_e}\nabla^2\phi'+\frac{N_A}{L_e}\nabla^2T', \quad (27)$$

$$w'=0,\ \frac{\partial w'}{\partial z}+\lambda_1\frac{\partial^2w'}{\partial z^2}=0,\ T'=0,\ \phi'=0\ \text{ at }\ z=0\,, \quad (28)$$

$$w'=0,\ \frac{\partial w'}{\partial z}-\lambda_2\frac{\partial^2w'}{\partial z^2}=0,\ T'=0,\ \phi'=0\ \text{ at }\ z=1\,. \quad (29)$$

The six unknowns u', v', w', p', T', and ϕ' can be reduced to three by operating on Eq.(25) multiplied by $\hat{\mathbf{e}}_\mathbf{z}$*.curl.curl* and also using Eq.(24), we get

$$\frac{1}{\sigma Va}\frac{\partial\nabla^2w'}{\partial t}-Da\nabla^4w'+\frac{\nabla^2w'}{1+\lambda}=Ra\nabla_H^2T'-Rn\nabla_H^2\phi', \quad (30)$$

where: $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ and $\nabla_H^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ are the two-dimensional Laplace operators.

NORMAL MODE ANALYSIS

The disturbances analyses by normal mode analysis are as follows

$$(w',T',\phi')=\left[W(z),\Theta(z),\Phi(z)\right]\exp(ilx+imy+st)\,, \quad (31)$$

where: s is growth rate; and l and m are the wave numbers along x and y directions, respectively.

Substituting Eq.(31) in Eqs.(25)-(29) and (31), we get

$$\left[Da(D^2-a^2)^2-\left(\frac{1}{1+\lambda}+\frac{s}{\sigma Va}\right)(D^2-a^2)\right]W-$$
$$-Ra\,a^2\Theta+Rn\,a^2\Phi=0\,, \quad (32)$$

$$W+\left(D^2+\frac{N_A}{L_e}D-\frac{2N_AN_B}{L_e}-a^2-s\right)\Theta-\frac{N_B}{L_e}D\Phi=0, \quad (33)$$

$$\frac{1}{\varepsilon}W-\frac{N_A}{L_e}(D^2-a^2)\Theta-\left(\frac{1}{L_e}(D^2-a^2)-\frac{s}{\sigma}\right)\Phi=0, \quad (34)$$

$$W=0,\ DW+\lambda_1D^2W=0,\ \Theta=0,\ \Phi=0\ \text{ at }\ z=0\,, \quad (35)$$

$$W=0,\ DW-\lambda_2D^2W=0,\ \Theta=0,\ \Phi=0\ \text{ at }\ z=1\,, \quad (36)$$

where, $D=\frac{d}{dz}$ and $a^2 = l^2 + m^2$ is the dimensionless wave number.

According to Chandrasekhar /19/, boundary conditions should be as follows:

1) free-free boundaries

$$W=D^2W=\Theta=\Phi=0\ \text{ at }\ z=0,1\,, \quad (37)$$

2) rigid-rigid boundaries

$$W=DW=\Theta=\Phi=0\ \text{ at }\ z=0,1\,, \quad (38)$$

3) rigid-free boundaries

$$W=DW=\Theta=\Phi=0\ \text{ at }\ z=0\,, \quad (39)$$

$$W=D^2W=\Theta=\Phi=0\ \text{ at }\ z=1\,. \quad (40)$$

The assumed solutions for W, Θ, and Φ, for all boundary conditions are taken as follows

- for free-free boundaries

$$W=W_0\sin\pi z,\ \Theta=\Theta_0\sin\pi z,\ \Phi=\Phi_0\sin\pi z\,, \quad (41)$$

- for rigid-rigid boundaries

$$W=W_0(z^2-2z^3+z^4),\ \Theta=\Theta_0(z-z^2),\ \Phi=\Phi_0(z-z^2), \quad (42)$$

- for rigid-free boundaries

$$W=W_0(3z^2-5z^3+2z^4),\ \Theta=\Theta_0(z-z^2),\ \Phi=\Phi_0(z-z^2)\,. \quad (43)$$

LINEAR STABILITY ANALYSIS FOR FREE-FREE BOUNDARIES

Substituting Eq.(41) in Eqs.(32)-(34) and integrating each term individually within limits $z = 0$ to $z = 1$, we get

$$\begin{bmatrix} Da\,J\left(J+\frac{i\omega}{1+\lambda}\right) & -Ra\,a^2 & Rn\,a^2 \\ 1 & -(J+i\omega) & 0 \\ \frac{1}{\varepsilon} & \frac{N_A}{L_e}J & \left(\frac{J}{L_e}+\frac{i\omega}{\sigma}\right)\end{bmatrix}\begin{bmatrix}W_0\\ \Theta_0\\ \Phi_0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}, \quad (44)$$

where: $J = \pi^2 + a^2$.

