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SORET AND DUFOUR EFFECTS ON THERMOSOLUTAL CONVECTION IN JEFFREY NANOFLUID IN THE PRESENCE OF POROUS MEDIUM

SORET I DUFUR UTICAJI NA TERMORASTVORLJIVU KONVEKCIJU U JEFFREY NANOFLUIDU SA POROZNOM SREDINOM

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Abstract

In this paper the thermosolutal instability of Jeffrey nanofluid in a porous medium is considered. The Navier-Stokes equations of motion of the fluid are modified under the impact of the Jeffrey parameter and nanoparticles. From the linear stability analysis, based upon normal modes analysis method, the dispersion relation accounting for the effect of various parameters is derived. The effects of Jeffrey parameter λ, solutal Rayleigh number R_S, medium porosity ε, nanoparticle Rayleigh number R_N, thermo-nanofluid Lewis number L_n, thermosolutal Lewis number L_e, modified diffusivity ratio N_A, Dufour parameter N_{CT}, and Soret parameter N_{TC}, are analysed analytically and presented graphically.

Ključne reči
- Jeffrey nanofluid
- nanofluid
- termorastvorljiva neravnoteža
- porozna sredina

Izvod

U radu se razmatra neravnotežna termorastvorljivost Jeffrey nanofluida u poroznoj sredini. Modifikovane su Navije-Stoksove jednačine kretanja fluida uticajem parametra Jeffrey-ja i nanočestica. Analizom linearne stabilnosti, a na bazi metode analize u normalnom modu, izvedena je relacija disperzije, kojom se uzimaju u obzir spomenuti parametri. Uticaji parametara: Jeffrey λ, Rejlejev broj rastvora R_S, poroznost sredine ε, Rejlejev broj nanočestica R_N, termički Luisov broj nanofluida L_n, Luisov broj nanorastvorljivosti L_e, modifikovan odnos difuznosti N_A, Dufur parametar N_{CT}, kao i Soret parametar N_{TC}, su analizirani analitički i predstavljeni su grafički.

INTRODUCTION

The natural environment is filled with many components. Thermosolutal instability problems associated with various kinds of fluids have been extensively considered. Veronis /16/ deliberated the problem of thermosolutal convection in a layer of fluid heated and saluted from below. These types of problems have many uses to diverse regions such as food processing, geophysics, astrophysics, limnology, oil reservoir modelling and engineering. The nanotechnology has fascinated several investigators and inventors by reason of its indefinite progress in the present period. A nanofluid is the suspension of nanoparticles in a base fluid, which was first utilised by Choi /4/. Nanoparticles used in a nanofluid usually have diameters below 100 nm. Due to their small size, nanoparticles fluidize easily inside the base fluid and as a consequence, the blockage of channels and erosion in channel walls are no longer a problem. Nanoparticle materials include oxides (Al_2O_3, CuO), metal carbides (SiC), nitrides (AlN, SiN), metals (Al, Cu), etc. As mentioned in literature, base fluids include water, ethylene, tri-ethylene-glycols, coolants, oils, lubricants, bio fluids and polymer solutions. Buongiorno /1/ analysed the convective transport in nanofluids and concluded that the absolute velocity of the nanoparticles is expressed as the sum of the base fluid velocity and a relative velocity. By applying this model, many have researched the criteria for the onset of thermal instability. The study of nanofluid in a porous medium has attracted numerous scientists, because of their uses in steam engine industries, fuel cells, medical, domestic refrigerators, heat exchangers, nuclear reactors, converters, and biomedical appliances. Several researchers have revealed that a certain type of nanofluid can be used to eliminate and terminate cancer cells without hurting the common tissues. Thermal convection and thermosolutal convection of nanofluids in a porous medium has been considered by numerous scientists. Sheu /13/ investigated the oscillatory instability of a nanofluid-saturated porous medium by regarding the nanofluid as a viscoelastic fluid. Rana et al. /11/ have examined the problem on the onset of thermosolutal instability in a layer of an elastico-viscous nanofluid in porous medium. Chand and Rana /3/ worked on the problem of thermal instability analysis of an elastico-viscous nanofluid layer. Pundir et al. /10/ analysed the effect of rotation on the thermosolutal convection in a visco-elastic nanofluid in the presence of a porous medium. Sharma et al. /12/ worked on the thermosolutal convection problem of an elastico-viscous nanofluid in a porous medium in the presence of rotation and magnetic field and derived that the magnetic field and the Taylor number have a stabilising effect for stationary convection, consecutively the solutal Rayleigh number, nanoparticle

Rayleigh number, thermonanofluid Lewis number, and the modified diffusivity ratio, have a destabilizing effect for stationary convection. The Jeffrey fluid is a non-Newtonian fluid, which shows linear viscoelastic feature, shear thinning characteristics, produces stress and high shear viscosity. The Jeffrey model is the simplest and common among the non-Newtonian fluids which have the time derivative rather than convective derivative. It has obtained universal consideration because of its significance in processing industries such as metal and polymer sheet, etc. Most of the investigations over Jeffrey fluid are related to stretching sheet and convective flow over a segment. Jeffrey /6/ functioned on the stability of a layer of physiological fluid heated from below. Hayat et al. /5/ analysed heat transfer in convective flow of Jeffrey nanofluid by vertical stretchable cylinder. Keeping in mind the many applications listed above, the main purpose in this paper is to investigate the thermosolutal convection in the Jeffrey nanofluid with porous medium which is heated from below. This transitory review of literature reveals that this type of problem was not there, hence, the present problem on thermosolutal convection in the Jeffrey nanofluid with porous medium has been investigated.

