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EVALUATION OF THERMAL STRESSES IN TRANSVERSELY ISOTROPIC PIEZOELECTRIC DISC WITH ROTATION AND INTERNAL PRESSURE

ODREĐIVANJE TERMIČKIH NAPONA U TRANSVERZALNO IZOTROPNOM PIJEZOELEKTRIČNOM ROTIRAJUĆEM DISKU SA UNUTRAŠNJIM PRITISKOM

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Keywords
- thermal stresses
- transversely isotropic disc
- piezoelectric
- internal pressure

Abstract

The paper deals with the analytical solution of transitional stresses in thin rotating disc composed of piezoelectric material under temperature and internal pressure. The stresses are evaluated in the rotating disc by using transition theory of Seth. The electric displacement relations and stresses are computed by using stress strain relations. The non-homogeneous differential equation is derived by substituting the obtained relations into the equilibrium equation. The formulated differential equation is solved with specified boundary conditions, applied pressure, electric displacement and stresses. Obtained results are exhibited graphically, analysed numerically and it is then concluded that transversely isotropic beryl is better than transversely isotropic magnesium material and transversely isotropic piezoelectric materials $BaTiO_4$ and PZT-4.

Ključne reči
- termički naponi
- transverzalno izotropan disk
- pijezoelektrični
- unutrašnji pritisak

Izvod

U radu je predstavljeno analitičko rešenje prelaznih napona kod tankog rotirajućeg diska sačinjenog od pijezoelektričnog materijala pod uticajem temperature i unutrašnjeg pritiska. Određivanje napona u rotirajućem disku je obavljeno primenom teorije prelaznih napona Seta. Relacije električnih pomeranja i napona su izvedene preko veza napona i deformacija. Nehomogena diferencijalna jednačina je izvedena smenom dobijenih relacija u jednačinu ravnoteže. Formirana diferencijalna jednačina se rešava uz posebne granične uslove, dejstvo pritiska, električnih pomeranja i napona. Dobijeni rezultati su predstavljeni grafički, analizirani su numerički, a zatim se izvodi zaključak da je transverzalno izotropni beril bolji materijal od transverzalno izotropnog magnezijuma, kao i transverzalno izotropnih pijezoelektričnih materijala $BaTiO_4$ i PZT-4.

INTRODUCTION

In the present era smart materials are of great importance in engineering and scientific research. Smart materials are defined as materials having properties which can be changed significantly by applications of external stimuli, for intense stress, by variation in temperature, moistness, pH, electric or magnetic fields. Examples of smart materials are piezoelectric material, shape memory alloys, shape memory polymer, pH sensitive polymer, magneto caloric material, pyroelectric material and thermo-piezoelectric material. The internal generation of an electric current has a result of mechanical force applications known as direct piezoelectric material. Material which exhibits piezoelectric effect is known as the piezoelectric material. Piezoelectric material is widely used in various sensing devices, waves, shock control devices, MEMS devices, navigation, medical instruments, household kitchen appliances and in smart structures. Most commonly piezoelectric materials that we use are quartz, barium titanate ($BaTiO_3$), Rochelle salt and polyvinylidene fluoride.

To ensure that piezoelectric instruments or appliances will work in various temperature conditions, it becomes necessary to include the temperature effect while developing the mathematical model of the scientific research problem. So electrical-thermal-mechanical coupling thermoelastic theories are developed by various authors. First of all, Mindlin /1/ developed the theory of piezoelectric thermoelastic materials. Later Mindlin /2/ derived the equation for thermal piezoelectric crystals under the effect of high frequency vibrations. Chandrasekharaiah /3/ considered the finite speed of thermal vibrations and extended the theory presented by Mindlin. Tauchert /4/ applied the thermoelastic theory of piezoelectric materials to composite plates. Eringen /5/ introduced electromagnetic effect in micropolar thermoelasticity. Eringen /6/ developed micropolar piezoelectricity. This theory can be applied to porous electric materials and various synthetic materials also. There is great use of this theory in intelligent structural systems, piezoelectric composite structural appliances, loudspeakers and ultrasonic transducers. Iesan /7/ presented the linear theory of piezoelectricity for microstretch piezoelectric materials and proved uniqueness and reciprocity theorems. Marin /8/ derived the expression for the solutions of elasticity problems concerned with dipolar porous materials. Migorski and Ochal /9/ discussed the dynamical bilateral problem for viscoelastic piezo thermoelastic medium with the adhesion effect. Sharma /10/ studied plane harmonic waves in an anisotropic piezo-electric thermoelastic solid. Othman and Ahmed /11/ discussed the rotational effect on the piezo thermoelastic medium under four different theories of thermoelasticity. Kumar and Sharma /12/ established the variational principle, uniqueness and reciprocity theorems in porous magneto piezo thermoelastic medium. Sharma and Radaković /13/ found an exact solution of elastic-plastic stresses in a thin rotating disc composed of piezoelectric material.