The eigenvalue to the system of linear Eqs.(44) is given as

$$Ra=-\frac{1}{a^2}\left\{\frac{Rn\,a^2\left(\frac{N_A J}{L_e}+\frac{J+i\omega}{\varepsilon}\right)}{\left(\frac{J}{L_e}+\frac{i\omega}{\sigma}\right)}+J\left(Da\,J+\frac{1}{1+\lambda}+\frac{Da\,i\omega}{\sigma Va}\right)\times\right.$$
$$\left.\times(J+i\omega)\right\}. \quad (45)$$

Stationary convection for free-free boundaries

For stationary convection $\omega = 0$ in Eq.(45), we obtain

$$Ra^S=\frac{Da(\pi^2+a^2)^3+(\pi^2+a^2)^2}{a^2(1+\lambda)}-\left(\frac{L_e}{\varepsilon}+N_A\right)Rn. \quad (46)$$

For the case when Da = 0, the critical wave number at the onset of instability is obtained by minimising the thermal Rayleigh-Darcy number Ra with respect to a. Thus, the critical wave number must satisfy

$$\left(\frac{\partial Ra}{\partial a^2}\right)_{a=a_c}=0.$$

Equation (46) gives

$$a_c=\pi. \quad (47)$$

On the other hand, when Da is large compared with unity, the critical wave number at the onset of instability is obtained by minimising the thermal Rayleigh-Darcy number Ra with respect to a. Thus, the critical wave number must satisfy

$$\left(\frac{\partial Ra}{\partial a^2}\right)_{a=a_c}=0.$$

Equation (46) gives $$a_c=\frac{\pi}{\sqrt{2}}. \quad (48)$$

LINEAR STABILITY ANALYSIS FOR RIGID-RIGID BOUNDARIES

Substituting Eq.(42) in Eqs.(32)-(34) and integrating each term individually within limits z = 0 to z = 1, after applying Galerkin first approximation, we get

$$\begin{bmatrix} 2Da(504+24a^2+a^4)+(12+a^2)\left(\frac{1}{1+\lambda}+\frac{s}{\sigma Va}\right) & -9Ra\,a^2 & 9Ra\,a^2 \\ 3 & -14(10+a^2+s) & 0 \\ \frac{3}{\varepsilon} & 14\frac{N_A}{L_e}(10+a^2) & \frac{14(10+a^2)}{L_e}+\frac{14s}{\sigma}\end{bmatrix}\times$$
$$\times\begin{bmatrix}W_0\\ \Theta_0\\ \Phi_0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}. \quad (49)$$

The eigenvalue to the system of linear Eqs.(49) is given as

$$Ra=\frac{27}{28a^2}\frac{\left[Da(504+24a^2+a^4)+12+a^2\left(\frac{1}{1+\lambda}+\frac{sDa}{\varepsilon\Pr}\right)\right](10+a^2+s)\times}{\left(10+a^2+\frac{sL_e}{\sigma}\right)}$$
$$\frac{\times\left(10+a^2+\frac{sL_e}{\sigma}\right)-\left(N_A(10+a^2)+\frac{L_e(10+a^2+s)}{\varepsilon}\right)}{\left(10+a^2+\frac{sL_e}{\sigma}\right)}. \quad (50)$$

Stationary convection for rigid-rigid boundaries

For stationary convection $\omega = 0$ in Eq.(50), we obtain

$$Ra^s=\frac{28}{27a^2}\left[(504+24a^2+a^4)Da+12+a^2\left(\frac{1}{1+\lambda}\right)\right]\times$$
$$\times(10+a^2)-\left(N_A+\frac{L_e}{\varepsilon}\right)Rn. \quad (51)$$

