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EFFECT OF ROTATION ON THERMOSOLUTAL CONVECTION IN JEFFREY NANOFLUID WITH POROUS MEDIUM

EFEKAT ROTACIJE PRI TERMO-RASTVORLJIVOJ KONVEKCIJI U JEFFREY NANOFLUIDU SA POROZNOM SREDINOM

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Abstract

In this paper thermosolutal convection of unsteady rotating Jeffrey nanofluid in porous medium is considered. The mathematical form of the problem comprises of equations of continuity, motion, concentration, and energy. To solve all these equations, we used normal mode techniques. The Brownian motion and thermophoresis has important effect on the nanofluid model. Analytical expressions for both non-oscillatory and oscillatory cases is derived when boundary surfaces are free-free. The effects of rotation (Taylor number T_a), Jeffrey parameter λ, solutal Rayleigh number R_s, medium porosity ε, nanoparticle Rayleigh number R_n, thermo nanofluid Lewis number L_n, thermosolutal Lewis number L_e, modified diffusivity ratio N_A, Dufour parameter N_{CT}, and Soret parameter N_{TC} are analysed analytically and presented graphically.

Izvod

U radu se razmatra termo-rastvorljiva konvekcija pri neravnomernoj rotaciji Jeffrey nanofluida u poroznoj sredini. Matematički oblik problema sastoji se od jednačina protoka, kretanja, koncentracije i energije. Za rešavanje svih ovih jednačine upotrebili smo metode u normalnom modu. Braunovo kretanje i termoforeza predstavljaju važan efekat u modelu nanofluida. Izvode se analitički izrazi za neoscilatorni i oscilatorni slučaj za slučaj graničnih površina koje su slobodne-slobodne (bez opterećenja i graničnih uslova). Uticaji rotacije (Tejlorov broj T_a), Džefri parametar λ, Rejlejev broj rastvora R_s, poroznost sredine ε, Rejlejev broj nanočestica R_n, termički Luisov broj nanofluida L_n, Luisov broj termorastvorljivosti L_e, modifikovan odnos difuznosti N_A, Dufur parametar N_{CT} i Soret parametar N_{TC} su analizirani analitički i predstavljeni grafički.

INTRODUCTION

Natural surrounding is full of diffusive constituents. Thermosolutal instability problems related to diverse types of fluids have been widely calculated. Veronis /18/ considered the problem of thermosolutal convection in a layer of fluid heated and soluted from below. Such problem has a vital sensation that has applications in different areas as, astrophysics, geophysics, limnology, food processing, engineering and oil reservoir modelling. Nanofluid is the suspension of nanoparticles in a regular fluid having diameter lower than 100 nm. The occurrence of the nanoparticles in the fluid improves the current thermal conductivity of the fluid and therefore boosts the heat transfer features. Choi /4/ was the first to introduce the term *nanofluid*. Nanoparticles are normally made up of metals, oxides, carbides or carbon nano tubes and regular fluids are like water, oil, bio-fluids, polymer solutions and other common fluids. The study of nanofluids in a porous medium has appealed several investigators due to their uses in locomotive industries, Fuel cells, pharmaceutical processes, domestic refrigerator, heat exchanger, nuclear reactors, transformers, biomedical appliances. Many scientists have established that certain type of nanofluids can be used to abolish and destroy cancer cells without injuring the normal tissues. Thermal convection and thermosolutal convection of nanofluids in a porous medium has been deliberated by various researchers. Buongiorno /1/ analysed the problem of convective transport in nanofluids. He carried forward the work of Choi /4/. Sheu /16/ worked on thermal instability in a porous medium layer saturated with a viscoelastic nanofluid. Rana et al. /12/ have investigated the problem on the onset of thermosolutal instability in a layer of an elastico-viscous nanofluid in porous medium and concluded that the Walters' (model B') elastico-viscous nanofluid behaves like an ordinary Newtonian nanofluid. Chand et al. /3/ worked on the problem thermal instability analysis of an elastico-viscous nanofluid layer. They concluded that the viscoelastic nanofluids are very appropriate in the cooling of nuclear reactors, cooling of power plants and computers, drug delivery to kill the cancer cells and tissues, etc. Pundir et al. /11/ have investigated rotation on the thermosolutal convection in visco-elastic nanofluid in the presence of porous medium and concluded that viscoelastic nanofluid behaves as a regular Newtonian nanofluid. Sharma et al. /15/ investigated thermosolutal convection of an elastico-viscous nanofluid in porous medium in the presence of rotation and magnetic field and derived that magnetic field and Taylor number have stabilizing effect for stationary convection, simultaneously solutal Rayleigh number, nanoparticle Rayleigh number, thermo nanofluid Lewis number and modified diffusivity ratio have destabilizing effect for stationary convection. The role of rotation is very important in fluid machinery, power plants, petroleum industry, biomechanics, mechanical engineering, geophysics, etc. Chandrasekhar /2/ scrutinized the effect of rotation on the Bénard convection which was later stretched. Microscopic work has been completed to analyse Jeffrey nanofluid model, it is reflected as a best model. Jeffrey /6/ worked on the stability of a layer of physiological fluids heated from below. Hayat et al. /5/ investigated heat transfer analysis in convective flow of Jeffrey nanofluid by vertical stretchable cylinder. Rana /14/ analysed the effects of rotation on a Jeffery nanofluid flow in a porous medium which is heated from below. He concluded that the rotation parameter has a stabilizing influence for both bottom/top-heavy patterns. This brief review of literature reflects that studies on such topics are lacking, hence present problem on effect of rotation on thermosolutal convection in Jeffrey nanofluid with porous medium has been worked upon in the present communication.

MATHEMATICAL MODEL

Here we consider a rotating horizontal layer of thickness d, angular velocity $\mathbf{\Omega}$ and in the presence of Jeffrey nanofluid situated between the plates $z = 0$ and $z = d$ (as shown in Fig. 1). The fluid layer is heated from below and working upwards direction with a gravity force $\boldsymbol{g} = (0, 0, \text{-g})$. Temperature T, concentration C and volumetric fraction φ of nanoparticle, at the upper boundary and lower boundary are taken to be: T_1 and T_0, C_1 and C_0, φ_1 and φ_0, respectively, with $T_0 > T_1$, $C_0 > C_1$, and $\varphi_0 > \varphi_1$.

