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THERMAL STRESS DISTRIBUTION IN A TUBE OF NATURAL RUBBER /POLYURETHANE MATERIAL AND SUBJECTED TO INTERNAL PRESSURE AND MECHANICAL LOAD

RASPODELA TERMIČKIH NAPONA U CEVOVODU OD PRIRODNE GUME/POLIURETANA KOJI JE OPTEREĆEN MEHANIČKI I UNUTRAŠNJIM PRITISKOM

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Keywords
- tube
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Abstract

This article deals with the study of thermal stress distribution in a tube made of natural rubber/polyurethane material and subjected to internal pressure and mechanical load. From the obtained results, it is noticed that natural rubber material of the tube requires higher pressure to yield at the internal surface in comparison to tube made of polyurethane, for the initial yielding stage. Moreover, the tube of natural rubber material requires maximum hoop stress at the external surface as compared to the tube of polyurethane material. Values of the hoop/radial stress also increase with increasing temperature/mechanical loads in the contraction/extension region of the tube. The tube of natural rubber material is more comfortable than that of polyurethane.

Ključne reči
- cevovod
- opterećenje
- naponi
- pritisak
- temperatura

Izvod

U ovom radu proučava se raspodela termičkih napona u cevovodu od prirodne gume/poliuretana, koji je opterećen mehanički i unutrašnjim pritiskom. Prema dobijenim rezultatima, primećuje se da je u slučaju materijala prirodne gume cevovoda potreban veći pritisak za pojavu tečenja na unutrašnjoj površini, u poređenju sa cevovodom od poliuretana. Osim toga, za cevovod od prirodne gume je potreban i maksimalan obimski napon na spoljašnjoj površini u poređenju sa cevovodom o poliuretana. Vrednosti obimskog/radijalnog napona takođe rastu sa povećanjem temperature/mehaničkog opterećenja u delu cevovoda sa kontrakcijom/izduženjem. Cevovod od materijala prirodne gume je u prednosti u odnosu na cevovod od poliuretana.

INTRODUCTION

Stress analyses in thick-walled tubes have attracted a lot of interest due to their important applications in engineering, petrochemical industry, agricultural irrigation, chemical industry, urban construction, and electric power industry. For structural use in bridges, piling pipe, piers, roads, building structures, etc., and also body transport in gas, steam, liquefied petroleum gas, etc. The analytical solutions of stress distribution are given for idealised elasto-plastic by Timoshenko /1/ and work hardening by Chadwick /2/ for homogeneous materials. Bland /3/ has analysed the problem of thick-walled tubes subjected to uniform pressure and thermal gradient. Gamer et al. /4/ achieved the analytical solution of stress distribution in rotating tube by using Tresca's yield condition. Bree /5/ has discussed plastic stress deformation in a closed tube due to the interaction and thermal stresses. Mufit et al. /6/ have investigated the thermal stress distribution in a heat generating tube with yield stress by using Tresca's yield condition and its associated flow rule. Xin et al. /7/ studied elastic-plastic stress distribution in a functionally graded thick-walled tube subjected to internal pressure by using the assumption of a uniform strain field within the representative volume element and the Tresca yield criterion. Matvienko et al. /8, 9/ investigated elastoplastic deformation of dispersion-hardened aluminium tube under external and internal/external pressure. After that, Matvienko et al. /10/ examined mathematical modelling in a tube from dispersion-hardened aluminium alloy with inhomogeneous temperature field. Furthermore, Matvienko et al. /11/ studied thermal effect in a tube made of aluminium and subjected to internal pressure. Qian et al. /12/ developed mechanical properties of highly efficient heat exchange tubes. Gupta et al. /13/ has investigated the elasto-plastic stress distribution in a cylindrical tube made of steel/copper material and subjected to internal pressure and mechanical load by using transition theory and generalised strain measure.

OBJECTIVE

In this research, our aim is to calculate the safety factor for a thick-walled cylindrical tube made of natural rubber/polyurethane material and subjected to internal pressure and thermo-mechanical load. Mathematical modelling is based on stress-strain relation and equilibrium equation. The effects of different pertinent parameters (i.e., temperature gradient, load and pressure) are considered for tube of natural rubber/ polyurethane material. The behaviour of stress distribution, pressure and temperature are investigated. In this study, we discuss the effect of stress distribution, pressure and temperature gradient in the contraction and the extension region of a thick-walled tube with mechanical loads by using transition theory and generalised strain measure.

MATERIAL USED

Polyurethanes are a large class of polymers that can be tailored to a wide range of applications, making a significant contribution to the construction, automotive, and electrical sectors. It is more commonly known for paints and liquid coatings, but applications can also vary from soft, flexible foams to rigid insulation. This broad range of applications is possible as there are both thermoplastic and thermosetting polyurethanes. At room temperature, Poisson's ratio $\nu = 0.39$ for polyurethane is given, /14/.

Natural rubber is an elastic substance obtained from the latex sap of trees, especially those trees which belong to the genera hevea and ficus. Technically speaking, natural rubber is an elastomer, also known by the names of India rubber, gum elastic, and caoutchouc. At room temperature, Poisson's ratio $\nu = 0.5$ for natural rubber, /15/.

MATHEMATICAL MODEL

We consider a thick-walled cylindrical tube made of natural rubber/polyurethane material, with internal radius a and external radius b ($a < b$), subjected to uniform pressure p, respectively. Let a uniform temperature Θ_0 be applied at the inner surface of the tube. Further, if we assume that there are no body forces, body couples and couple stresses on the tube, and only a steady deformation problem is considered, as shown in Fig. 1.

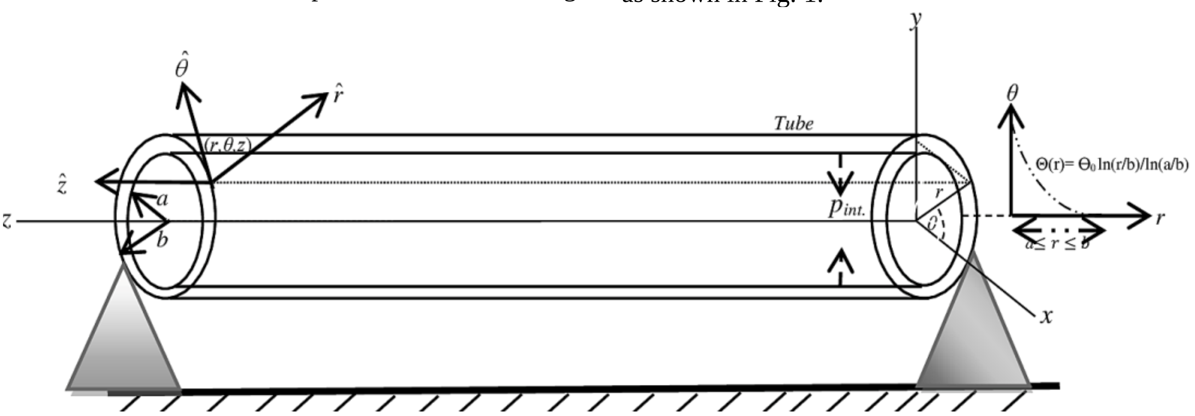


Figure 1. Geometrical configuration of tube made of polyurethane/natural rubber material.