MATHEMATICAL MODEL

Here we consider a horizontal layer with thickness d in the presence of Jeffrey nanofluid situated between plates $z = 0$ and $z = d$ (as shown in Fig. 1). The fluid layer is heated from below and working in upwards direction with a gravity force $\boldsymbol{g} = (0,0,-g)$. Temperature T, concentration C and volumetric fraction φ of nanoparticle, at upper and lower boundaries are taken to be T_1 and T_0, C_1 and C_0, φ_1 and φ_0, respectively, with $T_0 > T_1$, $C_0 > C_1$, and $\varphi_0 > \varphi_1$.

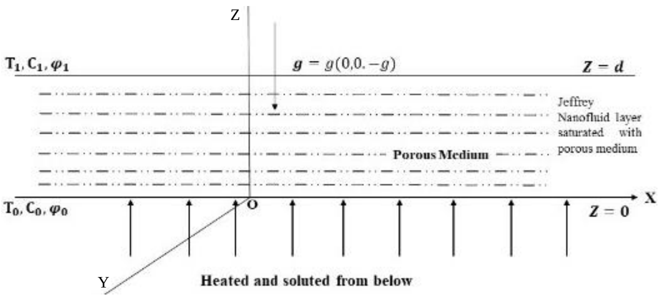

Figure 1. Physical configuration.

GOVERNING EQUATIONS

The governing equations for Jeffrey nanofluid in porous medium as given by Chandrasekhar /2/, Kuznetsov and Nield /7-9/, Pundir et al. /10/, Rana et al. /11/, and Sharma et al. /12/, are

$$\nabla.\boldsymbol{q} = 0, \tag{1}$$

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \boldsymbol{q}}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{q}.\nabla)\boldsymbol{q}\right] = -\nabla p + \rho \boldsymbol{g} - \frac{\mu}{k_1(1+\lambda)}\boldsymbol{q}, \tag{2}$$

where: ρ, μ, p, ε, $\boldsymbol{g}$, k_1, $\lambda = \lambda_1/\lambda_2$, and $\boldsymbol{q}(u,v,w)$ denote respectively the density, viscosity, pressure, medium porosity, acceleration due to gravity, coefficient of thermal conductivity, Jeffrey parameter (the ratio of stress relaxation-time parameter λ_1 to strain retardation-time parameter λ_2) and Darcy velocity vector.

Nanofluid density can be written as in Buongiorno /1/,

$$\rho = \varphi\rho_p + (1-\varphi)\rho_f, \tag{3}$$

where: φ is the volume fraction of nanoparticles; ρ_p is the density of nanoparticles; and ρ_f is the density of base fluid. Following Tzou /15, 16/, and Kuznetsov and Nield /7-9/, we approximate the density of the nanofluid by that of the base fluid, that is, we consider $\rho = \rho_f$.

Now, introducing the Boussinesq approximation for the base fluid, the specific weight, $\rho\boldsymbol{g}$ in Eq.(2) becomes

$$\rho\boldsymbol{g} \approx \left\{\varphi\rho_p + (1-\varphi)\left[\rho(1-\alpha_T(T-T_0) - \alpha_c(C-C_0))\right]\right\}\boldsymbol{g}, \tag{4}$$

where: α_T is coefficient of thermal expansion; and α_c is analogous to solute concentration.

If one introduces a buoyancy force, the equation of motion for Jeffrey nanofluid by using Boussinesq approximation and Darcy model for porous medium, Kuznetsov and Nield /7-9/, is given by

$$0 = -\nabla p + \left\{\varphi\rho_p + (1-\varphi)\left[\rho(1-\alpha_T(T-T_0) - \alpha_c(C-C_0))\right]\right\}\boldsymbol{g} - \\ -\frac{\mu}{k_1(1+\lambda)}\boldsymbol{q}. \tag{5}$$

For nanoparticles, the continuity equation given by Buongiorno /1/ is

$$\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\boldsymbol{q}.\nabla\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T, \tag{6}$$

where: D_B and D_T are the Brownian diffusion coefficient and thermophoresis diffusion coefficient, respectively.

For the nanofluid, the equation of thermal energy is given by

$$(\rho c)_m\frac{\partial T}{\partial t} + (\rho c)_f\,\boldsymbol{q}.\nabla T = k_m\nabla^2 T + \varepsilon(\rho c)_P \times \\ \times\left(D_B\nabla\varphi.\nabla T\,\frac{D_T}{T_1}\nabla T.\nabla T\right) + (\rho c)_f\, D_{TC}\nabla^2 C, \tag{7}$$

where: D_{TC} is Dufour diffusivity; k_m is thermal conductivity; $(\rho c)_P$ is the heat capacity of nanoparticles; and $(\rho c)_m$ is heat capacity of the fluid in porous medium. So,

$$\frac{\partial C}{\partial t} + \frac{1}{\varepsilon}\boldsymbol{q}.\nabla C = D_{SM}\nabla^2 C + D_{CT}\nabla^2 T, \tag{8}$$

where: D_{SM} and D_{CT} are the solute diffusivity of porous medium and Soret type diffusivity.