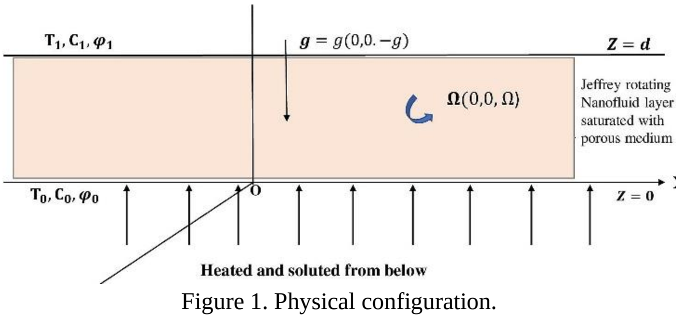

Figure 1. Physical configuration.

GOVERNING EQUATIONS

The governing equations for Jeffrey nanofluid in porous medium as given by Chandrasekhar /2/, Nield and Kuznetsov /7-10/, Pundir et al. /11/, Rana et al. /12-14/, and Sharma et al. /15/, are:

$$\nabla.\boldsymbol{q} = 0\,, \tag{1}$$

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \boldsymbol{q}}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{q}.\nabla)\boldsymbol{q}\right] = -\nabla p + \rho\boldsymbol{g} - \frac{\mu}{k_1(1+\lambda)}\boldsymbol{q} + \frac{2\rho}{\varepsilon}(\boldsymbol{q}\times\mathbf{\Omega})\,, \tag{2}$$

where: ρ, μ, p, ε, $\boldsymbol{g}$, k_1, $\lambda = \lambda_1/\lambda_2$, and $\boldsymbol{q}(u,v,w)$ denote respectively the density, viscosity, pressure, medium porosity, acceleration due to gravity, coefficient of thermal conductivity or thermal conductivity, Jeffrey parameter (which is the ratio of stress relaxation-time parameter, λ_1 to strain retardation-time parameter, λ_2), and Darcy velocity vector, in respect.

The density of nanofluid can be written as (Buongiorno /1/),

$$\rho = \varphi\rho_P + (1-\varphi)\rho_f\,, \tag{3}$$

where: φ is the volume fraction of nanoparticles; p_P is the density of nanoparticles; and ρ_f is the density of base fluid. Following Tzou /17/ and Nield and Kuznetsov /7-10/, we approximate the density of the nanofluid by that of the base fluid, that is, we consider $\rho = \rho_f$.

Now, introducing the Boussinesq approximation for the base fluid, the specific weight, $\rho\boldsymbol{g}$ in Eq.(2) becomes

$$\rho\boldsymbol{g} \approx \left(\varphi\rho_P + (1-\varphi)\left\{\rho\left(1-\alpha_T(T-T_0)-\alpha_c(C-C_0)\right)\right\}\right)\boldsymbol{g}\,, \tag{4}$$

where: α_T is the coefficient of thermal expansion; and α_c is analogous to solute concentration.

If one introduces a buoyancy force, the equation of motion for Jeffrey nanofluid by using Boussinesq approximation and Darcy model for porous medium (Kuznetsov and Nield /7-10/) is given by,

$$\frac{\rho}{\varepsilon}\frac{\partial}{\partial t}\boldsymbol{q} = -\nabla p + \left(\varphi\rho_P + (1-\varphi)\left\{\rho\left(1-\alpha_T(T-T_0)-\alpha_c(C-C_0)\right)\right\}\right)\boldsymbol{g} - \\ -\frac{\mu}{k_1(1+\lambda)}\boldsymbol{q} + \frac{2\rho}{\varepsilon}(\boldsymbol{q}\times\mathbf{\Omega})\,. \tag{5}$$

For nanoparticles, the continuity equation given by (Buongiorno /1/) is

$$\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\boldsymbol{q}.\nabla\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T\,, \tag{6}$$

where: D_B and D_T are the Brownian diffusion coefficient and thermophoresis diffusion coefficient, respectively.

For the nanofluid, the equation of thermal energy is given as

$$(\rho c)_m\frac{\partial T}{\partial t} + (\rho c)_f\,\boldsymbol{q}.\nabla T = k_m\nabla^2 T + \varepsilon(\rho c)_P \times \\ \times\left(D_B\nabla\varphi\nabla T + \frac{D_T}{T_1}\nabla T.\nabla T\right) + (\rho c)_f\,D_{TC}\nabla^2 C\,, \tag{7}$$

where: D_{TC} is Dufour diffusivity; k_m is thermal conductivity; $(\rho c)_P$ is the heat capacity of nanoparticles; and $(\rho c)_m$ is heat capacity of the fluid in porous medium.

The equation of conservation of solute concentration is given by

$$\frac{\partial C}{\partial t} + \frac{1}{\varepsilon}\boldsymbol{q}.\nabla C = D_{SM}\nabla^2 C + D_{TC}\nabla^2 T\,, \tag{8}$$

where: D_{SM} and D_{CT} are the solute diffusivity of porous medium and Soret type diffusivity, in respect.

The boundary conditions are given by:

$$w = 0,\quad T = T_0,\quad \varphi = \varphi_0,\quad C = C_0,\quad \text{at}\quad z = 0 \tag{9}$$

$$w = 0,\quad T = T_1,\quad \varphi = \varphi_1,\quad C = C_1,\quad \text{at}\quad z = 1. \tag{10}$$

Non-dimensional quantities: we introduce the following non-dimensional variables as:

$$(x^*, y^*, z^*) = \frac{(x,y,z)}{d}\,,\quad \boldsymbol{q}^* = \boldsymbol{q}\frac{d}{\kappa_m}\,,\quad t^* = \frac{t\kappa_m}{\sigma d^2}\,,\quad p^* = \frac{pk_1}{\mu\kappa_m}\,,$$

$$\varphi^* = \frac{\varphi - \varphi_0}{\varphi_1 - \varphi_0}, \quad T^* = \frac{T - T_1}{T_0 - T_1}, \quad C^* = \frac{C - C_1}{C_0 - C_1},$$

where: $\kappa_m = \dfrac{k_m}{(\rho c)_f}$, $\sigma = \dfrac{(\rho c)_m}{(\rho c)_f}$, are thermal diffusivity of the fluid and the thermal capacity ratio, respectively. Dropping the star (*) for simplification.