BASIC GOVERNING EQUATION

The components of displacement in cylindrical polar coordinates (r, θ, z) are given /16/:

$$u = r(1-\eta),\quad v - 0,\quad w = dz\,, \tag{1}$$

where: η is a function of r. The Almansi strain components are given /17, 18/:

$$\varepsilon_{rr} = \frac{1}{2}\left[1-(r\eta'+\eta)^2\right],\quad \varepsilon_{\theta\theta} = \frac{1}{2}\left[1-\eta^2\right],$$

$$\varepsilon_{zz} = \frac{1}{2}\left[1-(1-d)^2\right],\quad \varepsilon_{r\theta} = \varepsilon_{\theta z} = \varepsilon_{zr} = 0\,. \tag{2}$$

Stress-strain relations for isotropic material are given /19/:

$$\tau_{ij} = \lambda\delta_{ij}I_1 + 2\mu\varepsilon_{ij} - \xi\Theta\delta_{ij}\,,\quad (i,j=1,2,3)\,, \tag{3}$$

where: $I_1 = \varepsilon_{kk}$ is the first strain invariant. Substituting Eq.(2) into Eq.(3), we get

$$\tau_{rr} = \lambda\left\{1-\frac{1}{2}\left[(r\eta'+\eta)^2+\eta^2\right]\right\} + \mu\left[1-(r\eta'+\eta)^2\right] - \xi\Theta\,,$$

$$\tau_{\theta\theta} = \lambda\left\{1-\frac{1}{2}\left[(r\eta'+\eta)^2+\eta^2\right]\right\} + \mu(1-\eta^2) - \xi\Theta\,,$$

$$\tau_{zz} = \lambda\left\{1-\frac{1}{2}\left[(r\eta'+\eta)^2+\eta^2\right]\right\} - \xi\Theta = 0\,,$$

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = 0\,. \tag{4}$$

The stress equation of equilibrium is given as:

$$\frac{\partial\tau_{rr}}{\partial r} + \frac{(\tau_{rr}-\tau_{\theta\theta})}{r} = 0\,. \tag{5}$$

The temperature field satisfying the equation is given by the Laplace equation: $\nabla^2\Theta = \frac{1}{r}\frac{d}{dr}\left(r\frac{d\Theta}{dr}\right) = 0$, and $\Theta = \Theta_0$ at $r = a$; $\Theta = 0$ at $r = b$, where Θ_0 is a constant. Solving this equation, we get:

$$\Theta = \bar{\Theta}_0\ln(r/b)\,, \tag{6}$$

where: $\bar{\Theta}_0 = \Theta_0/\ln(a/b)$.

Boundary condition: the boundary conditions of tube for the contraction/extension region are taken as:

$$\tau_{rr} = -p_{int.}\ \text{ at }\ r = a,\ \text{ and }\ \tau_{rr} = l_0\ \text{ at }\ r = b, \tag{7}$$

where: l_0 is mechanical load applied at the external surface of tube made of polyurethane/natural rubber material.

Asymptotic solution at the transition points: substituting Eq. (4) into Eq.(5) and after integration, we get the following nonlinear integro-differential equation:

$$(\lambda+2\mu)\left[\eta^2+(r\eta'+\eta^2)\right] + 2\mu\int r\eta'^2 dr - (2\lambda+2\mu) + 2\xi\Theta = A_0 \tag{8}$$

where: A_0 is constant of integration. Now, differentiating Eq.(8) with respect to r and using Eq.(6), we get:

$$\left[T^2+\left(2+\frac{1}{2}C\right)T+2\right]\frac{d\eta}{dT} + \eta(1+T) - \frac{C\xi\Theta_0}{\mu} = 0\,, \tag{9}$$

where: $r\eta' = \eta T$ (T is the function of η, and η is the function of r), and $C = 2\mu/(\lambda+2\mu) = (1-2\nu)/(1-\nu)$ be the compressibility factor, and ν is Poisson's ratio. Transition point T from Eq.(9) is $T \to -1$ and $T \to \pm\infty$. The transition point $T \to -1$ corresponds to extension region, and $T \to \pm\infty$ corresponds to contraction region. The resultant force normal to the plane z = const. vanishes, i.e., $\int_a^b r\tau_{zz}dr = 0$. The deformation of the tube walls is determined by the magnitude of the applied pressure. If mechanical load is increased, stresses in the wall of the tube increase.

ANALYTICAL SOLUTION OF THE PROBLEM

(a) *Contraction in the tube*: transition point $T \to \pm\infty$, corresponds to contraction in the tube /13, 17, 18, 20-32/, we define the transition function Γ as:

$$\Gamma \cong (2-C) - \frac{C}{\mu}\left[\tau_{rr} + C\xi\Theta\right]. \tag{10}$$

Taking the logarithmic differentiation of Eq.(10) with respect to r and using Eq.(9), and after that by taking the asymptotic solution $T \to \pm\infty$ and integrating, we get:

$$\Gamma = Ar^{-C}, \tag{11}$$

where: A is a constant of integration. Comparing Eq.(10) and Eq.(11), we get

$$\tau_{rr} = \frac{\mu}{C}\left[(2-C) - Ar^{-C}\right] - c\xi\Theta_0\frac{\ln(b/r)}{\ln(b/a)}. \tag{12}$$

Yielding stress in tension is given /14/: $Y = \mu(1+\nu) = (3-2C)\mu/(2-C)$. Now substituting the value of yielding stress condition in Eq.(12), we get:

$$\tau_{rr} = \frac{(2-C)Y}{C(3-2C)}\left[(2-C) - Ar^{-C}\right] - c\xi\Theta_0\frac{\ln(b/r)}{\ln(b/a)}. \tag{13}$$