In this research, we have considered the piezo-electric effect and isotropic properties for the thermoelastic material. The dynamical problem is then solved by using analytical technique. The temperature distribution and stress components have been solved numerically. Resulting quantities are depicted graphically to explore the effect of piezoelectric parameter and internal pressure. In this paper, thermal stresses are computed in a rotating disc composed of transversely isotropic piezoelectric material under internal pressure by keeping in mind the concept of transition theory.

MATHEMATICAL FORMULATION

We consider a thin rotating disc having a and b as internal and external radii, respectively, and ω as angular velocity of the disc. A thin disc is assumed as we are discussing the state of plane stress, i.e., $T_{zz} = 0$. Displacements coordinates in polar form are given as:

$$u = r(1 - F);\ v = 0 \text{ and } w = dz$$

where: F is function of $r = \sqrt{(x^2 + y^2)}$; and d is a constant.

Components of strain are as follows:

$$e_{rr} = \frac{1}{n}[1-(r\beta+\beta)^n],\quad e_{\theta\theta} = \frac{1}{n}[1-\beta^n],$$
$$e_{zz} = \frac{1}{n}[1-(1-d)^n]. \tag{1}$$

Now, the stress-strain relations for this problem are

$$t_{rr} = c_{11}e_{rr} + (c_{11}-2c_{66})e_{\theta\theta} + c_{13}e_{zz} - e_{11}E_r - \beta_1\theta,$$
$$t_{\theta\theta} = (c_{11}-2c_{66})e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz} - e_{12}E_r - \beta_2\theta,$$
$$t_{zz} = t_{zr} = t_{r\theta} = t_{\theta z} = 0, \tag{2}$$

where: t_{rr} is radial stress; $t_{\theta\theta}$ is circumferential stress; $\beta_1 = \alpha_1 c_{11} + 2\alpha_2 c_{12}$; $\beta_2 = \alpha_1 c_{12} + 2\alpha_2(c_{22} + c_{33})$; α_1 is coefficient of linear thermal expansion across the axis of symmetry; α_2 is the quantity orthogonal to axis of symmetry; θ is the temperature given by $\theta_0\log(r/b)/\log(a/b)$, and θ_0 is constant.

The equation of equilibrium is given by

$$\frac{d}{dr}(rt_{rr}) - t_{\theta\theta} + \rho\omega^2 r^2 = 0. \tag{3}$$

Now,

$$\frac{d}{dr}(hrt_{rr}) = \frac{d}{dr}\left[h\left\{c_{11}\frac{r}{n}(1-(r\beta'+\beta)^n) + \frac{r}{n}(c_{11}-2c_{66})(1-\beta^n) + \frac{r}{n}c_{13}(1-(1-d)^n) - e_{11}\frac{r}{\eta_{11}}\left(\frac{1}{r} - e_{11}e_{rr} - e_{12}e_{\theta\theta}\right) - \beta_1\theta\right\}\right], \tag{4}$$

