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ELECTROHYDRODYNAMICS CONVECTION IN DIELECTRIC OLDROYDIAN NANOFLUID LAYER IN POROUS MEDIUM

ELEKTROHIDRODINAMIČKA KONVEKCIJA U DIELEKTRIČNOM OLDROYDOVSKOM NANOFLUIDU SA POROZNOM SREDINOM

Originalni naučni rad / Original scientific paper
UDK / UDC:

Rad primljen / Paper received: 24.05.2022

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Keywords
- Oldroydian model
- nanofluid
- convection
- electric field
- porous medium

Ključne reči
- Oldroydovski model
- nanofluid
- konvekcija
- električno polje
- porozna sredina

Abstract

The onset of thermal convection in an electrically conducting rheological nanofluid to include an external vertical AC electric field saturated by a homogeneous porous medium has been studied using linear stability theory by employing an Oldroydian model which incorporates the effects of the electric field, Brownian motion, thermophoresis, and rheological parameters for bottom heavy distribution of nanoparticles. The rheology of the nanofluid is described by the Oldroydian model for calculating the shear stresses from velocity gradients. Exact solutions of the eigenvalue problem for stress-free bounding surface are obtained analytically using Galerkin method and the Darcy Rayleigh number for onset of both stationary and oscillatory convection, obtained for bottom-heavy distribution of nanoparticles. It is found that the Deborah number has a stabilizing effect on the system, while strain retardation time parameter has a destabilizing effect on the oscillatory convection of the system. The effect of the Lewis number tends to stabilize the stationary convection and destabilizes oscillatory convection. The concentration Rayleigh number has a destabilizing effect on stationary convection and a stabilizing effect on the oscillatory convection. Medium porosity has a stabilizing effect on oscillatory convection and is destabilizing on stationary convection. The effect of Vadasz number on oscillatory convection is destabilizing. AC electric field has a destabilizing effect on both the stationary and oscillatory convection.

Izvod

U radu se razmatra pojava toplotne konvekcije u elektroprovodnom reološkom nanofluidu, u prisustvu spoljašnjeg vertikalnog naizmeničnog električnog polja, zasićenog homogenom poroznom sredinom, i to primenom teorije linearne stabilnosti uvrštavanjem Oldroydovskog modela, kojim se uvode efekti električnog polja, Braunovog kretanja, termoforeze, kao i reoloških parametara za raspodelu nanočestica tipa teškog repa. Reologija nanofluida je opisana Oldrojdovskim modelom za proračun napona smicanja preko gradijenta brzine. Tačna rešenja karakterističnih korena za graničnu površinu bez napona su dobijena analitički korišćenjem Galerkin metode i Darsi Rejlejevog broja za istovremeno stacionarno i oscilatorno strujanje, u slučaju raspodele nanočestica tipa teškog repa. Uočava se da Debora broj ima destabilizujući uticaj na sistem, dok vremenski parametar kašnjenja deformacije ima stabilizujući uticaj na oscilatorno strujanje sistema. Uticaj Liusovog broja teži ka stabilizaciji stacionarnog strujanja i destabilizaciji oscilatornog strujanja. Koncentracioni Rejlejev broj pokazuje destabilizujući uticaj na stacionarno strujanje i stabilizujući uticaj na oscilatorno strujanje. Poroznost sredine ima stabilizujući uticaj na oscilatorno strujanje i destabilizujući uticaj na stacionarno strujanje. Uticaj Vadazovog broja na oscilatorno strujanje je destabilizirajuće. Naizmenično električno polje ima destabilizirajući uticaj na stacionarno i na oscilatorno strujanje.

INTRODUCTION

Nanotechnology has attracted several new investigators and inventors because of its indefinite progress in the current period. Nanofluids are contrived by suspending nanoparticles in the range of 1 to 100 nm, which was first utilised by Choi /4/ in traditional heat transfer fluids such as water, bio-fluids, polymer solution, oil, and ethylene glycol. Nanofluids are used for a wide range of applications in chemical, biological, medical, electronics engineering and in many industrial sectors due to their enhanced characteristic in thermal conductivity. Jang and Choi /5/ inspected the part of Brownian motion in the improved thermal conductivity of nanofluids. Buongiorno /1/ communicated a model in which two effects - Brownian motion and thermophoresis are combined and used by many scholars to study the thermal convection in a nanofluid layer by spreading several features in saturated porous and non-porous medium.

Sheu /18/ has calculated the linear stability of convection in a viscoelastic nanofluid layer using Oldroydian model. Sheu /19/ has considered the thermal instability in a porous medium layer saturated with a viscoelastic nanofluid and

established that the oscillatory instability is possible in both bottom and top-heavy nanoparticle distributions. Sharma et al. /16/ have considered overstable convection in a visco-elastic nanofluid layer using Oldroydian model saturated by a Darcy-Brinkman porous medium in the presence of suspended dust particles. They have established that suspended dust particles destabilize the system. Presently they stretched this work to study the effect of rotation and found a stabilizing effect of rotation on the physical system.

Natural convection under AC/DC electric field of electrically improved heat transfer in fluids and likely applied uses has been studied by Jones /6/ and Chen et al. /3/. The problem of convective heat transfer through polarized dielectric liquids was analysed by Stiles et al. /21/. It is concluded that the convection pattern recognized by the electric field is somewhat like to the acquainted Bénard cells in common convection. Shivakumara et al. /20/ studied the consequences of velocity and temperature boundaries conditions on electro-thermal convection in a rotating dielectric fluid and concluded that AC electric field is to boost the heat transfer and to speed up the onset of convection. The dielectric nanofluid might be used in an electrical device for example instrument transformers, distribution transformers, regulating transformers, converter transformers, and power transformers.

Sharma et al. /17/ have analysed thermal convection in dielectric rheological nanofluid layer with AC electric field. By means of Maxwellian model to state the rheology of the nanofluid and acknowledge that the effect of the electric field and stress relaxation parameter are to destabilize both stationary and oscillatory modes for bottom-heavy distribution of nanoparticles. Sharma et al. /13/ investigated thermosolutal convection of an elastic-viscous nanofluid in porous medium in the presence of rotation and magnetic field and concluded that the magnetic field and Taylor number have stabilizing effect for stationary convection, simultaneously the solutal Rayleigh- and nanoparticle Rayleigh number, thermo-nanofluid Lewis number, and modified diffusivity ratio have a destabilizing effect for stationary convection. Sharma et al. /15/ studied the problem of Rivlin-Ericksen fluid in a Darcy-Brinkman porous medium in the presence of suspended particles with variable gravity. Sharma et al. /7, 14/ analysed thermosolutal convection in a Jeffrey nanofluid with porous medium and the effect of rotation on thermosolutal convection in Jeffrey nanofluid with a porous medium.

This brief review of literature reflects that the studies on such topics are lacking, hence, the present problem onset of thermal convection in an electrically conducting rheological nanofluid to include an external vertical AC electric field saturated by a homogeneous porous medium has been studied using linear stability theory by employing an Oldroydian model which incorporates the effects of electric field, Brownian motion, thermophoresis, and rheological parameters for bottom heavy distribution of nanoparticles.

