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THERMAL CONVECTIVE INSTABILITY IN A JEFFREY NANOFLUID SATURATING A POROUS MEDIUM: RIGID-RIGID AND RIGID-FREE BOUNDARY CONDITIONS

TERMIČKA NESTABILNOST KONVEKCIJE KOD POROZNE SREDINE ZASIĆENE JEFFREY NANOFLUIDOM: GRANIČNI USLOVI SU KRUTO-KRUTO I KRUTO-SLOBODNO

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Ključne reči
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Abstract

In this paper, the onset of stationary convection in a porous layer saturated with a thermally unstable Jeffery nanofluid is considered. The behaviour of the nanofluid is described by using a Jeffrey fluid model and the porous layer is assumed to adhere to Darcy's law. The momentum-balance equations for the fluid are modified by the Jeffrey parameter and nanoparticles. Linear stability analysis, the normal modes analysis, and Galerkin type weighted residual method (GWRM) techniques are used to calculate the dispersion relation for the Rayleigh number in terms of various parameters for rigid-rigid and rigid-free boundaries. The effects of the Rayleigh number of nanoparticles, Lewis number, modified diffusivity ratio, Jeffrey parameter, and porosity are investigated analytically and graphically.

Izvod

U ovom radu se razmatra uslov za stacionarnu konvekciju u poroznom sloju, koji je zasićen termički nestabilnim Jeffrey nanofluidom. Ponašanje nanofluida se opisuje primenom modela Jeffrey fluida, a porozni sloj se tretira prema zakonu Darcy-ja. Ravnotežne momentne jednačine za fluid se modifikuju Jeffrey parametrom i nanočesticama. Primenjene su metode: analiza linearne stabilnosti, analiza u normalnom modu, analiza težinskim ostatkom tipa Galerkin (GWRM), za proračun relacije disperzije za Rejlejev broj, u uslovima različitih parametara za granice kruto-kruto i kruto-slobodno. Uticaj Rejlejevog broja nanočestica, Lewis-ovog broja, modifikovanog odnosa difuznosti, Jeffrey-ovog parametra i poroznost su istraženi analitički i grafički.

INTRODUCTION

Non-Newtonian fluids are extensively utilised in many different industries and have significant applications in many different branches of science and technology, including the production of plastics, the polymer industry, textile and paper dyeing, food processing, geophysics, the chemical and biological industries. Examples of non-Newtonian fluids include engine oil, soap solutions, sauces, foam, paints, lubricants, and biological fluids like blood. The modelling of non-Newtonian fluids has produced a number of constitutive relations due to the importance of non-Newtonian fluids in contemporary technology and industry. The Jeffrey non-Newtonian fluid model is one of these constitutive relations. A linear model called the Jeffrey fluid model substitutes time derivatives for convective derivatives. Jeffrey /4/ investigated the stability of a fluid layer that had been heated from below. He came up with a numerical solution to a few issues with the stability of a layer in a compressible fluid as temperature rises. Chandrasekhar /3/ has provided a thorough literature assessment on thermal instability in a Newtonian fluid. The Jeffrey fluid model has been researched by numerous researchers and as a result, it is today regarded as the best fluid model to represent the behaviour of physiological and industrial fluids, /1, 5, 10-13, 16/.

The flow of a fluid through a homogenous and isotropic porous medium is governed by Darcy’s law that states that the usual viscous term in the momentum-balance equations is replaced by the resistance term, where the viscosity is the medium permeability, is the Jeffrey parameter and is the Darcian (filter) velocity of the Jeffrey fluid. The study of flow in porous layers has many real-world applications, including flow in molten earth cores, oil reservoirs, tires, ropes, cushions, chairs, and sand beds. Examples of naturally porous materials include sandstones, limestone, human lungs, bile ducts and gallbladders containing blood vessel stones. The convective flow in a porous material was researched by Lapwood /7/. The Rayleigh's instability of a thermal boundary layer in a flow through a porous media was explored by Wooding /21/. They discovered that the layer is stable under certain conditions, including a critical positive Rayleigh number for the system and a limited wave number for the critical neutral disturbance. Nield and Bejan /9/ worked on the problem of thermal convection in a porous medium.

The Buongiorno /2/ model-based investigation of hydrodynamic thermal convection issues in porous and non-porous media saturated by a nanofluid layer has attracted the attention of numerous researchers over the past ten years /1, 5-17/. Nanofluid is used in a wide range of industries, including the car industry, energy conservation, and nuclear reactors, etc. Nanoparticle suspensions are widely used in medical applications, such as cancer treatment. Numerous engineering applications, including geothermal energy recovery, crude oil extraction, groundwater pollution and thermal energy storage. Different authors /1, 6-8, 10, 13, 19, 20/ investigated the natural convection of a nanofluid using Buongiorno's model and they found that nanofluids are effective coolants because of their improved thermal conductivities.