The boundary conditions are given by

$$w = 0,\ T = T_0,\ \varphi = \varphi_0,\ C = C_0,\ \text{at}\ \ z = 0 \tag{9}$$

$$w = 0,\ T = T_1,\ \varphi = \varphi_1,\ C = C_1,\ \text{at}\ \ z = d. \tag{10}$$

We establish nondimensional variables as: $(x^*, y^*, z^*) = \frac{(x,y,z)}{d}$, $\boldsymbol{q}^* = \boldsymbol{q}\frac{d}{\kappa_m}$, $t^* = \frac{t\kappa_m}{\sigma d^2}$, $p^* = \frac{pk_1}{\mu\kappa_m}$, $\varphi^* = \frac{\varphi-\varphi_0}{\varphi_1-\varphi_0}$, $T^* = \frac{T-T_1}{T_0-T_1}$, $C^* = \frac{C-C_1}{C_0-C_1}$,

where: $\kappa_m = \frac{k_m}{(\rho c)_f}$ and $\sigma = \frac{(\rho c)_m}{(\rho c)_f}$ are thermal diffusivity of the fluid and the thermal capacity ratio, respectively. Dropping the star (*) in intended for simplification.

Equation (1) and Eqs.(5), (6), (7), (8) reduce in non-dimensional form

$$\nabla.\boldsymbol{q} = 0, \tag{11}$$

$$\frac{1}{\sigma V_a}\frac{\partial}{\partial t}\boldsymbol{q}=-\nabla p-\frac{1}{1+\lambda}\boldsymbol{q}-R_m\hat{k}-R_n\varphi\hat{k}+R_D T\hat{k}+\frac{R_s}{L_e}C\hat{k}\,, \quad (12)$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}\boldsymbol{q}.\nabla\varphi=\frac{1}{L_n}\nabla^2\varphi+\frac{N_A}{L_n}\nabla^2 T\,, \quad (13)$$

$$\frac{\partial T}{\partial t}+\boldsymbol{q}.\nabla T=\nabla^2 T+\frac{N_B}{L_n}\nabla\varphi.\nabla T+\frac{N_A N_B}{L_n}\nabla T.\nabla T+N_{CT}\nabla^2 C\,, \quad (14)$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}+\frac{1}{\varepsilon}\boldsymbol{q}.\nabla C=\frac{1}{L_e}\nabla^2 C+N_{TC}\nabla^2 T\,, \quad (15)$$

where dimensionless parameters are: the thermosolutal Lewis number $L_e=\frac{\kappa_m}{D_{SM}}$; thermosolutal-nanofluid Lewis number $L_n=\frac{\kappa_m}{D_B}$; density Rayleigh number $R_M=\frac{(\rho_P\varphi_0+\rho(1-\varphi_0))gk_1d}{\mu\kappa_m}$; nanoparticle Rayleigh number $R_N=\frac{(\rho_P-\rho)(\varphi_1-\varphi_0)gk_1d}{\mu\kappa_m}$; thermal Darcy-Rayleigh number $R_D=\frac{\rho\alpha_T(T_0-T_1)gk_1d}{\mu\kappa_m}$; solutal Rayleigh number $R_S=\frac{\rho\alpha_C(C_0-C_1)gk_1d}{\mu D_{SM}}$; Prandtl number $Pr=\frac{\mu}{\rho\kappa_m}$; modified diffusivity ratio $N_A=\frac{D_T(T_0-T_1)}{D_B T_1(\varphi_1-\varphi_0)}$; modified particle density increment $N_B=\frac{\varepsilon(\rho c)_P(\varphi_1-\varphi_0)}{(\rho c)_f}$; Soret parameter $N_{CT}=\frac{D_{CT}(C_0-C_1)}{\kappa_m(T_0-T_1)}$; Dufour parameter $N_{TC}=\frac{D_{TC}(T_0-T_1)}{\kappa_m(C_0-C_1)}$; Darcy number $D_a=\frac{k_1}{d^2}$; and Vadasz number $V_a=\frac{\varepsilon Pr}{D_a}$.

The dimensionless Boundary Conditions are

$$w=0\,,\ T=1\,,\ \varphi=0\,,\ C=1\,,\ \text{at}\ \ z=0 \quad (16)$$

$$w=0\,,\ T=0\,,\ \varphi=1\,,\ C=0\,,\ \text{at}\ \ z=1\,. \quad (17)$$

BASIC STATES AND ITS SOLUTIONS

Following Kuznetsov and Nield /7-9/, Sharma et al. /12/, and Sheu /13/, the basic state of nanofluid is assumed and does not depend on time and describes as:

$$\boldsymbol{q}(u,v,w)=0\ \Rightarrow\ u=v=w=0$$

$$p=p_b(z),\ C=C_b(z),\ T=T_b(z),\ \varphi=\varphi_b(z)\,. \quad (18)$$

The basic variable is represented by subscript b. Therefore, when the basic state defined in Eq.(18) is substituted into Eqs.(11)-(15), these equations reduce to

