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THERMOMECHANICAL RESPONSE OF A FUNCTIONALLY GRADED PRESSURIZED CYLINDER USING ITERATIVE TECHNIQUE

TERMOMEHANIČKI ODZIV CILINDRA POD PRITISKOM OD FUNKCIONALNOG KOMPOZITNOG MATERIJALA PRIMENOM ITERATIVNE METODE

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Keywords
- functionally graded materials
- pressure
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Abstract

In our research investigation for the performance of functionally graded (FGM) pressurized cylinder, we implement a novel iterative technique to examine the behaviour of thick hollow FG cylinder subjected to thermomechanical loading. Material properties, namely, Young's modulus and the coefficient of thermal expansion are varied exponentially from inner to outer surface of pressure vessel along radial direction, whereas Poisson's ratio and temperature are invariant. Iterative solutions for displacement and stresses are derived by solving the governing differential equation of isotropic functionally graded cylinder under plane strain and steady-state conditions. The novelty of the work is that an iterative technique is applied to Navier's equation under plane strain conditions, and it gives a good approximation to the solution of stresses and displacement field after two iterations. It may also be applicable to find the solutions for higher order nonlinear differential equations. The iterative method implemented is near to exact and fast with a reasonable number of iterations. Results obtained for radial stress, tangential stress, and displacement are presented graphically under different inner-outer pressure conditions and uniform temperature loading. Solutions depicted through graphs are used to present the comparison of stresses and displacement in FGM cylinder made up different metal-ceramic combinations, namely, aluminium (Al) - aluminium oxide (Al_2O_3), copper alloy (CA) - aluminium nitride (AlN), nickel (Ni) - silicon nitride (Si_3N_4), and pressurized cylinder made up of homogeneous material and plates, but few have attempted the two-dimensional solution. In this paper, we have developed solutions for two-dimensional pressurized cylinder made of functionally graded material with nonlinear variation of Young's modulus in the radial direction and its numerical analysis.

Ključne reči
- funkcionalni kompozitni materijali
- pritisak
- naponi
- pomeranje
- Jungov modul elastičnosti
- iterativna metoda

Izvod

Prema našim istraživanjima performansi cilindra pod pritiskom od funkcionalnog kompozitnog materijala (FGM), implementirali smo novu iterativnu metodu za analizu ponašanja debelozidog šupljeg FG cilindra, koji je opterećen termomehanički. Osobine materijala, zapravo, Jungov modul elastičnosti i koeficijent termičkog širenja se menjaju eksponencijalno od unutrašnje ka spoljnjoj površini posude pod pritiskom duž radijalnog pravca, gde su Poasonov koeficijent i temperatura su invarijante. Iterativna rešenja za pomeranja i napone dobijena su rešavanjem zadatih diferencijalnih jednačina cilindra od izotropnog funkcionalnog kompozitnog materijala u uslovima ravnih deformacija i ravnomernog opterećenja. Novina u radu je iterativna metoda primenjena na jednačinu Navijea u uslovima ravnih deformacija, koja daje dobru aproksimaciju rešenja polja napona i pomeranja nakon dve iteracije. Moguća je primena i u nalaženju rešenja nelinearnih diferencijalnih jednačina višeg reda. Implementirana iterativna metoda je veoma blizu tačnog rešenja i brza sa razumnim brojem iteracija. Dobijeni rezultati za radijalni napon, tangencijalni napon i pomeranje predstavljeni su grafički u uslovima različitog unutrašnjeg-spoljašnjeg pritiska i uniformnog temperaturskog opterećenja. Rešenja predstavljena grafički mogu da se upotrebe za poređenje napona i pomeranja FGM cilindra sačinjenog od različitih kombinacija metal-keramika, zapravo, aluminijum (Al) - aluminijum oksid (Al_2O_3), legura bakra (CA) - aluminijum nitrid (AlN), nikal (Ni) - silicijum nitrid (Si_3N_4), i za cilindar pod pritiskom od homogenog materijala i ploča, ali je bilo malo pokušaja za dvo-dimenzionalni problem. U ovom radu smo došli do rešenja za dvo-dimenzionalni cilindar po pritiskom od funkcionalnog kompozitnog materijala sa nelinearnom varijacijom Jangovog modula elastičnosti u radijalnom pravcu sa numeričkom analizom.