Equation (1) and Eqs.(5), (6), (7), (8) reduce in non-dimensional form:

$$\nabla . \boldsymbol{q} = 0, \tag{11}$$

$$\frac{1}{\sigma V_a}\frac{\partial}{\partial t}\boldsymbol{q} = -\nabla p - \frac{1}{1+\lambda}\boldsymbol{q} - R_m\hat{k} - R_n\varphi\hat{k} + R_D T\hat{k} + \frac{R_s}{L_e}C\hat{k} + \sqrt{T_a}\,(\boldsymbol{q}\times\hat{k}), \tag{12}$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\boldsymbol{q}.\nabla\varphi = \frac{1}{L_n}\nabla^2\varphi + \frac{N_A}{L_n}\nabla^2 T, \tag{13}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{q}.\nabla T = \nabla^2 T + \frac{N_B}{L_n}\nabla\varphi.\nabla T + \frac{N_A N_B}{L_n}\nabla T.\nabla T + N_{CT}\nabla^2 C, \tag{14}$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t} + \frac{1}{\varepsilon}\boldsymbol{q}.\nabla C = \frac{1}{L_e}\nabla^2 C + N_{TC}\nabla^2 T, \tag{15}$$

where dimensionless parameters are: thermosolutal Lewis number $L_e = \dfrac{\kappa_m}{D_{SM}}$; thermo nanofluid Lewis number $L_n = \dfrac{\kappa_m}{D_B}$; density Rayleigh number $R_m = \dfrac{(\rho_P\varphi_0 + \rho(1-\varphi_0))gk_1 d}{\mu\kappa_m}$; nanoparticle Rayleigh number $R_n = \dfrac{(\rho_P - \rho)(\varphi_1 - \varphi_0)gk_1 d}{\mu\kappa_m}$; thermal Darcy Rayleigh number $R_D = \dfrac{\rho\alpha_T(T_0 - T_1)gk_1 d}{\mu\kappa_m}$; solutal Rayleigh number $R_s = \dfrac{\rho\alpha_C(C_0 - C_1)gk_1 d}{\mu D_{SM}}$; Prandtl number $Pr = \dfrac{\mu}{\rho\kappa_m}$; modified diffusivity ratio $N_A = \dfrac{D_T(T_0 - T_1)}{D_B T_1(\varphi_1 - \varphi_0)}$; modified particle density increment $N_B = \dfrac{\varepsilon(\rho c)_P(\varphi_1 - \varphi_0)}{(\rho c)_f}$; Soret parameter $N_{TC} = \dfrac{D_{TC}(C_0 - C_1)}{\kappa_m(T_0 - T_1)}$; Dufour parameter $N_{CT} = \dfrac{D_{TC}(T_0 - T_1)}{\kappa_m(C_0 - C_1)}$; Taylor number $T_a = \left(\dfrac{2\Omega d^2\rho}{\mu}\right)^2$; Darcy number $D_a = \dfrac{k_1}{d^2}$; Vadasz number $V_a = \dfrac{\varepsilon Pr}{D_a}$.

The dimensionless boundary conditions are:

$$w = 0,\quad T = 1,\quad \varphi = 0,\quad C = 1,\quad \text{at}\quad z = 0, \tag{16}$$

$$w = 0,\quad T = 0,\quad \varphi = 1,\quad C = 0,\quad \text{at}\quad z = 1. \tag{17}$$

BASIC STATES AND ITS SOLUTIONS

Following Nield and Kuznetsov /7-10/, Sharma et al. /15/, and Sheu /16/. The basic state of nanofluid is assumed and does not depend on time and describes as:

$$\boldsymbol{q}(u,v,w) = 0 \;\Rightarrow u = v = w = 0,$$

$$p = p_b(z),\quad C = C_b(z),\quad T = T_b(z),\quad \varphi = \varphi_b(z). \tag{18}$$

The basic variable represented by subscript b.

Therefore, when the basic state defined in Eq.(18) is substituted into Eqs.(11)-(15), these equations reduce to:

$$0 = -\frac{d}{dz}p_b(z) - R_m - R_n\varphi_b(z) + R_D T_b(z) + \frac{R_s}{L_e}C_b(z), \tag{19}$$

$$\frac{d^2}{dz^2}\varphi_b(z) + N_A\frac{d^2}{dz^2}T_b(z) = 0, \tag{20}$$

$$\frac{d^2}{dz^2}T_b(z) + \frac{N_A}{L_n}\frac{d}{dz}\varphi_b(z)\frac{d}{dz}T_b(z) + \frac{N_A N_B}{L_n}\left(\frac{d}{dz}T_b(z)\right)^2 + N_{CT}\frac{d^2}{dz^2}C_b(z) = 0, \tag{21}$$

$$\frac{1}{L_e}\frac{d^2}{dz^2}C_b(z) + N_{TC}\frac{d^2}{dz^2}T_b(z) = 0. \tag{22}$$

Using boundary conditions Eqs.(16) and (17), the solution of Eq.(20) is given by

$$\varphi_b(z) = (1 - T_b)N_A + (1 - N_A)z. \tag{23}$$

Using boundary conditions Eqs.(16) and (17), the solution of Eq.(22) is given by

$$C_b(z) = (1 - T_b)N_{TC}L_e - (1 + N_{CT}L_e)z + 1. \tag{24}$$

Substituting the values of $\varphi_b(z)$ and $C_b(z)$, respectively, from Eqs.(23) and (24) in Eq.(21), we get

$$\frac{d^2}{dz^2}T_b(z) + \frac{(1-N_A)N_B}{L_n}\frac{d}{dz}(z) = 0. \tag{25}$$

The solution of differential equation Eq.(25) with boundary conditions in Eqs.(16) and (17) is

$$T_b(z) = e^{-\frac{(1-N_A)N_B z}{L_n}} \cdot \frac{\left[1 - e^{-\frac{(1-N_A)N_B(1-z)}{L_n}}\right]}{1 - e^{-\frac{(1-N_A)N_B}{L_n}}}. \tag{26}$$

According to Buongiorno /1/, for most nanofluids investigated so far $L_n/(\varphi_1 - \varphi_0)$ is large, of order 10^5-10^6, and since the nanoparticle fraction decrement $(\varphi_1 - \varphi_0)$ is not smaller than 10^{-3} which means L_n is large. Typical values of exponents in Eq.(20) are small.