Using second boundary condition in Eq.(7) into Eq.(13), we get $A = (2-C)b^C - \frac{l_0 C(3-2C)b^C}{Y(2-C)}$. Further, Eq.(13) becomes:

$$\tau_{rr} = \frac{(2-C)^2 Y}{C(3-2C)}\left[1-\left(\frac{b}{r}\right)^C\right] + l_0\left(\frac{b}{r}\right)^C - \alpha E(2-C)\Theta_0\frac{\ln(b/r)}{\ln(b/a)} \tag{14}$$

where: $C\xi = \alpha E(2-C)$, and $E = 2\mu(3-2C)/(2-C)$. Now using first boundary condition in Eq.(7) into Eq.(14), we get

$$p_{\text{int.}} = \frac{(2-C)^2 Y}{C(3-2C)}\left[(a/b)^{-C} - 1\right] - l_0(a/b)^{-C} + \alpha E(2-C)\Theta_0$$

$$\Rightarrow \frac{(2-C)^2 Y}{C(3-2C)} = \frac{p_{\text{int.}} + l_0(a/b)^{-C}}{\left\{(a/b)^{-C} - 1\right\}} - \frac{\alpha E(2-C)\Theta_0}{\left\{(a/b)^{-C} - 1\right\}}. \tag{15}$$

Substituting Eq.(15) into Eq.(14) and using Eq.(7), we get stress in the contraction region:

$$\tau_{rr} = \frac{p_{\text{int.}} + l_0(a/b)^{-C}}{\left[(a/b)^{-C} - 1\right]}\left[1-\left(\frac{b}{r}\right)^C\right] + l_0\left(\frac{b}{r}\right)^C - \alpha E(2-C)\times$$
$$\times\Theta_0\left[\frac{1-(b/r)^C}{(a/b)^{-C}-1} + \frac{\ln(b/r)}{\ln(b/a)}\right], \tag{16}$$

$$\tau_{\theta\theta} = \frac{p_{\text{int.}} + l_0(a/b)^{-C}}{\left[(a/b)^{-C} - 1\right]}\left[1-(1-C)\left(\frac{b}{r}\right)^C\right] + l_0(1-C)\left(\frac{b}{r}\right)^C +$$
$$+\alpha E(2-C)\Theta_0\left[\frac{1-(b/r)^C}{(a/b)^{-C}-1} + \frac{1-\ln(b/r)}{\ln(b/a)}\right]. \tag{17}$$

From Eq.(16) and Eq.(17), we get:

$$\tau_{\theta\theta} - \tau_{rr} = \frac{(p_{\text{int.}} + l_0)C(b/r)^C}{(a/b)^{-C} - 1} + \alpha E(2-C)\Theta_0\times$$
$$\times\left[\frac{2-(b/r)^C(2+C)}{(a/b)^{-C}-1} + \frac{1}{\ln(b/a)}\right]. \tag{18}$$

Initial yielding stage: from Eq.(18) it has been seen that $|\tau_{\theta\theta} - \tau_{rr}|$ are maximum at the inner surface (i.e., $r = a$), therefore yielding will take place at the outer surface of the tube and Eq.(18) becomes:

$$\left|\tau_{\theta\theta} - \tau_{rr}\right|_{r=a} = \frac{(p_{\text{int.}} + l_0)C(b/a)^C}{(a/b)^{-C} - 1} + \alpha E(2-C)\Theta_0\times$$
$$\times\left[\frac{2-(b/a)^C(2+C)}{(a/b)^{-C}-1} + \frac{1}{\ln(b/a)}\right] = Y,$$

where: Y is the yielding stress in the contraction region. The pressure required for initial yielding is given by:

$$P_i = \frac{p_{\text{int.}}}{Y} = \left|\frac{(a/b)^{-C} - 1}{C(b/a)^C}\right|\left(1 - \left|\alpha E(2-C)\Theta_0\left[\frac{2-(b/a)^C(2+C)}{(a/b)^{-C}-1} + \right.\right.\right.$$
$$\left.\left.\left. + \frac{1}{\ln(b/a)}\right]\right|\right) - \left|\frac{l_0}{Y}\right|. \tag{19}$$

Equations (16), (17), and (19), in non-dimensional form become:

$$\sigma_r = \frac{(P_i + L_0 R_0^{-C})(1-R^{-C})}{R_0^{-C} - 1} + L_0 R^{-C} - \Theta_1(2-C)\times$$
$$\times\left[\frac{1-R^{-C}}{R_0^{-C}-1} + \frac{\ln R}{\ln R_0}\right],$$

$$\sigma_\theta = \frac{(P_i + L_0 R_0^{-C})\left[1-(1-C)R^{-C}\right]}{R_0^{-C} - 1} + L_0(1-C)R^{-C} - \Theta_1(2-C)\times$$
$$\times\left[\frac{1-R^{-C}(1+C)}{R_0^{-C}-1} - \frac{1+\ln R}{\ln R_0}\right],$$

$$P_i = \left|\frac{R_0^{-C} - 1}{R_0^{-C} C}\right|\left|1 - \Theta_1(2-C)\left[\frac{2-R_0^{-C}(2+C)}{R_0^{-C}-1} - \frac{1}{\ln R_0}\right]\right| - \left|L_0\right|,$$
$$\forall C \neq 0, \tag{20}$$

where: $R = r/b$, $\sigma_r = \tau_{rr}/Y$, $\sigma_\theta = \tau_{\theta\theta}/Y$, $L_0 = l_0/Y$, $R_0 = a/b$, $\Theta_1 = \alpha E\Theta_0/Y$, $R_0 = a/b$, and $P_i = p_{\text{int.}}/Y$. When $C = 0$, Eq.(20) becomes:

$$\sigma_r = -(P_i + L_0)\frac{\ln R}{\ln R_0} - \frac{4\Theta_1\ln R}{\ln R_0} + L_0,$$

$$\sigma_\theta = -(P_i + L_0)\left(\frac{\ln R + 1}{\ln R_0}\right) - \frac{4\Theta_1(1+\ln R)}{\ln R_0} + L_0,$$

$$P_i = \left|\ln R_0\right| - \left|4\Theta_1\ln R_0\right| - \left|L_0\right|. \tag{21}$$

(b) *Extension in the tube*: the transition point $T \to -1$ corresponds to contraction in the tube /13, 17, 18, 20-32/. We define the transition function Γ as:

$$\Gamma \cong (2-C) - \frac{C}{\mu}[\tau_{rr} + C\xi\Theta]. \tag{22}$$

Taking the logarithmic differentiation of Eq.(22) with respect to r and using Eq.(7), and after that by taking the asymptotic solution $T \to \pm\infty$ and integrating, we get:

$$\Gamma = Br^{C/(1-C)}, \tag{23}$$

where: B is a constant of integration. Comparing Eq.(22) and Eq.(23), we get

$$\tau_{rr} = \frac{\mu}{C}\left[(2-C) - Br^{C/(1-C)}\right] - c\xi\Theta_0 \frac{\ln(b/r)}{\ln(b/a)}. \tag{24}$$