For the case when Da = 0, the critical wave number at the onset of instability is obtained by minimising thermal Rayleigh-Darcy number Ra with respect to a. Thus the critical wave number must satisfy

$$\left(\frac{\partial Ra}{\partial a^2}\right)_{a=a_c}=0.$$

Equation (51) gives $$a_c=3.31. \quad (52)$$

On the other hand when Da is large compared with unity, the critical wave number at the onset of instability is obtained by minimising thermal Rayleigh-Darcy number Ra with respect to a. Thus the critical wave number must satisfy

$$\left(\frac{\partial Ra}{\partial a^2}\right)_{a=a_c}=0.$$

Equation (51) gives $$a_c=3.12. \quad (53)$$

LINEAR STABILITY ANALYSIS FOR RIGID-FREE BOUNDARIES

Substituting Eq.(43) in Eqs.(32)-(34) and integrating each term individually within limits z = 0 to z = 1, after applying Galerkin first approximation, we get

$$\begin{bmatrix} 2Da(4536+432a^2+19a^4)+(216+19a^2)\left(\frac{1}{1+\lambda}+\frac{s}{\sigma Va}\right) & -39Ra\,a^2 & 39Ra\,a^2 \\ 13 & -14(10+a^2+s) & 0 \\ \frac{13}{\varepsilon} & 14\frac{N_A}{L_e}(10+a^2) & \frac{14(10+a^2)}{L_e}+\frac{14s}{\sigma}\end{bmatrix}\begin{bmatrix}W_0\\ \Theta_0\\ \Phi_0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}. \quad (54)$$

The eigenvalue to the system of linear Eq.(54) is given by

$$Ra=\frac{27}{28a^2}\frac{\left[Da(4536+432a^2+19a^4)+216+19a^2\left(\frac{1}{1+\lambda}+\frac{sDa}{\varepsilon Pr}\right)\right](10+a^2+s)\left(10+a^2+\frac{sL_e}{\sigma}\right)-\left(N_A(10+a^2)+\frac{L_e(10+a^2+s)}{\varepsilon}\right)}{\left(10+a^2+\frac{sL_e}{\sigma}\right)} \quad (55)$$

Stationary Convection for rigid-free boundaries

For stationary convection $\omega = 0$ in Eq.(55), we obtain

$$Ra = \frac{28}{507a^2}\left[(4536+432a^2+19a^4)Da + 216 + 19a^2\left(\frac{1}{1+\lambda}\right)\right](10+a^2) - \left(N_A + \frac{L_e}{\varepsilon}\right)Rn\,. \tag{56}$$

For the case when Da = 0, the critical wave number at the onset of instability is obtained by minimising thermal Rayleigh-Darcy number Ra with respect to wave number a. Thus, the critical wave number must satisfy

$$\left(\frac{\partial Ra}{\partial a^2}\right)_{a=a_c} = 0\,.$$

Equation (56), gives $a_c = 3.27\,.$ (57)

On the other hand, when Da is large compared with unity, the critical wave number at the onset of instability is obtained by minimising thermal Rayleigh-Darcy number Ra with respect to wave number a. Thus, the critical wave number must satisfy

$$\left(\frac{\partial Ra}{\partial a^2}\right)_{a=a_c} = 0\,.$$

Equation (56), gives $a_c = 2.67\,.$ (58)

RESULTS AND DISCUSSION

In this research paper, we have studied the stationary convection in the thermal instability of Jeffrey nanofluid layer saturated with a porous medium: Brinkman model. The effects of various parameters like: Darcy number, Jeffrey parameter, modified diffusivity ratio, Lewis number, porosity parameter and concentration Rayleigh number on stationary convection are analysed analytically and plotted graphically for free-free, rigid-rigid and rigid-free boundaries.

Figure 2 shows the graph of Ra with respect to wave number a for different values of Da = 0.1, 0.2, 0.3. Fixing other parameters as: λ = 0.2, N_A = 5, L_e = 1000, ε = 0.6, R_n = -1, it is clear from Fig. 2 that as Da goes on increasing there is increase in Ra. Thus, Da has a stabilising effect on stationary convection. Also, we have analysed that Da has a more stabilizing effect in the case of rigid-rigid boundaries. Thus, Da delays the onset of convection of the system.

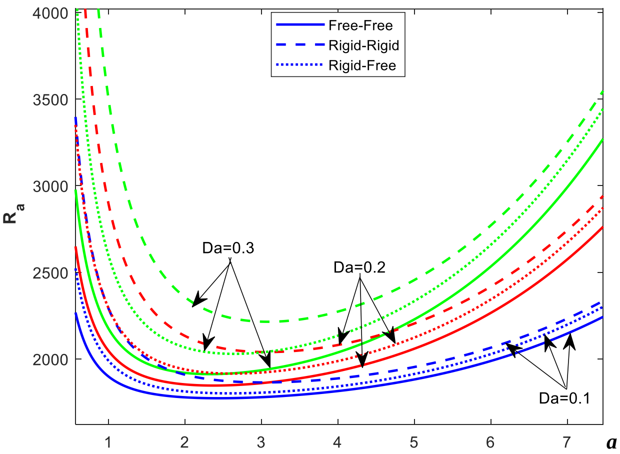

Figure 2. Variation of Rayleigh- with wave number for different values of Darcy number.

