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STATIC AND FREE VIBRATION ANALYSIS OF RIGID PAVEMENT RESTING ON WINKLER AND PASTERNAK FOUNDATIONS

STATIČKA I ANALIZA SLOBODNIH VIBRACIJA KRUTOG SLOJA POSTAVLJENOG NA VINKLER I PASTERNAK PODLOGAMA

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Keywords
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- finite element method (FEM)
- MATLAB®
- Winkler and Pasternak foundations
- static analysis
- modal analysis

Ključne reči
- kruti sloj
- metoda konačnih elemenata (MKE)
- MATLAB®
- Vinkler i Pasternak podloge
- statička analiza
- modalna analiza

Abstract

The main aim of this study is to find out the static and modal response of the rigid pavement resting on different soil models subjected to aircraft loading. The FEM employed in this work is validated by a closed-form numerical problem, which is in great accord with previous research findings with a maximum divergence of 1.76 %. Furthermore, it has been investigated how the static response and cyclic frequency change as a result of variations in the thickness and span length of pavement, the modulus of the subgrade, different boundary conditions, and element discretization. Limited previous research has been performed in both static- and free vibration analysis of rigid pavement with 6 degrees of freedom (DOF). Also, very few researchers have carried out such a variety of parametric studies.

Izvod

Glavni cilj u ovom radu je rešavanje statičkih i modalnih reakcija krutog sloja koji je postavljen na različitim modelima podloge, koji su opterećeni letelicama. MKE koja se koristi u radu se potvrđuje numeričkim problemom u zatvorenom obliku, što se u velikoj meri poklapa sa prethodnim istraživanjima sa maksimalnim odstupanjem od 1,76 %. Osim toga, istraživanja su pokazala kako se statička reakcija i frekvencija ciklusa menjaju kao rezultat varijacija u debljini i dužini opsega sloja, modulu agregata, različitim graničnim uslovima i diskretizaciji elemenata. Nedovoljna prethodna istraživanja su izvedena u statičkoj i u analizi slobodnih vibracija krute podloge sa 6 stepeni slobode (DOF). Takođe, mnogo manje istraživača je sprovelo toliko raznih parametarskih studija.

INTRODUCTION

The growth of a country's transportation infrastructure may be used to forecast its future development. One crucial component of the transportation infrastructure is the pavement. Pavements are a vital component of the urban transportation system and provide an effective way of transporting goods and services. The design of runway and highway pavement requires an examination of soils and paving materials, their load-bearing performance, and environmental conditions. The underlying subgrade provides the ultimate support for all pavements. Pavements are classified as flexible, rigid, or semi-flexible according to their rigidity compared to the subsoil /1/ and /2/. Pavements with a flexible surface are generally those with an asphalt concrete surface, whereas cement concrete is used to construct rigid pavement. The primary distinction between flexible and rigid pavement is how the load is distributed over the surface. Due to the flexural rigidity and high modulus of elasticity of rigid pavements, it spreads wheel loads across a larger area underneath and resists huge amounts of load through slab action. The most significant benefit of adopting rigid pavement is its endurance and capacity to retain its form in the face of traffic and adverse climatic conditions. The pavement-foundation system may be represented mathematically as a simply supported Euler-Bernoulli beam sitting on a linear visco-elastic foundation. The foundation elasticity is modelled as a series of continuous springs in this technique and has been used to study the interaction of soil and structure. The springs provide resistance in the vertical plane, restricting the deformation. Beams resting on elastic foundations have been widely employed in a variety of engineering problems and have applications in strip foundations, railroad lines, buildings, dams, marine engineering, and airport runways. In general, subgrade models such as Winkler, Filenenko-Borodich, Pasternak, Vlasov, Reissner, Hetenyi, and others play an essential role in the study of the bending of the beam on an elastic foundation. Prior to now, methods for designing rigid pavements relied on closed-form solutions obtained from static analyses of infinitely long plates resting on elastic foundations. Although the importance of more accurate dynamic pavement analysis has been acknowledged, the mathematical complexity involved meant that analytical solutions were only available for straightforward issues. Later, the limitations of analytical solutions were considerably reduced by the development of quick computers and efficient numerical approaches like the FEM. For studying stiff pavements with moving traffic or aviation loads, FEM has been one of the most effective mathematical methods.

Modal analysis has emerged as a key tool in the endeavour to determine, improve, and optimise the dynamic properties of engineered structures during the last two decades. In the absence of externally applied forces, free vibrations occur when a system oscillates under the action of forces inherent to the system, owing to internal deflection. The system will vibrate at one or more of its natural frequencies, which are determined by its system and mass distribution and are characteristics of the system dynamics. The structure may continue to reverberate and sustain structural damage if these natural frequencies coincide with frequencies caused by external loads (such as live-, wind-, and earthquake loads). Since modal factors like natural frequencies and mode shapes are crucial for comprehending the dynamic behaviour of the structure, research on the free vibrational properties of rigid pavement is required.

The assumption used in the study of beam bending on an elastic basis is that the reaction forces of the foundation are directly proportional to the deflection of the beam at each location. Winkler was the one who initially put up this hypothesis, /3/, and assumed that the springs' top ends are attached to an elastic membrane that is stretched under constant tension between two layers of springs, /4/, established interaction between different springs by embracing an elastic beam in a 2-D case and a plate in 3-D case. Shear interactions between the springs were introduced by /5-7/ and provided an algorithm for forecasting the dynamic properties of a thick plate on an elastic foundation to moving loads that is based on the finite element analysis technique. In /8/ authors developed a computer algorithm called FEBEF (finite element: beam on elastic foundation) that uses the finite element procedure to solve problems with beams on elastic foundations. In /9/ they provided a FEM for estimating the vibration characteristics of a uniform Timoshenko beam discretized into a number of elements with four DOF, each supported by a two-parameter elastic base. They illustrated that the current method offers a unified way to analyse the vibrations of the beam with any end conditions that rest on an elastic foundation. In /10/ they studied the modal analysis of plates sitting on a two-parameter elastic medium. The corresponding shape functions were obtained by solving a beam sitting on a two-parameter medium. The influence of various factors on mode shapes and natural frequencies was studied using parametric analysis. In /11/ they introduced the cubic displacement function of the governing differential equation to investigate the Euler-Bernoulli beam sitting on two-parameter soil by finite element technique. In /12/ they examined the differential quadrature approach. It was found that in the direction of line supports, differential quadrature was used, whereas, in the transfer domain perpendicular to them, the state space technique was used. In /13/ the authors proposed an improved solution algorithm for static analysis of rigid pavement discretized by finite and infinite beam elements based on the FEM.

During the last decade, there have been significant advancements in the two-parameter soil foundation design. It was observed that a new finite element formulation may be used to analyse the shallow and raft foundation response, overcoming the limitations of the Winkler model /14/. In /15/ they used the Hamilton variation principle to create the FEM solution for the analysis of free vibrations of elastic plates on elastic foundations using a two-parameter foundation model and Mindlin plate theory. In /16/ they implemented a mesh-free technique also known as the radical point interpolation technique for the analysis of a beam resting on 2-parameter soil. In /17/ they obtained a closed-form analytical solution of bending of the beam on an elastic foundation using singularity functions. In /18/ the authors employed the element-less Galerkin technique to investigate the free vibration of thin plates sitting on 2-parameter elastic foundations with all feasible classical constraints. In /19/ they conducted a free vibration analysis of a four-node Kirchoff rectangular element with three DOF per node resting on a Winkler model elastic foundation with varied boundary conditions for different thicknesses and foundation properties. In /20/ they performed the free vibration analysis of the Timoshenko beam on a two-parameter elastic foundation using the FEM. The two-nodded element was used for modelling with two DOF. In /21/ they provided a numerical model for analysing pavement vibration caused by a passing vehicle's dynamic load. The vehicle movement load was calculated using a quarter-vehicle model. Consideration was given to both the random and spatial properties of the load. In /22-25/ they performed static, free, and dynamic analysis on a curved thin-walled box-girder bridge. The authors employed finite element-based MATLAB® coding to calculate various stress resultants.