INTRODUCTION

Functionally graded materials (FGMs), also known as innovative composite materials are a class of futuristic engineering material as these innovative materials have an edge over traditional composite materials due to their ability to change material properties continuously and smoothly at the interface of two materials. Such ability of the material creates a strong bonding in layer composites to avoid material failure situations by reducing stress concentration at the interface layer. Therefore, many engineers and material scientists have been attracted towards functionally graded materials and started working on their wide possibilities of applications in engineering, medical science, defence sector, etc. In many applications of the aerospace and automobile industries, composite materials are used to build mechanical and thermal components, but such studies are restricted in terms of monitoring continuous behaviour of stress response with the variation in material change. The first idea about FGM was introduced in early 1980s by Japanese material scientists to sort out the problem of building a thermal component that can perform wisely at high-temperature differences during their space plane project, /1/. Reinforcement materials with the metal matrix are used to build many engineering components like a pressurized cylinder, rotating disk, cutting tool, internal combustion chambers, pistons, high-pressure storage tank, orthopaedic material, defence equipment, etc. Many researchers from different backgrounds are contributing their ideas on functionally graded materials to enhance interdisciplinary research and their applications. Thus, scientists have worked for analytical solutions of thermomechanical response for functionally graded materials. But due to the complex behaviour of reinforcement profile, geometric profile, and thermomechanical loading, researchers are working on semi-analytical and numerical methods to find the displacement and stresses. Çallıoğlu et al. /2/, conducted elastic-plastic stresses analysis in which a functionally graded disk is created by power metal processing method and observed that yielding of plasticity in the FG disk initiates at inner radial points. Kurşun and Topçu /3/ studied a hollow functionally graded disk with varying thickness along radial direction in power-law form and under the application of linear thermal loading. Sahni and Sahni /4/, have conducted an analysis of a thin annular disk with varying thickness and Young's modulus for elastic stresses and displacement subjected to outer pressure and angular velocity. Evci and Gülgeç /5/ obtained a direct solution for the stress and displacement fields, whereas Sharma and Kaur /6/ presented a numerical approach using the finite element method for stress analysis of FG cylinder. Mehta et. al /7/, using FEM, presented mathematical analysis for thermomechanical stresses and strains in a sandwich cylinder with inner layer tailored from functionally reinforced material and outer layer from composite material with uniform material properties. Yarimpabuç /8/ developed a mathematical model to examine thermomechanical stress in a thick-walled cylinder and sphere exposed under high thermal loading tailored from functionally graded material. Sahni et. al /9/, examined secondary creep deformations in pressurized FG cylinder. Their investigation presented the response of secondary creep stresses and secondary strains in a pressurized FG cylinder with linear, secondary, and exponential ceramic volume gradation in the aluminium metal matrix. Calderale et al. /10/, examined functionally graded disk with nonlinear thickness as a power law function and investigated the thermal response of concave disk under thermal loading. Sharma et al. /11/ and Sahni et al. /12/ determined stresses in elastic and plastic region for a functionally graded disk with radially varying thickness. The outcome of their studies validated the significant effect of variable thickness on behaviour of stresses in functionally graded disk. Paul and Sahni /13, 14/ conducted the study of mechanical behaviour for two-dimensional FGM pressure vessels of cylindrical and spherical design. Jalali and Shahriari /15/ implemented finite difference method (FDM), and Thawait et al. /16/ used finite element technique to investigate functionally graded disk with nonlinear variable thickness and material properties, namely, elasticity modulus and density, in radial direction. An orthotropic rotating functionally graded disk subjected to mechanical loading is analysed by researchers /17, 18/. They demonstrated the impact of the degree of anisotropy on elastic stresses and displacement. Kamdi and Lamba /19/ conducted a study of inverse thermoelastic problem for the dynamic response of functionally graded isotropic hollow cylinder tailored with alumina as the ceramic reinforcement material at inner radius and nickel as metal matrix material at outer radius specified to a uniform temperature loading distribution. A functionally graded disk with exponentially varying thickness is investigated for the stress response using Cauchy Euler iterative technique by Sandeep et al. /21/. The authors have compared numerical results for the three different ceramic-metal combinations for the disk. Paul and Sahni /26/ have used Fourier half range series to derive the solutions for displacement and stress fields along radial and circumferential directions. The effect of grading parameter on functionally graded cylinder under mechanical loading is examined by the authors. Kumaraswamy et al. /27/ have developed hybrid metal matrix composition (MMC) that focuses on the nickel alloy and its thermal properties. The different range of temperature are used to analyse the thermal conductivity and coefficient of thermal expansion for the metal matrix composite. Kaboglu et. al /28/, using digital image correlation, provided analysis of flexural mechanical properties for innovatively sandwich composite structures tailored with the materials, namely, balsa wood, tycor and polyethylene terephthalate, with the aid of four-point bending condition. Their study revealed various advantages of these three core materials over one another with regards to stiffness, shear strength, damage tolerance and sudden failure. An experimental work is done to bring out the effect of forced convection on aluminium alloy 6061, aluminium alloy 6061 MMC as-cast, and aluminium alloy 6061 post heat treatment conditions /29/. Analysis revealed that surface temperature on metal matrix composites is higher than base metal signalling higher thermal dissipation being perceived. Akinshilo /30/, applied Adomian Decomposition Method (ADM) for analysing the heat transfer in convection fins. The author considered temperature dependent conductivity and internal heat generation profiles in the study and examined the influence of parameters, namely, thermo geometric, internal heat generation, and thermal conductivity on the temperature and heat flow rate intensity of the fin. Deka et al. /31/ investigated thermoelastic stresses in FG disk and presented the solution for conductive-convective-radiative heat equation with internal heat generation. Researchers /32/ presented the study of hyperbolic shear deformation theory with five degrees of freedom for FG isotropic sandwich and laminated composite plates. The plates are subjected to parabolic transverse shear strains along its thickness profile with zero traction on its top and bottom surfaces. Hadji et al. /33/ and Hadji /34/ used a four-variable refined plate theory to investigate the free vibrations in functionally graded material rectangular plates with power law from varying profile forms for the volume fraction of material constituents in the thickness direction of FGM plates. The study outlines the effect of material distribution, aspect, and side-to-thickness ratio on the fundamental frequencies of FGM plates. Some researchers /35-36/ developed new five variable theory to examine free vibrations in innovatively graded sandwich plates and also in laminated composite plates considering the stretching and bending effect. These models, together with Hamiltonian principle, implement shear deformation theory of higher order to derive equation of motion to study transverse shear strains with the constraint of zero traction on the surface of the plates, in absence using shear correction factor. Bourada et al. /37/ studied the buckling analysis of isotropic and orthotropic plates under the in-plane loading by employing higher shear deformation theory with four variables in which instead of derivative terms in the displacement field, integral terms are used. The displacement field is obtained by considering the higher-order distributions of in-plane displacements within the plate thickness. Belifa et al. /38/ constructed an exact solution of bending and dynamic behaviour of FG rectangular plates with the aid of novel shear deformation theory of higher order. Zoubida et al. /39/ presented a refined theory for examining the static and free vibration in FG beam with necessary constraint of zero traction surface condition and shear strains varying in parabolic form along the thickness direction, in absence of shear correction factor. The model provides the solution for static and free vibration frequency of functionally graded beam with its material properties varying in the thickness direction. Hadji et al. /40/ studied the dynamic behaviour of FG beams to analyse the influence of volume fraction index and thickness-to-length ratio on the fundamental frequencies of FGM plates by introducing a new first-order shear deformation theory to derive the isotropic governing equations for axial and transverse deformations subjected to boundary conditions having the simple forms like in case of isotropic plates. Further, extending this study to FG sandwich beams using theory of hyperbolic shear deformation beam, Bouakkaz et al. /41/, while accounting for higher-order variation of transverse shear strain through the depth of the beam, considered varying material volume fraction in power law form and then demonstrated the effect on free vibrations in FG sandwich beams under the influence of variable volume gradation and thickness to length ratios. The FG sandwich beams are subjected to the zero traction boundary conditions on the surfaces, in absence of shear correction factors. Arefi et al. /43-44/ provided piezo-thermoelastic analysis for thick spherical and cylindrical FG shells composed of functionally graded piezoelectric (FGP) materials. They demonstrated the influence of non-homogeneity parameters on the electrical and mechanical components under the effect of electrical and thermomechanical loading. In electromechanical systems, FGP materials can be utilised as an actuator, sensor, or part of a piezo motor due to the smooth transition of characteristics along the radial direction. Rahimi et al. /45/ explored the application of FG piezo-electric cylinder as a sensor or an actuator in electromechanical systems. By using a novel concept described as the added energy, they have predicted the behaviours of piezoelectric structures, particularly those subjected to thermal loads. Further, for the purpose of angular velocity measure, a FGP hollow cylinder mounted to the rotating devices is used. As a result of the findings, it is possible to forecast how a FGP rotating cylinder would behave when it is subjected to different angular velocities. Applying a modified form of the combination of Adomian's decomposition and successive approximation method, Arefi /46/ has studied a nonlinear analysis for functionally graded piezoelectric cylinder subjected to electric, thermal, mechanical loads. They have also compared the obtained results with linear analysis for the same FGP cylinder. By using the shear deformation theory of order three, Mohammadi et al. /47/ have examined two-dimensional thermoelastic response of cylindrical pressure vessels made of functionally graded carbon nanotube-reinforced composite and using the rule of mixture, properties of pressure vessels are calculated for various reinforcement patterns. They have investigated the impact of various important elements on the thermoelasticity behaviour, namely, volume fraction for the carbon nanotubes and the Pasternak coefficients related with elasticity. The thermo-elastic study of a clamped-clamped axisymmetric functionally graded cylinder under internal pressure is carried out by Arefi and Rahimi /48/. The authors have derived the principle differential equations using the Hamilton principle and first order shear deformation theory (FSDT). Arefi et al. /49/ have analysed a cylindrical shell reinforced with graphene nanoplatelets (GPLs) and subjected to thermomechanical loads using the shear deformation theory. For the purpose of evaluation of attributes of composite material with uniform symmetric and asymmetric reinforcement distributions for nanoplatelet material, Halpin-Tsai micromechanical model and rule of mixtures are implemented. Analysis of cementitious material in form of variable ratio for the replacement of ceramsite sand to silica sand is presented by Wang et al. /50/ for extrusion-based three-dimensional printability. The layer of polyvinyl alcohol on ceramsite sands prevents shrinkage and microcracks caused by water absorption. By coordinating the new qualities with the continuous printing technique, an optimal cementitious material is discovered. Based on the optimum lightweight combination, cubic and beam elements with four different types of inner hollow structures are created and 3D printed. Using the Pasternak theory, Arefi et al. /51/ examined two-dimensional thermoelastic analysis of FG thick-walled cylinders exposed to thermomechanical loading conditions. The displacement field is described by shear deformation theory for the first-order. The energy technique and Euler equations are used to get the fundamental governing differential equations of the system. Using the theory of shear deformation, Arefi et al. /52/ analysed the thermal elasticity static problem of containers of cylindrical geometry with the functionality to operate under high pressure and tailored with FG carbon nanotube reinforced composite. Arefi and Rahimi /53/ conducted the study of two-dimensional electro elasticity for an internally pressurized FG piezoelectric cylinder. The major applications of shells, cylindrical shells are discussed as pressure vessels, reactors, heat exchanger, other nuclear and chemical equipment by Arefi and Rahimi /54/. In this era of integrative research, scientists and engineers from science and engineering branches are collaboratively engaged in exploring new and advanced real-world applications of these innovative materials in their respective fields. For designing of engineering components, determination of appropriate is imperative for accomplishing the desired functionality from the engineering component subjected to thermomechanical loading. This initiates the main motivation for our study and formulation of this paper.