MATHEMATICAL FORMULATION OF PROBLEM

An infinitely extending electrically conducting horizontal layer of an incompressible non-Newtonian Oldroydian nanofluid of thickness d heated from below is considered by taking vertical gravity force field $\boldsymbol{g}(0,0,-g)$. A Cartesian frame of reference is occupied with the origin at the lower boundary and the z-axis vertically upwards. The lower and upper boundaries are maintained at constant, but different temperatures T and volumetric fraction of nanoparticles ϕ are taken to be T_0 and ϕ_0 at $z = 0$ and T_1 and ϕ_1 at $z = d$ ($T_0 > T_1$ and $\phi_1 > \phi_0$). This dielectric nanofluid layer is subjected to a uniform vertical AC electric field. The electrical circuit is taken at the lower surface against which other potentials are measured as the root mean square value of electric field. Thermo-physical properties of the nanofluid are constant for the analytical formulation but these properties are not constant and strongly depend upon volume fraction of the nanoparticles.

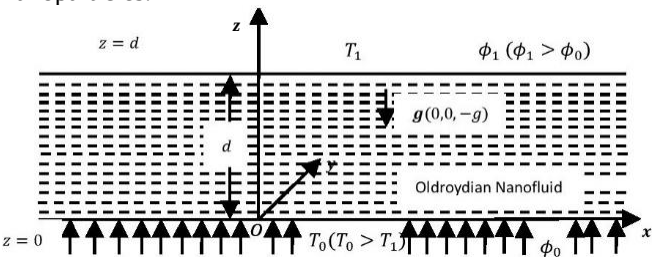

Figure 1. Physical configuration.

GOVERNING EQUATIONS

The basic hydrodynamic equations that govern the physical problem using Lapwood /8/, Chandrasekhar /2/, Buongiorno /1/, Roberts /12/, and Oldroyd /11/ are:

$$\nabla.\boldsymbol{q_D} = 0, \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial}{\partial t}+\frac{1}{\varepsilon}\boldsymbol{q_D}.\nabla\right]\boldsymbol{q_D} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla p+\left(\phi\rho_p+(1-\phi)\times\right.\right.$$
$$\left.\left.\times\rho_f\left\{1-\beta(T-T_1)\right\}\right)\boldsymbol{g}+\boldsymbol{f_e}\right]-\frac{\mu}{k_1}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\boldsymbol{q_D}, \tag{2}$$

where: $\boldsymbol{f_e}$ is the electrical origin force which is given by

$$\boldsymbol{f_e} = \rho_e\boldsymbol{E}-\frac{1}{2}\boldsymbol{E}^2\nabla K+\frac{1}{2}\left(\rho\frac{\partial K}{\partial t}\boldsymbol{E}^2\right),$$

where: ρ_e is the density of charge; K is the electric constant; $\boldsymbol{E}$ is the electric field. The term $\rho_e\boldsymbol{E}$ is the force due to a free charge known as Coulomb force. The term $-\boldsymbol{E}^2\nabla K/2$ depends on the gradient of K, known as dielectrophoretic force. Due to the dielectric constant K and electrical conductivity, the free charge is prevented for a long time, so relaxation appears in the presence of the electric field in most dielectric fluids at standard power-line frequencies. Thus, dielectric loss produced at these frequencies becomes very low that it makes no contribution to the temperature field. Therefore, the term $\rho_e\boldsymbol{E}$ is neglected as compared to the term $-\boldsymbol{E}^2\nabla K/2$ for most dielectric fluids.

The modified pressure term is

$$P = p-\frac{1}{2}\left(\rho\frac{\partial K}{\partial t}\boldsymbol{E}^2\right), \tag{3}$$

where: p is hydrodynamical pressure.

Assuming free charge density to be very small, the relevant Maxwell equations /9/ are

$$\nabla.(K\boldsymbol{E}) = 0, \quad \nabla\times\boldsymbol{E} = 0. \tag{4}$$

In view of Eq.(4), $\boldsymbol{E}$ can be expressed as

$$\boldsymbol{E} = -\nabla\varphi\,. \tag{5}$$

It is also assumed that the dielectric constant K can be expressed as (Yadav et al. /22/)

$$K = K_0[1-\gamma(T-T_1)], \tag{6}$$

where: $\gamma > 0$, is the coefficient of the dielectric constant, assumed to be small, $0 < \gamma\Delta T << 1$.

Thus, the modified equations of motion for Oldroydian nanofluid saturating a porous medium in the presence of the electric field become

$$\frac{\rho_f}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial}{\partial t}+\frac{1}{\varepsilon}\boldsymbol{q_D}.\nabla\right]\boldsymbol{q_D} = \left(1+\lambda\frac{\partial}{\partial t}\right)\Big[-\nabla p+\Big(\phi\rho_p+(1-\phi)\times$$
$$\times\rho_f\{1-\beta(T-T_1)\}\Big)\boldsymbol{g}-\frac{1}{2}(\boldsymbol{E}.\boldsymbol{E})\nabla K\Big]-\frac{\mu}{k_1}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\boldsymbol{q_D}. \tag{7}$$

The equation of continuity for the nanoparticles is

$$\left[\frac{\partial}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{q_D}.\nabla)\right]\phi = D_B\nabla^2\phi+\left(\frac{D_T}{T_1}\right)\nabla^2 T\,, \tag{8}$$

where: D_B is the Brownian diffusion coefficient; and D_T is the thermophoresis diffusion coefficient.

The equation of heat energy of nanofluid saturating a porous medium is

$$(\rho c)_m\frac{\partial T}{\partial t}+(\rho c)_f\,\boldsymbol{q_D}.\nabla T = k_m\nabla^2 T+\varepsilon(\rho c)_p\times$$
$$\times\left[D_B\nabla\phi.\nabla T+\left(\frac{D_T}{T_1}\right)\nabla T.\nabla T\right], \tag{9}$$

where: $\boldsymbol{q}_D$, p, ε, λ, ϕ, ρ_f, ρ_p, β, μ, k_1, λ_0, k_m, φ, $(\rho c)_f$ are the Darcy velocity, pressure, porosity, relaxation time, nanoparticles volume fraction, density of base fluid, density of nanoparticles, coefficient of volume expansion, coefficient of viscosity, medium permeability, retardation time, coefficient of thermal conductivity, root mean square value of electric potential, heat capacity of fluid in porous medium, and the heat capacity of nanoparticles, respectively.

Hence, boundary conditions appropriate to the problem are

$$\left.\begin{aligned} &w=0,\ \frac{\partial^2 w}{\partial z^2}=0,\ \frac{\partial\varphi}{\partial z}=0,\ T=T_0,\ \phi=\phi_0\ \text{ at }\ z=0\\ &w=0,\ \frac{\partial^2 w}{\partial z^2}=0,\ \frac{\partial\varphi}{\partial z}=0,\ T=T_1,\ \phi=\phi_1\ \text{ at }\ z=d \end{aligned}\right\}. \tag{10}$$

Using the non-dimensional variables $(x^*,y^*,z^*) = \frac{(x,y,z)}{d}$, $t^* = \frac{t\alpha_m}{\sigma d^2}$, $\boldsymbol{q}_{\boldsymbol{D}}^* = \frac{\boldsymbol{q_D}\,d}{\alpha_m}$, $p^* = \frac{pk_1}{\mu\alpha_m}$, $\phi^* = \frac{\phi-\phi_0}{\phi_1-\phi_0}$, $\varphi^* = \frac{\varphi}{\gamma E_0\Delta T d}$, $T^* = \frac{T-T_1}{T_0-T_1}$, $\boldsymbol{E}^* = \frac{\boldsymbol{E}}{\gamma E_0\Delta T d}$, $K^* = \frac{K}{K_0}$,

where: $\sigma = (\rho c)_m/(\rho c)_f$ and $\alpha_m = k_m/(\rho c)_f$ are heat capacity ratio and thermal diffusivity of the porous medium, in respect.