$$0=-\frac{d}{dz}p_b(z)-R_m-R_n\varphi_b(z)+R_D T_b(z)+\frac{R_s}{L_e}C_b(z)\,, \quad (19)$$

$$\frac{d^2}{dz^2}\varphi_b(z)+N_A\frac{d^2}{dz^2}T_b(z)=0\,, \quad (20)$$

$$\frac{d^2}{dz^2}T_b(z)+\frac{N_A}{L_n}\frac{d}{dz}\varphi_b(z)\frac{d}{dz}T_b(z)+\frac{N_A N_B}{L_n}\left(\frac{d}{dz}T_b(z)\right)^2+$$
$$+N_{CT}\frac{d^2}{dz^2}C_b(z)=0\,, \quad (21)$$

$$\frac{1}{L_e}\frac{d^2}{dz^2}C_b(z)+N_{TC}\frac{d^2}{dz^2}T_b(z)=0\,. \quad (22)$$

Using boundary conditions, Eqs.(16) and (17), the solution of Eq.(20) is given by

$$\varphi_b(z)=(1-T_b)N_A+(1-N_A)z\,. \quad (23)$$

Using boundary conditions, Eqs.(16) and (17), the solution of Eq.(22) is given by

$$C_b(z)=(1-T_b)N_{TC}L_e-(1+N_{CT}L_e)z+1\,. \quad (24)$$

Substituting the values of $\varphi_b(z)$ and $C_b(z)$, respectively, from Eq.(23) and Eq.(24) in Eq.(21), we get

$$\frac{d^2}{dz^2}T_b(z)+\frac{(1-N_A)N_B}{L_n}\frac{d}{dz}T_b(z)=0\,. \quad (25)$$

The solution of differential Eq.(25) with boundary conditions in Eqs.(16) and (17) is

$$T_b(z)=e^{-\frac{(1-N_A)N_B z}{L_n}}\left[\frac{1-e^{-\frac{(1-N_A)N_B(1-z)}{L_n}}}{1-e^{-\frac{(1-N_A)N_B}{L_n}}}\right]. \quad (26)$$

According to Buongiorno /1/, for most nanofluids investigated so far $L_n/(\varphi_1-\varphi_0)$ is large, of order 10^5–10^6 and since the nanoparticle fraction decrement $(\varphi_1-\varphi_0)$ is not smaller than 10^{-3} which means L_n is large. Typical values of exponents in Eq.(26) are small.

By expanding the exponential function into the power series and retaining up to the first order and negligible other higher order terms (*i.e.*, $e^{-x}=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots\approx 1-x$ and so, to a good approximation for the solution.

$$T_b=1-z\,,\ \ C_b=1-z\,,\ \text{and}\ \ \varphi_b=z\,, \quad (27)$$

these results are identical with the results obtained by Kuznetsov and Nield /7-9/, Sharma et al. /12/, and Sheu /13/.

PERTURBATION SOLUTIONS

We introduce small perturbations on the basic state for investigating the stability of the system and write

$$q(u,v,w)=0+q'(u,v,w)\,,\ T=(1-z)+T'\,,\ C=(1-z)+C'\,,$$
$$\varphi=z+\varphi'\,,\ p=p_b+p'\,. \quad (28)$$

Using Eq.(28) in Eqs.(11) to (15), linearizing the resulting equations by ignoring nonlinear terms that are a product of prime quantities and dropping the primes (′) for convenience, the following equations are obtained

$$\nabla.\boldsymbol{q}=0\,, \quad (29)$$

$$\frac{1}{\sigma V_a}\frac{\partial}{\partial t}\boldsymbol{q}=-\nabla p-\frac{1}{1+\lambda}\boldsymbol{q}-R_N\varphi\hat{k}+R_D T\hat{k}+\frac{R_S}{L_e}C\hat{k}\,, \quad (30)$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}w=\frac{1}{L_n}\nabla^2\varphi+\frac{N_A}{L_n}\nabla^2 T\,, \quad (31)$$

$$\frac{\partial T}{\partial t}-w=\nabla^2 T+\frac{N_B}{L_n}\left(\frac{\partial T}{\partial z}-\frac{\partial\varphi}{\partial z}\right)-2\frac{N_A N_B}{L_n}\frac{\partial T}{\partial z}+N_{CT}\nabla^2 C\,, \quad (32)$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}-\frac{1}{\varepsilon}w=\frac{1}{L_e}\nabla^2 C+N_{TC}\nabla^2 T\,, \quad (33)$$

and boundary conditions are

$$w=0,\ T=0,\ \varphi=0,\ C=0,\ \text{at}\ z=0\ \text{and}\ z=1. \quad (34)$$

Note that the parameter R_M is not involved in Eqs.(29) to (33), it is just a measure of the basic static pressure gradient.

Operating the Eq.(30) with $\hat{k}.curl.curl$, we get (i.e., making use of the result $curl.curl = grad.div - \nabla^2$)

$$\left(\frac{1}{\sigma V_a}\frac{\partial}{\partial t}+\frac{1}{1+\lambda}\right)\nabla^2 w+R_N\nabla_H^2\varphi-R_D\nabla_H^2 T+\frac{R_S}{L_e}\nabla_H^2 C=0, \quad (35)$$

where: $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$; and $\nabla_H^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplace operator on the horizontal plane.