Many researchers /1, 5, 10-11, 14, 16, 18, 19/ have researched thermal convection in a viscoelastic nanofluid layer saturating a porous media and they discovered that viscoelastic nanofluids have applications in a variety of automotive sectors and biomedical engineering. The primary goal of this research is to investigate the impact of Jeffrey parameter and other parameters in a porous layer saturated in a nanofluid heated from below, taking into consideration the numerous applications of viscoelastic nanofluid as mentioned above. On the commencement of stationary convection, an analytical/graphical analysis of the thermal instability of a porous layer saturated with a Jeffrey nanofluid is conducted for rigid-rigid and rigid-free boundaries. The above problem is the extension of the work of Rana and Gautam /10/. To the author's knowledge, no research has yet been done on this issue.

MATHEMATICAL FORMULATION OF THE PROBLEM

Consider an infinite horizontal layer of Jeffrey nanofluid of thickness d bounded by planes $z = 0$ and $z = d$ and heated from below (see Fig. 1). Temperature T and volumetric fraction of nanoparticles ϕ at $z = 0$ and $z = d$ are assumed to take constant values T_0, ϕ_0, and T_1, ϕ_1 ($T_0 > T_1$ and $\phi_1 > \phi_0$), respectively. The physical system is permeated by the gravity force $\mathbf{g} = \mathbf{g}(0, 0, -g)$.

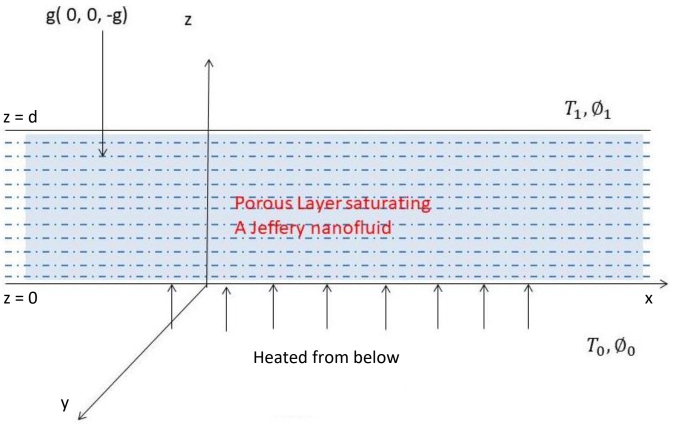

Figure 1. Physical sketch of the problem.

GOVERNING EQUATIONS

For an incompressible fluid, the mass-balance equation is

$$\nabla.\mathbf{q_D} = 0\,, \tag{1}$$

where: $\mathbf{q_D}$ is the flow velocity of nanofluid.

The modified momentum-balance equation of Jeffrey nanofluid in a porous layer after applying the Boussinesq approximation /1, 5, 10-13, 16/, is:

$$\frac{\rho_f}{\varepsilon}\left(\frac{\partial \mathbf{q_D}}{\partial t}+\frac{1}{\varepsilon}(\mathbf{q_D}.\nabla)\mathbf{q_D}\right) = -\nabla p - \frac{\mu}{k_1(1+\lambda_3)}\mathbf{q_D} + \left[\phi\rho_P+(1-\phi)\rho_f\{1-\alpha(T-T_1)\}\right]\mathbf{g}\,, \tag{2}$$

where: $\lambda_3 = \lambda_1/\lambda_2$, λ_1, λ_2, ρ_f, ρ_P, T, μ, k_1, and ε denote the Jeffrey parameter (accounting for viscoelasticity), stress relaxation-time parameter, strain relaxation-time parameter, fluid density, fluid pressure, fluid temperature, fluid viscosity, medium permeability and medium porosity, in respect.