Figure 3 shows the graph of Ra with respect to wave number a, for different values of λ = 0.2, 0.5, 0.8. Fixing other parameters as: Da = 0.1, N_A = 5, L_e = 1000, ε = 0.6, R_n = -1, it is clear from the figure that Ra goes on decreasing with increase in λ. Thus, λ has a destabilizing effect on stationary convection, and it is also clear from the figure that it has more destabilizing effect in the case of free-free boundaries. Thus, λ enhances the onset of convection of the system.

Figure 4 shows the graph of Ra with respect to wave number a for different values of N_A = 1, 5, 10. Fixing other parameters as Da = 0.1, λ = 0.2, L_e = 1000, ε = 0.6, R_n = -1, it is clear from Fig. 4 that Ra goes on increasing with an increase in N_A. Thus, N_A has a stabilizing effect, and it is also clear from the figure that it has more stabilizing effect in the case of rigid-rigid boundaries. Thus, N_A delays the onset of convection of the system.

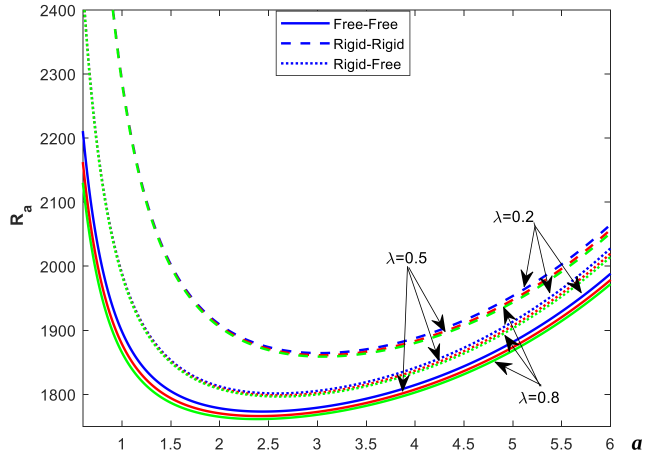

Figure 3. Variation of Rayleigh- with wave number for different values of Jeffrey parameter.

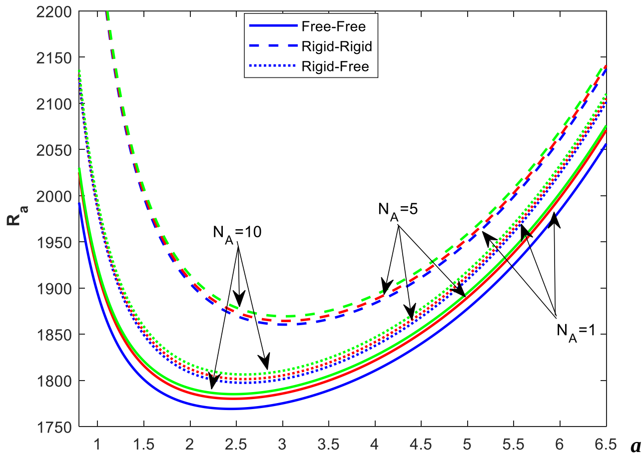

Figure 4.Variation of Rayleigh with wave number for different values of modified diffusivity ratio.

Figure 5 shows the graph of Ra with respect to wave number a for different values of L_e = 100, 500, 1000. Fixing other parameters as: Da = 0.1, λ = 0.2, N_A = 5, ε = 0.6, R_n = -1, it is clear from the figure that as Ra goes on increasing with increase in L_e, thus, it has stabilizing effect on stationary convection and Fig. 5 demonstrates that L_e has more stabilizing effect in the case of rigid-rigid boundaries. Thus, L_e delays the onset of convection of the system.