The yielding stress in tension is given /14/: $Y = \mu(1+\nu) = (3-2C)\mu/(2-C)$. Now substituting the value of yielding stress condition in Eq.(24), we get

$$\tau_{rr} = \frac{(2-C)Y}{C(3-2C)}\left[(2-C) - Ar^{C/(1-C)}\right] - c\xi\Theta_0 \frac{\ln(b/r)}{\ln(b/a)}. \tag{25}$$

Using second boundary condition in Eq.(7) into Eq.(25), we get $B = (2-C)b^{-C/(1-C)} - \dfrac{l_0 C(3-2C)b^{-C/(1-C)}}{Y(2-C)}$. Further, Eq.(25) becomes:

$$\tau_{rr} = \frac{(2-C)^2 Y}{C(3-2C)}\left[1 - \left(\frac{r}{b}\right)^{C/(1-C)}\right] + l_0\left(\frac{r}{b}\right)^{C/(1-C)} - \alpha E(2-C)\Theta_0 \frac{\ln(b/r)}{\ln(b/a)}. \tag{26}$$

Now using first boundary condition in Eq.(7) into Eq.(26), we get

$$p_{\text{int.}} = \frac{(2-C)^2 Y}{C(3-2C)}\left[(a/b)^{C/(1-C)} - 1\right] - l_0 (a/b)^{C/(1-C)} + \alpha E(2-C)\Theta_0$$

$$\Rightarrow \frac{(2-C)^2 Y}{C(3-2C)} = \frac{p_{\text{int.}} + l_0 (a/b)^{C/(1-C)}}{(a/b)^{C/(1-C)} - 1} - \frac{\alpha E(2-C)\Theta_0}{(a/b)^{C/(1-C)} - 1}. \tag{27}$$

Substituting Eq.(27) into Eq.(26) and using Eq.(5), we get stress in the contraction region:

$$\tau_{rr} = \frac{p_{\text{int.}} + l_0 (a/b)^{C/(1-C)}}{(a/b)^{C/(1-C)} - 1}\left[1 - \left(\frac{r}{b}\right)^{C/(1-C)}\right] + l_0\left(\frac{r}{b}\right)^{C/(1-C)} - \alpha E(2-C)\Theta_0\left[\frac{1-(r/b)^{C/(1-C)}}{(a/b)^{C/(1-C)} - 1} + \frac{\ln(b/r)}{\ln(b/a)}\right], \tag{28}$$

$$\tau_{\theta\theta} = \frac{p_{\text{int.}} + l_0 (a/b)^{C/(1-C)}}{(a/b)^{C/(1-C)} - 1}\left[1 - \frac{1}{1-C}\left(\frac{r}{b}\right)^{C/(1-C)}\right] + \frac{l_0}{1-C} \times \left(\frac{r}{b}\right)^{C/(1-C)} - \alpha E(2-C)\Theta_0\left[\frac{1-(r/b)^{C/(1-C)}}{(a/b)^{C/(1-C)} - 1} + \frac{1-\ln(b/r)}{\ln(b/a)}\right]. \tag{29}$$

From Eq.(28) and Eq.(29), we get:

$$\tau_{\theta\theta} - \tau_{rr} = \frac{(p_{\text{int.}} + l_0)C(r/b)^{C/(1-C)}}{(1-C)\left\{1-(a/b)^{C/(1-C)}\right\}} + \alpha E(2-C)\Theta_0 \times \left[\frac{(r/b)^{C/(1-C)}(2-C) - (2-2C)}{(1-C)\left\{(a/b)^{C/(1-C)} - 1\right\}} + \frac{1}{\ln(b/a)}\right]. \tag{30}$$

Initial yielding stage: from Eq.(30) it has been seen that $|\tau_{\theta\theta} - \tau_{rr}|$ are maximum at the outer surface (i.e., $r = b$), therefore yielding will take place at the outer surface of the tube in the extension region and Eq.(30) becomes:

$$\left|\tau_{\theta\theta} - \tau_{rr}\right|_{r=b} = \frac{(p_{\text{int.}} + l_0)C}{(1-C)\left\{1-(a/b)^{C/(1-C)}\right\}} + \alpha E(2-C)\Theta_0 \times \left[\frac{C}{(1-C)\left\{(a/b)^{C/(1-C)} - 1\right\}} + \frac{1}{\ln(b/a)}\right] = Y^*,$$

where: Y^* is the yielding stress in the extension region. The pressure required for the initial yielding is given by:

$$P_i = \frac{p_{\text{int.}}}{Y^*} = \left|\frac{\left\{1-(a/b)^{C/(1-C)}\right\}(1-C)}{C}\left(1 - \alpha E(2-C)\Theta_0 \times \left[\frac{C}{(1-C)\left\{(a/b)^{C/(1-C)} - 1\right\}} + \frac{1}{\ln(b/a)}\right]\right)\right| - \left|\frac{l_0}{Y^*}\right|. \tag{31}$$

Equations (28), (29), and (31), in non-dimensional form become:

$$\sigma_r^* = \frac{P_i + L_0 R_0^{C/(1-C)}}{R_0^{C/(1-C)} - 1}\left[1 - R^{C/(1-C)}\right] + \Theta_1(2-C) \times \left[\frac{1-R^{C/(1-C)}}{R_0^{C/(1-C)} - 1} - \frac{\ln R}{\ln R_0}\right] + L_0 R^{C/(1-C)},$$

$$\sigma_\theta^* = \frac{P_i + L_0 R_0^{C/(1-C)}}{R_0^{C/(1-C)} - 1}\left[1 - \left(\frac{1}{1-C}\right)R^{C/1-C}\right] + \frac{L_0 R^{C/1-C}}{1-C} - \Theta_1(2-C)\left[\frac{1-R^{C/(1-C)}\left(\frac{1}{1-C}\right)}{R_0^{C/(1-C)} - 1} + \frac{1+\ln R}{\ln R_0}\right],$$

and

$$P_i^* = \left|\frac{\left\{1-R_0^{C/(1-C)}\right\}(1-C)}{C}\left(1 - \Theta_1(2-C) \times \left[\frac{C}{(1-C)\left\{R_0^{C/(1-C)} - 1\right\}} - \frac{1}{\ln R_0}\right]\right)\right| - |L_0|, \quad \forall C \neq 0, \tag{32}$$

when $C = 0$, Eq.(32) becomes:

$$\sigma_r^* = -(P_i + L_0)\left(\frac{\ln R}{\ln R_0}\right) - \frac{4\Theta_1 \ln R}{\ln R_0} + L_0,$$