Let k_B, k_f, k_p, ρ_f, μ_f and d_p denote the Boltzmann's constant, thermal conductivities of base fluid, thermal conductivities of nanoparticles, base fluid density, base fluid viscosity, and nanoparticles diameter, respectively. The Brownian diffusion coefficient D_B and thermophoretic diffusion coefficient D_T are defined respectively, as:

$$D_B = \frac{k_B T}{3\pi\mu_f d_p} \quad \text{and} \quad D_T = \frac{\mu_f}{\rho_f}\frac{0.26k_f}{(2k_f+k_p)}\phi\,.$$

The momentum-balance equation of nanoparticle /1-15/ is given by

$$\left[\frac{\partial}{\partial t}+\frac{q_D.\nabla}{\varepsilon}\right]\phi = D_B\nabla^2\phi+\frac{D_T}{T_1}\nabla^2 T\,. \tag{3}$$

The energy-balance equation is given by

$$(\rho c)_m\frac{\partial T}{\partial t}+(\rho c)_f\, q_D.\nabla T = k_m\nabla^2 T+\varepsilon(\rho c)_P \times \left[D_B\nabla\phi.\nabla T+\frac{D_T}{T_1}\nabla T.\nabla T\right], \tag{4}$$

where: k_m is thermal conductivity of porous medium; and $(\rho c)_f$ is the heat capacity of fluid.

In non-dimensional form Eqs.(1)-(4) can be written by omitting the dashes (′) for convenience as:

$$\nabla.q_D = 0\,, \tag{5}$$

$$\left(\frac{1}{V_a}\frac{\partial}{\partial t}+\frac{1}{1+\lambda_3}\right)w = -\nabla p - R_m\hat{e}_z + R_a T\hat{e}_z - R_n\phi\hat{e}_z\,, \tag{6}$$

$$\frac{1}{\sigma}\frac{\partial\phi}{\partial t}+\frac{1}{\varepsilon}q_D.\nabla\phi = \frac{1}{\mathrm{Le}}\nabla^2\phi+\frac{N_A}{\mathrm{Le}}\nabla^2 T\,, \tag{7}$$

$$\frac{\partial T}{\partial t}+q_D.\nabla T = \nabla^2 T+\frac{N_B}{\mathrm{Le}}\nabla\phi.\nabla T+\frac{N_A N_B}{\mathrm{Le}}\nabla T.\nabla T\,. \tag{8}$$

Here, we have used the non-dimensional variables:

$$(x',y',z') = \left(\frac{x,y,z}{d}\right),\; (u',v',w') = \left(\frac{u,v,w}{\kappa_m}\right)d\,,\; t' = \frac{t\kappa_m}{\sigma d^2}\,,$$

$$p' = \frac{pk_1}{\mu\kappa_m}\,,\; \phi' = \frac{\phi-\phi_0}{\phi_1-\phi_0}\,,\; T' = \frac{T-T_1}{T_0-T_1}\,,$$

where: $\kappa_m = k_m/(\rho c)_f$ is the thermal diffusivity of the base fluid; $\sigma = (\rho c)_p/(\rho c)_f$ is thermal capacity ratio; the Prandtl number is $\mathrm{P_r} = \mu/\rho_f\kappa_m$; Darcy's number is $\mathrm{D_a} = k_1/d^2$; the Vadasz number is $\mathrm{V_a} = \varepsilon\,\mathrm{P_r}/\mathrm{D_a}$, the Rayleigh number is $\mathrm{R_a} = \rho_f g\beta dk(T_0 - T_1)/\mu_f\kappa_m$; nanoparticle's Rayleigh number is $\mathrm{R_n} = (\rho_p - \rho_f)(\phi_1 - \phi_0)gk_1d/\mu\kappa_m$; modified particle density increment is $N_B = \varepsilon(\rho c)_p(\phi_1 - \phi_0)/(\rho c)_f$; the Lewis number is $\mathrm{Le} = \kappa_m/D_B$; the modified diffusivity ratio is $N_A = D_T(T_0 - T_1)/D_BT_1(\phi_1 - \phi_0)$; the basic density Rayleigh number is

$\mathrm{R_m} = (\rho_p\phi_0 + \rho_f(1-\phi_0))gk_1 d/\mu\kappa_m$; $\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is a Laplacian operator; and $\nabla_H^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is a horizontal Laplacian operator.

The dimensionless boundary conditions are

$$w=0,\ T=T_0,\ \phi=\phi_0,\ \text{at}\ \ z=0\ \ \text{and}$$
$$w=0,\ T=T_1,\ \phi=\phi_1,\ \text{at}\ \ z=d\,. \tag{9}$$

STEADY STATE SOLUTIONS

Following Buongiorno /2/, Nield and Kuznetsov /8/, and Sheu /18, 19/, the basic state of the nanofluid is assumed and does not depend on time and is describes as:

$$q_D(u,v,w)=0 \ \Rightarrow\ u=v=w=0\,,$$
$$p=p_b(z),\ \ T=T_b(z),\ \ \phi=\phi_b(z)\,. \tag{10}$$