According to the literature, previous researchers made a good attempt to perform static and free vibration analysis of rigid pavement using a variety of methodologies. However, these procedures either provide closed-form solutions that are much too complicated for engineers to employ in reality, or analysis was carried out using a finite element approach with fewer DOF. Furthermore, very few scholars have carried out such a broad variety of parametric analyses on both static as well as free vibration behaviour of rigid pavement on Winkler and Pasternak soil foundation models. The current study reveals different key parameters that influence the static analysis and modal characteristics of a rigid pavement. The following are the main objectives of this study.

- Modelling 1-D finite beam elements for rigid pavement in a computationally efficient manner.
- Static response analysis of a rigid concrete pavement resting on Winkler and Pasternak soil model exposed to aircraft loads.
- Free vibration analysis of rigid pavement resting on Winkler and Pasternak soil model.

FEM FORMULATION FOR RIGID PAVEMENT RESTING ON WINKLER AND PASTERNAK SOIL-FOUNDATION MODEL

Geometry of the rigid pavement

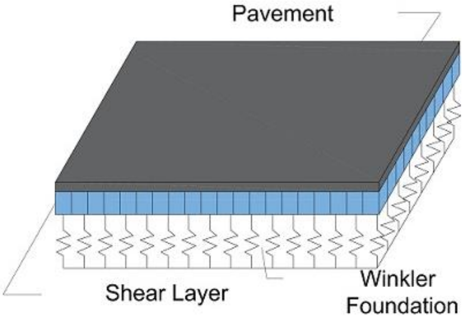


Figure 1. Rigid pavement resting on a two-parameter elastic foundation.

Figure 1 shows a rigid pavement sitting on a two-parameter elastic foundation consisting of two-layer which are the shear layer and the Winkler foundation (subgrade layer). To perform 1-D finite element analyses, pavement is discretized into small 3-noded flexure beam elements with 6 DOF (3 translations and 3 rotations). The element is formulated by a local Cartesian coordinate system (LCS) and natural coordinate ζ is used to define the global coordinate system (GCS). The value of ζ is -1, 0, and 1, on the left node, central node, and right node of the element.

Stress-strain relationship

Displacements are represented in both LCS and GCS. As in LCS, displacements are expressed as,

$$\bar{\delta}=[u,v,w,\theta_x,\theta_y,\theta_z]^T \, . \tag{1}$$

In the above equation u, v, and w represent translation in the local x, y, and z-axes; θ_x denotes torsional angle, θ_y and θ_z represent rotation in the y and z-axes.

The displacements mentioned above are also expressed in the GCS as,

$$\delta=[U,V,W,\varphi_x,\varphi_y,\varphi_z] \, . \tag{2}$$

In the above equation, U, V, and W represent translations, and φ_x, φ_y, and φ_z represent rotations in the x, y, and z directions, respectively.

The general form of the stress vector is given by

$$\bar{\sigma}=[N_x,Q_y,Q_z,M_T,M_y,M_z]^T \, . \tag{3}$$

In the above equation, N_x stands for the axial force, Q_y and Q_z are shear forces, M_T is the pure torsional moment, and M_y and M_z are the primary bending moments.

The general form of the strain vector is given by

$$\varepsilon=[\varepsilon_x,\varepsilon_{yx},\varepsilon_{zx},\psi_{\theta x},\psi_{yx},\psi_{zx}]^T \, . \tag{4}$$

The generalised form of the rigidity matrix D is given as:

$$\mathrm{D}=\begin{bmatrix} EA & & & & & \\ & GA_{sy} & & & 0 & \\ & & GA_{sz} & & & \\ & & & GJ_T & & \\ & 0 & & & E_1I_y & \\ & & & & & E_1I_z \end{bmatrix} . \tag{5}$$

In the equation above, E stands for Young's modulus of elasticity, G for shear modulus, A for cross-sectional area, A_{sy} and A_{sz} for effective shear areas in the y and z directions, J_T for torsional moment of inertia, and I_y and I_z for principal flexural moments of inertia around the y and z axes. The axial and flexural effects of the beam element only need C_0 continuity, and the shape functions N are provided as follows:

$$N_i=\frac{1}{2}(\xi^2+\xi_0) \quad \text{for } i=1 \text{ and } 3$$
$$N_i=(1-\xi^2) \quad \text{for } i=2 \, , \tag{6}$$

where: $\xi_0 = \xi\xi_i$.

Finite element formulation of Winkler and Pasternak soil foundation model

In this study, the soil layer below the rigid pavement is modelled by a two-parameter Pasternak soil model in which the subgrade is represented by a set of vertical springs, and the base course layer is represented by the Pasternak layer of finite thickness H. Considering a beam in equilibrium condition, the foundation model with external load $q(x)$ is given by the following differential equation:

$$q(x)=-G_pbH\frac{d^2w}{dx^2}+kbw \, , \tag{7}$$

where: G_p is the shear modulus of the Pasternak layer or shear layer; w is the vertical deflection of the beam; and k is the modulus the of subgrade. The differential equation of the beam is given by:

$$EI\frac{d^4w}{dx^4}=\bar{P}(x) \, , \tag{8}$$

$$\bar{P}(x)=P(x)-q(x) \, , \tag{9}$$

where: $P(x)$ is a load on the surface; and $q(x)$ is foundation bearing pressure. From the above three equations, we can define the equilibrium condition of the beam as

$$EI\frac{d^4w}{dx^4}-G_pbH\frac{d^2w}{dx^2}+kbw=P(x) \, . \tag{10}$$

The system's strain energy U may be written as follows:

$$U=\frac{1}{2}\int_0^L\left(EI\left(\frac{d^2w}{dx^2}\right)\cdot\left(\frac{d^2w}{dx^2}\right)-G_pbH\left(\frac{dw}{dx}\right)\cdot\left(\frac{dw}{dx}\right)+kbw\cdot w\right)dx \, . \tag{11}$$

The above equation may be represented in the stiffness matrix format

$$\left[[k_1]+[k_2]+[k_3]\right]\{\delta\}=\{f\} \, , \tag{12}$$

where: δ and f are element nodal displacement and element nodal force; $[k_1]$, $[k_2]$, and $[k_3]$ are element stiffness matrices of rigid pavement, shear layer, and subgrade layer, respectively, and these matrices are derived from the equations,

$$[k_1]=\int_0^L\left(\left[\frac{d^2N}{dx^2}\right]EI\left[\frac{d^2N}{dx^2}\right]\right)dx \, , \tag{13}$$

$$[k_2]=\int_0^L\left(\left[\frac{dN}{dx}\right]G_pbH\left[\frac{dN}{dx}\right]\right)dx \, , \tag{14}$$

$$[k_3]=\int_0^L\left(\left[N\right]kb\left[N\right]\right)dx \, . \tag{15}$$

For the formulation of the Winkler foundation model, the foundation is modelled as a set of springs and interactions between the springs are not considered. In other words, the shear layer is not present in the Winkler foundation. Thus, the stiffness matrix of the shear layer (k_2) in Eq.(12) is equal to zero in the formulation of Winkler foundation.