$$\frac{d}{dr}(hrt_{rr}) = h\left[r(r\beta'+\beta)^{n-1}\frac{d}{dr}(r\beta'+\beta)\left\{-c_{11} - \frac{e_{11}^2}{\eta_{11}}\right\} + \frac{1}{n}[1-(r\beta'+\beta)^n]\left\{c_{11} + \frac{e_{11}^2}{\eta_{11}}\right\} + r\beta^{n-1}\frac{d\beta}{dr}\left\{(c_{11}-2c_{66}) + \frac{e_{11}e_{12}}{\eta_{11}}\right\} + \frac{1}{n}(1-\beta^n)\right.$$
$$\left.\left\{(c_{11}-2c_{66}) + \frac{e_{11}e_{12}}{\eta_{11}}\right\} + \frac{1}{n}c_{13}(1-(1-d)^n - \beta_1\theta - \beta_1 r\frac{d\theta}{dr}\right], \tag{5}$$

$$t_{\theta\theta} = \frac{1}{n}(c_{11}-2c_{66})(1-(r\beta'+\beta)^n) + \frac{c_{11}}{n}(1-\beta)^n + \frac{c_{13}}{n}(1-(1-d)^n) - \frac{e_{12}}{\eta_{11}}\left(\frac{1}{r} - e_{11}e_{rr} - e_{12}e_{\theta\theta}\right) - \xi\theta = \frac{1}{n}(1-(r\beta'+\beta)^n)\times$$
$$\times\left\{(c_{11}-2c_{66}) + \frac{e_{11}e_{12}}{\eta_{11}}\right\} + \frac{1}{n}(1-\beta^n)\left\{c_{11} + \frac{e_{12}^2}{\eta_{11}}\right\} + \frac{c_{13}}{n}(1-(1-d)^n) - \frac{e_{12}}{\eta_{11}}\frac{1}{r} - \beta_2\theta. \tag{6}$$

Putting the value of $\frac{d}{dr}(rt_{rr})$ and $t_{\theta\theta}$ in Eq.(5), we have

$$\frac{d}{dr}(rt_{rr}) - t_{\theta\theta} + \rho\omega^2 r^2 = r(r\beta'+\beta)^{n-1}\frac{d}{dr}(r\beta'+\beta)\left\{-c_{11} - \frac{e_{11}^2}{\eta_{11}}\right\} + \frac{1}{n}[1-(r\beta'+\beta)^n]\left\{c_{11} + \frac{e_{11}^2}{\eta_{11}}\right\} - (c_{11}-2c_{66}) + \frac{e_{11}e_{12}}{\eta_{11}} +$$
$$+ r\beta^{n-1}\frac{d\beta}{dr}\left\{(c_{11}-2c_{66}) + \frac{e_{11}e_{12}}{\eta_{11}}\right\} + \frac{1}{n}(1-\beta^n)\left\{(c_{11}-2c_{66}) + \frac{e_{11}e_{12}}{\eta_{11}} - c_{11} - \frac{e_{12}^2}{\eta_{11}}\right\} + \frac{e_{12}}{\eta_{11}}\frac{1}{r} - \beta_1 r\frac{d\theta}{dr} + \rho\omega^2 r^2, \tag{7}$$

where: $r\beta' = \beta P$.

So, the equations are

$$r(r\beta'+\beta)^{n-1}\frac{d}{dr}(r\beta'+\beta) = \beta^n P(1+P)^n + \beta^{n+1}P(1+P)^{n-1}\frac{dP}{d\beta} \tag{8}$$

$$\frac{1}{n}(1-(r\beta'+\beta)^n) = \frac{1}{n}[1-\beta^n(1+P)^n], \tag{9}$$

and

$$r = \beta^{n-1}\frac{d\beta'}{dr} = r\beta^{n-1}\frac{\beta P}{r} = \beta^n P. \tag{10}$$

By utilizing Eqs.(8)-(10) in Eq.(7), we obtain:

$$-\frac{dP}{d\beta}\left[c_{11}\beta^{n+1}P(1+P)^{n-1} + \beta^{n+1}P(1+P)^{n-1}\frac{e_{11}^2}{\eta_{11}}\right] = [1-\beta^n(1+P)^n]\left[\frac{e_{11}}{n\eta_{11}}(e_{11}-e_{12}) + 2c_{66}\right] + P\beta^n\left[c_{11} - 2c_{66} + \frac{e_{11}e_{12}}{\eta_{11}}\right] + \frac{1}{n}(1-\beta^n)\times$$
$$\times\left[-2c_{66} + \frac{e_{12}}{\eta_{11}}(e_{11}-e_{12})\right] + \beta^n P(1+P)\left[c_{11} + \frac{e_{11}^2}{\eta_{11}}\right] + \frac{1}{r}\frac{e_{12}}{\eta_{11}} - \frac{\beta_1}{r}\frac{\theta_0}{\log(a/b)} + \rho\omega^2 r^2. \tag{11}$$