The basic variable is represented by subscript b. Therefore, when the basic state defined in Eq.(10) is substituted into Eqs.(5)-(8), these equations reduce to:

$$0=-\frac{d}{dz}p_b(z)-R_m\hat{e}_z+R_aT_b(z)\hat{e}_z-R_n\phi_b(z)\hat{e}_z\,, \tag{11}$$

$$\frac{d^2}{dz^2}\phi_b(z)+N_A\frac{d^2}{dz^2}T_b(z)=0\,, \tag{12}$$

$$\frac{d^2}{dz^2}T_b(z)+\frac{N_A}{\mathrm{Le}}\frac{d}{dz}\phi_b(z).\frac{d}{dz}T_b(z)+\frac{N_AN_B}{\mathrm{Le}}\left(\frac{d}{dz}T_b(z)\right)^2=0\,. \tag{13}$$

Using boundary conditions Eq.(9) in Eq.(12), we get

$$\phi_b(z)=(1-T_b)N_A+(1-N_A)z\,. \tag{14}$$

Substituting Eq.(14) in Eq.(13), we get

$$\frac{d^2}{dz^2}T_b(z)+\frac{N_A}{\mathrm{Le}}(1-N_A)\frac{d}{dz}T_b(z)+\frac{N_AN_B}{\mathrm{Le}}\left(\frac{d}{dz}T_b(z)\right)^2=0\,.$$

Neglecting the higher degree term, we get

$$\frac{d^2}{dz^2}T_b(z)+\frac{N_A(1-N_A)}{\mathrm{Le}}\frac{d}{dz}T_b(z)=0\,. \tag{15}$$

The solution of differential Eq.(15) with boundary condition Eq.(9) is

$$T_b(z)=\frac{e^{\frac{-N_B(1-N_A)z}{\mathrm{Le}}}\left[1-e^{\frac{-N_B(1-N_A)(1-z)}{\mathrm{Le}}}\right]}{1-e^{\frac{-N_B(1-N_A)}{\mathrm{Le}}}}\,. \tag{16}$$

According to the Buongiorno /2/ hypothesis, the approximation solution for Eqs.(14) and (16) are given as

$$T_b=1-z\quad \text{and}\quad \phi_b=z\,. \tag{17}$$

These results are identical with the result obtained by Buongiorno /2/, Nield and Kuznetsov /8/, Sheu /18, 19/, and Sharma et al. /11-17/.

PERTURBATION SOLUTIONS

For the examination of the stability of the system, a small perturbation to the basic state is introduced as

$$q_D(u,v,w)=0+q_D{}^*(u,v,w)\,,\quad p=p_b+p^*\,,$$
$$T=T_b+T^*,\quad \phi=\phi_b+\phi^*\,. \tag{18}$$

Using Eq.(18) in Eqs.(5) to (8), linearizing the resulting equations by neglecting nonlinear terms, we obtain the non-dimensional perturbed equations as

$$\nabla.q_D^*=0\,, \tag{19}$$

$$\left(\frac{1}{\mathrm{V_a}}\frac{\partial}{\partial t}+\frac{1}{1+\lambda_3}\right)q_D^*=-\nabla p^*+R_aT^*\hat{e}_z-R_n\phi^*\hat{e}_z\,, \tag{20}$$

$$\frac{1}{\sigma}\frac{\partial\phi^*}{\partial t}+\frac{q_D^*}{\varepsilon}=\frac{1}{\mathrm{Le}}\nabla^2\phi^*+\frac{N_A}{\mathrm{Le}}\nabla^2T^*\,, \tag{21}$$

$$\frac{\partial T^*}{\partial t}-q_D^*=\nabla^2T^*+\frac{N_B}{\mathrm{Le}}(\nabla T^*-\nabla\phi^*)-\frac{2N_AN_B}{\mathrm{Le}}\nabla T^*\,. \tag{22}$$

and the boundary conditions are

$$w^*=T^*=\phi^*=0\quad \text{at}\quad z=0\quad \text{and}\quad z=1\,. \tag{23}$$

Operating Eq.(20) with $\hat{e}_z.curl.curl.q_D^*$, we get

$$\left(\frac{1}{\mathrm{Va}}\frac{\partial}{\partial t}+\frac{1}{1+\lambda_3}\right)\nabla^2w^*-\mathrm{R_a}\,\nabla_H^2T^*+R_n\nabla_H^2\phi^*=0\,. \tag{24}$$