The non-dimensional forms of Eqs.(1) and (5)-(9) are (asterisk is removed for convenience):

$$\nabla.\boldsymbol{q_D} = 0\,, \tag{11}$$

$$\frac{1}{V_a}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{\sigma}\frac{\partial}{\partial t}+\frac{1}{\varepsilon}\boldsymbol{q_D}.\nabla\right]\boldsymbol{q_D} = \left(1+\lambda_1\frac{\partial}{\partial t}\right)\Big[-\nabla p-R_n\phi\hat{\boldsymbol{e}}_{\boldsymbol{z}}-$$
$$-R_m\hat{\boldsymbol{e}}_{\boldsymbol{z}}+R_D T\hat{\boldsymbol{e}}_{\boldsymbol{z}}-R_e\frac{\partial\varphi}{\partial z}\Big]-\left(1+\lambda_2\frac{\partial}{\partial t}\right)\boldsymbol{q_D}, \tag{12}$$

$$\frac{1}{\sigma}\frac{\partial\phi}{\partial t}+\frac{1}{\varepsilon}\boldsymbol{q_D}.\nabla\phi = \frac{1}{L_e}\nabla^2\phi+\frac{N_A}{L_e}\nabla^2 T\,, \tag{13}$$

$$\frac{\partial T}{\partial t}+\boldsymbol{q_D}.\nabla T = \nabla^2 T+\frac{N_B}{L_e}\nabla\phi.\nabla T+\frac{N_A N_B}{L_e}\nabla T.\nabla T\,, \tag{14}$$

$$\boldsymbol{E} = -\nabla\varphi\,, \tag{15}$$

$$K = [1-\gamma T(T_0-T_1)], \tag{16}$$

where: non-dimensional parameters are: $\lambda_1 = \lambda\alpha_m/\sigma d^2$ is the Deborah number; $\lambda_2 = \lambda_0\alpha_m/\sigma d^2$ is strain-retardation time parameter; $P_r = \mu/\rho_f\alpha_m$ is the Prandtl number; $D_r = k_1/d^2$ is Darcy number; $V_a = \varepsilon P_r/D_r$ is Vadasz number; $L_e = \alpha_m/D_B$ is Lewis number; $R_D = \rho_f g\beta d k_1(T_0 - T_1)/\mu\alpha_m$ is the Darcy Rayleigh number; $R_m = [\phi_0\rho_p + (1 - \phi_0)\rho_f]gdk_1/\mu\alpha_m$ is basic density Rayleigh number; $R_n = (\rho_p - \rho_f)(\phi_1 - \phi_0)gdk_1/\mu\alpha_m$ is the concentration Rayleigh number; $N_A = D_T(T_0 - T_1)/D_B T_1$ ($\phi_1 - \phi_0$) is modified diffusivity ratio; $N_B = \varepsilon(\rho c)_p(\phi_1 - \phi_0)/(\rho c)_f$ is modified particle-density increment; and $R_e = K\gamma^2 E_0^2(T_0 - T_1)^2 k_1 d^2/\mu\alpha_m$ is AC electric Rayleigh number.

In terms of non-dimensional form, boundary conditions Eq.(10) transform to

$$\left.\begin{aligned} &w=0,\ \frac{\partial^2 w}{\partial z^2}=0,\ \frac{\partial\varphi}{\partial z}=0,\ T=1,\ \phi=0\ \text{ at }\ z=0\\ &w=0,\ \frac{\partial^2 w}{\partial z^2}=0,\ \frac{\partial\varphi}{\partial z}=0,\ T=0,\ \phi=1\ \text{ at }\ z=1 \end{aligned}\right\}. \tag{17}$$

BASIC STATE SOLUTION

The basic state is given as

$$\left.\begin{aligned} &\boldsymbol{q_D} = (u,v,w) = (0,0,0),\ p = p_b(z),\ T = T_b(z),\\ &\phi = \phi_b(z),\ K = K_b(z),\ \boldsymbol{E} = \boldsymbol{E_b}(z),\ \varphi = \varphi_b(z) \end{aligned}\right\}. \tag{18}$$

When no motion is present, Eqs.(13) and (14) require the temperature and the volumetric fraction of nanoparticles to satisfy the equations

$$\frac{d^2\phi_b}{dz^2}+N_A\frac{d^2 T_b}{dz^2} = 0\,, \tag{19}$$

$$\frac{d^2 T_b}{dz^2}+\frac{N_B}{L_e}\frac{d\phi_b}{dz}\frac{dT_b}{dz}+\frac{N_A N_B}{L_e}\frac{dT_b}{dz}\frac{dT_b}{dz} = 0\,. \tag{20}$$

Using the boundary conditions Eq.(17), Eq.(19) can be integrated to give

$$\phi_b(z) = -N_A T_b+(1-N_A)z+N_A\,. \tag{21}$$

Substituting ϕ_b from Eq.(21) into Eq.(20), we get

$$\frac{d^2 T_b}{dz^2}+\frac{(1-N_A)}{L_e}\frac{dT_b}{dz} = 0\,. \tag{22}$$

Equation (22) along with boundary condition Eq.(17) gives the solution as

$$T_b = e^{-(1-N_A)N_B z/L_e}\left(\frac{1-e^{-(1-N_A)N_B(1-z)/L_e}}{1-e^{-(1-N_A)N_B/L_e}}\right). \tag{23}$$

As suggested by Buongiorno /1/, for most of the nanofluid investigated so far, permissible values of L_e range from 10^2 to 10^3. Then, the terms of second and higher order in the expansion of exponential function in Eq.(23) are neglected as they are small, and the best approximate initial stationary state solutions are given as

$$T_b = 1 - z,\ \phi_b = z,\ K_b = 1 + \gamma \nabla T z,\ \boldsymbol{E_b} = \frac{E_0}{\gamma \Delta T (1 + \gamma \Delta T z)},$$

$$\varphi_b = -\frac{E_0}{(\gamma \Delta T)^2} \log(1 + \gamma \Delta T z), \quad (24)$$

where: $\Delta T = (T_0 - T_1)$; and $E_0 = -\gamma \Delta T \varphi / \log(1 + \gamma \Delta T)$ is the root mean square value of the electric field at $z = 0$.

PERTURBATION EQUATIONS

Let the basic state as described by Eq.(24) be slightly disturbed by superimposing infinitesimal disturbances to the state variables so that

$$\boldsymbol{q_D} = (0,0,0) + \boldsymbol{q'_D}(u', v', w'),\ T = T_b + T',\ \phi = \phi_b + \phi',$$
$$p = p_b + p',\ K = K_b + K',\ \boldsymbol{E} = \boldsymbol{E_b} + \boldsymbol{E'},\ \varphi = \varphi_b + \varphi', \quad (25)$$

where: $\boldsymbol{q'_D}(u,v,w)$, T', ϕ', p', K', $\boldsymbol{E'}$, and φ' are perturbations in the nanofluid velocity, temperature, volumetric fraction, pressure, dielectric constant, electric field, and electric potential, respectively.