NORMAL MODE ANALYSIS

Disturbances by normal mode analysis are as follows:

$$[W,T,C,\varphi]=[W(z),\Theta(z),\Gamma(z),\Phi(z)]\exp(ik_x x+ik_y y+nt) \quad (36)$$

where: n is growth rate; and k_x and k_y are the wave numbers along x and y directions, respectively.

Using Eq.(36) in Eqs.(31), (32), (33), and (35), we get

$$\left[\left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)(D^2-a^2)\right]W+R_D a^2\Theta+\frac{R_S}{L_e}a^2\Gamma-R_N a^2\Phi=0 \quad (37)$$

$$\frac{1}{\varepsilon}W-\frac{N_A}{L_n}(D^2-a^2)\Theta+\left[\frac{n}{\sigma}-\frac{(D^2-a^2)}{L_n}\right]\Phi=0, \quad (38)$$

$$W+\left[(D^2-a^2)+\frac{N_B}{L_n}D-2\frac{N_AN_B}{L_n}D-n\right]\Theta+N_{CT}(D^2-a^2)\Gamma-$$
$$-\frac{N_B}{L_n}D\Phi=0, \quad (39)$$

$$\frac{1}{\varepsilon}W+N_{TC}(D^2-a^2)\Theta+\left[\frac{(D^2-a^2)}{L_e}-\frac{n}{\sigma}\right]\Gamma=0, \quad (40)$$

where: $D = d/dz$; and $a^2 = k_x^2 + k_y^2$ is the dimensionless wave number.

We have applied stress-free conditions for a free surface. Now the disappearing of shear stresses tangent to the surface and the continuity equation gives the boundary conditions for free-free boundary, as

$$W = D^2W = \Gamma = \Theta = \Phi = 0 \text{ at } z = 0 \text{ and } z = 1. \quad (41)$$

LINEAR STABILITY ANALYSIS AND DISPERSION RELATION

The eigen functions $f_i(z)$ corresponding to the eigenvalue problem, Eqs.(37)-(40), are $f_i = \sin(\pi z)$.

Considering solutions W, Θ, Γ, Φ of the form

$$W = W_0\sin(\pi z),\ \Theta = \Theta_0\sin(\pi z),\ \Gamma = \Gamma_0\sin(\pi z),$$
$$\text{and } \Phi = \Phi_0\sin(\pi z), \quad (42)$$

which satisfies boundary conditions Eq.(41). Substituting solution Eq.(42) into Eqs.(37)-(40) and integrating each equation from $z = 0$ to $z = 1$, we obtain the following matrix equations

$$\begin{bmatrix} \left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)J^2 & a^2R_D & -a^2\frac{R_S}{L_e} & a^2R_N \\ \frac{1}{\varepsilon} & \frac{N_A}{L_n}J^2 & 0 & \frac{J^2}{L_n}+\frac{n}{\sigma} \\ -1 & J^2+n & N_{CT}J^2 & 0 \\ -\frac{1}{\varepsilon} & N_{TC}J^2 & \frac{J^2}{L_e}+\frac{n}{\sigma} & 0 \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Gamma_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where: $J^2 = \pi^2 + a^2$ is the total wave number.

The non-trivial solution of the above matrix requires that

$$R_D=\frac{1}{(J^2\sigma\varepsilon+n\varepsilon L_e-\sigma L_e N_{CT}J^2)}\left\{\frac{\varepsilon}{a^2}\left[\left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)J^2\right]\times\right.$$
$$\times\left[(J^2+n)(\sigma J^2+nL_e)-\sigma L_e N_{CT}N_{TC}J^4\right]+\sigma R_S\left[\varepsilon N_{TC}J^2-\right.$$
$$\left.-(J^2+n)\right]-\frac{R_N\sigma}{(\sigma J^2+nL_n)}\left[\left((J^2+n)L_n+\varepsilon N_A J^2\right)(\sigma J^2+nL_e)-\right.$$
$$\left.\left.-\sigma L_e N_{CT}J^4(L_nN_{TC}+N_A)\right]\right\}. \quad (43)$$

THE STATIONARY CONVECTION

For stationary convection $n = 0$ in Eq.(43), we obtain

$$R_D=\frac{1}{(\varepsilon-L_eN_{CT})}\left\{\frac{\varepsilon(\pi^2+a^2)}{a^2}\left[\frac{(\pi^2+a^2)}{1+\lambda}\right][1-L_eN_{CT}N_{TC}]+\right.$$
$$\left.+R_S[\varepsilon N_{TC}-1]-R_N[(L_n+\varepsilon N_A)-L_eN_{CT}(L_nN_{TC}+N_A)]\right\}. \quad (44)$$

The thermal Darcy-Rayleigh number revealed from Eq. (44) is a function of a, λ, ε, N_{TC}, N_{CT}, L_e, L_n, N_A, R_S, R_N.