The transition points of the above equation are given by $P \to \pm\infty$ and $P \to -1$ and the boundary conditions are

$$t_{rr} = -p \quad \text{at} \quad r = a \quad \text{and} \quad t_{rr} = 0 \quad \text{at} \quad r = b\,. \tag{12}$$

Transition from elastic to plastic state: according to the transition theory /10-26/, a material in the elastic state changes to plastic at critical points $P \to \pm\infty$. For calculating the stresses, the transition function is taken as

$$R = t_{rr} + B + \beta_1 \theta\,, \tag{13}$$

where: B is constant.

Taking the logarithmic differentiation of Eq.(13), we get

$$\frac{d}{dr}(\log R) = \frac{(1-\beta^n(1+P)^n)\left[\frac{e_{11}(e_{11}-e_{12})}{nr\eta_{11}} - \frac{2c_{66}}{r}\right] + \beta^n P \frac{e_{11}e_{12}}{r\eta_{11}} + \frac{(1-\beta^n)}{nr}\left[\frac{e_{12}(e_{11}-e_{12})}{\eta_{11}} - 2c_{66}\right] + \frac{1}{r^2 \frac{e_{12}}{\eta_{11}} - \frac{2}{r}\frac{\beta_1\theta_0}{\log(a/b)} + \rho\omega^2 r} - \frac{e_{11}e_{12}}{r\eta_{11}} P\beta(1-\beta)^{n-1} - \frac{\beta_1}{r}\frac{\theta_0}{\log(a/b)}}{\frac{1}{n}(1-\beta^n(1+P)^n)\left(c_{11} + \frac{e_{11}^2}{\eta_{11}}\right) + \frac{1}{n}\left\{(1-\beta^n)\left(c_{11} - 2c_{66} + \frac{e_{11}e_{12}}{\eta_{11}}\right)\right\} + \frac{1}{n}c_{13}(1-(1-d)^n) - \frac{e_{11}}{\eta_{11}} - \beta_1\theta + B - \beta_1\theta}\,. \tag{14}$$

Taking $P \to \pm\infty$ in Eq.(14) and integrating, we obtain

$$R = Ar^M\,, \tag{15}$$

where: $M = \frac{e_{11}(e_{11}-e_{12}) - 2c_{66}\eta_{11}}{\eta_{11}c_{11} + e_{11}^2}$.

From Eqs.(3) and (15), the expressions for transitional stresses are as follows:

$$t_{rr} = Ar^M - B - \beta_1\theta\,,$$
$$t_{\theta\theta} = \rho\omega^2 r^2 + (M+1)Ar^M + \frac{\beta_1\theta_0}{\log(a/b)} - B - \beta_1\theta\,. \tag{16}$$

By using Eqs.(14) and (18), we have

$$A = \frac{p}{b^M - a^M}\,, \quad B = \frac{b^M p}{b^M - a^M} - \beta_1\theta\,. \tag{17}$$

Using Eqs.(16) and (17), transitional stresses are

$$t_{rr} = \frac{p(r^M - b^M)}{b^M - a^M}\,, \quad t_{\theta\theta} = \rho\omega^2 r^2 + \frac{p(M+1)r^M}{b^M - a^M} - \frac{\beta_1\theta_0}{\log(a/b)} - \frac{b^M p}{b^M - a^M}\,. \tag{18}$$

From Eq.(18), we have Tresca's yield criterion as

$$|t_{rr} - t_{\theta\theta}| = \left|\frac{-pMr^M}{b^M - a^M} - \rho\omega^2 r^2 + \frac{\beta_1\theta_0}{\log(a/b)}\right|\,. \tag{19}$$