Using these perturbations given by Eq.(25) and the linear stability theory (i.e., by neglecting the terms of higher powers than the first, and products of perturbations) in Eqs.(11)-(16), the resulting linearized non-dimensional perturbed equations are:

$$\left[\left(1 + \lambda_2 \frac{\partial}{\partial t}\right) + \frac{1}{\sigma V_a}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\right]\nabla^2 w' - \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \times$$
$$\times \left[-R_n \nabla_H^2 \phi' + R_D \nabla_H^2 T' + R_e \nabla_H^2 T' - R_e \nabla_H^2 \frac{\partial \varphi'}{\partial z}\right] = 0, \quad (26)$$

$$\frac{1}{\sigma}\frac{\partial \phi'}{\partial t} + \frac{w'}{\varepsilon} = \frac{1}{L_e}\nabla^2 \phi' + \frac{N_A}{L_e}\nabla^2 T', \quad (27)$$

$$\frac{\partial T'}{\partial t} - w' = \nabla^2 T' + \frac{N_B}{L_e}\left(\frac{\partial T'}{\partial z} - \frac{\partial \phi'}{\partial z}\right) - \frac{2 N_A N_B}{L_e}\frac{\partial T'}{\partial z}, \quad (28)$$

$$\frac{\partial T'}{\partial z} - \nabla^2 \varphi' = 0. \quad (29)$$

The boundary conditions Eq.(17) for the infinitesimal perturbations become

$$\left.\begin{aligned} &w' = 0,\ \frac{\partial^2 w'}{\partial z^2} = 0,\ \frac{\partial \varphi'}{\partial z} = 0,\ T' = 1,\ \phi' = 0 \ \text{ at } z = 0 \\ &w' = 0,\ \frac{\partial^2 w'}{\partial z^2} = 0,\ \frac{\partial \varphi'}{\partial z} = 0,\ T' = 0,\ \phi' = 1 \ \text{ at } z = 1 \end{aligned}\right\}, \quad (30)$$

where: $\nabla_H^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplacian operator on the horizontal plane; and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the three-dimensional Laplacian operator.

NORMAL MODE ANALYSIS

For the system of Eqs.(26)-(29), the analysis can be made in terms of two-dimensional periodic waves of assigned wave numbers. Thus, we ascribe to the quantities describing the dependence on x, y, and t of the form $\exp(ik_x x + ik_y y + st)$, where k_x and k_y are the wave numbers in x- and y-direction, respectively, and $a^2 = k_x^2 + k_y^2$ is the resultant wave number, s is growth rate, which in general is a complex constant.

The above consideration allows to suppose that perturbation quantities w', T', ϕ', and φ' are of the form

$$\left.\begin{aligned} w' &= W \exp(ik_x x + ik_y y + st) \\ T' &= \Theta \exp(ik_x x + ik_y y + st) \\ \phi' &= \Phi \exp(ik_x x + ik_y y + st) \\ \varphi' &= \Psi \exp(ik_x x + ik_y y + st) \end{aligned}\right\}. \quad (31)$$

Using Eq.(31), the set of partial differential equations Eqs.(26)-(29), reduces to ordinary differential equations as:

$$\left[(1 + \lambda_2 s) + \frac{1}{\sigma V_a}(1 + \lambda_1 s)s\right](D^2 - a^2)W - (1 + \lambda_1 s)[a^2 R_n \Phi -$$
$$-\alpha^2 (R_D + R_e)\Theta + a^2 R_e D\Psi] = 0, \quad (32)$$

$$\frac{s}{\sigma}\Phi + \frac{W}{\varepsilon} = \frac{1}{L_e}(D^2 - a^2)\Phi + \frac{N_A}{L_e}(D^2 - a^2)\Theta, \quad (33)$$

$$s\Theta - W = (D^2 - a^2)\Theta + \frac{N_B}{L_e}(D\Theta - D\Phi) - \frac{2 N_A N_B}{L_e} D\Theta, \quad (34)$$

$$D\Theta - (D^2 - a^2)\Psi = 0, \quad (35)$$

where: $D = d/dz$; and $a = \sqrt{(k_x^2 + k_y^2)}$ is the dimensionless resultant wave number.

Boundary conditions Eq.(30) using Eq.(31) become:

$$W = D^2 W = \Theta = \Phi = D\Psi = 0 \ \text{ at } z = 0 \text{ and } z = 1. \quad (36)$$

Therefore, the trial functions of lowest mode satisfying the boundary conditions Eq.(36) are chosen as

$$W = A_1 \sin \pi z,\ \Theta = A_2 \sin \pi z,\ \Phi = A_3 \sin \pi z,\ \Psi = A_4 \cos \pi z, \quad (37)$$

where: A_1, A_2, A_3, and A_4 are constants.

Substituting the trial functions given by Eq.(37) in Eqs.(32)-(35) and integrating by parts using the condition of orthogonality and boundary conditions Eq.(36), the following matrix equation is obtained

$$\begin{bmatrix} \left\{(1 + \lambda_2 s) + \frac{1}{\sigma V_a}(1 + \lambda_1 s)s\right\} J & -a^2(1 + \lambda_1 s)(R_D + R_e) & a^2(1 + \lambda_1 s)R_n & -a^2 \pi (1 + \lambda_1 s)R_e \\ 1 & -J - s & 0 & 0 \\ \frac{1}{\varepsilon} & \frac{N_A J}{L_e} & \frac{J}{L_e} + \frac{s}{\sigma} & 0 \\ 0 & -\pi & 0 & -J \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (38)$$

where: $J = (\pi^2 + a^2)$.

The non-trivial solution of the above matrix equation requires the determinant of the coefficients to vanish, which yields the Darcy thermal Rayleigh number,

$$R_D=\frac{(\pi^2+a^2+s)(\pi^2+a^2)}{a^2(1+\lambda_1 s)}\left[(1+\lambda_2 s)+\frac{s(1+\lambda_1 s)}{\sigma V_a}\right]-\frac{a^2}{\pi^2+a^2}R_e-\frac{\sigma L_e}{\sigma(\pi^2+a^2)+sL_e}\left[\frac{\pi^2+a^2+s}{\varepsilon}+\frac{(\pi^2+a^2)N_A}{L_e}\right]R_n. \quad (39)$$

Equation (39) is the dispersion relation accounting for the effect of the Vadasz number V_a, analogous electric Rayleigh number R_e, nanofluid Lewis number L_e, modified diffusivity ratio N_A, and concentration Rayleigh number R_n.

LINEAR STABILITY CONVECTION

(A) Stationary convection

For the validity of principle of exchange of stabilities (i.e., steady case), we have $s = 0$ ($s = r + i\omega = 0 \Rightarrow r = \omega = 0$) at marginal stability.

Putting $s = 0$ in Eq.(39), we get the Darcy thermal Rayleigh number at which marginally stable steady mode exists as

$$R_D^s=\frac{(\pi^2+a^2)^2}{a^2}-\frac{a^2}{\pi^2+a^2}R_e-\left(\frac{L_e}{\varepsilon}+N_A\right)R_n, \quad (40)$$

which expresses the nanofluid Darcy thermal Rayleigh number R_D^s for stationary convection as a function of dimensionless wave number a, electric Rayleigh number R_e, nanofluid Lewis number L_e, modified diffusivity ratio N_A, concentration Rayleigh number R_n, and medium porosity ε. It is clear from Eq.(40) that R_D^s is independent of stress relaxation time λ_1, strain retardation time λ_2 for stationary modes, since these vanish with the vanishing of s (growth rate).