Boundary conditions

The different boundary conditions are shown in following equations.

Fixed support $\quad u = v = w = 0, \quad (16)$

$\theta_x = \theta_y = \theta_z = 0. \quad (17)$

Pinned support $\quad u = v = w = 0, \quad (18)$

$\theta_x = 0. \quad (19)$

Linear roller support $\quad v = w = 0, \quad (20)$

$\theta_x = 0. \quad (21)$

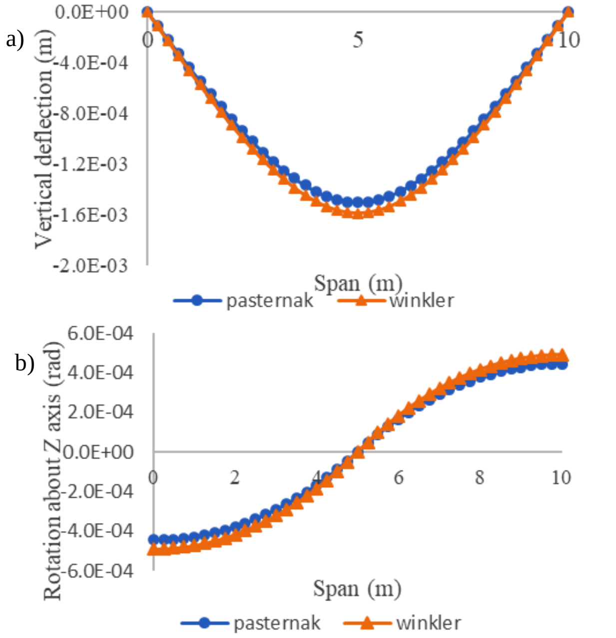
RESULTS AND DISCUSSION

Validation of static response

In order to validate the above formulation for both the Winkler- and the Pasternak soil model, one beam on elastic soil problem is taken from /11/ who obtained a closed-form analytical solution of the beams resting on an elastic foundation. The properties of the beam, soil, and loading are given in Table 1. Some critical reaction parameters as vertical deflection, rotations about the *z*-axis, and stress resultants as bending moment, and shear force are shown in Fig. 2a-d.

Table 1. Various sectional properties of beam and soil.

Sectional property	Value
Size of the beam (L, B, t) (m)	10, 1.0, 0.5
Area (A, A_{sy}, A_{sz}) (m^2)	0.5, 10, 5
Youngs modulus of beam, E (N/m^2)	3.605E+10
Rigidity modulus of beam, G (N/m^2)	1.567E+10
Poisson's ratio (ν)	0.15
Shear modulus of shear layer, G_p (kN/m^2)	7000
Height, H (m)	1
Subgrade modulus, k (kN/m^3)	10000
Number of elements	20
Load, P (kN)	100



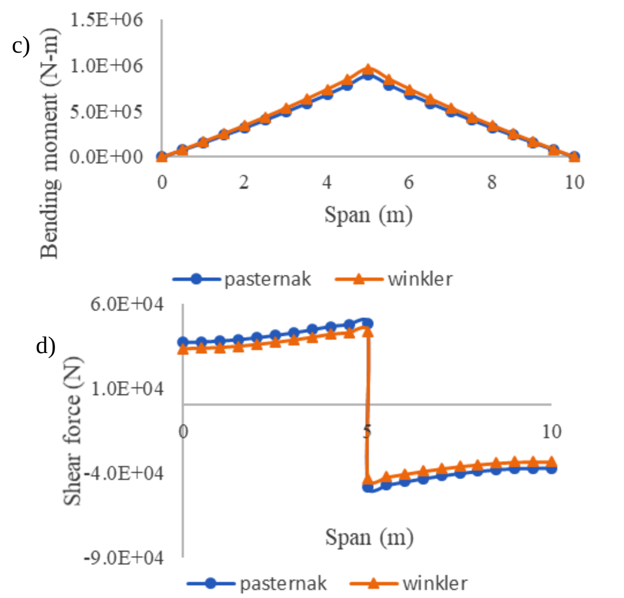


Figure 2a-d. Span length vs. vertical deflection, rotation about *z* axis, bending moment, and shear force, of rigid pavement resting on Winkler and Pasternak soil foundation.

Table 2. Variation of results of previous and present study.

	Central deflection (mm)		Central moment (kNm)	
	Winkler	Pasternak	Winkler	Pasternak
Previous study	1.558	1.506	94.714	89.274
Authors	1.586	1.501	96.14	89.52
Error (%)	1.76	0.33	1.48	0.27

Table 2 shows the deviation between the previous and present study from which we can conclude that the numerical findings are in great agreement with the work of /11/, with a maximum divergence of 1.76 % in central deflection and 1.48 % in the central moment, both in the case of Winkler foundation, therefore, verifying the finite element approach used in the current study.

PRESENT PROBLEM FOR STATIC ANALYSIS

In this numerical problem, a rigid pavement of length 10 m on the Pasternak soil model subjected to aircraft loading is taken. The pavement is discretized into 20 elements. The main landing gear is responsible for carrying 90 % of overall aircraft weight, while the nose gear is responsible for carrying the remaining 10 %. Because of this, the load that is carried by each gear in the main twin assembly is roughly 343 kN, /11/. In this case, the load is assumed to be at the centre of the pavement, and for boundary conditions one end is assumed to be pinned and the end as roller support. The different sectional properties are enlisted in Table 3.

Table 3. Sectional properties of rigid pavement on two-parameter soil.

Sectional property	Value
Size of pavement (L, B, t) (m)	10, 10, 0.4
Area (A, A_{sy}, A_{sz}) (m^2)	4, 4, 100
Young's modulus of pavement E (N/m^2)	2.9E+10
Rigidity modulus of pavement G (N/m^2)	1.26E+10
Moment of inertia I (m^4)	5.33E-02
Poisson's ratio (ν)	0.15
Shear modulus of shear layer G_p (N/m^2)	1E+07
Height H (m)	1
Subgrade modulus k (N/m^3)	1E+07

Figure 4a-d illustrates several important response characteristics and the main stress resultants.

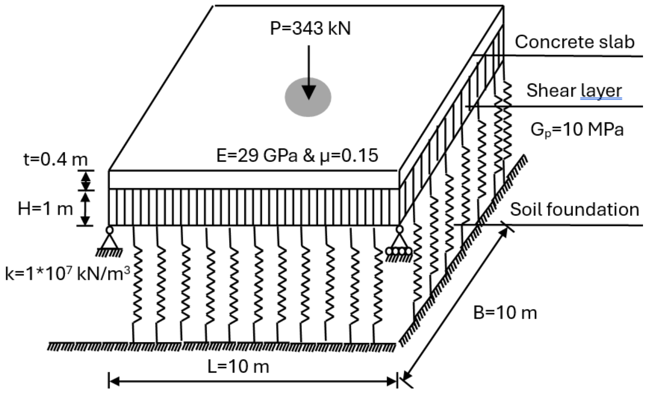


Figure 3. Rigid pavement on two-parameter soil subjected to aircraft loading.