From Eq.(19) it is evaluated that $|t_{rr} - t_{\theta\theta}|$ yields maximal value at $r = a$, resulting as initial yielding stress,

$$|t_{rr} - t_{\theta\theta}|_{r=a} = \left|\frac{-pMa^M}{b^M - a^M} - \rho\omega^2 a^2 - \frac{\beta_1\theta_0}{\log(a/b)}\right| = \gamma\,, \tag{20}$$

and the pressure required for initial yielding in piezoelectric material is given by the expression

$$\frac{p}{\gamma} = \frac{(b^M - a^M)\left(1 - \frac{\rho a^2\omega^2}{\gamma} + \frac{\beta_1\theta_0/\gamma}{\log(a/b)}\right)}{Ma^M}\,. \tag{21}$$

Also, it has been analysed that at $r = b$, Eq.(19) yields a fully-plastic yielding stress as

$$|t_{rr} - t_{\theta\theta}|_{r=b} = \left|\frac{-pMb^M}{b^M - a^M} - \rho\omega^2 b^2 + \frac{\beta_1\theta_0}{\log(a/b)}\right| = \gamma_1\,, \tag{22}$$

and pressure necessary for fully-plastic state in piezoelectric material is given as

$$p_f = \frac{p}{\gamma_1} = \left(1 - \frac{\rho\omega^2 b^2}{\gamma_1}\right)\log\left(\frac{a}{b}\right)\,. \tag{23}$$

Now, the non-dimensional form of all the parameters is defined as

$$R = \frac{r}{b},\ R_0 = \frac{a}{b},\ \sigma_{r1} = \frac{t_{rr}}{\gamma},\ \sigma_{\theta 1} = \frac{t_{\theta\theta}}{\gamma},\ \sigma_{r2} = \frac{t_{rr}}{\gamma_1},\ \sigma_{\theta 2} = \frac{t_{\theta\theta}}{\gamma_1},$$
$$p_i = \frac{p}{\gamma},\ p_f = \frac{p}{\gamma_1},\ \Omega = \frac{\rho\omega^2 a^2}{\gamma},\ \Omega_1 = \frac{\rho\omega^2 b^2}{\gamma_1},\ \frac{\theta_0}{\gamma} = \vartheta\,. \tag{24}$$

The non-dimensional form of Eq.(21) becomes

$$p_i = \frac{\left(1 - \Omega - \frac{\beta_1\vartheta}{\log(R_0)}\right)(1 - R_0^M)}{MR_0^M}\,, \tag{25}$$

and of Eq.(23) as

$$p_f = (1 - \Omega_1)\log(R_0)\,. \tag{26}$$

From Eq.(18) the non-dimensional forms of transitional stresses are given as

$$\sigma_{r1} = \frac{t_{rr}}{\gamma} = \frac{p_i(R^M - 1)}{(1 - R_0^M)}\,, \quad \sigma_{\theta 1} = \frac{t_{\theta\theta}}{\gamma} = \Omega_2\frac{R^2}{R_0^2} + \frac{p_i((M+1)R^M - 1)}{1 - R_0^M} - \frac{\beta_1\vartheta}{\log(R_0)}(1 + \log R)\,. \tag{27}$$

When $M \to 0$, then non-dimensional form of Eqs.(27) is

$$\sigma_{r2} = \frac{-p_f \log R}{\log R_0}\,,$$
$$\sigma_{\theta 2} = \frac{-p_f(1 + \log R)}{\log R_0} + \Omega_2\frac{R^2}{R_0^2}\,. \tag{28}$$

NUMERICAL DISCUSSION

For finding the impact of pressure and temperature on transversely isotropic piezoelectric materials ($BaTiO_4$ and PZT-4) and transversely isotropic materials (magnesium and beryl) the Figs. 1 and 2 are drawn for pressure with different radii ratios. It is observed that pressure for initial yielding gradually decreases with increasing value of radii ratio. It has a maximum value at the inner surface of the disc. Also, the pressure value at inner surface of the disc increases with increasing value of temperature and angular velocity. It is also observed that pressure is maximal for transversely isotropic piezoelectric material $BaTiO_4$ and minimal for transversely isotropic beryl.