In this research study, thermal and mechanical response of a pressurized cylinder made up of functionally graded material is examined with varying material attributes, namely, Young's modulus and thermal expansion coefficient along radial direction with an exponential law. Using iterative scheme, Navier's equations for displacement and stress fields are obtained for uniform temperature distribution in the radial direction. The numerical solution for the material response in terms of stresses and displacement are depicted graphically for different FGM materials and are further compared with homogenous material. The novelty of the work is that it gives a good approximation to the solution even after two iterations. It may also be applicable to find the solutions for the higher order nonlinear equation. At $n = 0$, we can validate the formulated iterative equations with Cauchy Euler equations which provides a direct solution of the homogeneous equation. We have considered two iterations because of more complex and time consumption /42/. From our calculation of iterations, for number of iterations $n = 0$, 1, and 2, there is a difference in terms of computing time, but the percentage of relative error as observed in our calculations is approximately equal and hence there is no need to go ahead with further iterations as it does not affect our final results.

Problem description

We consider a problem of a thick axisymmetric functionally graded (FG) cylinder having inner and outer radii a and b, respectively. The FG cylinder is further subjected to plane strain and steady-state conditions. This FG cylinder is tailored with a fixed Poisson's ratio ν whereas, the modulus of elasticity $E(r)$ and thermal expansion coefficient $\alpha(r)$ are varying exponentially along the radial direction as, /20/,

$$\alpha(r) = \alpha_M e^{-m_2(r-b)}, \tag{1}$$

$$E(r) = E_M e^{-m_1(r-a)}, \tag{2}$$

where: radial parameters, $a \le r \le b$; E_M is constant for Young's modulus at inner radius; and α_M is coefficient of thermal expansion at outer radius. The indices of material gradation parameter and thermal expansion are denoted by m_1 and m_2, respectively, and are expressed as,

$$m_1 = \frac{b}{b-a}\log\left(\frac{E(a)}{E(b)}\right), \quad m_2 = \frac{b}{b-a}\log\left(\frac{\alpha(a)}{\alpha(b)}\right).$$

Navier equation for the isotropic functionally graded cylinder with an exclusion of the influence of centrifugal body force is formulated as, /24/,

$$\frac{d\sigma_r}{dr} + \frac{1}{r}(\sigma_r - \sigma_\theta) = 0, \tag{3}$$

where: σ_r and σ_θ are radial and tangential stresses along the radial direction of the FG cylinder, respectively.