$$\sigma_\theta^* = -(P_i + L_0)\left(\frac{\ln R + 1}{\ln R_0}\right) - \frac{4\Theta_1(1+\ln R)}{\ln R_0} + L_0,$$

$$P_i^* = |\ln R_0| - |4\Theta_1 \ln R_0| - |L_0|. \tag{33}$$

VALIDATION OF RESULTS

By taking $\Theta_1 \to 0$ into Eqs.(20)-(21) and Eqs.(32)-(33), we get:

$$\sigma_r = \frac{(P_i + L_0 R_0^{-C})(1-R^{-C})}{R_0^{-C} - 1} + L_0 R^{-C},$$

$$\sigma_\theta = \frac{P_i + L_0 R_0^{-C}}{R_0^{-C} - 1}\left[1 - (1-C)R^{-C}\right] + L_0(1-C)R^{-C},$$

$$P_i = \left|\frac{R_0^{-C} - 1}{R_0^{-C} C}\right| - |L_0|, \quad \forall C \neq 0$$

and

$$\sigma_r = -(P_i + L_0)\frac{\ln R}{\ln R_0} + L_0,$$

$$\sigma_\theta = -(P_i + L_0)\left(\frac{\ln R + 1}{\ln R_0}\right) + L_0,$$

$$P_i = |\ln R_0| - |L_0|, \quad \forall C = 0, \tag{34}$$

in the contraction region.

$$\sigma_r^* = \frac{P_i + L_0 R_0^{C/(1-C)}}{R_0^{C/(1-C)} - 1}\left[1 - R^{C/(1-C)}\right] + L_0 R^{C/(1-C)},$$

$$\sigma_\theta^* = \frac{P_i + L_0 R_0^{C/(1-C)}}{R_0^{C/(1-C)} - 1}\left[1 - \left(\frac{1}{1-C}\right) R^{C/(1-C)}\right] + \frac{L_0 R^{C/(1-C)}}{1-C}$$

$$P_i^* = \left|\frac{\left\{1 - R_0^{C/(1-C)}\right\}(1-C)}{C}\right| - |L_0|, \quad \forall C \neq 0$$

and

$$\sigma_r^* = -(P_i + L_0)\frac{\ln R}{\ln R_0} + L_0,$$

$$\sigma_\theta^* = -(P_i + L_0)\left(\frac{\ln R + 1}{\ln R_0}\right) + L_0,$$

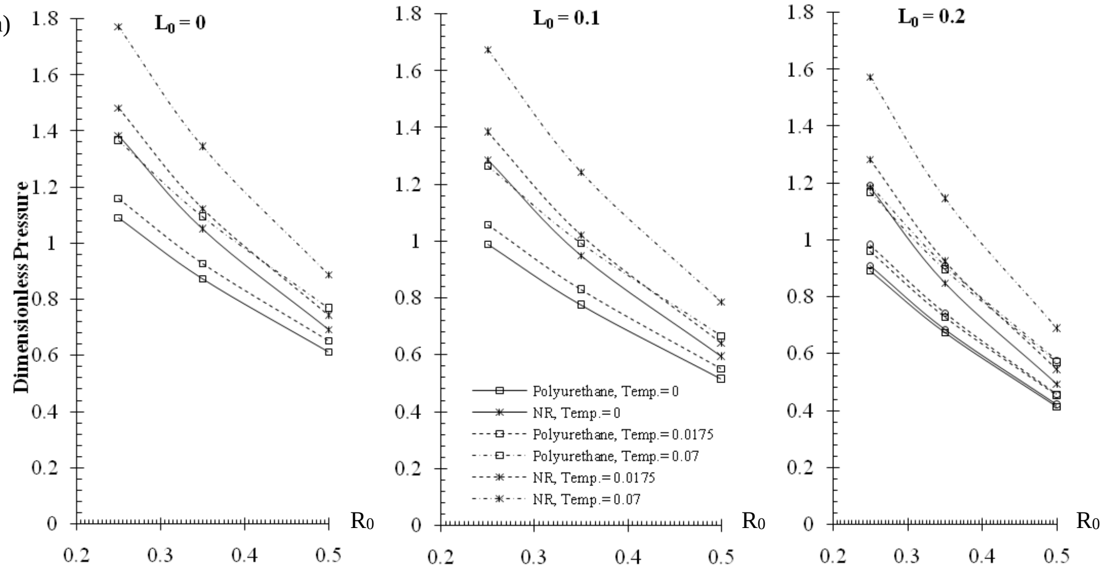
$$P_i^* = |\ln R_0| - |L_0|, \quad \forall C = 0, \tag{35}$$

in the extension region. The present results obtained from Eqs.(34)-(35) are the same as given by Gupta et al. /13/ in the contraction/extension region. Therefore, the present results are correct and authenticate the validity of the derived solutions. In the present study, we discuss the new addition in thermal condition at the inner surface of the tube.

NUMERICAL RESULTS AND DISCUSSION

To see the combined effect of stress distribution and pressure in a cylindrical tube made of polyurethane, (say C = 0.361 or ν = 0.39) /14/, and natural rubber (NR) (say ν = 0.5) /15/, based upon the following numerical values has been taken: L_0 = 0, 0.1, 0.2; Θ_1 = 0, 0.0175, 0.07; a = 1 and b = 2, respectively. In Fig. 2, curves are drawn between dimensionless pressure required for initial yielding stage versus radii ratio $R_0 = a/b$ for the contraction/extension region and having mechanical loads L_0 = 0, 0.1, 0.2 and temperature Θ_1 = 0, 0.0175, respectively. It is observed that the tube made of natural rubber material requires higher pressure to yield at the internal surface as compared to the tube made of polyurethane, for the initial yielding stage. Further, the value of pressure increases with increasing temperature Θ_1 = 0.0175 and decreases with increasing mechanical load (say L_0 = 0.1 and 0.2) at the internal surface of a tube made of natural rubber and also of polyurethane material for the initial stage. Moreover, the thermomechanical loaded tube made of polyurethane material requires higher pressure in the contraction region as compared to the extension region at the initial yielding stage. Figure 3 portrays in order to demonstrate the behaviour of dimensionless stress distribution versus radii ratio $R = r/b$ in the contraction/extension region and having mechanical loads (i.e., L_0 = 0, 0.1, 0.2) and temperature Θ_1 = 0, 0.0175, respectively. It is observed that the tube of natural rubber material requires maximum hoop stress at the external surface in comparison to tube of polyurethane material. Further, the values of the hoop/radial stress also increase with increasing temperature Θ_1 = 0.175 as well as the mechanical load L_0 = 0.1, 0.2 in the contraction/extension region of the tube. Moreover, polyurethane material tube requires maximum hoop stress at the external surface in the extension region as compared to the contraction region.