Figures 3 and 4 are drawn for circumferential stresses with radii ratios of different temperature values. It is noticed from Fig. 3 that circumferential stresses are tensile in nature and these stresses are maximal at internal surface of the disc with angular velocity Ω = 10 and temperature values 0.1, 0.3, 0.5, and with increase in angular velocity Ω = 20 the circumferential stresses show remarkable increase. These stresses are the highest in case of $BaTiO_4$ and the lowest in the case of beryl.

From Fig. 5 it is observed that fully-plastic stresses are tensile and maximal at inner surface of the disc. Also, these stresses have the highest value for angular velocity Ω = 30.

Figures 6 and 7 are drawn for angular velocity with different radii ratios, different pressure and temperature values. It is observed from Fig. 6 that angular velocity enhances with increasing ratios of radii and has maximum value at the outer surface of the disc and has the highest value for transversely isotropic piezoelectric $BaTiO_4$ from Fig. 7. It is seen that angular velocity shows remarkable increase with the increasing value of pressure.

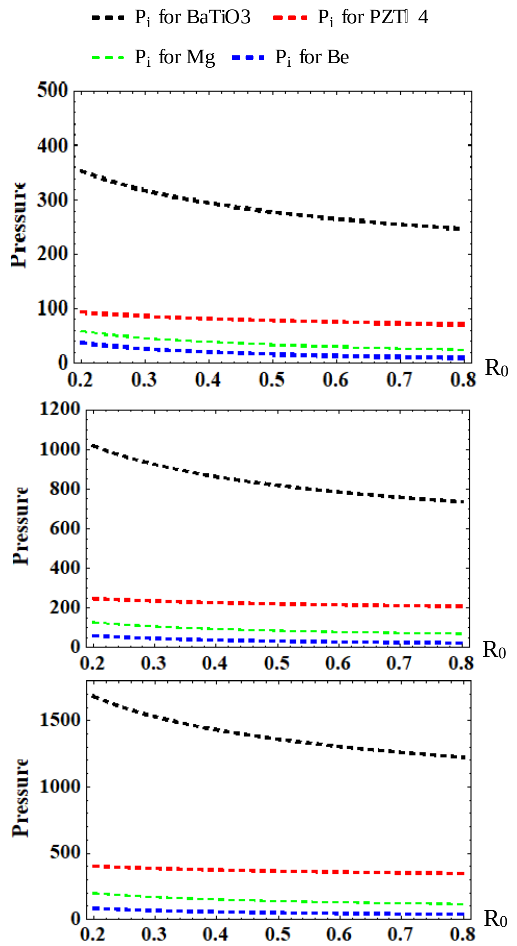
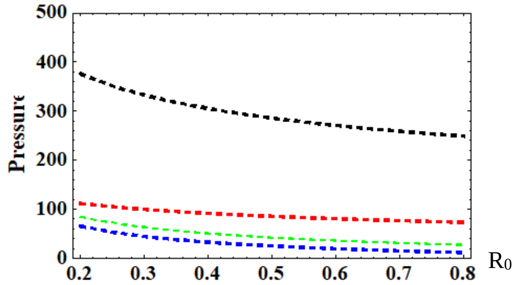


Figure 1. Pressure for initial yielding and fully-plastic state of transversely isotropic piezoelectric material and transv. Isotr. material with ang. velocity 10 and temp. 0.1, 0.3 and 0.5, respectively.



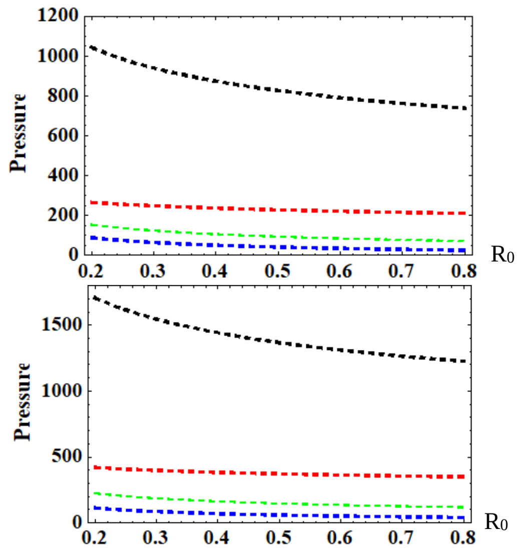


Figure 2. Pressure for initial yielding and fully-plastic state of transversely isotropic piezoelectric material and transversely isotropic material with angular velocity 20 and temperature 0.1, 0.3, and 0.5, respectively.