The compatibility relation or the strain-displacement relation for the cylinder with radial strain $\varepsilon_r(r)$, tangential strain $\varepsilon_\theta(r)$, and radial displacement $u(r)$ can be considered as, /22/,

$$\varepsilon_r(r) = \frac{du(r)}{dr}, \quad \text{and} \quad \varepsilon_\theta(r) = \frac{u(r)}{r}. \tag{4}$$

By employing Hooke's law, the relationship between stresses and strains under a plane strain condition is presented as, /23/,

$$\sigma_{rr} = (\lambda + 2\mu)\varepsilon_{rr} + \lambda\varepsilon_{\theta\theta} - (3\lambda + 2\mu)\alpha(r)T, \tag{5}$$

$$\sigma_{\theta\theta} = (\lambda + 2\mu)\varepsilon_{\theta\theta} + \lambda\varepsilon_{rr} - (3\lambda + 2\mu)\alpha(r)T, \tag{6}$$

where: λ and μ are Lame's constants and are defined as

$$\lambda = \frac{\nu E(r)}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E(r)}{2(1+\nu)}.$$

The required necessary surface constraints in terms of internal-external pressure boundary condition can be specified as p and q, respectively, and defined as,

$$\text{at} \quad r = a, \quad \sigma_r = -p \quad \text{and at} \quad r = b, \quad \sigma_r = -q. \tag{7}$$

On solving Eqs.(1)-(6), the equation of equilibrium in the second order non-homogeneous differential equation form for the displacement can be formulated as,

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = m_1\frac{du}{dr} + \frac{m_1\nu}{1-\nu}\frac{u}{r} - \left(\frac{1+\nu}{1-\nu}\right)(m_1 + m_2)\alpha_0 T e^{-m_2(r-a)}. \tag{8}$$

Proposed iterative technique

An ordinary differential equation (ODE) in its generic form can be expressed as, /22/,

$$\psi_1(s(x)) = t(x) + \psi_2(s(x)), \tag{9}$$

under the necessary constraints, $B\left(s, \frac{ds}{dx}\right) = 0$,

where: $s(x)$ and $t(x)$ are unknown and known functions, in respect. In Eq.(9), ψ_1 is linear operator and ψ_2 is linear or nonlinear operator. But the main requirement in this iterative technique is that ψ_1 must be linear but sometimes it is also possible to add some linear parts of ψ_1 into ψ_2, as needed. In this problem, ψ_1 shows linear part and follows Cauchy Euler form, and ψ_2 is formed of other remaining linear parts, and function $t(x)$ in the ODE. Here, iteration is considered for the function $s(x)$ only.

In this iterative scheme, an initial solution $s_0(x)$ is considered as a complementary function of Eq.(9) and can be obtained by solving Eq.(10) as,

$$\phi_1(s(x)) = 0, \tag{10}$$

specified under initial boundary condition, $B\left(s_0,\dfrac{ds_0}{dx}\right)=0$.

To further get the refinement in the solution $s_0(x)$, the succeeding iteration $s_1(x)$ is evaluated using Eq.(11) as,

$$\psi_1(s_1(x))=t(x)+\psi_2(s_0(x))$$

with the modified boundary condition as,

$$B\left(s_1,\frac{ds_1}{dx}\right)=0\,. \tag{11}$$

By reiterating this procedure for $(n + 1)$ terms, Eq.(11) can be formulated in the iteration form as,

$$\psi_1(s_{n+1}(x))=t(x)+\psi_2(s_n(x))\,, \tag{12}$$

with given condition, $B\left(s_{n+1},\dfrac{ds_{n+1}}{dx}\right)=0$, $n = 0,1,2,3, \ldots$,

where: $s_j(x)$, $(j = 1,2,3, \ldots)$ presents a separate solution to Eq. (11), but one can obtain a refined value of the solution by performing higher number of iterations. The succeeding iteration shows a good improvement over the previous iteration.

Implementation of the proposed iterative technique

On the basis of undertaking problem with the reference of Eq.(9), here $s(x) = u(r)$ and $s_{n+1}(x) = u_{n+1}(r)$, the variable x is replaced by r. Further, by applying iterative technique from Eq.(12) to Eq.(8), the governing differential equation can be expressed as,

$$\frac{d^2u_{n+1}}{dr^2}+\frac{1}{r}\frac{du_{n+1}}{dr}-\frac{u_{n+1}}{r^2}=m_1\frac{du_n}{dr}+\frac{m_1\nu}{1-\nu}\frac{u_n}{r}-\left(\frac{1+\nu}{1-\nu}\right)(m_1+m_2)\alpha_0Te^{-m_2(r-a)}\,, \tag{13}$$

specific to the boundary condition $B\left(u_{n+1},\dfrac{du_{n+1}}{dr}\right)=0$,

where: $\psi_1(u_{n+1}(r))=\dfrac{d^2u_{n+1}}{dr^2}+\dfrac{1}{r}\dfrac{du_{n+1}}{dr}-\dfrac{u_{n+1}}{r^2}$, and

$\psi_2(u_n(r))=m_1\dfrac{du_n}{dr}+\dfrac{m_1\nu}{1-\nu}\dfrac{u_n}{r}-\left(\dfrac{1+\nu}{1-\nu}\right)(m_1+m_2)\alpha_0Te^{-m_2(r-a)}$

represent linear parts in left- and right-hand side, respectively, in Eq.(13).