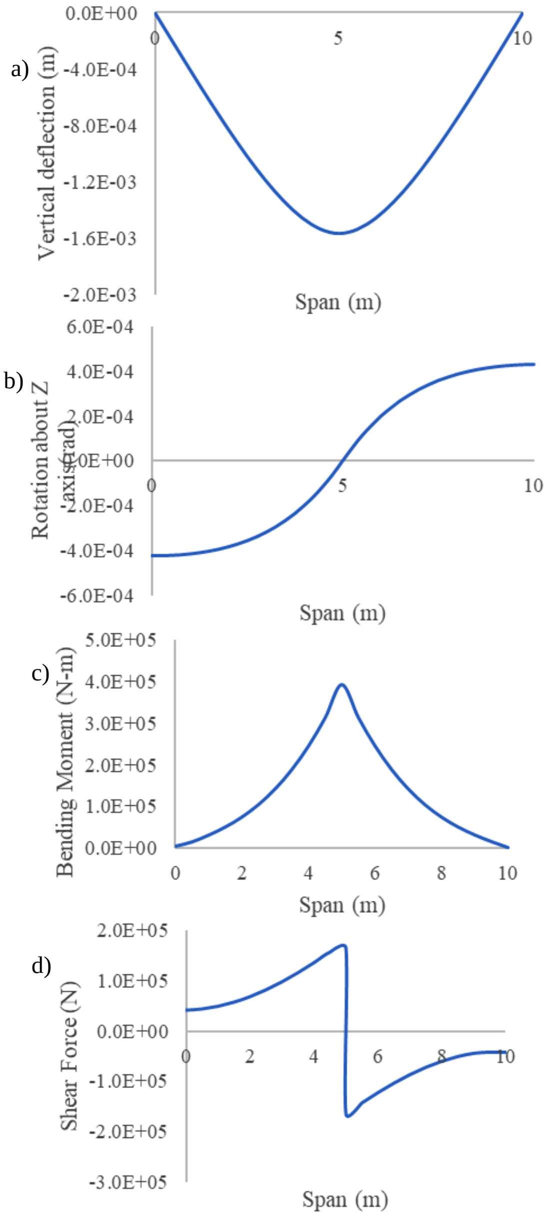


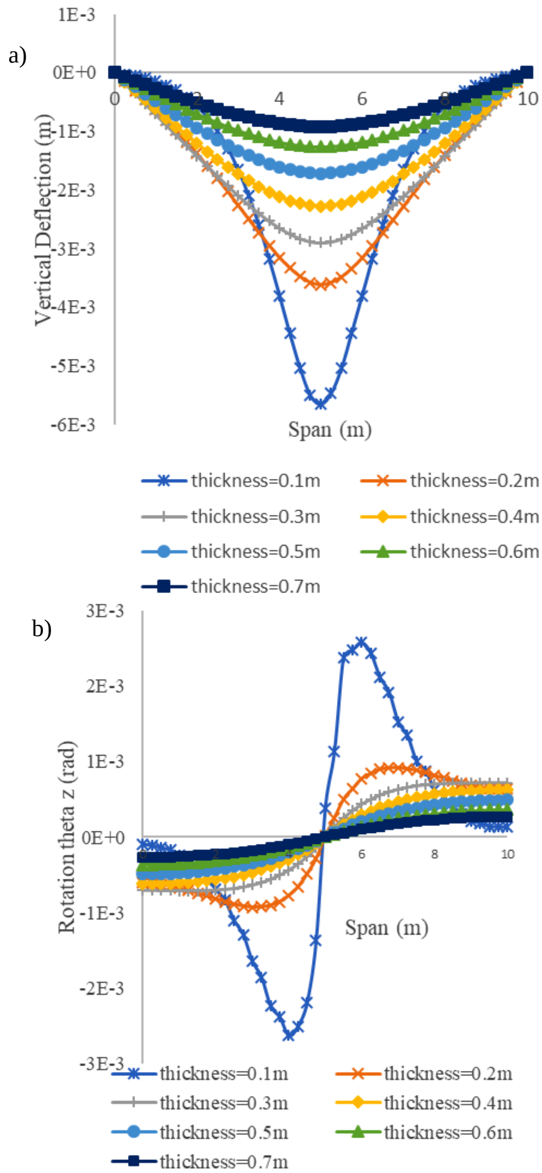
Figure 4a-d. Span length vs. vertical deflection, rotation about z axis, bending moment, and shear force, of rigid pavement resting on two-parameter soil.

PARAMETRIC STUDY FOR STATIC ANALYSIS

As the Pasternak foundation model simulates the underlying soil medium better and is theoretically more attractive than the one-parameter (Winkler) foundation model, a parametric analysis is undertaken on the above rigid pavement based on a Pasternak soil model to identify parameters impacting the cyclic frequency of the rigid pavement: thickness of pavement (t); Young's modulus of pavement (E); modulus of subgrade (k); loading position.

Thickness of pavement (t)

The impact of thickness ranging from 0.1 to 0.7 m on the response parameters and stress results of rigid pavement with a span of 10 m and simply supported ends are shown in Fig. 5a-d. It is quite visible that by increasing the thickness of pavement both vertical deflection and rotation about the z-axis are reduced with a maximal reduction of 36.10 % in vertical deflection and 64.59 % in rotation about the z-axis.



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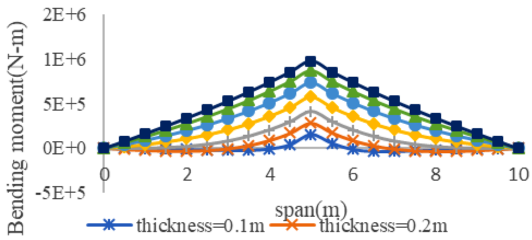

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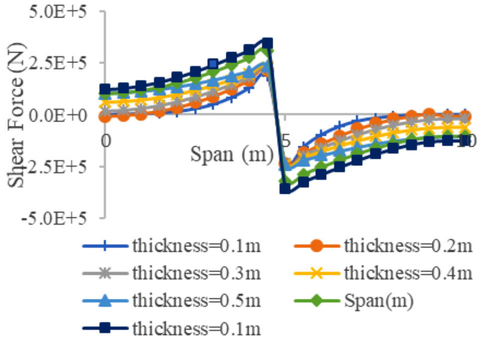


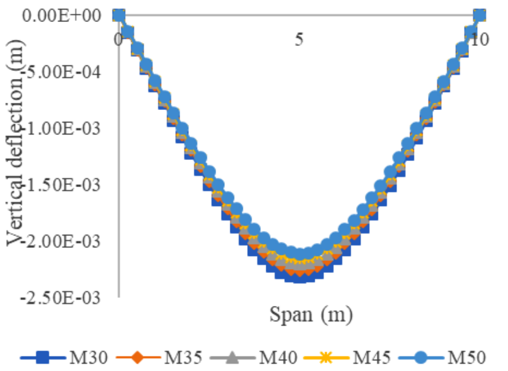
Figure 5a-d. Span length vs. vertical deflection, rotation, bending moment, and shear force with varying thickness.