By expanding the exponential function into the power series and retaining up to the first order and negligible other higher order terms (i.e., $e^{-x} = 1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \ldots \approx 1 - x$) and so, to a good approximation for the solution,

$$T_b = 1 - z,\quad C_b = 1 - z \quad \text{and} \quad \varphi_b = z. \tag{27}$$

These results are identical with the results obtained by Kuznetsov and Nield /7-10/, Sharma et al. /15/, and Sheu /16/.

PERTURBATION SOLUTIONS

We introduce small perturbation on the basic state for investigating the stability of the system and write

$$\boldsymbol{q}(u,v,w) = 0 + \boldsymbol{q}'(u,v,w),\quad T = (1-z) + T',\quad C = (1-z) + C',$$

$$\varphi = z + \varphi',\quad p = p_b + p'. \tag{28}$$

Using Eq.(28) in Eqs.(11) to (15), linearizing the resulting equations by neglecting nonlinear terms that are product of prime quantities and dropping the primes (′) for convenience, the following equations are obtained:

$$\nabla . \boldsymbol{q} = 0, \tag{29}$$

$$\frac{1}{\sigma V_a}\frac{\partial}{\partial t}\boldsymbol{q}=-\nabla p-\frac{1}{1+\lambda}\boldsymbol{q}-R_n\varphi\hat{k}+R_D T\hat{k}+\frac{R_s}{L_e}C\hat{k}+\sqrt{T_a}\,(\boldsymbol{q}\times\hat{k}), \quad (30)$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}w=\frac{1}{L_n}\nabla^2\varphi+\frac{N_A}{L_n}\nabla^2 T, \quad (31)$$

$$\frac{\partial T}{\partial t}-w=\nabla^2 T+\frac{N_B}{L_n}\left(\frac{\partial T}{\partial z}-\frac{\partial\varphi}{\partial z}\right)-2\frac{N_A N_B}{L_n}\frac{\partial T}{\partial z}+N_{CT}\nabla^2 C, \quad (32)$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}-\frac{1}{\varepsilon}w=\frac{1}{L_e}\nabla^2 C+N_{TC}\nabla^2 T, \quad (33)$$

and boundary conditions are:

$w = 0$, $T = 0$, $\varphi = 0$, $C = 0$ at $z = 0$ and $z = 1$. (34)

Note that the parameter R_m is not involved in Eqs.(29) to (33), it is just a measure of the basic static pressure gradient. Operating Eq.(30) with $\hat{k}$.curl.curl, we get (i.e., making use of result curl.curl = grad.div – ∇^2),

$$\left(\frac{1}{\sigma V_a}\frac{\partial}{\partial t}+\frac{1}{1+\lambda}\right)\left[\left(\frac{1}{\sigma V_a}\frac{\partial}{\partial t}+\frac{1}{1+\lambda}\right)\nabla^2 w+R_n\nabla_H^2\varphi-R_D\nabla_H^2 T-\frac{R_s}{L_e}\nabla_H^2 C\right]+T_a\frac{\partial^2 w}{\partial z^2}=0, \quad (35)$$

where: $\frac{1}{\sigma V_a}\frac{\partial}{\partial t}+\frac{1}{1+\lambda}=\sqrt{T_a}$; $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$; and $\nabla_H^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplace operator on the horizontal plane.

NORMAL MODE ANALYSIS

The disturbances analysing by normal mode analysis as follows:

$$[w,T,C,\varphi]=[W(z),\Theta(z),\Gamma(z),\Phi(z)]\exp(ik_x x+ik_y y+nt), \quad (36)$$

where: n is growth rate; and k_x and k_y are wave numbers along x and y directions, respectively.

Using Eq.(36) in Eqs.(31), (32), (33), and (35), we get

$$\left[\left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)^2(D^2-a^2)+T_a D^2\right]W+\left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)R_D a^2\Theta+\left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)\frac{R_s}{L_e}a^2\Gamma-\left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)R_n a^2\Phi=0, \quad (37)$$

$$\frac{1}{\varepsilon}W-\frac{N_A}{L_n}(D^2-a^2)\Theta+\left[\frac{n}{\sigma}-\frac{(D^2-a^2)}{L_n}\right]\Phi=0, \quad (38)$$

$$W+\left[(D^2-a^2)+\frac{N_B}{L_n}D-2\frac{N_A N_B}{L_n}D-n\right]\Theta+N_{CT}(D^2-a^2)\Gamma-\frac{N_B}{L_n}D\Phi=0, \quad (39)$$

$$\frac{1}{\varepsilon}W-N_{TC}(D^2-a^2)\Theta+\left[\frac{(D^2-a^2)}{L_e}-\frac{n}{\sigma}\right]\Gamma=0, \quad (40)$$

where: $D = d/dz$; and $a^2 = k_x^2 + k_y^2$ is the dimensionless ensuing wave number, and the boundary conditions in view of normal mode are:

$W = D^2W = \Gamma = \Theta = \Phi = 0$ at $z = 0$ and $z = 1$. (41)

LINEAR STABILITY ANALYSIS AND DISPERSION RELATION

The eigen function $f_i(z)$ corresponding to the eigen value problem Eqs.(37)-(41) are $f_j = \sin(\pi z)$.