NORMAL MODE AND STABILITY ANALYSIS

The disturbances analysis by normal mode analysis is as follows:

$$[w^*,T^*,\phi^*]=[W(z),\Theta(z),\Phi(z)]\exp(ilx+imy+\omega t)\,. \tag{25}$$

Using Eq.(25) in Eqs.(22) to (24), we get

$$\left(\frac{\omega}{\mathrm{Va}}+\frac{1}{1+\lambda_3}\right)(D^2-a^2)W+a^2\,\mathrm{R_a}\,\Theta-a^2\,\mathrm{R_n}\,\Phi=0\,, \tag{26}$$

$$\frac{W}{\varepsilon}-\frac{N_A}{\mathrm{Le}}(D^2-a^2)\Theta-\left\{\frac{1}{\mathrm{Le}}(D^2-a^2)-\frac{\omega}{\sigma}\right\}\Phi=0\,, \tag{27}$$

$$W+\left\{\frac{N_B}{\mathrm{Le}}D+(D^2-a^2)-\frac{2N_AN_B}{\mathrm{Le}}D-\omega\right\}\Theta-\frac{N_B}{\mathrm{Le}}D\Phi=0\,, \tag{28}$$

where: $D = d/dz$; and $a^2 = l^2 + m^2$ is the dimensionless resultant wave number.

The set of differential Eqs.(26) to (28) together with the boundary conditions Eq.(23) constitute a characteristic value problem for Rayleigh number $\mathrm{R_a}$ and given value of the other parameters λ_3, R_n, ε, Le, N_A, N_B, $\mathrm{V_a}$, whose solutions have to be obtained.

RIGID-RIGID BOUNDARIES

We confine our analysis to the one-term Galerkin approximation. Appropriate trial functions satisfying the boundary conditions, which are now

$$W=0,\ \ \Theta=0,\ \ \Phi=0,\ \ DW=0\quad \text{at}\quad z=0$$
$$W=0,\ \ \Theta=0,\ \ \Phi=0,\ \ DW=0\quad \text{at}\quad z=1\,. \tag{29}$$

LINEAR STABILITY ANALYSIS

We assume the solution to W, Θ, and Φ is of the form

$$W=W_0(1-z)^2z^2\,,\ \ \Theta=\Theta_0z(1-z)\,,\ \ \Phi=\Phi_0z(1-z)\,, \tag{30}$$

which satisfies boundary conditions Eq.(29).

Substituting solution Eq.(30) into Eqs.(26) to (28), integrating each equation from $z = 0$ to $z = 1$, and performing some integrations by parts, we obtain the following matrix equation:

$$\begin{bmatrix} \left(\frac{\omega}{\mathrm{V_a}}+\frac{1}{1+\lambda_3}\right)(24+2a^2) & -9a^2\,\mathrm{R_a} & 9a^2\,\mathrm{R_n} \\ \frac{3}{\varepsilon} & 14\frac{N_A}{\mathrm{Le}}(10+a^2) & \frac{14(10+a^2)}{Le}+\frac{14\omega}{\sigma} \\ 3 & -\{14(10+a^2)+14\omega\} & 0 \end{bmatrix}\times$$

$$\times\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{30}$$

The above matrix equation has a non-trivial solution, if

$$\begin{vmatrix} \left(\frac{\omega}{\mathrm{V_a}}+\frac{1}{1+\lambda_3}\right)(24+2a^2) & -9a^2\,\mathrm{R_a} & 9a^2\,\mathrm{R_n} \\ \frac{3}{\varepsilon} & 14\frac{N_A}{\mathrm{Le}}(10+a^2) & \left\{\frac{14(10+a^2)}{Le}+\frac{14\omega}{\sigma}\right\} \\ 3 & -\{14(10+a^2)+14\omega\} & 0 \end{vmatrix}=0 \tag{32}$$

which implies that

$$\mathrm{R_a}=\frac{28\left(\frac{\omega}{\mathrm{V}_a}+\frac{1}{1+\lambda_3}\right)(12+a^2)(10+a^2+\omega)}{27a^2}-\frac{\left(\frac{(10+a^2)+\omega}{\varepsilon}+\frac{N_A}{\mathrm{Le}}(10+a^2)\right)\mathrm{R_n}}{\frac{(10+a^2)}{Le}+\frac{\omega}{\sigma}}. \tag{33}$$

Non oscillatory convection

For the case of steady-state (i.e., the principle of exchange of stability), we put $\omega = 0$ in Eq.(33) and obtain

$$\mathrm{R_a}+\left(\frac{\mathrm{Le}}{\varepsilon}+N_A\right)\mathrm{R_n}=\frac{28}{27a^2}\left[(12+a^2)\left(\frac{1}{1+\lambda_3}\right)\right](10+a^2). \tag{34}$$

Equation (34) is the required dispersion relation according for the effect of the Jeffrey parameter, Lewis number, nanoparticle's Rayleigh number, modified diffusivity ratio, and medium porosity on the onset of thermal instability in a porous layer saturating a Jeffrey nanofluid.