Young's modulus of pavement (E)

IRC 44:2008 specifies that a minimum M40 grade of concrete must be used for the building of standard concrete pavements, an M30 grade of concrete is recommended for rural roads, and an M50 for high-performance concrete pavements. According to IS 456:2000 the modulus of elasticity of concrete (E) in MPa is given as (IRC 44:2008):

$$E = 5000\sqrt{f_{ck}}\ , \tag{22}$$

where: f_{ck} is characteristic compressive strength of concrete.

a)


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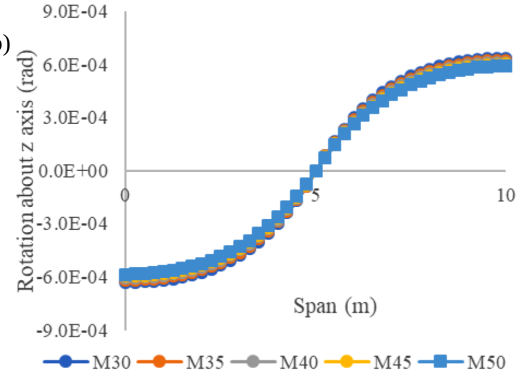

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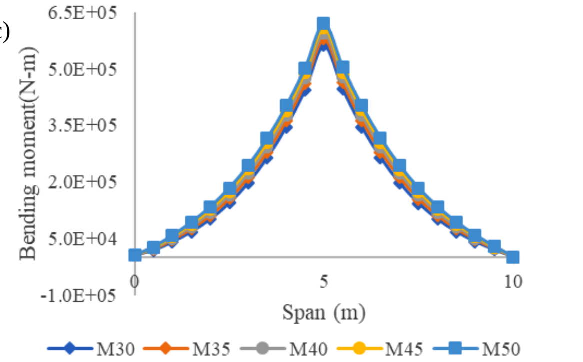

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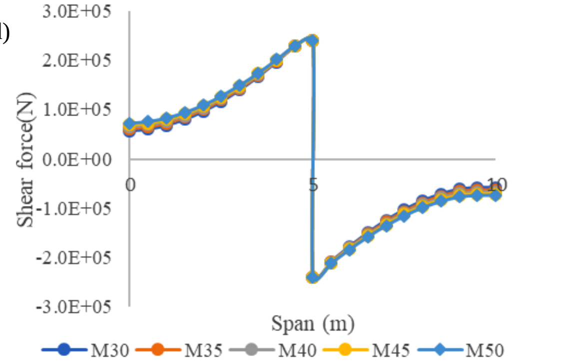
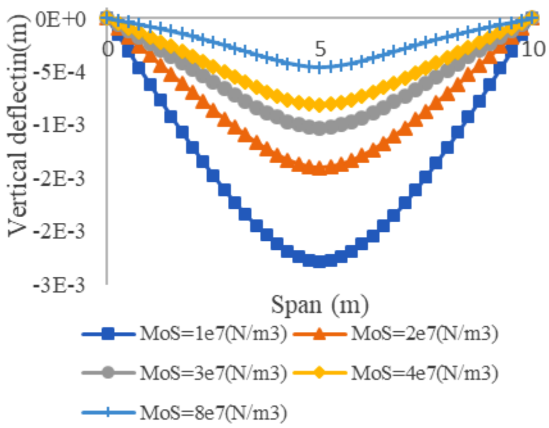


Figure 6a-d. Span length vs. vertical deflection, rotation, bending moment, and shear force with varying modulus of elasticity (E) of pavement.

Figure 6a-d shows the response parameters and stress resultants for different concrete grades of rigid pavement with a span length of 10 m and thickness of 0.4 m. It is apparent that both the vertical deflection and bending moment tend to decrease as the grade of concrete is increased with a maximum decrease of 2.67 % and 9.53 %, in respect when the grade of concrete varies from M30 to M35.

Modulus of subgrade (k)

Figure 7a-d reveals the effect of the modulus of subgrade (k) on vertical deflection, rotation, bending moment, and shear force, respectively. It is quite visible that on increasing both parameters, DOF and stress resultants decrease. The modulus of subgrade induces the maximal reduction of 43.15 % on vertical deflection, 43.40 % percent reduction on rotation, 24.80 % reduction on bending moment, and 2.28 % reduction on the shear force.

a)


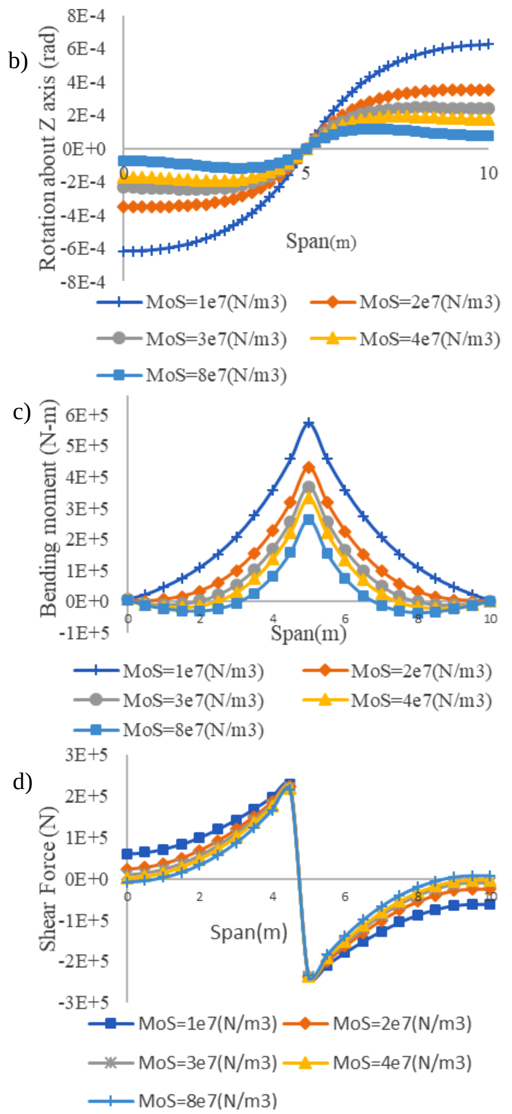
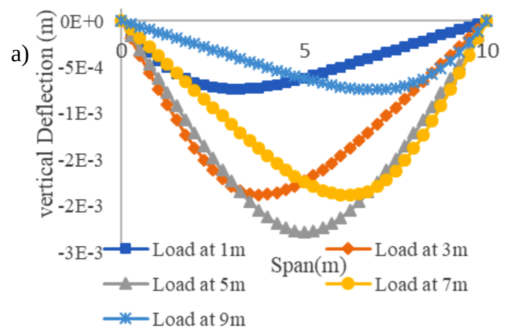


Figure 7a-d. Span length vs. vertical deflection, rotation, bending moment, and shear force with varying modulus of subgrade.

Position of the load

The load has been applied at different positions on the pavement along the length. Figure 8a-d illustrates the effect of loading position along the length on vertical deflection, rotation about the *z*-axis, bending moment, and shear force. Vertical deflection and bending moment are seen maximum when the load is exactly at the centre of the pavement while shear force is maximum when the load is applied near ends.



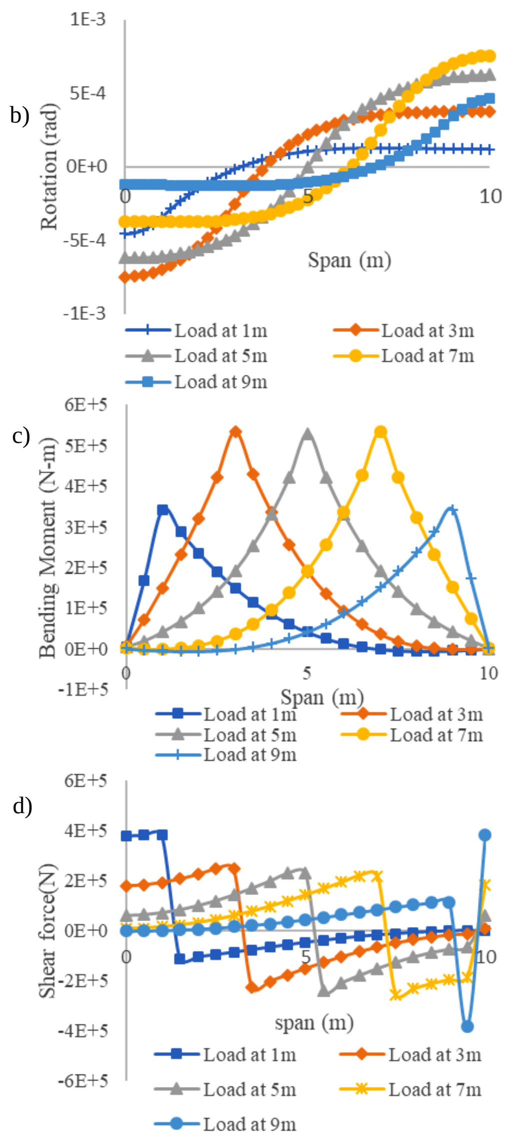


Figure 8a-d. Span length vs. vertical deflection, rotation, bending moment, and shear force with varying load location from the left end along the length.