(a)



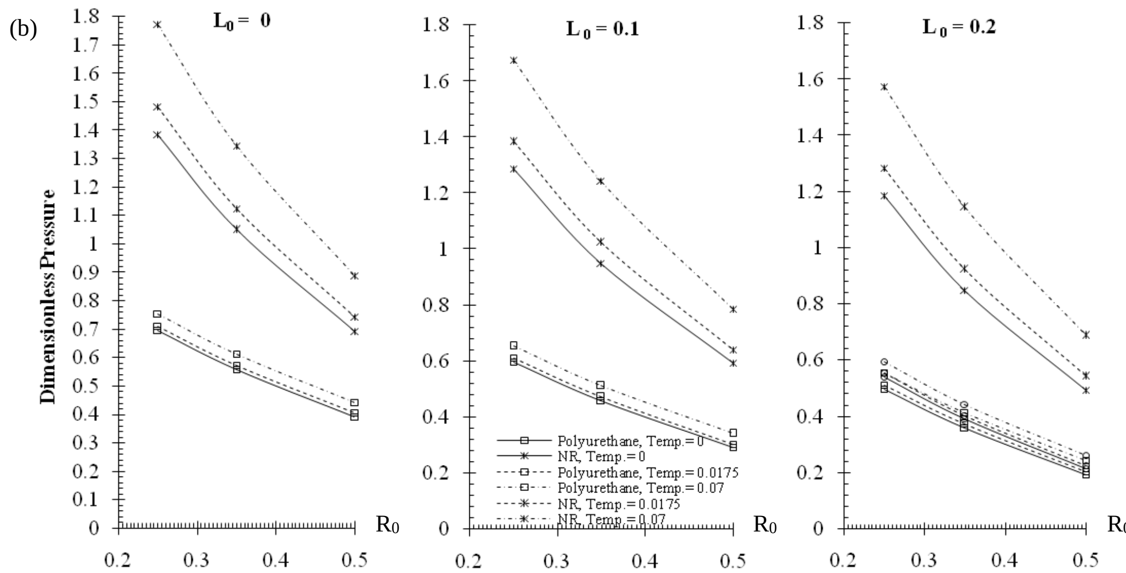


Figure 2. Graphical comparisons between dimensionless pressure required for initial yielding stage vs. radii ratio $R_0 = a/b$ in the (a) contraction region; (b) extension region.

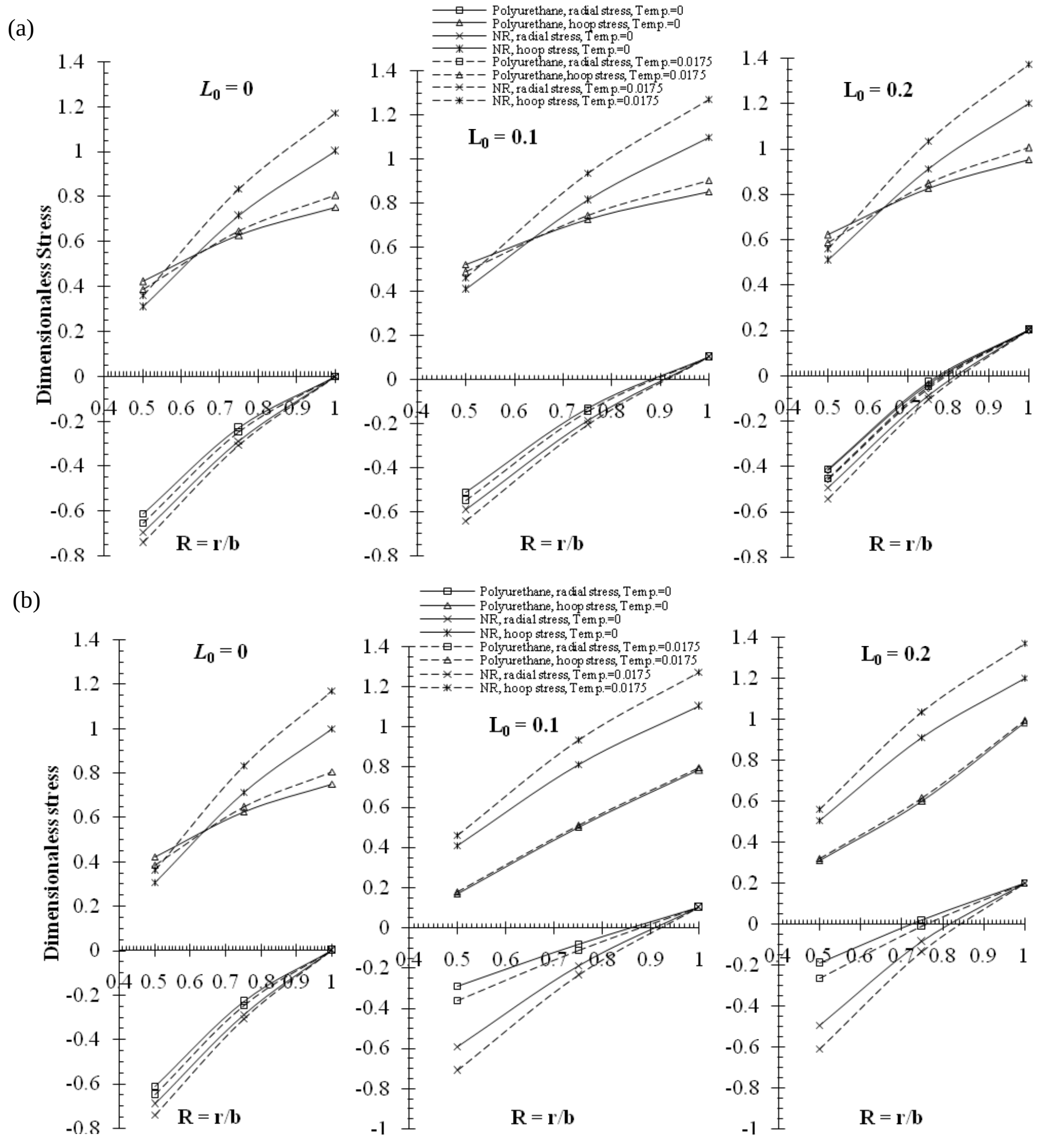


Figure 3. Dimensionless stress distribution vs. radii ratio $R = r/b$ for initial yielding stage with: (a) contraction; (b) extension region.