Thus, the initial solution for Eq.(13) is,

$$\psi_1(u_0)=0\,, \tag{14}$$

with the boundary conditions as

$$\left(\frac{1-\nu}{(1+\nu)(1-2\nu)}E(r)\frac{du_0}{dr}+\frac{\nu E(r)}{(1+\nu)(1-2\nu)}\frac{u_0}{r}-\frac{E(r)\alpha(r)}{1-2\nu}T\right)_{r=a}=-p$$

and

$$\left(\frac{1-\nu}{(1+\nu)(1-2\nu)}E(r)\frac{du_0}{dr}+\frac{\nu E(r)}{(1+\nu)(1-2\nu)}\frac{u_0}{r}-\frac{E(r)\alpha(r)}{1-2\nu}T\right)_{r=b}=-q$$

The general solution of the initial problem Eq.(14) can be obtained as,

$$u_0=\frac{A_1}{r}+A_2r\,. \tag{15}$$

Using above necessary constraint (boundary) conditions, A_1 and A_2 can be evaluated as,

$$A_1=\frac{a^2b^2}{(t_2-t_1)(b^2-a^2)}\left(\frac{q}{E(b)}-\frac{p}{E(a)}+(\alpha(a)-\alpha(b))t_3T\right), \tag{16}$$

$$A_2=\left(\frac{-p}{E(a)}+t_3\alpha(\alpha)T-\left(\frac{t_2-t_1}{a^2}\right)A_1\right)\frac{1}{t_1+t_2}\,. \tag{17}$$

The first iteration u_1 can be obtained by solving Eq.(13) for $n = 0$ as,

$$\frac{d^2u_1}{dr^2}+\frac{1}{r}\frac{du_1}{dr}-\frac{u_1}{r^2}=m_1\frac{du_0}{dr}+\frac{m_1\nu}{1-\nu}\frac{u_0}{r}-\left(\frac{1+\nu}{1-\nu}\right)(m_1+m_2)\alpha_0Te^{-m_2(r-a)}\,, \tag{18}$$

specified to boundary conditions $B\left(u_1,\dfrac{du_1}{dr}\right)=0$, and has a solution as,

$$u_1=\frac{A_3}{r}+A_4r+\frac{1-2\nu}{1-\nu}m_1A_1+\frac{m_1}{3(1-\nu)}A_2r^2+\frac{1}{m_2^2}\left(-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a}\right)e^{-m_2r}+\frac{1}{m_2^3}\left(-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a}\right)\frac{e^{-m_2r}}{r}\,, \tag{19}$$

where: constants of integration A_3 and A_4, can be found as,

$$A_3=\frac{a^2b^2(\delta_1-\delta_2)}{(t_2-t_1)(b^2-a^2)}\,,\quad A_4=\left(\delta_1-\frac{A_3(t_2-t_1)}{a^2}\right)\frac{1}{(t_1+t_2)}\,.$$

Applying the same process, the second iteration can be obtained as,

$$u_2=\frac{A_5}{r}+A_6r+\frac{1-2\nu}{1-\nu}m_1A_3+\frac{m_1^2\nu(1-2\nu)}{2(1-\nu)^2}A_1r\log r+\frac{m_1A_4}{3(1-\nu)}r^2+\frac{(2-\nu)m_1^2A_2}{24(1-\nu)^2}r^3+\left(-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a}\right)\left(\frac{(m_2-m_1)(m_2r+1)}{m_2^4r}\times e^{-m_2r}+\left(\frac{2\nu-1}{1-\nu}\right)\frac{m_1}{m_2^2}\left(I_1(r)+\frac{I_2(r)}{m_2}\right)\right), \tag{20}$$

and constants of integration A_5 and A_6 are calculated as,

$$A_5=\frac{a^2b^2(\delta_3-\delta_4)}{(t_2-t_1)(b^2-a^2)}\,,\quad A_6=\left(\delta_3-\frac{A_5(t_2-t_1)}{a^2}\right)\frac{1}{(t_1+t_2)}\,.$$

With the continuation of the above mentioned process, we evaluate $u_3(r)$ which is found as same as that of $u_2(r)$. Hence, the solution accuracy may not be improved by considering increment in the total number of iterations. Substituting $u(r) = u_2(r)$ in Eqs.(4)-(6), radial and tangential strains and stresses, respectively, are obtained as,

$$\varepsilon_{rr}=-\frac{A_5}{r^2}+A_6+\frac{m_1^2\nu(1-2\nu)(1+\log r)A_1}{2(1-\nu)^2}+\frac{2m_1rA_4}{3(1-\nu)}+\frac{1}{8}\frac{(2-\nu)m_1^2A_2r^2}{(1-\nu)^2}+\left(-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a}\right)\left(-\frac{m_2-m_1}{m_2^4}\times\left(m_2^2+\frac{m_2}{r}+\frac{1}{r^2}\right)+\frac{m_1(2\nu-1)}{m_2^2(1-\nu)}\left(I_1'(r)+\frac{1}{m_2}I_2'(r)\right)\right), \tag{21}$$

$$\varepsilon_{\theta\theta}=\frac{A_5}{r^2}+A_6+\frac{(1-2\nu)}{(1-\nu)^2}\frac{m_1}{r}A_3+\frac{m_1^2\nu(1-2\nu)}{2(1-\nu)^2}A_1\log r+\frac{m_1A_4}{3(1-\nu)}r+\frac{(2-\nu)m_1^2A_2}{24(1-\nu)^2}r^2+\left(-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a}\right)\times$$
$$\times\left(\frac{(m_2-m_1)(m_2r+1)e^{-m_2r}}{m_2^4r^2}+\left(\frac{2\nu-1}{1-\nu}\right)\frac{m_1}{m_2^2r}\left(I_1(r)+\frac{I_2(r)}{m_2}\right)\right), \tag{22}$$

$$\sigma_{rr}=\frac{(1-\nu)E(r)}{(1+\nu)(1-2\nu)}\left(-\frac{A_5}{r^2}+A_6+\frac{m_1^2\nu(1-2\nu)(1+\log r)A_1}{2(1-\nu)^2}+\frac{2m_1rA_4}{3(1-\nu)}+\frac{1}{8}\frac{(2-\nu)m_1^2A_2r^2}{(1-\nu)^2}+\left(-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a}\right)\times\right. \tag{23}$$
$$\left.\times\left(-\frac{m_2-m_1}{m_2^4}\left(m_2^2+\frac{m_2}{r}+\frac{1}{r^2}\right)+\frac{m_1(2\nu-1)}{m_2^2(1-\nu)}\left(I_1'(r)+\frac{1}{m_2}I_2'(r)\right)\right)\right)+\frac{\nu E(r)}{(1+\nu)(1-2\nu)}\left(\frac{A_5}{r^2}+A_6+\frac{1-2\nu}{1-\nu}\frac{m_1}{r}A_3+\frac{m_1^2\nu(1-2\nu)}{2(1-\nu)^2}A_1\log r+\right.$$
$$\left.+\frac{m_1A_4}{3(1-\nu)}r+\frac{(2-\nu)m_1^2A_2}{24(1-\nu)^2}r^2+\left(-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a}\right)\left(\frac{(m_2-m_1)(m_2r+1)e^{-m_2r}}{m_2^4r^2}+\left(\frac{2\nu-1}{1-\nu}\right)\frac{m_1}{m_2^2r}\left(I_1(r)+\frac{I_2(r)}{m_2}\right)\right)\right)-\frac{E(r)\alpha(r)T}{1-2\nu},$$