Eigen value problem for the undamped system

Investigations on free vibrational properties of the rigid pavement have been carried out because modal parameters as natural frequencies and mode shapes are vital for understanding the structure's dynamic behaviour. The generalised equation of motion for an undamped free-vibrating system may be written as

$$M\ddot{\delta}+K\delta=0\,. \tag{23}$$

After applying boundary conditions, [K] and [M] are the global stiffness and mass matrices.

Under the assumption that the natural mode of vibration has harmonic motion, the solution can be written as:

$$\{\delta\}=\{X\}\sin(\omega t+\varphi)\,, \tag{24}$$

where: $\{X\}$ represents nodal amplitude; ω is natural frequency (rad/s); and φ represents the phase angle.

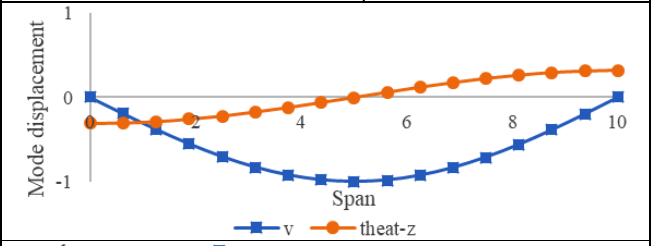
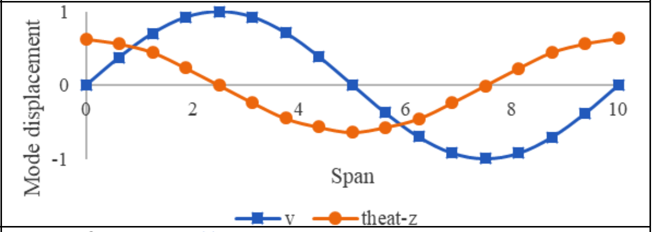
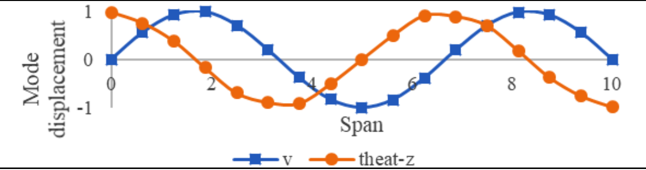
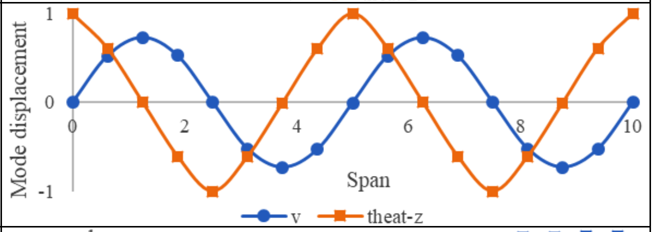
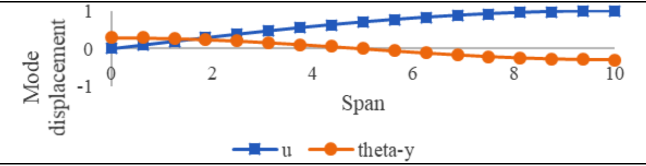
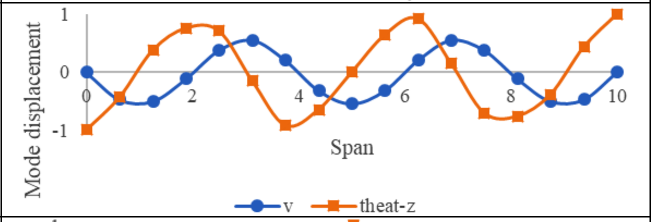
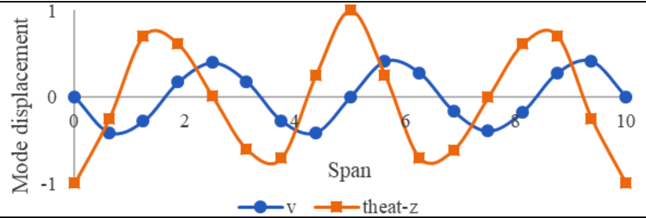
On substituting Eq.(24) in Eq.(22), the generalised eigen value problem can be written as:

$$\left\{[K]-\omega^2[M]\right\}\{X\}=0\,. \tag{25}$$

Using a typical eigen solver, Eq.(24) is solved to derive natural frequencies and mode shapes of the rigid pavement resting on the Winkler and Pasternak soil models.

The same numerical problem as described previously is used to perform the modal analysis of rigid pavement on the Winkler and Pasternak soil models. Tables 4 and 5 show mode shape diagrams and cycle frequency data for the first ten modes of rigid pavement resting on Winkler and Pasternak soil models, respectively. Multiple modes may be associated with a certain frequency. However, the dominant mode is the most significant. Figure 9 shows the variation of cyclic frequency of the rigid pavement resting on both the Winkler and Pasternak soil model.

Table 4. Frequency and mode shapes for rigid pavement resting on Winkler soil model.

Mode	DOF	Cyclic frequency (Hz)	Mode shape
1	v (1st vertical mode)	3.16	
2	v (2nd vertical mode)	7.03	
3	v (3rd vertical mode)	13.09	
4	Theta *z* (1st rotation about *z*-axis mode)	20.11	
5	Theta *y* (1st axial mode)	26.49	
6	Theta *z* (2nd rotation about *z*-axis mode)	27.74	
7	Theta *z* (3rd rotation about *z*-axis mode)	35.91	