Considering solutions W, Θ, Γ, Φ of the form:

$$W = W_0\sin(\pi z),\ \Theta = \Theta_0\sin(\pi z),\ \Gamma = \Gamma_0\sin(\pi z),\ \Phi = \Phi_0\sin(\pi z). \quad (42)$$

Substituting Eqs.(42) into Eqs.(37)-(40) and integrating each equation from $z = 0$ to $z = 1$, we obtain the following matrix equations

$$\begin{bmatrix} \left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)J^2+\pi^2 T_a & -\left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)a^2 R_D & -\left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)a^2\frac{R_s}{L_e} & -\left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)a^2 R_n \\ \frac{1}{\varepsilon} & \frac{N_A}{L_n}J^2 & 0 & \frac{J^2}{L_n}+\frac{n}{\sigma} \\ -1 & J^2+n & N_{CT}J^2 & 0 \\ -\frac{1}{\varepsilon} & N_{TC}J^2 & \frac{J^2}{L_e}+\frac{n}{\sigma} & 0 \end{bmatrix}\begin{bmatrix} W_0\\ \Theta_0\\ \Gamma_0\\ \Phi_0\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \quad (43)$$

where: $J^2 = \pi^2 + a^2$ is the total wave number.

The linear system Eq.(43) has a non-trivial solution if and only if

$$R_D=\frac{1}{(J^2\sigma\varepsilon+n\varepsilon L_e-\sigma L_e N_{CT}J^2)}\left\{\frac{\varepsilon}{a^2}\left[\left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)J^2+\frac{\pi^2 T_a}{\left(\frac{n}{\sigma V_a}+\frac{1}{1+\lambda}\right)}\right]\left[(J^2+n)(\sigma J^2+nL_e)-\sigma L_e N_{CT}N_{TC}J^4\right]+R_s\sigma\left[\varepsilon N_{TC}J^2-(J^2+n)\right]-\frac{R_n\sigma}{(\sigma J^2+nL_n)}\left[\left((J^2+n)L_n+\varepsilon N_A J^2\right)(\sigma J^2+nL_e)-\sigma L_e N_{CT}J^4(L_n N_{TC}+N_A)\right]\right\}, \quad (44)$$

where: $J^2 = \pi^2 + a^2$.

THE STATIONARY CONVECTION

For the case of steady state, put $n = 0$ in Eq.(44), we obtain

$$R_D=\frac{1}{(\varepsilon-L_eN_{CT})}\left\{\frac{\varepsilon(\pi^2+a^2)}{a^2}\left[\frac{(\pi^2+a^2)}{1+\lambda}+\pi^2(1+\lambda)T_a\right]\left[1-L_eN_{CT}N_{TC}\right]+R_s\left[\varepsilon N_{TC}-1\right]-R_n[(L_n+\varepsilon N_A)-L_eN_{CT}(L_nN_{TC}+N_A)]\right\}. \quad (45)$$

The thermal Darcy Rayleigh number revealed from Eq.(45) is a function of: a, λ, ε, N_{TC}, N_{CT}, L_e, L_n, N_A, R_s, R_n, T_a.

In the non-appearance of the Soret (N_{TC}) and Dufour (N_{CT}) parameters, Eq.(45) reduces to

$$R_D=\frac{1}{1+\lambda}\frac{(\pi^2+a^2)^2}{a^2}+\frac{\pi^2(1+\lambda)(\pi^2+a^2)T_a}{a^2}-\frac{R_s}{\varepsilon}-R_n\left(N_A+\frac{L_n}{\varepsilon}\right). \quad (46)$$

The critical wave number obtained by minimizing R_D with respect to a^2, i.e., satisfying $\partial R_D/\partial a^2 = 0$, is

$$a_{cri}^2=\pi^2\sqrt{\frac{1+(1+\lambda)^2T_a}{2\pi^2-1}}.$$

Now, the critical thermal Darcy Rayleigh number for steady onset is

$$(R_D)_{cri}=\frac{1}{(1+\lambda)}\left[\frac{\sqrt{2\pi^2-1}}{\sqrt{1+(1+\lambda)^2T_a}}+1\right]+\frac{\pi^2(1+\lambda)\sqrt{T_a}}{\sqrt{1+(1+\lambda)^2}}\times$$
$$\times\left\{\sqrt{(2\pi^2-1)}+\sqrt{1+(1+\lambda)^2T_a}\right\}-\frac{R_s}{\varepsilon}-R_n\left(N_A+\frac{L_n}{\varepsilon}\right). \quad (47)$$

Special cases

In the absence of rotation, $T_a = 0$, then Eq.(46) becomes

$$R_D=\frac{1}{1+\lambda}\frac{(\pi^2+a^2)^2}{a^2}-\frac{R_s}{\varepsilon}-R_n\left(N_A+\frac{L_n}{\varepsilon}\right). \quad (48)$$

In the absence of Jeffrey parameter and rotation (i.e., $\lambda = 0$ and $T_a = 0$), then Eq.(46) becomes

$$R_D=\frac{(\pi^2+a^2)^2}{a^2}-\frac{R_s}{\varepsilon}-R_n\left(N_A+\frac{L_n}{\varepsilon}\right). \quad (49)$$

In the absence of nanoparticles (i.e., $\lambda = 0$, $R_n = 0$, and $N_A = 0$), then Eq.(46) becomes

$$R_D=\frac{(\pi^2+a^2)^2}{a^2}+\frac{\pi^2(\pi^2+a^2)T_a}{a^2}-\frac{R_s}{\varepsilon}. \quad (50)$$

THE OSCILLATORY CONVECTION

Put $n = in_i$ in Eq.(44), we have

$$R_D=\frac{1}{(J^2\sigma\varepsilon+in_i\varepsilon L_e-\sigma L_eN_{CT}J^2)}\left\{\frac{\varepsilon}{a^2}\left[\left(\frac{in_i}{\sigma V_a}+\frac{1}{1+\lambda}\right)J^2+\frac{\pi^2T_a}{\left(\frac{in_i}{\sigma V_a}+\frac{1}{1+\lambda}\right)}\right][(J^2+in_i)(\sigma J^2+in_iL_e)-\sigma L_eN_{CT}N_{TC}J^4]+\right.$$
$$\left.+R_s\sigma\left[\varepsilon N_{TC}J^2-J^2-in_i\right]-\frac{R_n\sigma}{(\sigma J^2+in_iL_n)}\left[\left((J^2+in_i)L_n+\varepsilon N_AJ^2\right)(\sigma J^2+in_iL_e)-\sigma L_eN_{CT}J^4(L_nN_{TC}+N_A)\right]\right\}. \quad (51)$$