$$\sigma_{\theta\theta}=\frac{(1-\nu)E(r)}{(1+\nu)(1-2\nu)}\left(\frac{A_5}{r^2}+A_6+\frac{1-2\nu}{1-\nu}\frac{m_1}{r}A_3+\frac{m_1^2\nu(1-2\nu)}{2(1-\nu)^2}A_1\log r+\frac{m_1A_4}{3(1-\nu)}r+\frac{(2-\nu)m_1^2A_2}{24(1-\nu)^2}r^2+\left(-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a}\right)\times\right.$$
$$\left.\times\left(\frac{(m_2-m_1)(m_2r+1)e^{-m_2r}}{m_2^4r^2}+\left(\frac{2\nu-1}{1-\nu}\right)\frac{m_1}{m_2^2r}\left(I_1(r)+\frac{I_2(r)}{m_2}\right)\right)\right)+\frac{\nu E(r)}{(1+\nu)(1-2\nu)}\left(-\frac{A_5}{r^2}+A_6+\frac{m_1^2\nu(1-2\nu)(1+\log r)A_1}{2(1-\nu)^2}+\frac{2m_1rA_4}{3(1-\nu)}+\right.$$
$$\left.+\frac{1}{8}\frac{(2-\nu)m_1^2A_2r^2}{(1-\nu)^2}+\left(-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a}\right)\left(-\frac{m_2-m_1}{m_2^4}\left(m_2^2+\frac{m_2}{r}+\frac{1}{r^2}\right)+\frac{m_1(2\nu-1)}{m_2^2(1-\nu)}\left(I_1'(r)+\frac{1}{m_2}I_2'(r)\right)\right)\right)-\frac{E(r)\alpha(r)T}{1-2\nu}. \tag{24}$$

NUMERICAL RESULTS AND DISCUSSION

In our analysis, we have examined a thick FGM cylinder having internal radius of a = 0.2 m, external radius of b = 1 m, internal pressure p = 100 MPa, and external pressure q = 10 MPa. The study focuses on the thermomechanical stresses and displacement response of three different material combinations of metals (inner material) and ceramics (outer material). Material properties of metal and ceramic materials considered are shown in Table 1. In this table, Poisson's ratio for metal and ceramic are ν_M and ν_C, in respect, and Poisson's ratio ν for FGMs are assumed as $\nu = (\nu_M + \nu_C)/2$. Homogenous disk made up of aluminium (Al) is considered with Young's modulus 70 GPa, coefficient of thermal expansion 23.1×10^{-6} 1/°C), and Poisson's ratio 0.32.

Table 1. FGM Materials and their properties /24, 25/.

FGM	$E(r)$ [GPa] (inner-outer)	$\alpha(r)$ [×10^{-6} 1/°C] (inner-outer)	m_1	m_2	ν
Al - Al_2O_3	70-393	23.1-4.5	-2.156	2.044	0.27
Copper alloy - AlN	130-318	18.4-5.3	-1.118	1.555	0.29
Ni - Si_3N_4	199.5-348.4	17-3.27	-0.697	2.060	0.26

Young's modulus is a property of a material that is independent and material specific. It measures material's capacity to sustain an applied load with the least amount of deformation. In choosing materials, the Young's modulus is a crucial factor. The coefficient of thermal expansion is a measure of tendency of the material to undergo dimensional change with respect to variability in temperature. With a coefficient of a small value indicating less tendency for change in dimension, it measures the fractional change in dimension per degree change in temperature at constant pressure. Figures 1 and 2 represent Young's modulus and the thermal expansion coefficient for considered functionally graded and homogeneous material, respectively. The purpose is to restrict the elastic deformations as small as possible. Therefore, Young's modulus plays a crucial role in choosing materials for the intended application. The amalgamation of Young's modulus with the sectional properties helps in deciphering the occurrence of elemental deformation subjected to specified thermomechanical loading conditions. Hence, Young's modulus with its aid in evaluating the stiffness of FGMs, has a prominent part to play in the design and analysis of an engineering component. In our study, as radial values move from inner to outer radii, for a functionally graded cylinder, Young's modulus increases while the thermal expansion coefficient decreases, whereas for a cylinder of homogeneous material, namely aluminium, these material properties remain constant throughout the radial points.

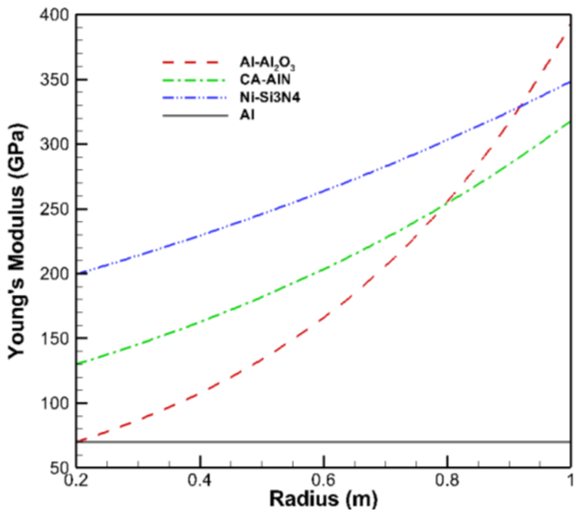

Figure 1. Young's modulus.