The critical wave number at the onset of instability is obtained by minimizing $\mathrm{R_a}$ with respect to a^2, thus the critical wave number must satisfy

$$\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c}=0\,.$$

Equation (34) gives

$$a_c=3.31\,. \tag{35}$$

RIGID-FREE BOUNDARIES

We confine our analysis to the one-term Galerkin approximation. Appropriate trial functions satisfying the boundary conditions are now

$$W=0\,,\ \ \Theta=0\,,\ \ \Phi=0\,,\ \ DW=0\,,\ \text{ at }\ z=0$$
$$W=0\,,\ \ \Theta=0\,,\ \ \Phi=0\,,\ \ D^2W=0\,,\ \text{ at }\ z=1\,. \tag{36}$$

LINEAR STABILITY ANALYSIS

We assume the solution to W, Θ, and Φ is of the form

$$W=W_0z^2(1-z)(3-2z),\ \Theta=\Theta_0 z(1-z),\ \Phi=\Phi_0 z(1-z), \tag{37}$$

which satisfies boundary conditions Eq.(36).

Substituting solution Eq.(37) into Eqs.(26) to (28), and integrating each equation from $z = 0$ to $z = 1$, and performing some integrations by parts, we obtain the following matrix equation:

$$\begin{bmatrix} \left(\frac{\omega}{\mathrm{V_a}}+\frac{1}{1+\lambda_3}\right)\left(\frac{12}{35}+\frac{19a^2}{630}\right) & -\frac{13}{420}a^2\,\mathrm{R_a} & \frac{13}{420}a^2\,\mathrm{R_n} \\ \frac{13}{420\varepsilon} & \frac{N_A}{\mathrm{Le}}\left(\frac{1}{3}+\frac{a^2}{30}\right) & \frac{1}{\mathrm{Le}}\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30\sigma} \\ \frac{13}{420} & -\left\{\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30}\right\} & 0 \end{bmatrix}\times$$
$$\times\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{38}$$

The above matrix equation has a non-trivial solution if

$$\begin{vmatrix} \left(\frac{\omega}{\mathrm{V_a}}+\frac{1}{1+\lambda_3}\right)\left(\frac{12}{35}+\frac{19a^2}{630}\right) & -\frac{13}{420}a^2\,\mathrm{R_a} & \frac{13}{420}a^2\,\mathrm{R_n} \\ \frac{13}{420\varepsilon} & \frac{N_A}{\mathrm{Le}}\left(\frac{1}{3}+\frac{a^2}{30}\right) & \frac{1}{\mathrm{Le}}\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30\sigma} \\ \frac{13}{420} & -\left\{\left(\frac{1}{3}+\frac{a^2}{30}\right)+\frac{\omega}{30}\right\} & 0 \end{vmatrix}=0 \tag{39}$$

which implies that

$$\mathrm{R_a}=\frac{28}{507a^2}\left(\frac{\omega}{\mathrm{V_a}}+\frac{1}{1+\lambda_3}\right)(216+19a^2)\{(10+a^2)+\omega\}-\frac{\frac{(10+a^2)+\omega}{\varepsilon}+\frac{N_A(10+a^2)}{\mathrm{Le}}}{\frac{(10+a^2)}{\mathrm{Le}}+\frac{\omega}{\sigma}}\mathrm{R_n}\,. \tag{40}$$

Non oscillatory convection

For the case of steady-state (i.e., the principle of exchange of stability), we put $\omega = 0$ in Eq.(47) and obtain

$$\mathrm{R_a}+\left(\frac{\mathrm{Le}}{\varepsilon}+N_A\right)\mathrm{R_n}=\frac{28}{507a^2}\left[(216+19a^2)\left(\frac{1}{1+\lambda_3}\right)\right](10+a^2). \tag{41}$$

Equation (41) is the required dispersion relation according for the effect of the Jeffrey parameter, Lewis number, nanoparticle's Rayleigh number, modified diffusivity ratio and medium porosity on the onset of thermal instability in a porous layer saturating a Jeffrey nanofluid.