In the non-appearance of the Dufour (N_{TC}) and Soret (N_{CT}) parameters, Eq.(44) reduces to

$$R_D=\frac{1}{1+\lambda}\frac{(\pi^2+a^2)^2}{a^2}-\frac{R_s}{\varepsilon}-R_N\left(N_A+\frac{L_n}{\varepsilon}\right). \quad (45)$$

The critical wave number obtained by minimizing R_D with respect to a^2, i.e., satisfying $\partial R_D/\partial a^2 = 0$, is

$$a_c^2=\pi^2. \quad (46)$$

Now, the critical thermal Darcy-Rayleigh number for steady onset is

$$(R_D)_c=\frac{1}{(\varepsilon-L_eN_{CT})}\left\{\frac{4\varepsilon\pi^2}{(1+\lambda)}[1-L_eN_{CT}N_{TC}]+R_S[\varepsilon N_{TC}-1]-\right.$$
$$\left.-R_N[(L_n+\varepsilon N_A)-L_eN_{CT}(L_nN_{TC}+N_A)]\right\}. \quad (47)$$

Special cases:

In the absence of Jeffrey, Dufour, and Soret parameters (i.e., $\lambda = N_{TC} = N_{CT} = 0$) then, Eq.(44) becomes

$$R_D=\frac{(\pi^2+a^2)^2}{a^2}-\frac{R_S}{\varepsilon}-R_N\left(N_A+\frac{L_n}{\varepsilon}\right), \quad (48)$$

which is identical with the result derived by Kuznetsov and Nield, /7/.

In the absence of Jeffrey parameter and nanoparticles (i.e., $\lambda = 0$, $R_N = L_n = N_A = 0$) then, Eq.(48) becomes

$$R_D=\frac{(\pi^2+a^2)^2}{a^2}-\frac{R_S}{\varepsilon}, \quad (49)$$

and the corresponding critical thermal Darcy-Rayleigh number for steady onset in the absence of the stable solute gradient parameter R_S, is

$$R_D=4\pi^2-R_N\left(N_A+\frac{L_n}{\varepsilon}\right), \quad (50)$$

which is identical with the results derived by Sheu /13/ and Chand and Rana /3/.

RESULTS AND DISCUSSION

The critical thermal Darcy-Rayleigh number on the onset of stationary convection is given by Eq.(47) and depends on

Jeffrey parameter and takes a different value compared to the one obtained for ordinary Newtonian fluid.

The critical wave number a_c, defined by Eq.(46) at the onset of stationary convection coincides with those reported by Tzou /14, 15/, Kuznetsov and Nield /7/, and Chand and Rana /3/. Note that this critical value does not depend on any thermophysical property of the nanofluid. Consequently, the interweaving behaviours of Brownian motion and thermophoresis of nanoparticles does not change the cell size at the onset of steady instability and the critical cell size a_c is identical to the well-known result for Bénard instability with a regular fluid, Chandrasekhar /2/.

It is noted that the absence of the Dufour and Soret parameters N_{TC} and N_{CT} and nanoparticles, one recovers the well-known result that the critical thermal Darcy-Rayleigh number is equal to $4\pi^2$ as obtained by Sheu /13/. Thus, the combined effect of Brownian motion and thermophoresis of nanoparticles on the critical Rayleigh number is reflected in the third term in Eq.(47). For the case of bottom-heavy distribution of nanoparticles ($\varphi_1 < \varphi_0$ and $\rho_p > \rho$), which corresponds to negative values of R_N, the value of the critical Rayleigh number for the nanofluid is larger than that for an ordinary fluid, that is, convection sets earlier in an ordinary fluid than in a nanofluid with bottom-heavy distribution of nanoparticles. This implies that thermal conductivity of this kind of nanofluid is higher than that of ordinary fluids.

The dispersion relation Eq.(44) is analysed mathematically. Graphs are plotted by giving some numerical values to the parameters to represent the stability characteristics.

Figure 2 shows the variation of thermal Darcy-Rayleigh number with respect to the non-dimensional wave number for three different values of nanoparticles Rayleigh number R_N = -0.8, -0.9, -1, and for fixed permissible values of N_A = 5, ε = 0.4, L_e = 500, R_S = 200, L_n = 500, λ = 0.6, N_{CT} = 1, N_{TC} = 0.1.

It is depicted from the graphs for cases of thermal Darcy-Rayleigh number decreasing with the increase in nanoparticles Rayleigh number which causes the destabilizing.

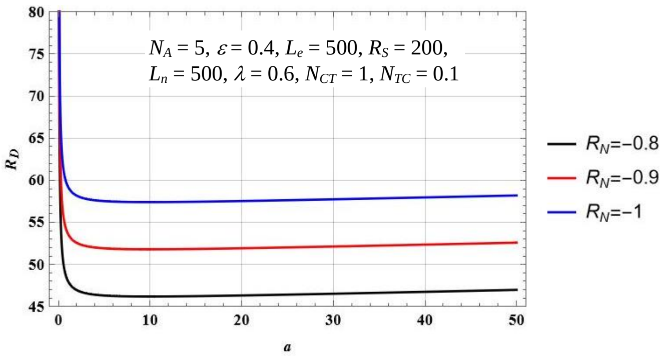

Figure 2. Variation of thermal Darcy-Rayleigh number with the wave number for different nanoparticle Rayleigh number.

Figure 3 shows the variation of thermal Darcy-Rayleigh number with wave number for different values of porosity, and it has been found that the Rayleigh number increases with increase in the value of porosity, thus porosity stabilizing the stationary convection.