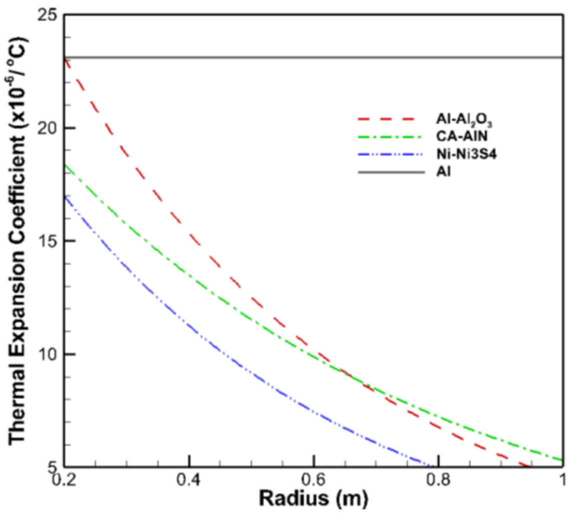

Figure 2. Thermal expansion coefficient.

From Fig. 3, it can be observed that the distribution of radial displacement for each material of the cylinder is varying with increasing magnitude as radial points move from internal radius to outer radius. Homogenous material (Al) is having a high magnitude of displacement in comparison to the considered FGM materials. Due to the lower value of Young's modulus, material attains more flexibility and under internal pressure it expands more as compared to others. Radial displacement in FGM cylinders is found to be of lesser magnitude as compared to the homogeneous cylinder. It can be noticed that the displacement in cylinder made up of Al-aluminium oxide (Al_2O_3) is lowest at inner radius, whereas the cylinder with nickel (Ni)-silicon nitride (Si_3N_4) combination has the lowest magnitude at the exterior radial surface of the cylinder. In Fig. 4, the radial stress for the cylinder of Ni-Si_3N_4 is highly compressive than other functionally graded cylinders which means that this material is required to have more amount of force than others to resist the uniform pressure. The magnitude of radial stresses in FGMs are decreasing towards the outer radius under uniform pressure as it requires less amount of pressure to maintain force equilibrium at the outer surface.

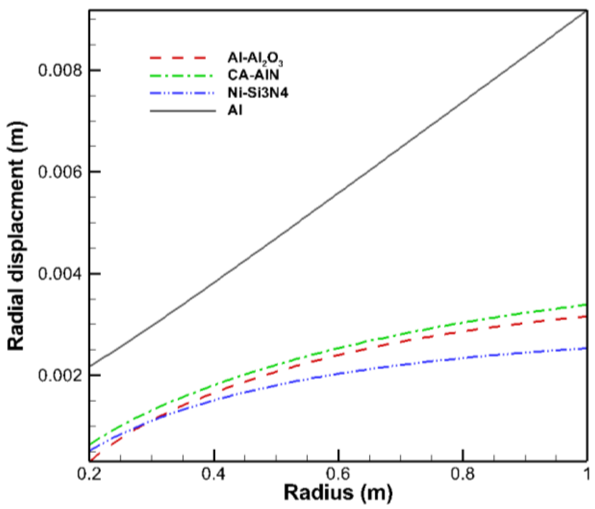

Figure 3. Radial displacement at T = 300 °C.

Amongst FGM material combinations, FGM cylinder made up of copper alloy (CA)-aluminium nitride (AlN) is found to have lower magnitude of radial stress throughout the radial points.

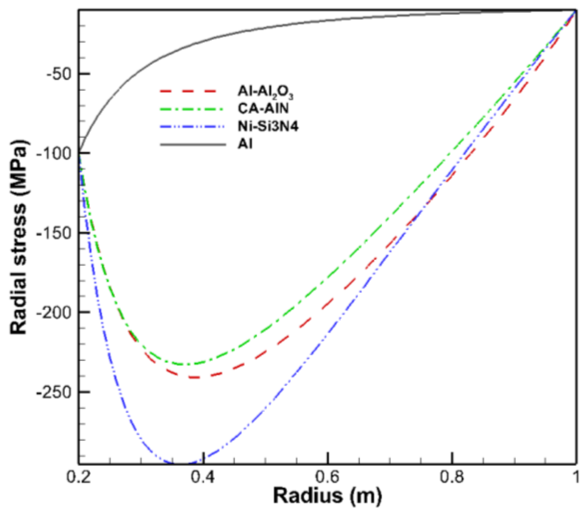

Figure 4. Radial stress T = 300 °C.

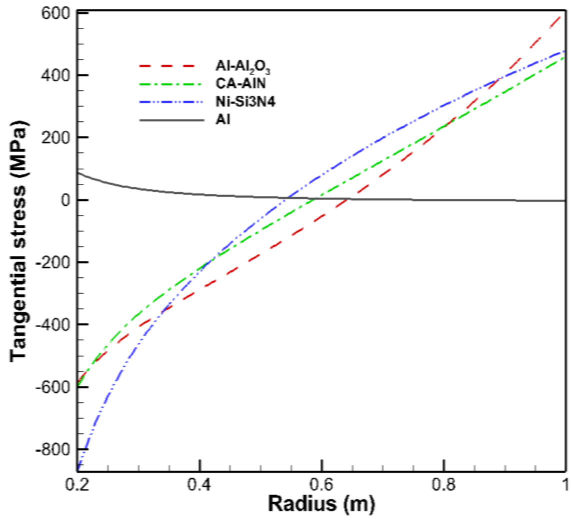

Figure 5. Tangential stress at T = 300 °C.

Figure 5 presents tangential stresses in homogeneous and FGM cylinders under thermomechanical loading conditions. Tangential stress plays a crucial role in designing engineering components as it is a primary reason for causing a fracture deformation. From Fig. 5, it can be noticed that for Al-Al_2O_3, the tangential stress is highly tensile towards outer radius in comparison with other materials. On the other hand, tangential stress for Ni-Si_3N_4 is more compressive among all considered materials, and that may happen due to the requirement of more resisting force to balance distortion generated by radial stress. It can be noted from Fig. 5, FGM cylinder made up of Ni-Si_3N_4 has higher magnitude at inner radial points, whereas FGM cylinder with Al-Al_2O_3 combination has higher magnitude at outer radius. The tangential stress for pure metal Al is showing almost linear behaviour which means resisting the distortion effect generated by radial stress is significantly invariant. In the design of engineering components, an appropriate analysis of stresses is conducted as the high magnitude of stresses beyond the material strength leads to material failure, causing catastrophic failure. Such failure can lead to a situation of loss of life, or resources, and damage the environment. Under the influence of high temperature loading and pressure conditions, analysis of radial and tangential stresses can warrant the life as well as the longevity of an engineering component.