Let us take Lewis number (L_n) and Prandtl number (Pr) approach to infinity with negligible Dufour (N_{TC}) and Soret (N_{CT}) parameters, and heat capacity ratio (σ) as unity, then we obtain Eq.(51) as

$$R_D=\frac{1}{\varepsilon(J^2+in_iL_e)}\left\{\frac{\varepsilon}{a^2}\left[\frac{J^2}{1+\lambda}+\pi^2(1+\lambda)T_a\right](J^2+in_i)(J^2+in_iL_e)-R_s(J^2+in_i)-\frac{R_n}{(J^2+in_iL_n)}\left[\left((J^2+in_i)L_n+\varepsilon N_AJ^2\right)(J^2+in_iL_e)\right]\right\}. \quad (52)$$

After separating real and imaginary parts of Eq.(52), we get Eq.(52) in the form: $R_D=\Delta_1+in_i\Delta_2$, (53)

where: $\Delta_1=\frac{J^2}{a^2}\left[\frac{J^2}{1+\lambda}+\pi^2(1+\lambda)T_a\right]-\frac{R_s(J^4+n_i^2L_e)}{\varepsilon(J^4+n_i^2L_e^2)}-\frac{R_n[J^4(L_n+\varepsilon N_A)+n_i^2L_n^2]}{\varepsilon(J^4+n_i^2L_n^2)}$, and

$$\Delta_2=\frac{1}{a^2}\left[\frac{J^2}{1+\lambda}+\pi^2(1+\lambda)T_a\right]-\frac{R_sJ^2(1-L_e)}{\varepsilon(J^4+n_i^2L_e^2)}-\frac{R_nJ^2L_n(1-L_n-\varepsilon N_A)}{\varepsilon(J^4+n_i^2L_n^2)}.$$

With oscillatory onset $\Delta_2 = 0$ and $n_i \neq 0$, this gives the dispersion relation of the form: $a_1(n_i^2)^2+a_2(n_i^2)+a_3=0$, (54)

where: $a_1=\frac{\varepsilon L_e^2L_n^2}{a^2}\left[\frac{J^2}{1+\lambda}+\pi^2(1+\lambda)T_a\right]$; $a_2=\frac{\varepsilon}{a^2}\left[\frac{J^2}{1+\lambda}+\pi^2(1+\lambda)T_a\right](L_e^2+L_n^2)J^4-R_s(1-L_e)L_n^2J^2+R_nL_n(1-L_n-\varepsilon N_A)$;

$$a_3=\frac{\varepsilon}{a^2}\left[\frac{J^2}{1+\lambda}+\pi^2(1+\lambda)T_a\right]J^8-R_s(1-L_e)J^6+R_nL_n(1-L_n-\varepsilon N_A)J^6.$$

Then, Eq.(53) gives

$$R_D^{osc}=\frac{J^2}{a^2}\left[\frac{J^2}{1+\lambda}+\pi^2(1+\lambda)T_a\right]-\frac{R_s(J^4+n_i^2L_e)}{\varepsilon(J^4+n_i^2L_e^2)}-\frac{R_n[J^4(L_n+\varepsilon N_A)+n_i^2L_n^2]}{\varepsilon(J^4+n_i^2L_n^2)}. \quad (55)$$

We find the oscillatory neutral solution from Eq.(55) by procedure as follows: first find the roots of n_i^2, Eq.(54). If there are no positive roots (i.e., if roots are negative, or in complex form) then oscillatory instability is not possible. If there are positive roots, the critical thermal Rayleigh number for oscillatory convection can be derived by numerically minimizing Eq.(55) with respect to wave number, after substituting various values of physical parameters for n_i^2 of Eq. (54) to determine their effects on the onset of oscillatory convection (Sheu /16/).

RESULTS AND DISCUSSION

The Eq.(46) expresses for stationary thermal Darcy Rayleigh number computed as a function of Jeffrey parameter $\lambda = \lambda_1/\lambda_2$, solutal Rayleigh number R_s, medium porosity ε, nanoparticle Rayleigh number R_n, thermo nanofluid Lewis number L_n, modified diffusivity ratio N_A, and Taylor number T_a. Whereas Eq.(55) expresses for oscillatory thermal Darcy Rayleigh number computed as a function of Jeffrey parameter $\lambda = \lambda_1/\lambda_2$, solutal Rayleigh number R_s, medium porosity ε, nanoparticle Rayleigh number R_n, thermo nanofluid Lewis number L_n, thermosolutal Lewis number L_e, modified diffusivity ratio N_A and Taylor number T_a.

We observe the nature of $\frac{\partial R_D}{\partial \lambda_1}$, $\frac{\partial R_D}{\partial \lambda_2}$, $\frac{\partial R_D}{\partial T_a}$, $\frac{\partial R_D}{\partial R_s}$, $\frac{\partial R_D}{\partial R_n}$, $\frac{\partial R_D}{\partial L_n}$, $\frac{\partial R_D}{\partial \varepsilon}$, $\frac{\partial R_D}{\partial L_e}$, and $\frac{\partial R_D}{\partial N_A}$ analytically. Equations (46) and (55) give results analytically.

Graph 1 (Fig. 2) shows that the Rayleigh number R_D decreases with increase in relaxation parameter λ_1 implying that relaxation parameter λ_1 has a destabilizing effect on the system.

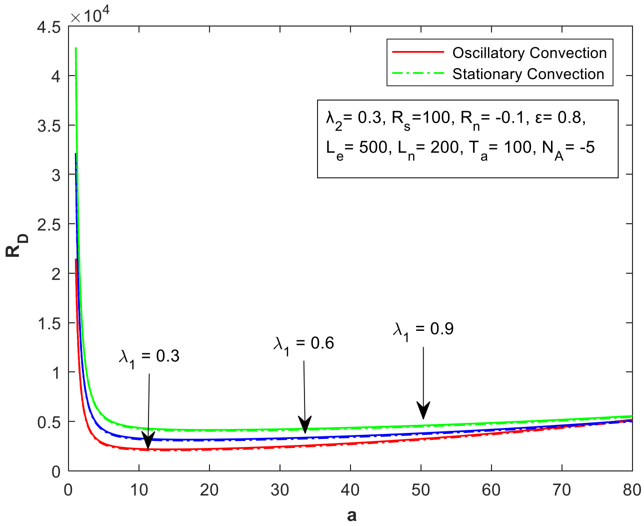

Figure 2. Variation of Rayleigh number with wave number for different values of stress relaxation-time parameter, λ_1.