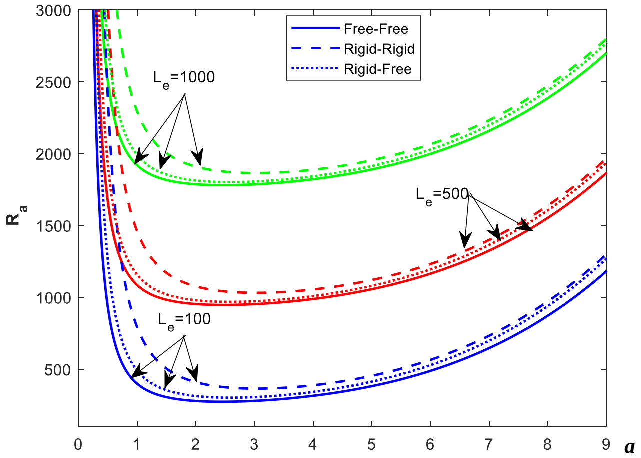

Figure 5. Variation of Rayleigh- with wave number for different values of Lewis number.

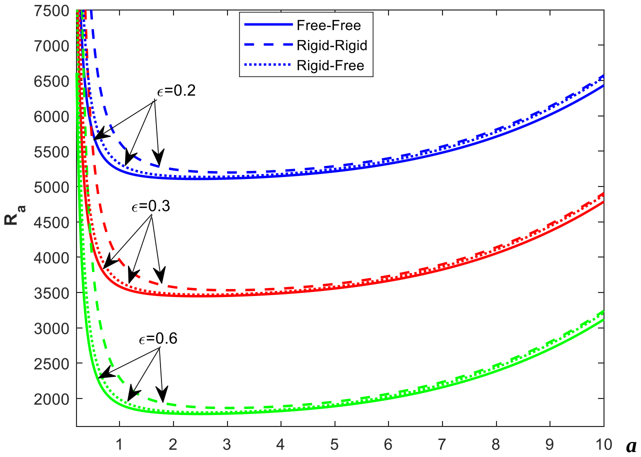

Figure 6. Variation of Rayleigh- with wave number for different values of porosity parameter.

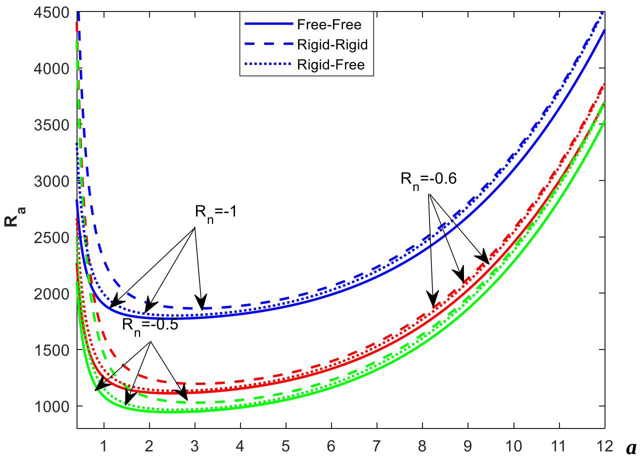

Figure 7. Variation of Rayleigh- with wave number for different values of concentration Rayleigh number.

Figure 6 shows the graph of Ra with respect to wave number a for different values of ε = 0.2, 0.3, 0.6. Fixing other parameters as: Da = 0.1, λ = 0.2, N_A = 5, L_e = 1000, R_n = -1, it is clear from the figure that as Ra goes on decreasing within increase in ε. Thus, ε shows a destabilizing effect and it is also clear from Fig. 6 that it has more destabilizing effect in the case of free-free boundaries. Thus, ε enhances the onset of convection of the system.

Figure 7 shows the graph of Ra with respect to wave number a for different values of R_n = -0.1, -0.6, -0.5. Fixing other parameters as: Da = 0.2, λ = 0.2, N_A = 5, L_e = 1000, ε = 0.6, it is clear from the figure that as Ra goes on decreasing with increase in R_n. Thus, R_n has a destabilizing effect, and it is also clear from Fig. 7 that R_n has more destabilizing effect in the case of free-free boundaries. Thus, R_n enhances the onset of convection of the system.

CONCLUSION

In this article, we use linear stability analysis to make the following key conclusion:

(i) Da, N_A, L_e have stabilizing influence on the system.

(ii) λ ε, R_n produce impact on the system in such a way that they enhance the onset of convection.

(iii) In case of rigid-rigid boundaries, the system has greater stabilizing impact rather than at free-free/rigid-free boundaries.

(iv) Parameters as: Da, N_A, L_e have a more destabilising effect on stationary convection in case of free-free boundaries, as compared to rigid-rigid/rigid-free boundaries.

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