The minimum value of R_D^s is obtained by putting $\partial R_D^s/\partial a^2 = 0$, and which on simplification implies that

$$a^4\left(1+\frac{\pi^2}{a^2}\right)^3\left(1-\frac{\pi^2}{a^2}\right)=\pi^2 R_e. \quad (41)$$

Therefore, the critical wave number a_c shows a substantial increase when the electric Rayleigh number R_e increases and is independent of nanoparticles.

To study the effects of the electric field R_e, the nanofluid Lewis number L_e, the modified diffusivity ratio N_A, and the concentration Rayleigh number R_n, on stationary convection, we examine the behaviour of $\partial R_D^s/\partial R_e$, $\partial R_D^s/\partial L_e$, $\partial R_D^s/\partial N_A$, and $\partial R_D^s/\partial R_n$ analytically. From Eq.(40) we obtain

$$\frac{\partial R_D^s}{\partial R_e}=-\frac{a^2}{(\pi^2+a^2)}, \quad (42)$$

which is always negative for all wave numbers. Thus, AC electric field has a destabilizing effect on the system.

Equation (40) further yields

$$\frac{\partial R_D^s}{\partial L_e}=-\frac{R_n}{\varepsilon}, \quad \frac{\partial R_D^s}{\partial N_A}=-R_n. \quad (43)$$

It is clear from Eq.(43) for the bottom-heavy particles (for negative value of R_n) both negative nanofluid Lewis number L_e and modified diffusivity ratio N_A stabilize the system for the value of R_n.

Equation (40) also depicts that

$$\frac{\partial R_D^s}{\partial R_n}=-\left(\frac{L_e}{\varepsilon}+N_A\right), \quad (44)$$

which is always negative for $(L_e/\varepsilon + N_A) > 0$, since the value of N_A is taken in the range of -1 to -25, and L_e in the range of 100-400. Thus R_D^s decreases with increase in R_n implying thereby the destabilizing effect of R_n on the onset of stationary convection.

(B) Oscillatory convection

Let us write the growth s as $s = r + i\omega$, where r and ω are real. For oscillatory convection, $s \neq 0$ and $r = 0$, i.e., $s = i\omega \neq 0$.

Putting $s = i\omega$ in Eq.(39), we get

$$R_D=\Delta_1+i\omega\Delta_2, \quad (45)$$

where: Δ_1 and Δ_2 are given by

$$\Delta_1=\frac{(\pi^2+a^2)(\pi^2+a^2+\omega^2\lambda_1)}{a^2(1+\omega^2\lambda_1)}-\frac{\omega^2}{\sigma V_a}\left(1+\frac{\pi^2}{a^2}\right)-\frac{a^2\omega^2 L_e}{(\pi^2+a^2)^3\sigma^2+(\pi^2+a^2)\omega^2L_e^2}R_e-\frac{\sigma L_e((\pi^2+a^2)^2\sigma+\omega^2L_e)}{\varepsilon((\pi^2+a^2)^2\sigma^2+\omega^2L_e^2)}R_n-$$
$$-\frac{\sigma^2(a^2R_e+(\pi^2+a^2)N_AR_n)(\pi^2+a^2)}{(\pi^2+a^2)^2\sigma^2+\omega^2L_e^2}-\frac{\omega^2\lambda_2(1-\lambda_1(\pi^2+a^2))(\pi^2+a^2)}{a^2(1+\omega^2\lambda_1^2)}, \quad (46)$$

and

$$\Delta_2=-\frac{\sigma L_e(\sigma-L_e)(\pi^2+a^2)}{\varepsilon((\pi^2+a^2)^2\sigma^2+\omega^2L_e^2)}R_n+\frac{\sigma L_eN_A(\pi^2+a^2)}{(\pi^2+a^2)^2\sigma^2+\omega^2L_e^2}R_n+\frac{(\pi^2+a^2)((\pi^2+a^2)+\omega^2\lambda_1)}{a^2(1+\omega^2\lambda_1^2)}\left(\lambda_2+\frac{1}{\sigma V_a}\right)+$$
$$+\frac{(\pi^2+a^2)(1-\lambda_1(\pi^2+a^2))}{a^2(1+\omega^2\lambda_1^2)}\left(1-\frac{\omega^2\lambda_1}{\sigma V_a}\right). \quad (47)$$

Equating real and imaginary parts of Eq.(45) we get $R_D = \Delta_1$ which, on simplification, gives value of the Darcy thermal Rayleigh number for oscillating modes as

$$R_D=\frac{(\pi^2+a^2)(\pi^2+a^2+\omega^2\lambda_1)}{a^2(1+\omega^2\lambda_1^2)}-\frac{\omega^2}{\sigma V_a}\left(1+\frac{\pi^2}{a^2}\right)-\frac{a^2\omega^2 L_e}{(\pi^2+a^2)^3\sigma^2+(\pi^2+a^2)\omega^2L_e^2}R_e-\frac{\sigma L_e((\pi^2+a^2)^2\sigma+\omega^2L_e)}{\varepsilon((\pi^2+a^2)^2\sigma^2+\omega^2L_e^2)}R_n-$$
$$-\frac{\sigma^2(a^2R_e+(\pi^2+a^2)N_AR_n)(\pi^2+a^2)}{(\pi^2+a^2)^2\sigma^2+\omega^2L_e^2}-\frac{\omega^2\lambda_2(1-\lambda_1(\pi^2+a^2))(\pi^2+a^2)}{a^2(1+\omega^2\lambda_1^2)}, \quad (48)$$

and

$$\sigma^2a^2\lambda_1^2L_e^2V_aR_n(\pi^2+a^2)(1+\omega^2\lambda_1^2)-\sigma^2a^2L_eV_aR_n(\sigma-\varepsilon N_A)(\pi^2+a^2)(1+\omega^2\lambda_1^2)+\varepsilon\sigma^2(-\sigma\lambda_1V_a+\omega^2\lambda_1^2)(\pi^2+a^2)^4+$$
$$+\varepsilon\sigma^2(\pi^2+a^2)^4+\varepsilon\omega^2L_e^2(\pi^2+a^2)^2+\varepsilon\sigma\omega^2L_e^2V_a(\lambda_2-\lambda_1)(\pi^2+a^2)^2+\varepsilon\sigma^3V_a(\pi^2+a^2)^3+\varepsilon\sigma\omega^2L_e^2V_a(\pi^2+a^2)+$$
$$+\varepsilon\lambda_1^2\omega^4L_e^2(\pi^2+a^2)^2+\varepsilon\sigma^3\lambda_2V_a(\pi^2+a^2)^4+\varepsilon\sigma^3\lambda_1\lambda_2\omega^2V_a(\pi^2+a^2)^3+\varepsilon\sigma\lambda_1\lambda_2\omega^4L_e^2(\pi^2+a^2)=0. \quad (49)$$

(C) Overstability convection

Here we consider the possibility of whether instability may occur as overstability. Since for overstability, we wish to determine the critical Darcy thermal Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find the conditions for which Eq.(39) will admit of solutions with ω real.