Graph 2 (Fig. 3) shows that the Rayleigh number R_D increases with increase in retardation parameter λ_2 implying that retardation parameter λ_2 has a stabilizing effect on the system.

Graph 3 (Fig. 4) shows that the Rayleigh number R_D increases with increase in Taylor number T_a which implies that Taylor number T_a has a stabilizing effect on the system.

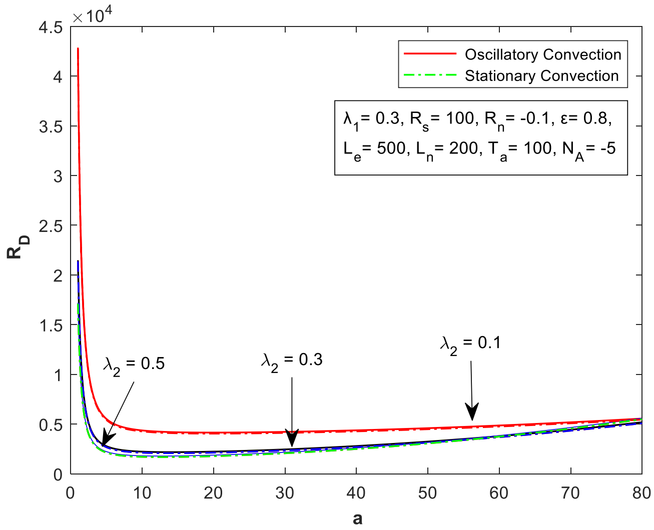

Figure 3. Variation of Rayleigh number with wave number for different values of strain retardation-time parameter, λ_2.

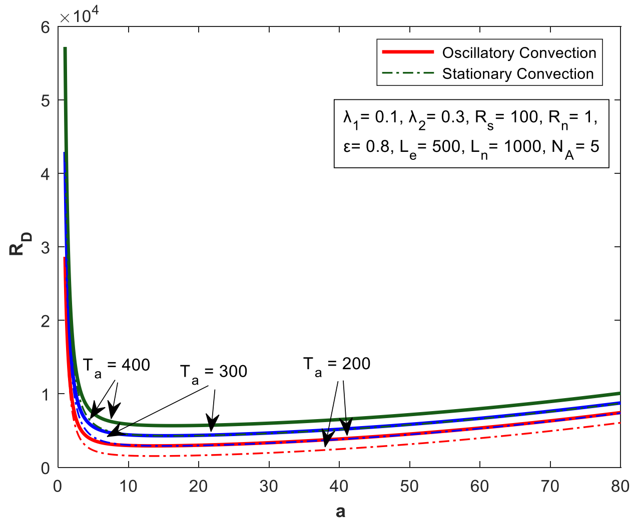

Figure 4. Variation of Rayleigh number with wave number for different values of Taylor number, T_a.

Graph 4 (Fig. 5) shows that Rayleigh number R_D increases with solutal Rayleigh number R_s implying that solutal Rayleigh number R_s has a stabilizing effect on the system.

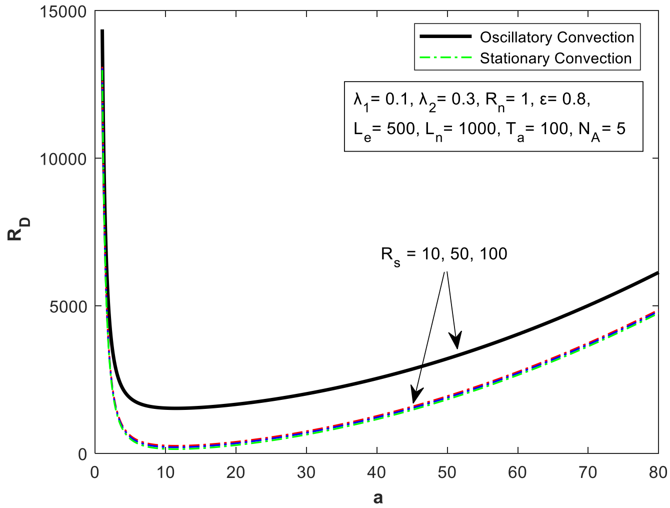

Figure 5. Variation of Rayleigh number with wave number for different values of solutal Rayleigh number, R_s.

Graph 5 (Fig. 6) shows that the Rayleigh number R_D decreases with increase in nanoparticle Rayleigh number R_n which implies that nanoparticle Rayleigh number R_n has a destabilizing effect on the system.

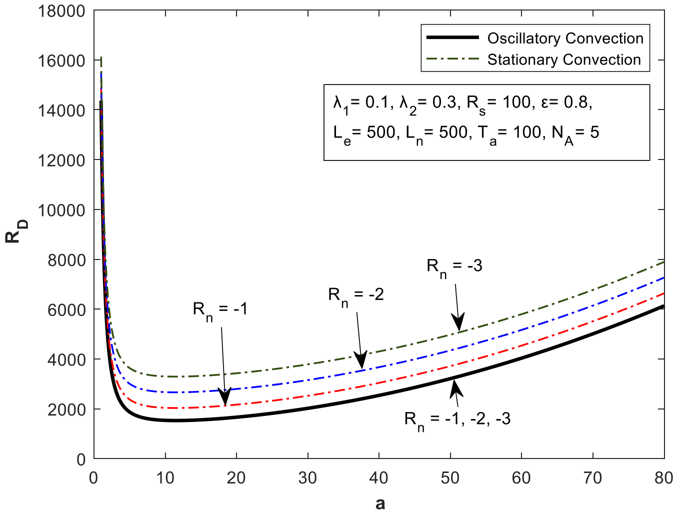

Figure 6. Variation of Rayleigh number with wave number for different values of nanoparticle Rayleigh number, R_n.

Graph 6 (Fig. 7) shows that Rayleigh number R_D increases with thermo nanofluid Lewis number L_n which implies that thermo nanofluid Lewis number L_n has a stabilizing effect on the system.