CONCLUSION

In our research study, the implementation of a novel iterative technique on a thermomechanical stress distribution problem and a semi-analytical solution for the radial displacement and stress for a thick-walled functionally graded cylinder is presented. Materials properties such as Young's modulus and coefficient of thermal expansion are varying exponentially along the radial direction. Three FGM cylinders with different metal-ceramic (inner-outer) combinations, namely, aluminium (Al)-aluminium oxide (Al_2O_3), copper alloy (CA)-aluminium nitride (AlN) and nickel (Ni)-silicon nitride (Si_3N_4) are scrutinized for the analysis of the thermomechanical response of innovatively graded cylinders. Displacement and stress fields obtained in cylinders for these materials are further compared with pressurized cylinder made up of homogenous material (Al). The novelty of the work is that an iterative technique is applied to Navier's equation under plane strain conditions, and it gives a good approximation to the solution of stresses and displacement field after two iterations. It may also be applicable to find solutions for higher order nonlinear differential equations. From our study, the following findings are concluded.

- Using the iterative approach, at $n = 0$, we can validate the equations with Cauchy Euler equations, which provides a direct solution of the homogeneous equation.
- We have considered two iterations because of more complexity and time consumption, /42/. However, from our calculations at every iteration, $n = 0$, 1, and 2, there is a difference in terms of computational time but the percentage of relative error as observed in our calculations is approximately equal, and hence, there is no need to go ahead with further iterations as it does not affect our final results. This study also provides a comparative discussion that may help an engineer to choose the appropriate material for the pressure vessel design.
- Radial displacement in FGM cylinders is found to be of lesser magnitude as compared to homogeneous cylinder. It can be observed that the displacement in FGM cylinder with nickel (Ni)-silicon nitride (Si_3N_4) combination has the lowest magnitude towards the outer radial points.
- The magnitude of radial stresses in FGMs are decreasing towards the outer radius under uniform pressure as it requires less amount of pressure to maintain force equilibrium at the outer surface.
- In comparison with other considered materials, the FGM cylinder made up of Ni-Si_3N_4 is found to have lower magnitude of radial stress throughout the radial points.
- FGM cylinder made up of Ni-Si_3N_4 has higher magnitude at inner radial points, whereas FGM cylinder with the Al-Al_2O_3 combination has higher magnitude at outer radius.
- The outcome of our study shows that the thermoelastic response of the stresses and displacement in the cylinder are highly dependent on the material combination that is chosen for metal matrix and ceramic reinforcement. Also, the inhomogeneity parameter and thermal expansion coefficient have significant role in the stress distribution.
- Besides component inspections and hand calculations, a computer modelling method enables the engineer to analyse stresses in the cylinder.

Appendix

$$t_1=\frac{1-\nu}{(1+\nu)(1-2\nu)},\ t_2=\frac{\nu}{(1+\nu)(1-2\nu)},\ t_3=\frac{1}{1-2\nu},$$

$$k=-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a},\ \alpha_1=\frac{(1-2\nu)m_1A_1}{1-\nu},\ \alpha_2=\frac{m_1A_2}{3(1-\nu)},$$

$$\alpha_3=\frac{1}{m_2^2}\left(-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a}\right),\ \alpha_4=\frac{1}{m_2^3}\left(-\frac{1+\nu}{1-\nu}\alpha_0(m_1+m_2)Te^{m_2a}\right)$$

$$\lambda_1=\frac{(1-2\nu)m_1A_3}{1-\nu},\ \lambda_2=\frac{\nu m_1\alpha_1}{2(1-\nu)},\ \lambda_3=\frac{m_1A_4}{3(1-\nu)},\ \lambda_4=\frac{(2-\nu)m_1\alpha_2}{8(1-\nu)},$$

$$\lambda_5=\frac{k-m_1m_2\alpha_3}{m_2^2},\ \lambda_6=\frac{m_1\nu\alpha_3}{1-\nu}-m_1m_2\alpha_4,\ \lambda_7=\frac{(2\nu-1)m_1\alpha_4}{1-\nu},$$

$$\delta_1=-t_1\left(2\alpha_2a-m_2\alpha_3e^{-m_2a}-\alpha_4\left(\frac{m_2}{a}+\frac{1}{a^2}\right)e^{-m_2a}\right)-t_2\left(\frac{\alpha_1}{a}+\alpha_2a+\frac{\alpha_3}{a}e^{-m_2a}+\frac{\alpha_4}{a^2}e^{-m_2a}\right)-\frac{p}{E(a)}+t_3\alpha(a)T,$$

$$\delta_2=-t_1\left(2\alpha_2b-m_2\alpha_3e^{-m_2b}-\alpha_4\left(\frac{m_2}{b}+\frac{1}{b^2}\right)e^{-m_2b}\right)-t_2\left(\frac{\alpha_1}{b}+\alpha_2b+\frac{\alpha_3}{b}e^{-m_2b}+\frac{\alpha_4}{b^2}e^{-m_2b}\right)-\frac{q}{E(b)}+t_3\alpha(b)T,$$

$$\delta_3=-t_1\left(\lambda_2(1+\log a)+2\lambda_3a+3\lambda_4a^2+\lambda_5\left(-m_2e^{-m_2a}\left(1+\frac{1}{am_2}\right)+e^{-m_2a}\left(\frac{-1}{m_2a^2}\right)\right)+\lambda_6\left(\frac{dI_1(r)}{dr}\right)_a+\lambda_7\left(\frac{dI_2(r)}{dr}\right)_a\right),$$

$$\delta_4=-t_1\left(\lambda_2(1+\log b)+2\lambda_3b+3\lambda_4b^2+\lambda_5\left(-m_2e^{-m_2b}\left(1+\frac{1}{bm_2}\right)+e^{-m_2b}\left(\frac{-1}{m_2b^2}\right)\right)+\lambda_6\left(\frac{dI_1(r)}{dr}\right)_b+\lambda_7\left(\frac{dI_2(r)}{dr}\right)_b\right).$$

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