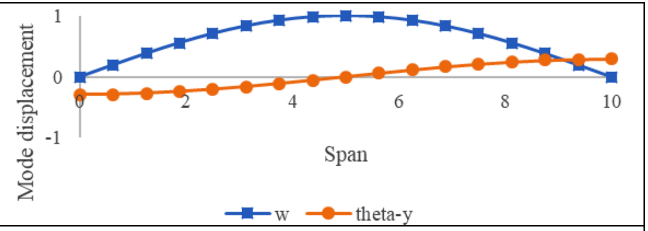
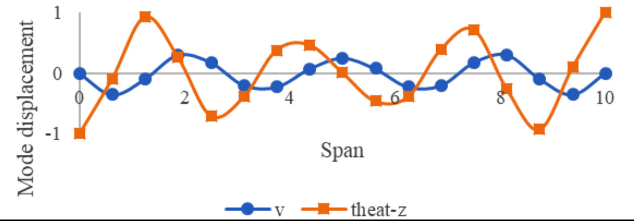
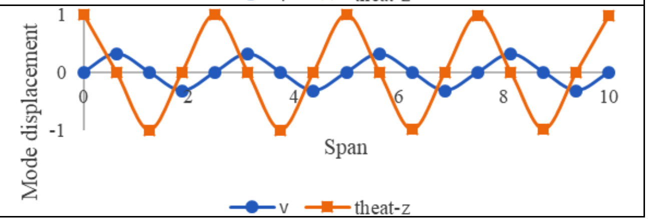
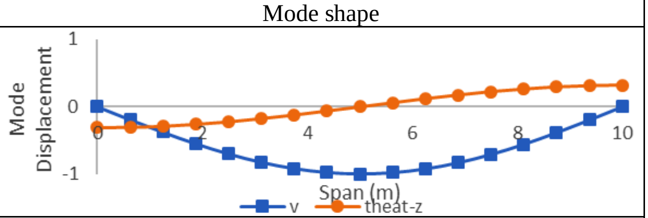
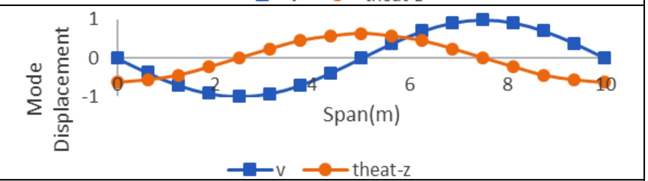
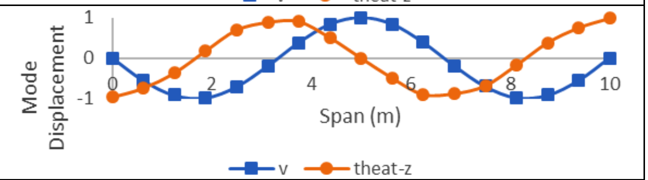
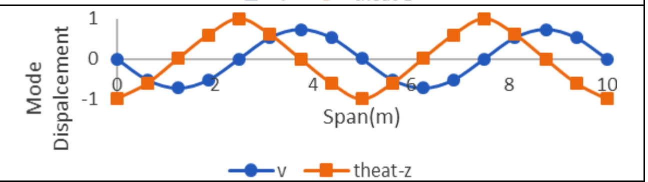
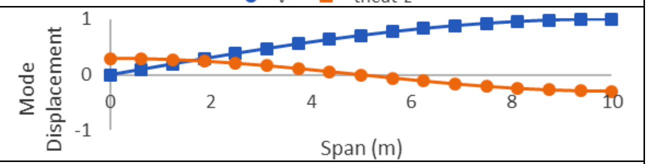
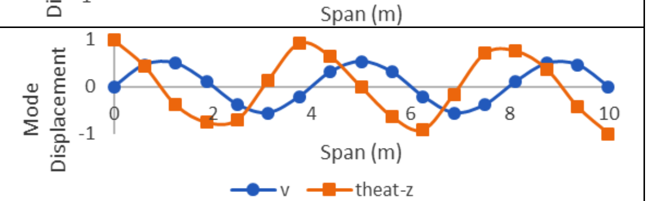
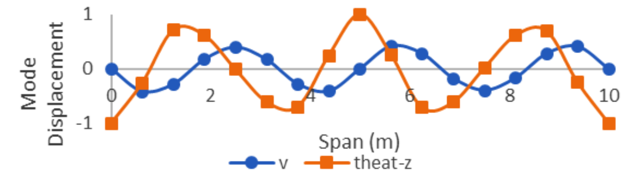
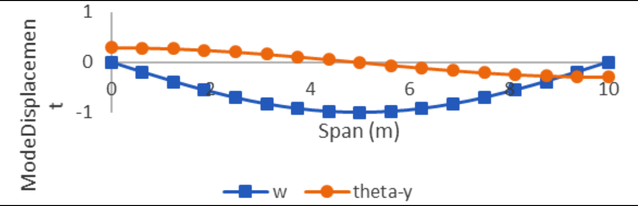
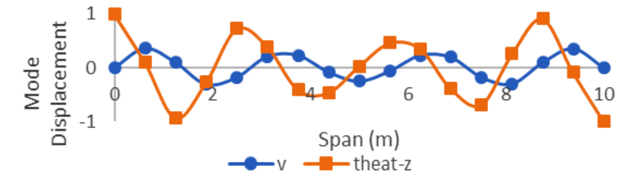
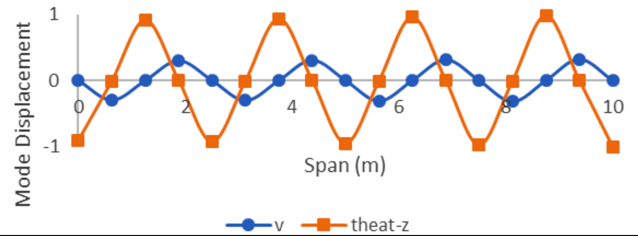
8	w (1st lateral mode)	44.35	
9	Theta *z* (4th rotation about *z*-axis mode)	44.37	
10	Theta *z* (5th rotation about *z*-axis mode)	49.96	

Table 5. Frequency and mode shapes for rigid pavement resting on two-parameter soil model.

Mode	DOF	Cyclic frequency (Hz)	Mode shape
1	v (1st vertical mode)	3.32	
2	v (2nd vertical mode)	7.38	
3	v (3rd vertical mode)	13.75	
4	Theta z (1st rotation about z-axis mode)	21.13	
5	u (1st axial mode)	27.60	
6	Theta z (2nd rotation about z-axis mode)	29.15	

7	Theta *z* (3rd rotation about *z*-axis mode)	37.74	
8	w (1st lateral mode)	46.21	
9	Theta *z* (4th rotation about *z*-axis mode)	46.63	
10	Theta *z* (5th rotation about *z*-axis mode)	52.51	

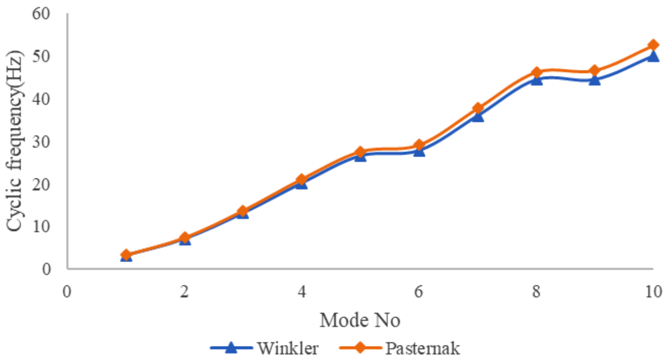


Figure 9. Variation of cyclic frequency for Winkler and Pasternak soil model.

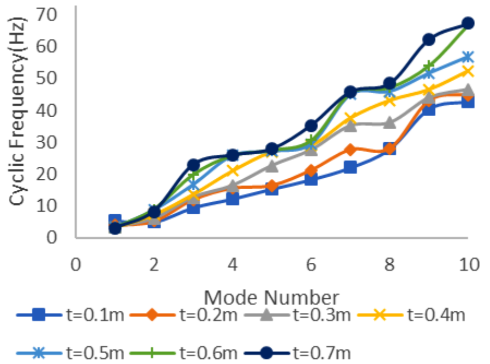


Figure 10. Variation of cyclic frequency with pavement thickness.

PARAMETRIC STUDY FOR MODAL ANALYSIS

To identify different parameters impacting the cyclic frequency of the rigid pavement, a parametric analysis is undertaken on the above rigid pavement based on a two-parameter soil model.

Effect of thickness of pavement (t)

The impact of thickness ranging from 0.1 to 0.7 m on the first ten natural frequencies of rigid pavement with a span of 10 m and simply supported ends are shown in Fig. 10. It's worth noting that, with the exception of the first vertical mode, the fundamental frequencies tend to rise with increasing thickness. It is important to note that the 3rd vertical mode, the 1st rotation about the *z*-axis mode, and the 3rd rotation about the *z*-axis mode seem to rise more than the other modes. As the thickness increases from 0.1 to 0.2 m, a maximum rise of 29.70 % is seen in the 2nd rotation about the *z*-axis mode. Therefore, it is possible to state that the fundamental frequencies start to rise as thickness increases.