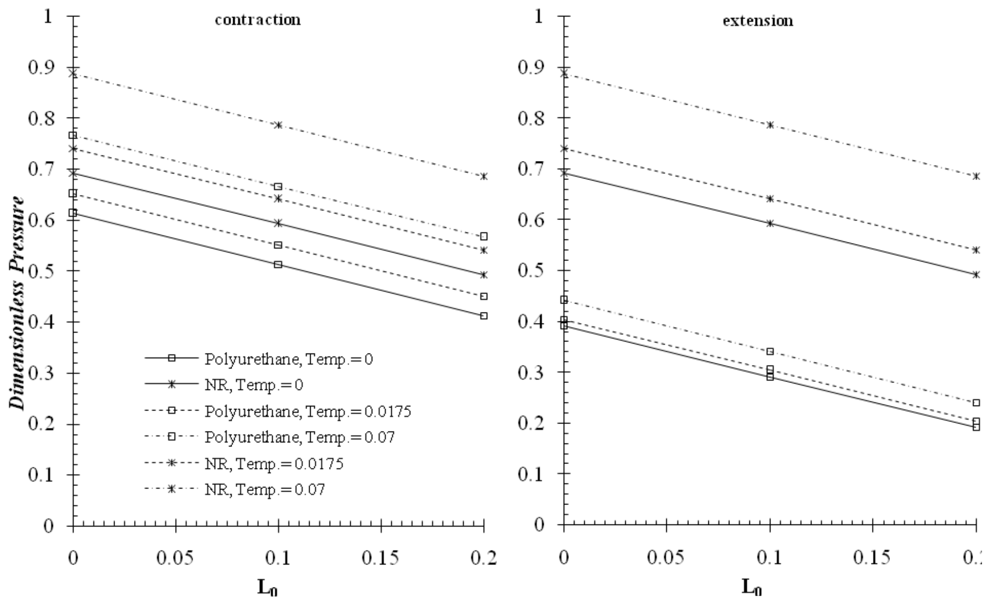


Figure 4. Graphical comparisons between dimensionless pressure vs. mechanical load at R_0 = 0.5 for initial yielding stage along the contraction/extension region.

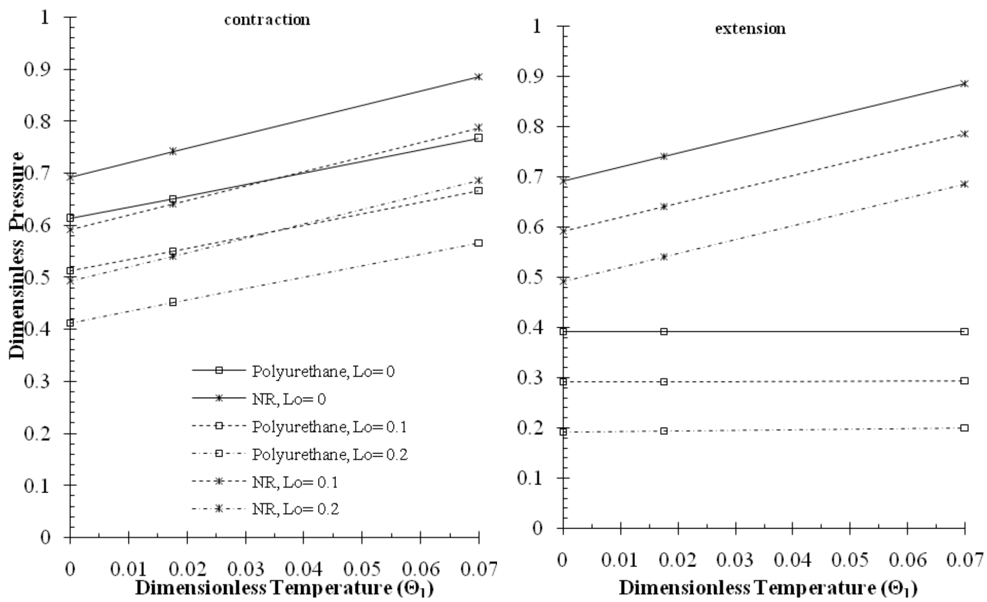


Figure 5. Graphical comparisons between dimensionless pressure vs. temperature for initial yielding stage at R_0 = 0.5 along the contraction/extension region.

Figure 4 is prepared to illustrate the behaviour of dimensionless pressure versus mechanical load and having temperature Θ_1 = 0, 0.0175, for the initial yielding stage at R_0 = 0.5. It is shown that in the contraction/extension region, the value of pressure decreases with increasing mechanical load (say L_0 = 0.1, 0.2), and increases with increasing temperature Θ_1 = 0.175, for the initial yielding stage. With increasing mechanical load (say L_0 = 0.1, 0.2), the tube of natural rubber requires maximum pressure in comparison to tube made of polyurethane material. Figure 5 demonstrates the behaviour of dimensionless pressure vs. temperature with mechanical load L_0 = 0, 0.1, 0.2 for the initial yielding stage at R_0 = 0.5. In the contraction region, the value of pressure increases with increasing temperature (say Θ_1 = 0, 0.0175, 0.07) for the initial yielding stage. Furthermore, for a tube of polyurethane material, the value of pressure neither increases nor decreases at load L_0 = 0. With the introduction of mechanical load (say L_0 = 0.1, 0.2), the values of pressure decrease in the contraction/extension region.

CONCLUSIONS

The main findings can be concluded as follows:

- The natural rubber material tube requires higher dimensionless pressure to yield at the internal surface in comparison to the tube made of polyurethane, for the initial yielding stage.
- The value of pressure increases with increasing temperature and decreases with increasing mechanical load applied at the internal surface of a tube made of natural rubber/ polyurethane material for the initial stage.
- The natural rubber tube requires maximum hoop stress at the external surface as compared to the polyurethane tube.

– Values of hoop/radial stress also increase with increasing temperature/mechanical loads in the contraction/extension region of the tube.
– In the contraction/extension region, the value of pressure decreases with increasing mechanical loads, and increases with increasing temperature for the initial yielding stage.
– The natural rubber tube is more comfortable than that of polyurethane.