Figure 4 shows the variations of thermal Darcy-Rayleigh number with wave number a for three different values of thermosolutal Lewis number L_e = 500, 1000, 1500, as plotted and it is found that the thermal Darcy-Rayleigh number increases with the increase in thermosolutal Lewis number, so the thermosolutal Lewis number has stabilizing effect on stationary convection.

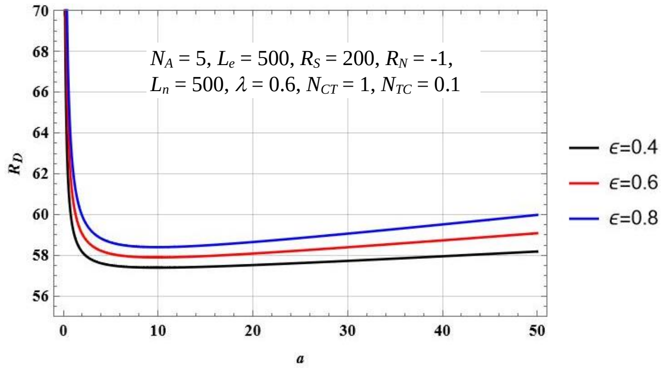

Figure 3.Variation of thermal Darcy-Rayleigh number with the wave number for different values of medium porosity.

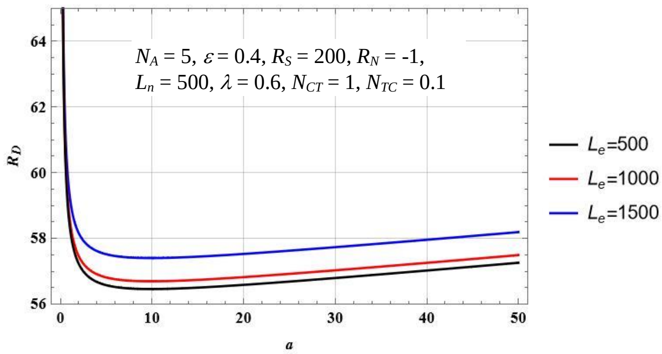

Figure 4. Variation of thermal Darcy-Rayleigh number with the wave number for different values of thermosolutal Lewis number.

Figure 5 represents the variation of thermal Darcy-Rayleigh number R_D with wave number a for different values of Jeffrey parameter λ = 0.5, 0.7, 0.9, and it decreases with the increase in Jeffrey parameter λ = 0.5, 0.7, 0.9, which implies that Jeffrey parameter has a destabilizing effect on stationary convection.

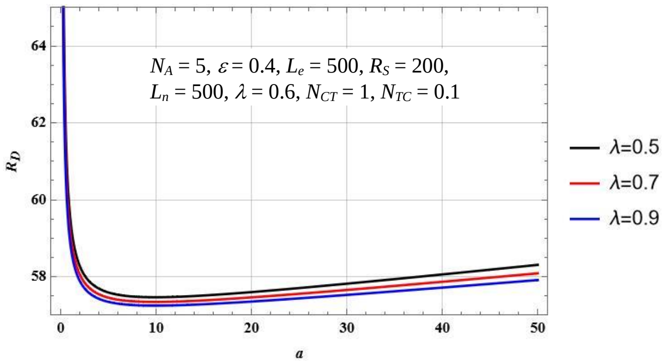

Figure 5. Variation of thermal Darcy-Rayleigh number with the wave number for different values of Jeffrey parameter.

From Figure 6, the graphs show that with an increase in the values of thermo-nanofluid Lewis number, the thermal Darcy-Rayleigh number increases for stationary convection. Hence, the thermo-nanofluid Lewis number stabilizes the physical system for stationary mode. This happens so as for both Brownian motion of the nanoparticles increases with increase in thermo-nanofluid Lewis number.

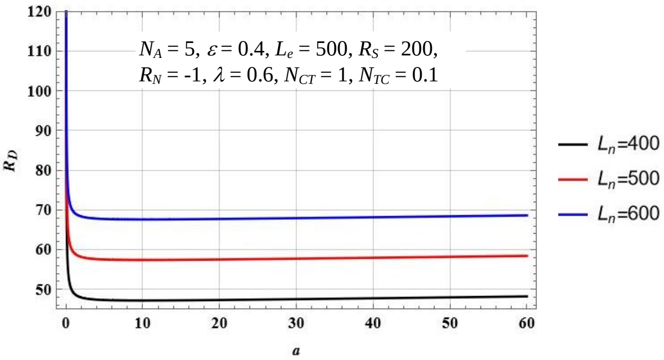

Figure 6. Variation of thermal Darcy-Rayleigh number with the wave number for different thermo-nanofluid Lewis number.

Figure 7 shows the variation of the thermal Darcy-Rayleigh number for stationary convection with respect to the non-dimensional wave number for three different values of the modified diffusivity ratio N_A = 5, 10, 15. The graph shows that with the increase in modified diffusivity ratio, the thermal Darcy-Rayleigh number increases for the stationary convection which has a stabilizing effect on the system.