The critical wave number is at the onset of instability is obtained by minimizing $\mathrm{R_a}$ with respect to a^2, thus the critical wave number must satisfied

$$\left(\frac{\partial R_a}{\partial a^2}\right)_{a=a_c}=0\,.$$

Equation (41) gives

$$a_c=3.27\,. \tag{42}$$

RESULT AND DISCUSSIONS

In this paper, we have analysed the effects of Jeffrey parameter, Lewis number, nanoparticle's Rayleigh number, modified diffusivity ratio and medium porosity on the onset of stationary convection by considering Jeffrey nanofluids in the presence of rigid-rigid and rigid-free boundary conditions. We have analysed their effects analytically and presented graphically.

Figure 2 shows the graph of R_{as} w.r.t. wave number a for different value of λ_3 = 0.3, 0.5, 0.9 by fixing the other parameters as N_A = 10, Le = 1000, ε = 0.6, R_n = –1. It is clear from Fig. 2 that within increase in the value of λ_3, R_{as} goes on decreasing, and hence shows the destabilising effect on stationary convection. It is also clear from Fig. 2 that rigid-rigid boundary condition has more destabilising impact on stationary convection as compared to rigid-free boundary conditions. Thus, λ_3 enhance the onset of convection.

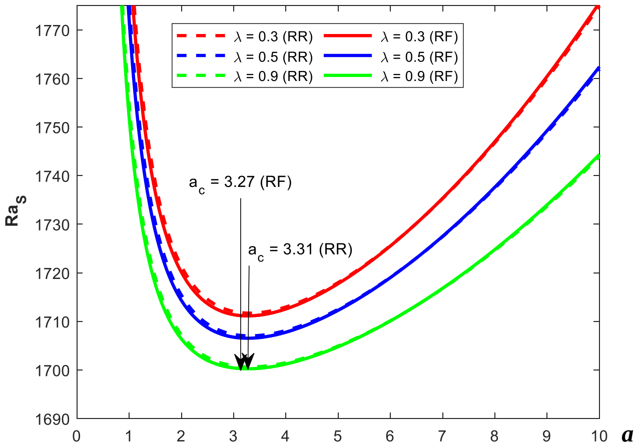

Figure 2. Variation of stationary Rayleigh number R_{as} with wave number a for different value of Jeffrey parameter λ_3.

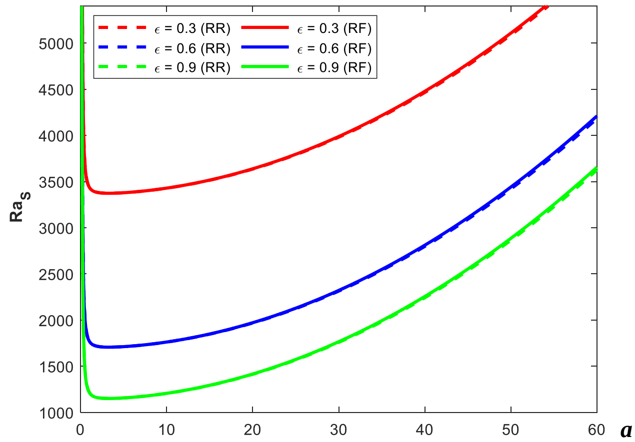

Figure 3. Variation of stationary Rayleigh number R_{as} with wave number a for different value of medium porosity ε.

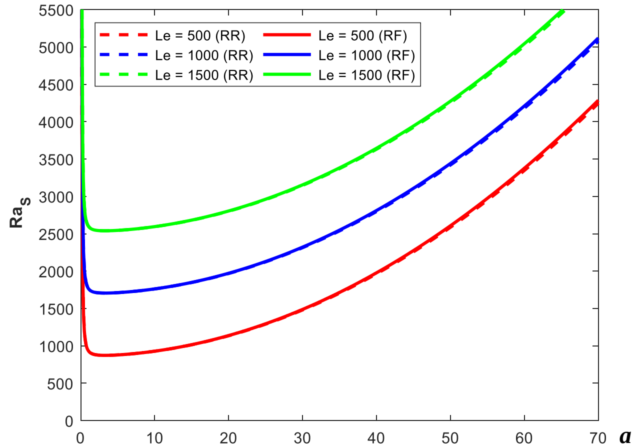

Figure 4. Variation of stationary Rayleigh number R_{as} with wave number a for different value of Lewis number Le.

Figure 3 shows the graph of R_{as} w.r.t. wave number a for different value of ε = 0.3, 0.6, 0.9 by fixing the other parameters as N_A = 10, Le = 1000, R_n = –1, λ_3 = 0.5. It is clear from Fig. 3 that within increase in the value of ε, R_{as} goes on decreasing, and hence shows the destabilising effect on stationary convection. It is also clear from Fig. 3 that rigid-rigid boundary condition has more destabilising impact on stationary convection as compared to rigid-free boundary conditions. Thus, ε also enhances the onset of convection.