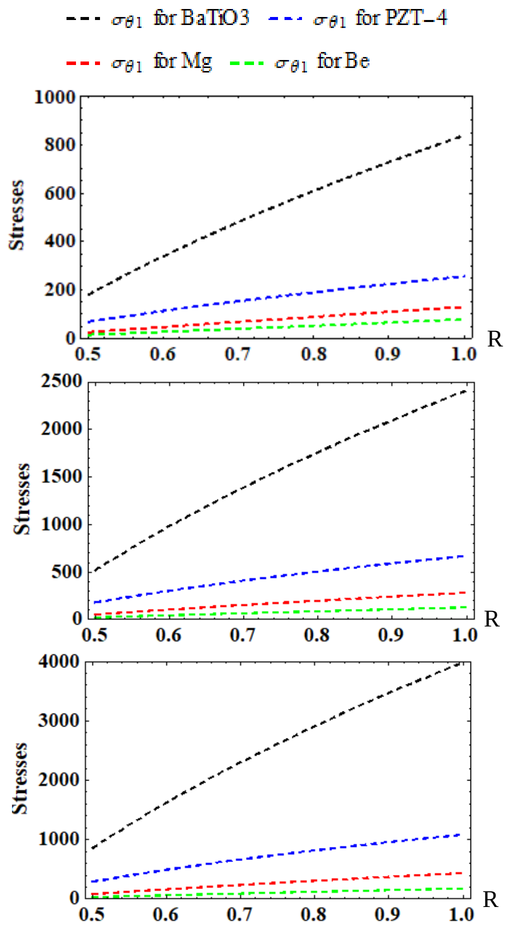


Figure 3. Circumferential stresses for piezoelectric and isotropic material steel with angular velocity 10 and temp. 0.1, 0.3, and 0.5, in respect.

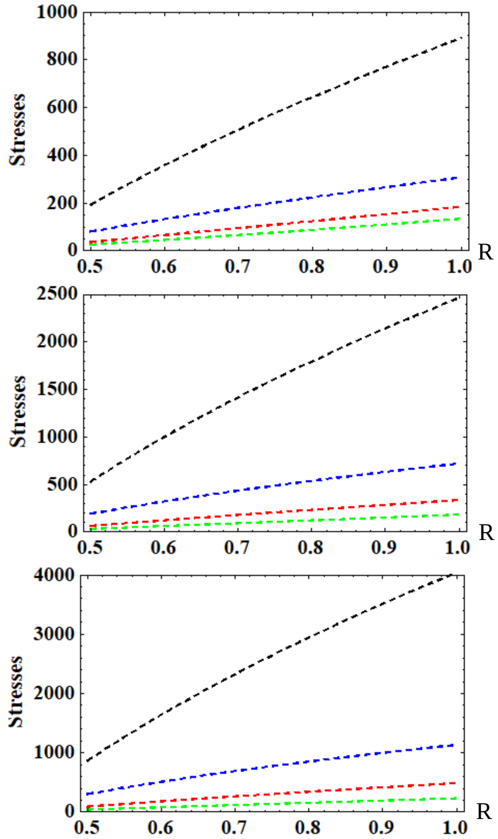


Figure 4. Circumferential stresses for piezoelectric and isotropic material steel with angular velocity 20 and temp. 0.1, 0.3, and 0.5, in respect.