The Eq.(49) on simplification gives a dispersion relation (relation between growth rate ω and wave number a) of the form

$$a_1(\omega^2)^2 + a_2\omega^2 + a_3 = 0\,, \tag{50}$$

where,

$$a_1 = \varepsilon\lambda_1^2 L_e^2(\pi^2+a^2)^2 + \varepsilon\sigma\lambda_1\lambda_2 L_e^2(\pi^2+a^2), \tag{51}$$

$$a_2 = \sigma^2 a^2\lambda_1^2 L_e^2 V_a R_n(\pi^2+a^2) - \lambda_1^2\sigma^2 a^2 L_e V_a R_n(\sigma-\varepsilon N_A)(\pi^2+a^2) + \varepsilon\sigma^2\lambda_1^2(\pi^2+a^2)^4 + \varepsilon L_e^2(\pi^2+a^2)^2 + \\ + \varepsilon\sigma L_e^2 V_a(\lambda_2-\lambda_1)(\pi^2+a^2)^2 + \varepsilon\sigma L_e^2 V_a(\pi^2+a^2) + \varepsilon\sigma^3\lambda_1\lambda_2 V_a(\pi^2+a^2)^3\,, \tag{52}$$

$$a_3 = \sigma^2 a^2 L_e^2 V_a R_n(\pi^2+a^2) - \sigma^2 a^2 L_e V_a R_n(\sigma-\varepsilon N_A)(\pi^2+a^2) - \varepsilon\sigma^3\lambda_1 V_a(\pi^2+a^2)^4 + \varepsilon\sigma^2(\pi^2+a^2)^4 + +\varepsilon\sigma^3 V_a(\pi^2+a^2)^3 + \\ + \varepsilon\sigma^3\lambda_2 V_a(\pi^2+a^2)^4\,. \tag{53}$$

The expression of Darcy thermal Rayleigh number for overstability is given by

$$R_D^0 = \frac{(\pi^2+a^2)(\pi^2+a^2+\omega^2\lambda_1)}{a^2(1+\omega^2\lambda_1^2)} - \frac{\omega^2}{\sigma V_a}\left(1+\frac{\pi^2}{a^2}\right) - \frac{a^2\omega^2 L_e}{(\pi^2+a^2)^3\sigma^2+(\pi^2+a^2)\omega^2 L_e^2}R_e - \frac{\sigma L_e((\pi^2+a^2)^2\sigma+\omega^2 L_e)}{\varepsilon((\pi^2+a^2)^2\sigma^2+\omega^2 L_e^2)}R_n - \\ - \frac{\sigma^2(a^2 R_e+(\pi^2+a^2)N_A R_n)(\pi^2+a^2)}{(\pi^2+a^2)^2\sigma^2+\omega^2 L_e^2} - \frac{\omega^2\lambda_2(1-\lambda_1(\pi^2+a^2))(\pi^2+a^2)}{a^2(1+\omega^2\lambda_1^2)}\,. \tag{54}$$

Here, the value of ω^2 is calculated using Eq.(50). In the absence of electric field, i.e., $R_e = 0$, Eq.(54) reduces to

$$R_D^0 = \frac{(\pi^2+a^2)(\pi^2+a^2+\omega^2\lambda_1)}{a^2(1+\omega^2\lambda_1^2)} - \frac{\omega^2}{\sigma V_a}\left(1+\frac{\pi^2}{a^2}\right) - \frac{\sigma L_e((\pi^2+a^2)^2\sigma+\omega^2 L_e)}{\varepsilon((\pi^2+a^2)^2\sigma^2+\omega^2 L_e^2)}R_n - \frac{\sigma^2(\pi^2+a^2)^2 N_A R_n}{(\pi^2+a^2)^2\sigma^2+\omega^2 L_e^2} - \\ - \frac{\omega^2\lambda_2(1-\lambda_1(\pi^2+a^2))(\pi^2+a^2)}{a^2(1+\omega^2\lambda_1^2)}\,, \tag{55}$$

which is in good agreement with the earlier results given by Sheu /18/.

Further, when the nanoparticles are not considered, i.e., $R_n = 0$, $N_A = 0$, Eq.(55) gives

$$R_D^0 = \frac{(\pi^2+a^2)(\pi^2+a^2+\omega^2\lambda_1)}{a^2(1+\omega^2\lambda_1^2)} - \frac{\omega^2}{\sigma V_a}\left(1+\frac{\pi^2}{a^2}\right) - \frac{\omega^2\lambda_2(1-\lambda_1(\pi^2+a^2))(\pi^2+a^2)}{a^2(1+\omega^2\lambda_1^2)}\,, \tag{56}$$

which is again in good agreement with the earlier results given by Robert /12/.

Further, in the absence of stress relaxation time parameter λ_1 and strain retardation time parameter λ_2, Eq.(56) reduces to

$$R_D^0 = \frac{(\pi^2+a^2)^2}{a^2} - \frac{\omega^2}{\sigma V_a}\left(1+\frac{\pi^2}{a^2}\right), \tag{57}$$

which is in good agreement with the earlier results given by Lapwood /8/.

NUMERICAL DISCUSSION

Expressions of thermal Rayleigh number for both stationary and oscillatory motions are presented in Eqs.(40) and (54), respectively. Equation (50) is quite complicated to find the analytical roots and obtain non-dimensional wave numbers so as to find critical thermal Rayleigh numbers for overstable motions, which only occur for positive values of growth rate ω. Thus, numerical roots of Eq.(50) are obtained using software *Scientific WorkPlace*® for bottom-heavy configuration. The variation of Rayleigh number with respect to wave-number has been plotted using Eq.(54) for oscillatory case and Eq.(40) for stationary case, whereas experimental values and fixed permissible values of dimensionless parameters are the same as those taken by Buongiorno, Roberts, Yadav and Sharma, and many others to investigate the effects of stress-relaxation time parameter, strain retardation time, porosity, modified diffusivity ratio, Vadasz number, Lewis number, concentration Rayleigh number, heat capacity ratio, and electric Rayleigh number, i.e., $\lambda_1 = 0.8$, $\lambda_2 = 0.3$, $\varepsilon = 0.6$, $N_A = -5$, $V_a = 3$, $L_e = 200$, $R_n = -0.1$, $\sigma = 1.5$, $R_e = 100$. The stationary thermal Rayleigh number is found to be independent of stress relaxation time and strain retardation time, since it vanishes with the vanishing of s (growth rate). Thus, the viscoelastic (Oldroydian) nanofluid behaves like a regular (Newtonian) nanofluid. Nield and Kuznetsov /10/ have shown the possibility of oscillatory motions to set in only for the bottom-heavy nanoparticle distributions.

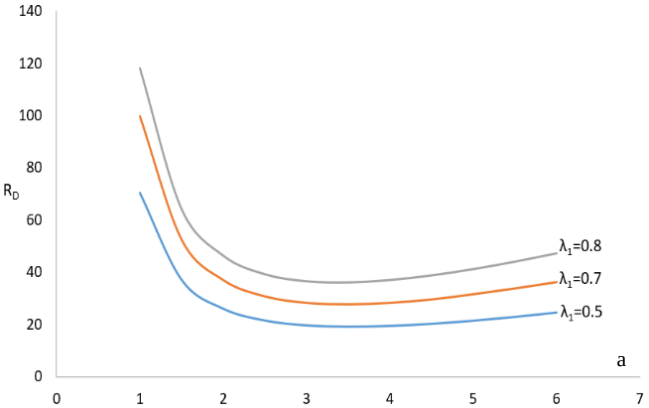

Figure 2. Variations of oscillatory thermal Rayleigh number for 3 different values of the Deborah number $\lambda_1 = 0.5, 0.7, 0.8$.