REFERENCES

1. Timoshenko, S.P., Goodier, J.N., Theory of Elasticity, Third Ed., Mc Graw-Hill Book Co. New York, London, 1951.
2. Chadwick, P. (1963), *Compression of a spherical shell of work-hardening material*, Int. J. Mech. Sci. 5(2): 165-182. doi: 10.1016/0020-7403(63)90020-1
3. Bland, D.R. (1956), *Elastoplastic thick-walled tubes of work-hardening material subject to internal and external pressures and to temperature gradient*s, J. Mech. Phys. Solids, 4(4): 209-229. doi: 10.1016/0022-5096(56)90030-8
4. Gamer, U., Lance, R.H. (1983), *Stress distribution in a rotating elastic-plastic tube*, Acta Mechanica, 50: 1-8. doi: 10.1007/BF01170437
5. Bree, J. (1989), *Plastic deformation of a closed tube due to interaction of pressure stresses and cyclic thermal stresses*, Int. J Mech. Sci. 31(11-12): 865-892. doi: 10.1016/0020-7403(89)90030-1
6. Gulgec, M., Orcan, Y. (2000), *Elastic-plastic deformation of a heat generating tube with temperature dependent yield stress*, Int. J. Eng. Sci. 38(1): 89-106. doi: 10.1016/S0020-7225(99)00014-2
7. Xin, L., Dui G, Yang, S.Y., Liu, Y. (2016), *Elastic-plastic analysis for functionally graded thick-walled tube subjected to internal pressure*, Adv. Appl. Math. Mech. 8(2): 331-352. doi: 10.4208/aamm.2014.m841
8. Matvienko, O.V., Daneyko, O.I., Kovalevskaya, T.A. (2018), *Elastoplastic deformation of dispersion-hardened aluminum tube under external pressure*, Russ. Phys. J. 61: 1520-1528. doi: 10.1007/s11182-018-1565-5
9. Matvienko, O.V., Daneyko, O.I., Kovalevskaya, T.A. (2019), *Elastoplastic deformation of dispersion-hardened aluminum tube under external and internal pressure*, Russ. Phys. J. 62: 720-728. doi: 10.1007/s11182-019-01769-x
10. Matvienko, O.V., Daneyko. O.I., Kovalevskaya, T.A. (2020), *Mathematical modeling of plastic deformation of a tube from dispersion-hardened aluminum alloy in an inhomogeneous temperature field*, Crystals, 10(12): 1103. doi: 10.3390/cryst10121103
11. Matvienko, O.V., Daneyko O.I., Kovalevskaya. T.A. (2021), *Stress-strain state of a tube of heterophase alloy subjected to internal pressure in an inhomogeneous temperature field*, Bull. Russ. Acad. Sci. Phys., 85: 791-797. doi: 10.3103/S1062873821070157
12. Qian, C., Wu, Z., Wen, S., et al. (2020), *Study of the mechanical properties of highly efficient heat exchange tubes*, Materials, 13(2): 382. doi: 10.3390/ma13020382
13. Gupta, K., Thakur, P., Bhardwaj, R.K. (2022), *Elasto-plastic stress analysis in a tube made of isotropic material and subjected to pressure and mechanical load*, Mech. Solids, 57(3): 617-628. doi: 10.3103/S002565442203013X
14. Tsukinovsky, D., Zaretsky, E., Rutkevich, I. (1997), *Material behavior in plane polyurethane-polyurethane impact with velocities from 10 to 400 m/sec*, Journal de Physique IV Proc. EDP Sciences 07 (C3): C3-335-C3-339. doi: 10.1051/jp4:1997359
15. https://polymerdatabase.com/polymer%20physics/Poisson%20Table.html
16. Seth, B.R. (1962), *Transition theory of elastic-plastic deformation, creep and relaxation*, Nature, 195: 896-897. doi: 10.1038/195896a0
17. Seth, B.R. (1963), *Elastic-plastic transition in shells and tubes under pressure*, ZAMM J Appl. Math. Mech. 43(7-8): 345-351. doi: 10.1002/zamm.19630430706
18. Seth, B.R. (1970), *Transition conditions: The yield condition*, Int. J Non-Liner Mech. 5(2): 279-285. doi: 10.1016/0020-7462(70)90025-9
19. Sokolnikoff, I.S., Mathematical Theory of Elasticity, 2nd Ed., McGraw-Hill Inc., New York, 1953.
20. Sethi, M., Thakur, P., Singh, H.P. (2019), *Characterization of material in a rotating disc subjected to thermal gradient by using Seth transition theory*, Struct. Integ. Life, 19(3): 151-156.
21. Thakur, P., Shahi, S., Singh, S.B., Sethi, M. (2019), *Elastic-plastic stress concentrations in orthotropic composite spherical shells subjected to internal pressure*, Struct. Integ. Life, 19(2): 73-77.
22. Thakur, P., Sethi, M. (2019), *Lebesgue measure in an elasto-plastic shell*, Struct. Integ. Life, 19(2): 115-120.
23. Temesgen, A.G., Singh, S.B., Thakur, P. (2020), *Modelling of elastoplastic deformation of transversely isotropic rotating disc of variable density with shaft under a radial temperature gradient*, Struct. Integ. and Life, 20(2): 113-121.
24. Thakur, P., Kumar N., Sukhvinder (2020), *Elasto-plastic density variation in a deformable disk*, Struct. Integ. and Life, 20(1): 27-32.
25. Thakur, P., Gupta, N., Gupta, K., Sethi, M. (2020), *Elastic-plastic transition in an orthotropic material disk*, Struct. Integ. and Life, 20(2): 169-172.
26. Thakur, P., Chand, S., Sukhvinder et al. (2020), *Density parameter in a transversely and isotropic disc material with rigid inclusion*, Struct. Integ. and Life, 20(2): 159-164.
27. Sethi, M. Thakur P. (2020), *Elastoplastic deformation in an isotropic material disk with shaft subjected to load and variable density*, J Rubber Res. 23: 69-78. doi: 10.1007/s42464-020-00038-8
28. Thakur, P., Gupta N., Sethi, M., Gupta K. (2020), *Effect of density parameter in a disk made of orthotropic material and rubber*, J Rub. Res. 23(3): 193-201. doi: 10.1007/s42464-02 0-00049-5
29. Kumar, N., Thakur, P. (2021), *Thermal behaviour in a rotating disc made of transversely isotropic material with rigid shaft*, Struct. Integr. Life, 21(3): 217-223.
30. Thakur, P., Sethi, M., Gupta, N., Gupta, K. (2021), *Thermal effects in rectangular plate made of rubber, copper and glass materials*, J Rubber Res. 24(1): 147-155. doi: 10.1007/s42464-020-00080-6
31. Thakur, P., Sethi, M., Kumar, N., et al. (2021), *Thermal effects in a rotating disk made of rubber and magnesium materials and having variable density*, J Rubber Res. 24(3): 403-413. doi: 10.1007/s42464-021-00107-6
32. Thakur, P., Kumar, N., Gupta, K. (2022), *Thermal stress distribution in a hyperbolic disk made of rubber/brass material*, J Rub. Res. 25(1): 27-37. doi: 10.1007/s42464-022-00147-6