Figure 4 shows the graph of R_{as} w.r.t. wave number a for different values of Le = 500, 1000, 1500 by fixing the other parameters as N_A = 10, ε = 0.6, R_n = –1, λ_3 = 0.5. It is clear from Fig. 4 that within increase in the value of Le, R_{as} goes on increasing, and hence shows the stabilising effect on stationary convection. It is also clear from Fig. 4 that rigid-free boundary condition has more stabilising impact on stationary convection as compared to rigid-rigid boundary conditions. Thus, Le delays the onset of convection.

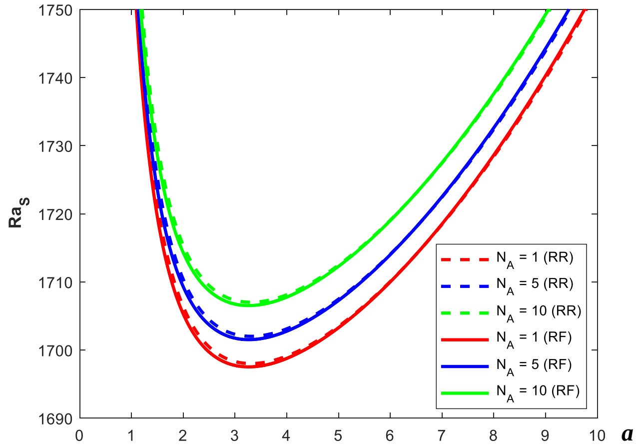

Figure 5. Variation of stationary Rayleigh number R_{as} with wave number a for different value of modified diffusivity ratio N_A.

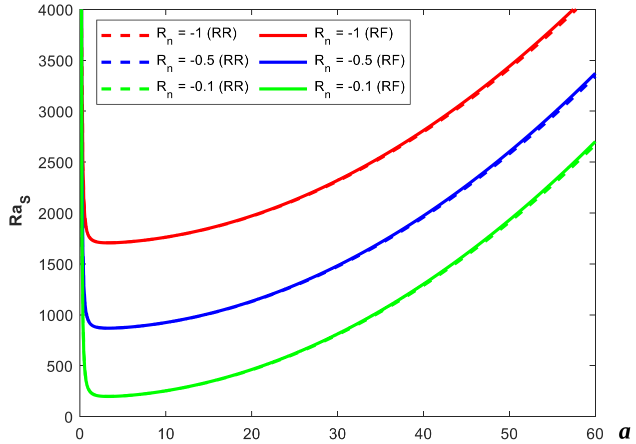

Figure 6. The variation of stationary Rayleigh number R_{as} vs. wave number a for different value of nanoparticles Rayleigh number R_n.

Figure 5 shows the graph of R_{as} w.r.t. wave number a for different values of N_A = 1, 5, 10 by fixing the other parameters as λ_3 = 0.5, Le = 1000, ε = 0.6, R_n = –1. It is clear from Fig. 5 that within increase in the value of N_A, R_{as} goes on increasing, and hence shows the stabilising effect on stationary convection. It is also clear from Fig. 5 that rigid-free boundary condition has more stabilising impact on stationary convection as compared to rigid-rigid boundary conditions. Thus, N_A delays the onset of convection.

Figure 6 shows the graph of R_{as} w.r.t. wave number a for different values of $R_n = -1, -0.5, -0.1$ by fixing the other parameters as $\lambda_3 = 0.5$, $N_A = 10$, Le = 1000, $\varepsilon = 0.6$. It is clear from Fig. 6 that within increase in the value of R_n, R_{as} goes on decreasing, and hence shows the destabilising effect on stationary convection. It is also clear from Fig. 6 that rigid-free boundary condition has more destabilising impact on stationary convection as compared to rigid-rigid boundary conditions. Thus, R_n accelerate the onset of convection.

CONCLUSIONS

In this paper, we have analysed the stationary convection in the thermal instability of Jeffrey nanofluid in a porous medium: rigid-rigid and rigid-free boundary conditions. For this analysis we have utilised the GWR method.

We have drawn the following conclusions:

- Jeffrey parameter λ_3, nanoparticle's Rayleigh number R_n and medium porosity ε have destabilising effects on stationary convection.
- Lewis number Le and modified diffusivity ratio N_A, both have stabilising impact on stationary convection.
- It is found that in case of rigid-free boundary condition, the system remains more stable rather than at rigid-rigid boundary condition.

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