Figure 2 shows the variation of thermal Rayleigh number for oscillatory convection with respect to non-dimensional wave number for three different values of Deborah number, $\lambda_1 = 0.5, 0.7, 0.8$, accounting for the stress relaxation time and

for fixed permissible values of $\varepsilon = 0.6$, $\lambda_2 = 0.3$, $N_A = -5$, $V_a = 3$, $R_n = -0.1$, $L_e = 200$, $\sigma = 1.5$, $R_e = 100$. The graph shows that the value of thermal Rayleigh number increases with increase in stress relaxation time, implying thereby a stabilizing effect of stress relaxation time on the system.

The effect of the strain-retardation time parameter λ_2 on the Darcy thermal Rayleigh number for the exchange of stabilities is displayed in Fig. 3. It is found that with increase in strain-retardation time, the oscillatory thermal Rayleigh number decreases, implying thereby that the onset of convection in viscoelastic nanofluid in a porous medium is postponed. Thus, the strain-retardation time λ_2 has a destabilizing effect on the system.

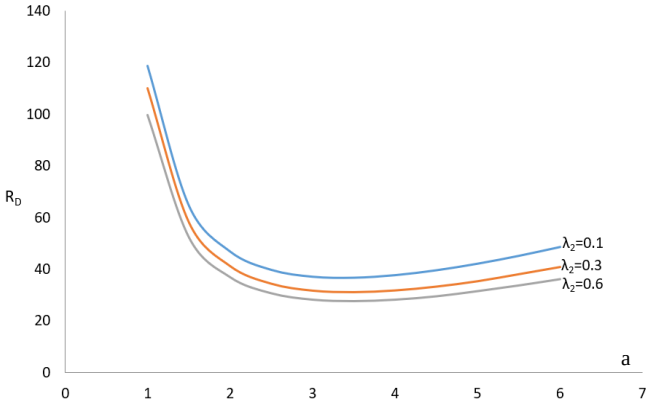

Figure 3. Variations of oscillatory thermal Rayleigh number for 3 different values of retardation parameter $\lambda_2 = 0.1, 0.3, 0.6$.

In Fig. 4, the effect of Lewis number L_e on the neutral curves is displayed. It is found from the graphs that with an increase in Lewis number, the Darcy thermal Rayleigh number increases for stationary convection, whereas the effect on oscillatory thermal Rayleigh number is not influenced. Hence, the nanofluid thermal Rayleigh number stabilizes the physical system for stationary convection. This happens also for the Brownian motion of nanoparticles that increases with the increase in the Lewis number.

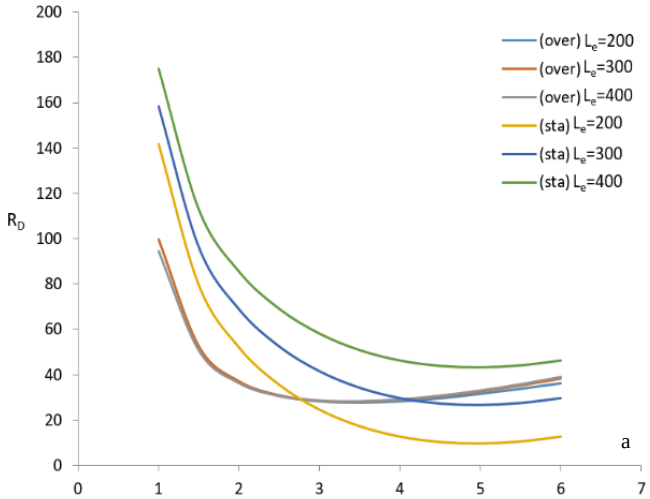

Figure 4. Variations of thermal Rayleigh number for 3 different values of Lewis number $L_e = 200, 300, 400$.

Figure 5 shows the variation of thermal Rayleigh number for oscillatory and stationary convection with respect to the non-dimensional wave number for three different values of concentration Rayleigh number $R_n = -0.1, -0.5, -0.9$ and for fixed permissible values of $\varepsilon = 0.6$, $\lambda_1 = 0.8$, $\lambda_2 = 0.3$, $N_A = -5$, $V_a = 3$, $L_e = 200$, $\sigma = 1.5$, $R_e = 100$. It is depicted from the graphs that for cases of stationary convection, the Darcy thermal Rayleigh number decreases with increase in concentration Rayleigh number which causes the destabilizing effect, and for the oscillatory convection, Darcy thermal Rayleigh number increases with the increase in concentration Rayleigh number thereby stabilizing the physical system for bottom-heavy (R_n is negative) nanoparticles distribution.

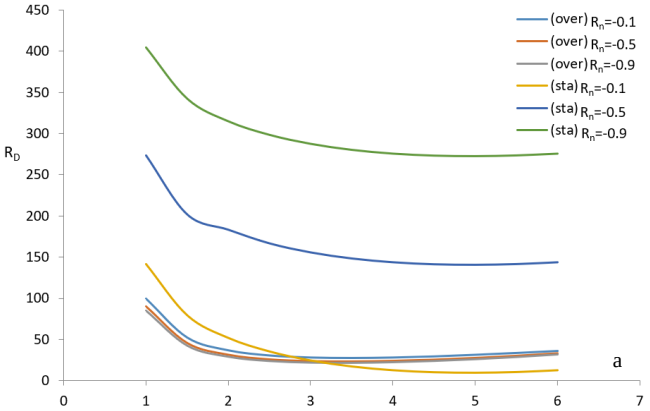

Figure 5. Variations of thermal Rayleigh number for 3 different values of concentration Rayleigh number $R_n = -0.1, -0.5, -0.9$.

In Fig. 6 the variation of thermal Rayleigh number for oscillatory and stationary convection with respect to the non-dimensional wave number for three different values of the modified diffusivity ratio $N_A = -5, -10, -20$, and for fixed permissible values of $\varepsilon = 0.6$, $\lambda_1 = 0.8$, $\lambda_2 = 0.3$, $R_n = -0.1$, $V_a = 3$, $L_e = 200$, $\sigma = 1.5$, $R_e = 100$. The graphs depict that with the increase in modified diffusivity ratio, the thermal Rayleigh number increases for stationary convection, while there is no significant effect on thermal Rayleigh number for the case of over-stability.

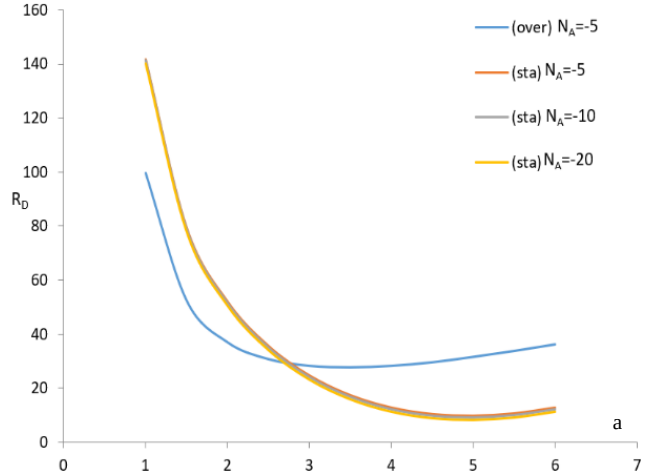

Figure 6. Variations of thermal Rayleigh number for 3 different values of modified diffusivity ratio $N_A = -5, -10, -20$.