Effect of span length (L)

The impact of span length on the free vibration characteristics of rigid pavements has long been recognised. Figure 11 depicts the effect of span length on the first ten natural frequencies of simply-supported rigid pavement with span lengths of 5, 10, 15, and 20 m and a thickness of 0.4 m resting on a two-parameter soil foundation with the shear modulus of 1E+7 N/m^2 and modulus of subgrade of 1E+7 N/m^3. The natural frequencies for both DOF, i.e., translation and rotation, decrease with increasing pavement span length. It is important mentioning that all the modes have a stronger propensity to change as the span changes. All the modes decrease by more than 50 % with a maximum decrease of 63.49 % observed in mode 2 as the span changes from 5 to 10 m. As a result, it is possible to infer that span length significantly influences the natural frequencies of rigid pavements and should be considered in the parametric analysis.

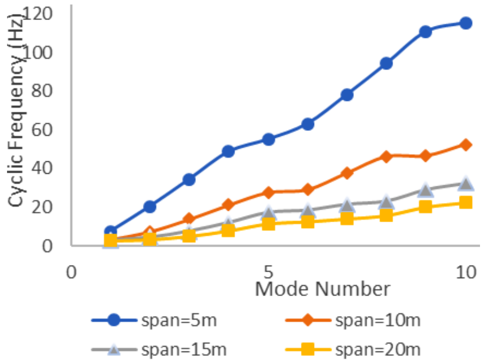


Figure 11. Variation of cyclic frequency with pavement span length.

Young's modulus of pavement (E)

Figure 12 shows the variation in cyclic frequency for different grades of concrete grades of rigid pavement with a span length of 10 m and thickness of 0.4 m. Apparently, the cyclic frequency tends to increase as the grade of concrete is increased with a maximum increase of 3.94 % when the grade of concrete varies from M30 to M35 for the first axial mode.

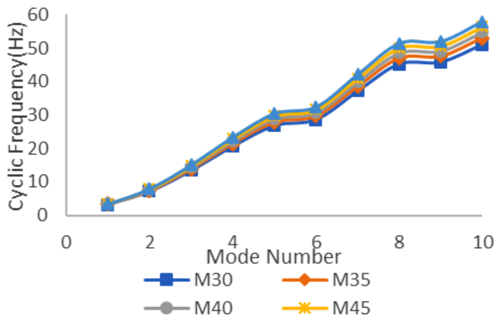


Figure 12. Variation of the cyclic frequency with concrete grade.

Effect of modulus of subgrade (k)

Figure 13 displays the effects of a subgrade modulus varying from 1E+07 to 8E+07 N/m^3 on the first ten natural frequencies of a rigid pavement that is simply supported and rests on a two-parameter soil foundation with a span of 10 m. As seen, the fundamental frequencies generally tend to rise as the modulus of subgrade increases. It is worth noting that the first, second, and third vertical modes seem to rise more than other modes. The first vertical mode has a maximum rise of 37.34 % when the subgrade's modulus changes from 4E+07 to 8E+07 N/m^3. Furthermore, the cyclic frequency rises dramatically with each node. Therefore, it can be concluded that the modulus of the subgrade must be considered in the parametric study since it significantly affects the natural frequencies of rigid pavement.

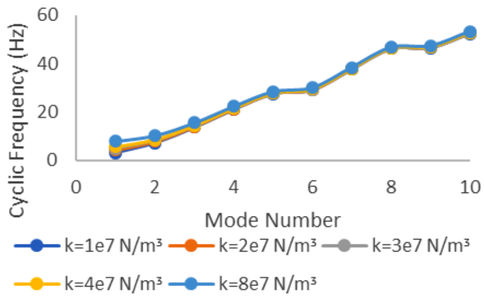


Figure 13. Variation of cyclic frequency with modulus of subgrade, *k*.

Effect of boundary condition

This segment examines the impact of boundary conditions on the modal response of the rigid pavement with a span of 10 m and thickness of 0.4 m. Different boundary conditions for fixed, pinned, and linear roller support are enlisted in the previous section. Figure 14 shows the first 10 cyclic frequencies for three different boundary condition cases: cantilever, simply supported, and fixed support of the rigid pavement resting on a two-parameter soil foundation with the shear modulus of 1E+7 N/m^2 and modulus of subgrade 1E+7 N/m^3. The fundamental frequency is clearly rising in all DOF modes for all boundary conditions. For most modes, cyclic frequencies are lowest in the cantilever case and highest in the fixed case. The cyclic frequency of the fixed boundary condition reaches 52.51 Hz for the 10th mode, which is the highest for all examples studied in this parametric study.

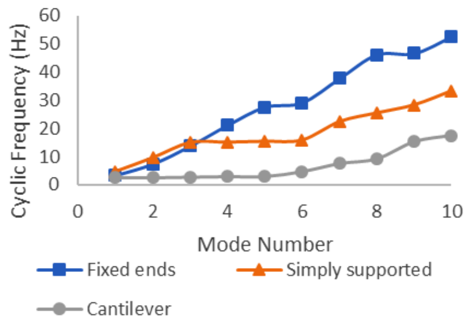


Figure 14. Variation of the cyclic frequency with different boundary conditions.

Effect of element discretization

Figure 15 shows the variance of first ten cycle frequencies for a simply supported rigid pavement with a span of 10 m and a thickness of 0.4 m. Variations in the number of elements have been used as the foundation for the study. The findings reveal two noteworthy trends. First, the cyclic frequency values generally increase with each mode, and second, element discretization seems to have no major effect on cyclic frequency. All modes have values that are almost the same for each element.

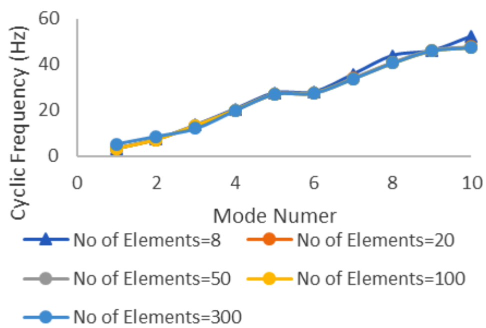


Figure 15. Variation of cyclic frequency with the number of elements.

CONCLUSION

A MATLAB® programme developed based on the mathematically efficient 3-noded 1-D element was used to model the rigid pavement resting on both Winkler and Pasternak soil model. Using the finite element analysis, the static and free vibration was carried out for both models. Validation is based on the deflections and moments of the beam under central load. The numerical findings are in great agreement with the work of earlier researchers, with a maximal divergence of 1.76 %, therefore verifying the finite element approach used in the current study. The parametric study on static analysis revealed that the increase in thickness induces a decrease of degrees of freedom and an increase in bending moment and a reduction in shear force with a maximum of 36.10 % in deflection, 64.59 % in rotation reduction about the *z*-axis, 87.95 % increase in bending moment, and 13.82 % increase in shear force.

Free vibration analysis of rigid pavement on both Winkler and Pasternak soil model is performed, and mode shape diagrams and cyclic frequency data for the first ten modes of the pavement are presented. The parametric study on free vibration indicated that fundamental frequencies increase with increasing thickness and decrease with an increasing span length of the rigid pavement. It is found that the cyclic frequencies increase with increasing both moduli of subgrade and the same trend is followed by Young's modulus of concrete pavement. Also, different boundary conditions are used and showed that the fixed condition case has the highest values of cyclic frequency, while the cantilever boundary condition has the lowest cyclic frequencies for all modes.

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