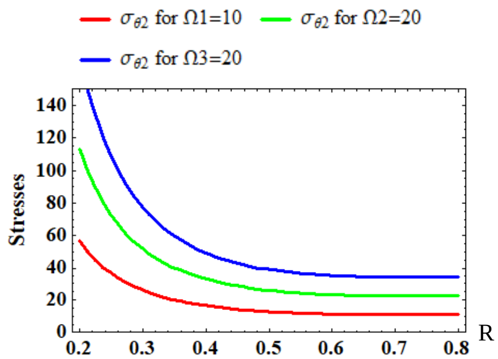
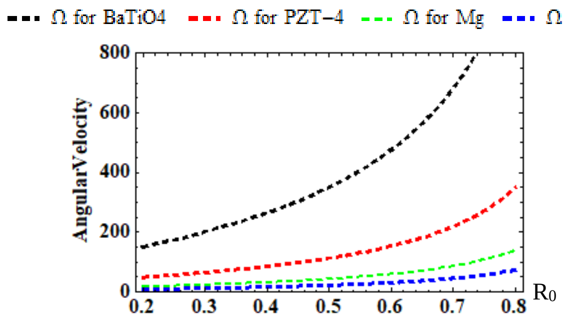


Figure 5. Fully-plastic stresses for piezoelectric and isotropic material steel with angular velocity 10, 20, and 30, in respect.



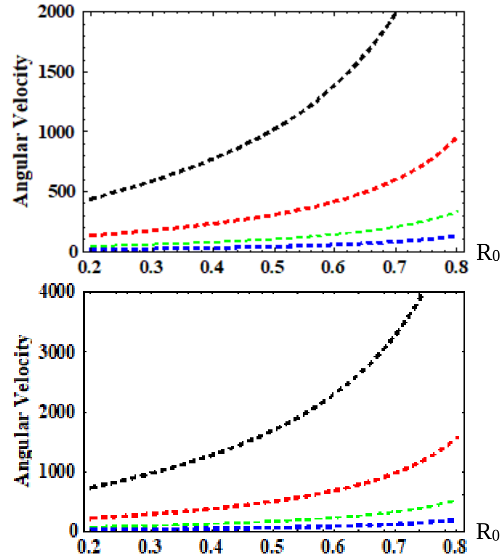


Figure 6. Angular velocity in initial yielding and fully-plastic state with pressure 10 and temperature 0.1, 0.3, and 0.5, respectively.

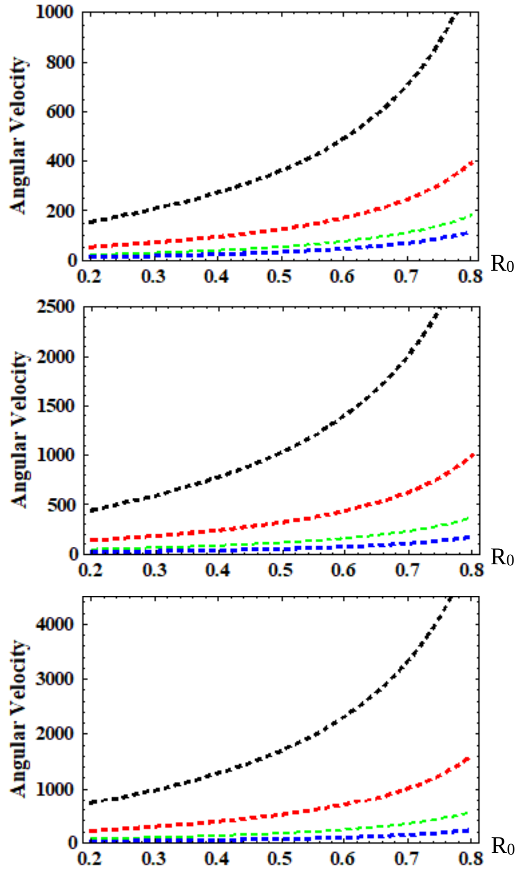


Figure 7. Angular velocity in initial yielding and fully plastic state with pressure 20 and temperature 0.1, 0.3, and 0.5, respectively.

CONCLUSION

On the basis of all graphs and numerical calculations it is concluded that the transversely isotropic material beryl is safe for designing of the transversely isotropic piezoelectric rotating disc. This is because of the reason that the pressure essential for initial yielding is less, also circumferential stresses are minimal for beryl in both transitional and fully plastic states. In case of other materials considered in this problem, the initial pressure and circumferential stresses in elastic and fully-plastic state are high. So, these materials are not safe for designing as compared to beryl.

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