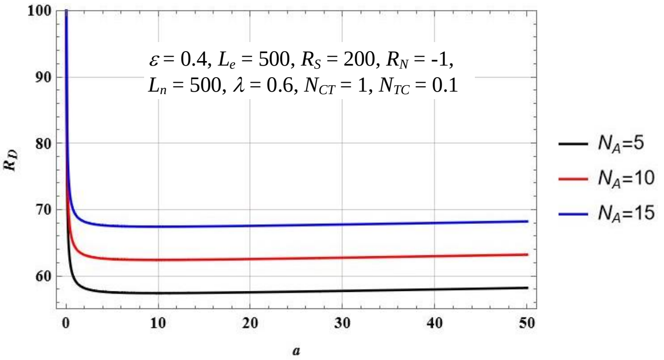

Figure 7. Variation of thermal Darcy-Rayleigh number with the wave number for different values of Modified diffusivity ratio

Figure 8 show variations of the thermal Darcy-Rayleigh number with wave number a for three different values of the Soret parameter, namely N_{CT} = 5, 10, 15, which shows that thermal Rayleigh-Darcy number decreases with the increase in Soret parameter. Thus, Soret parameter has destabilizing effect on the stationary convection.

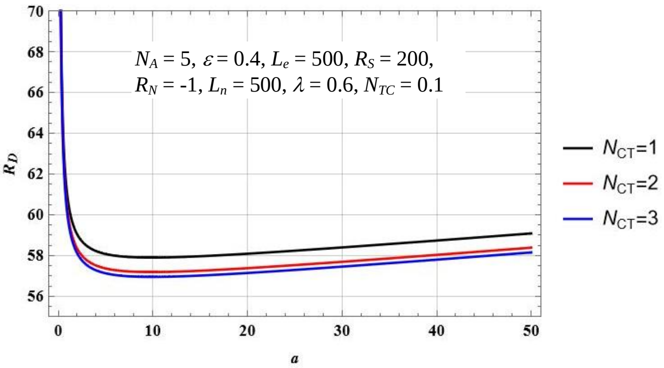

Figure 8. Variation of thermal Darcy-Rayleigh number with the wave number for different values of Soret parameter.

Figure 9 shows the variations of thermal Darcy-Rayleigh number with wave number a for three different values of Dufour parameter, namely N_{TC} = 0.1, 0.2, 0.3, which shows that the thermal Darcy-Rayleigh number increases with the increase in Dufour parameter. Thus, Dufour parameter has a stabilizing effect on stationary convection.

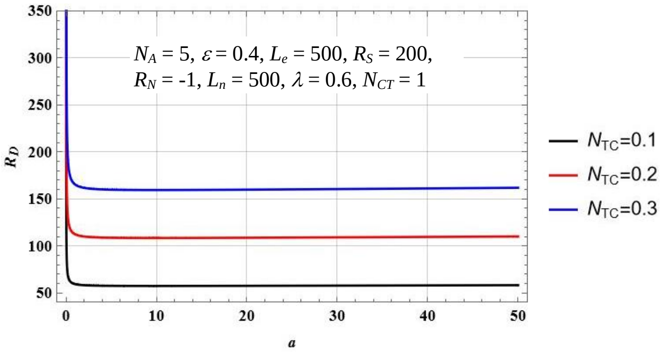

Figure 9. Variation of thermal Darcy-Rayleigh number with the wave number for different values of Dufour parameter.

Figure 10 shows variations of thermal Darcy-Rayleigh number with wave number a for three different values of the solutal Rayleigh number R_S = 200, 400, 600, as plotted and it is observed that the thermal Darcy-Rayleigh number increases with increase in solutal Rayleigh number so the solutal Rayleigh number has a stabilizing effect on stationary convection.

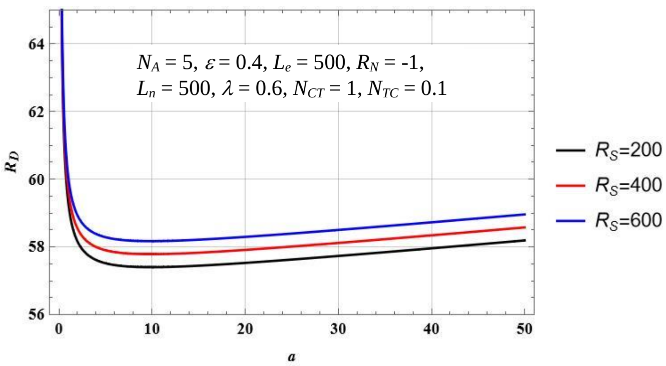

Figure 10. Variation of thermal Darcy-Rayleigh number with the wave number for different values of solutal Rayleigh number.

CONCLUSIONS

The onset of thermosolutal convection of nanofluid in porous medium in the presence of Jeffrey parameter is investigated by using linear stability analysis. The principal conclusions of the present study are given below:
- Nanoparticles Rayleigh number and Soret parameter have destabilizing effect on stationary convection.
- The Jeffery parameter has a destabilizing effect on stationary convection.
- Medium porosity, thermo-nanofluid Lewis number, Dufour parameter, solutal Rayleigh number, thermosolutal Lewis number, and modified diffusivity ratio have a stabilizing effect on stationary convection.
- The solutal Rayleigh number has a stabilizing effect on stationary convection.

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