The effect of medium porosity ε on the Darcy thermal Rayleigh number is displayed in Fig. 7. It is found that with increase in medium porosity, the oscillatory thermal Rayleigh number increases thus stabilizing the system, and for stationary convection medium porosity has a destabilizing effect on the system.

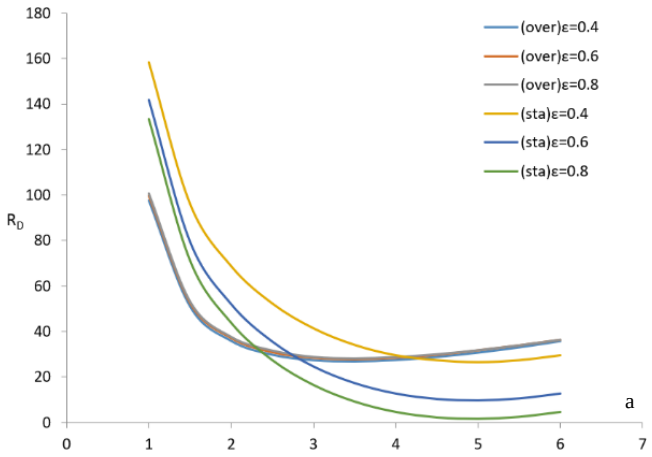

Figure 7. Variations of thermal Rayleigh number for 3 different values of medium porosity ε = 0.4, 0.6, 0.8.

Figure 8 illustrates the effect of Vadasz number V_a on the stability curves for the onset of overstability convection. It is observed from the graphs that the increase in Vadasz number decreases the thermal Rayleigh number, depicting thereby a destabilizing effect of Vadasz number on the physical system.

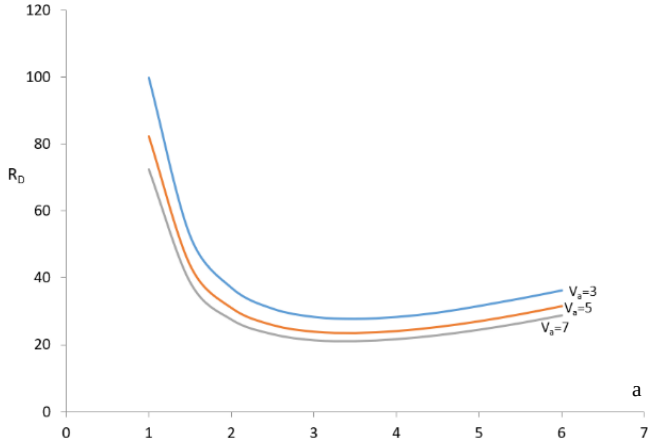

Figure 8. Variations of thermal Rayleigh number for 3 different values of Vadasz number V_a = 3, 5, 7.

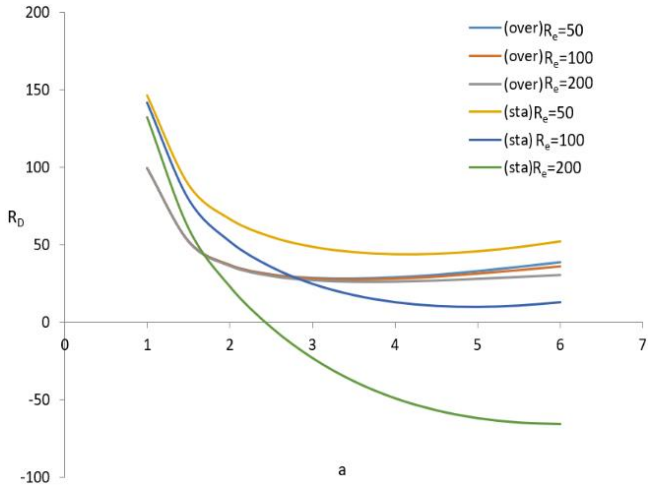

Figure 9. Variations of thermal Rayleigh number for 3 different values of AC electric Rayleigh number R_e = 50, 100, 200.

Figure 9 shows the variation of thermal Rayleigh number for oscillatory and stationary convection with respect to the non-dimensional wave number for three different values of the AC electric Rayleigh number R_e = 50, 100, 200. It is found from the graphs that with the increase in the AC electric Rayleigh number, the thermal Rayleigh number decreases for stationary convection, whereas the effect on the oscillatory thermal Rayleigh number is not influenced. Hence, the thermal Rayleigh number destabilizes the physical system for stationary convection.

CONCLUSIONS

Onset of thermal convection in an electrically conducting rheological nanofluid to include an external vertical AC electric field saturated by a homogeneous porous medium has been studied using linear stability theory by employing an Oldroydian model which incorporates the effects of electric field, Brownian motion, thermophoresis, and rheological parameters for bottom-heavy distribution of nanoparticles. Principal conclusions of the present study are given below:

- Deborah number has a stabilizing effect on the system, while strain retardation time parameter has a destabilizing effect on the oscillatory convection;
- the effect of Lewis number (non-dimensional parameter accounting for Brownian motion parameter D_B) tends to stabilize stationary convection and destabilize oscillatory convection;
- the concentration Rayleigh number has a destabilizing effect on stationary convection and a stabilizing effect on oscillatory convection;
- medium porosity has a stabilizing effect on oscillatory convection and destabilizing on stationary convection;
- the effect of Vadasz number on oscillatory convection is destabilizing;
- AC electric field has a destabilizing effect on both stationary and oscillatory convection.

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June 11-13, 2024	VAL5, 5th Int. Conf. on Material and Component Performance under Variable Amplitude Loading	Dresden, Germany	Flyer link
June 19-21, 2024	Fatigue 2024, 9th Engineering Integrity Society Int. Conf. on Durability & Fatigue	Cambridge, UK	http://fatigue2024.com/
July 7-10, 2024	ICEFA X, 10th Int. Conf. on Engineering Failure Analysis	Athens, Greece	elsevier.com/events/conferences/international-conference-on-engineering-failure-analysis
July 15-17, 2024	ICMM8, 8th Int. Conf. on Material Modelling	London, UK	https://www.lboro.ac.uk/research/icmm8/
August 26-30, 2024	ECF24, 24th European Conf. on Fracture, and Summer School	Zagreb, Croatia	www.ecf24.eu
September 3-5, 2024	DyCOMP (Int. Conf. on the Dynamic Behaviour of Composites) + ECCCS (European Conf. on Crashworthiness of Composite Structures)	Enna, Sicily, Italy	https://dycomp2024.cimne.com/
September 10-12, 2024	ESIS TC3, CP 2024, The 8th Int. Conference on Crack Paths	Rimini, Italy	https://www.crackpaths.org/
November 5-8, 2024	2024 Int. Symposium on Structural Integrity (ISSI2024)	Songshan Lake, Dongguan, China	link
November 17-19, 2024	12th Annual Conf. of the Society of Structural Integrity and Life (DIVK12)	Belgrade, Serbia	https://www.divk12.com/
November 18-21, 2024	2nd IQ-ICF Conf. in Africa and Middle East Processing, performance, and failure analysis of engineering materials	Luxor, Egypt	https://www.icf-egypt2024.org/