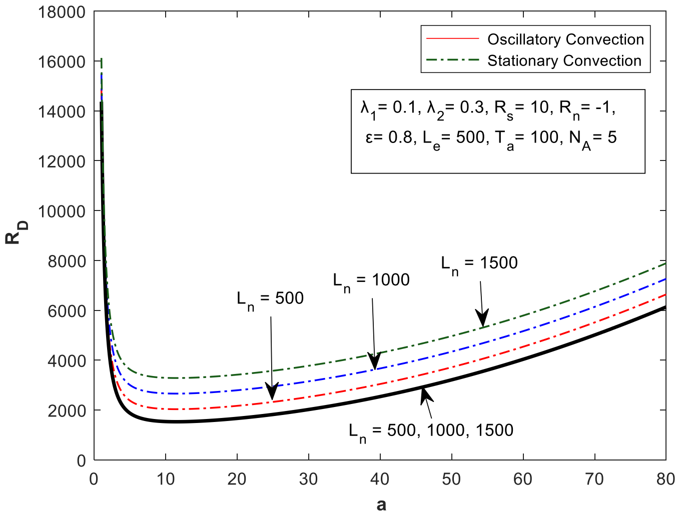

Figure 7. Variation of Rayleigh number with wave number for different values of thermo nanofluid Lewis number, L_n.

Graph 7 (Fig. 8) shows that Rayleigh number R_D increases with modified diffusivity ratio N_A which implies that modified diffusivity ratio N_A has a stabilizing effect on the system.

Graph 8 (Fig. 9) shows that the Rayleigh number R_D decreases with increase in medium porosity ε which implies that medium porosity ε has a destabilizing effect on the system.

Graph 9 (Fig. 10) shows that the Rayleigh number R_D increases with increase in thermosolutal Lewis number L_e which implies that only in oscillatory convection the thermosolutal Lewis number L_e has a stabilizing effect.

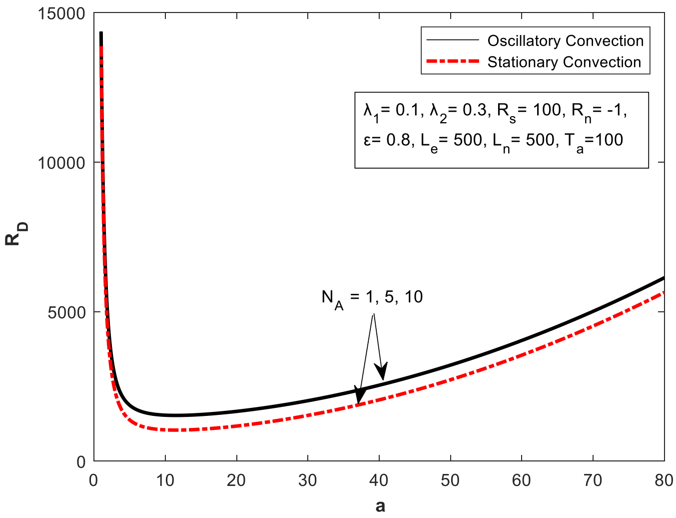

Figure 8. Variation of Rayleigh number with wave number for different values of modified diffusivity ratio, N_A.

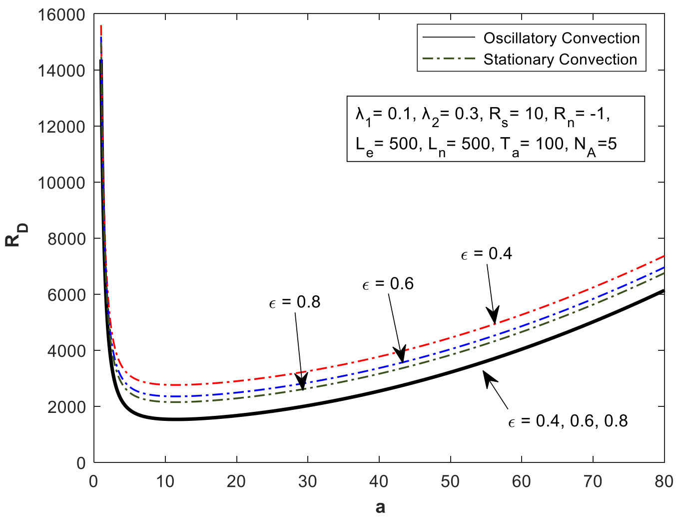

Figure 9. Variation of Rayleigh number with wave number for different values of medium porosity, ε.

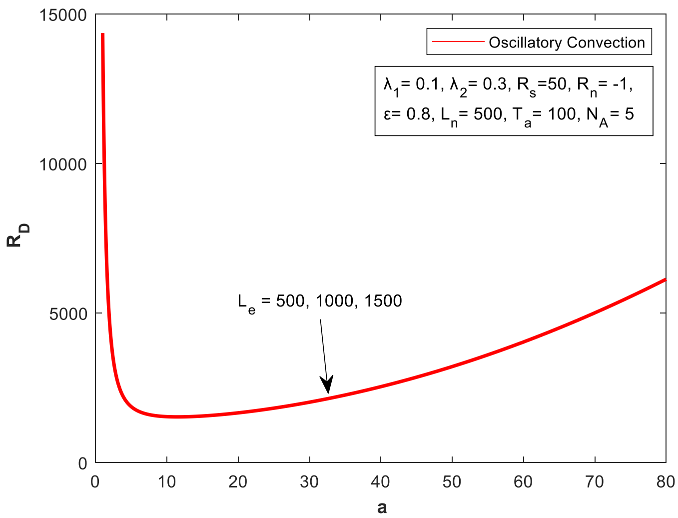

Figure 10. Variation of Rayleigh number with wave number for different values of thermosolutal Lewis number, L_e.

CONCLUSION

The onset of thermosolutal convection of Jeffrey nanofluid in a porous medium in the presence of rotation is investigated by using linear stability analysis. We draw the main conclusions as the following:

- The Taylor number has a stabilizing effect for both stationary and oscillatory convections.
- The retardation parameter, solutal Rayleigh number, thermo nanofluid Lewis number, and modified diffusivity ratio have stabilizing effects for both stationary and oscillatory convection.
- Thermosolutal Lewis number has a stabilizing effect only for oscillatory convection.
- The relaxation parameter, medium porosity, and nanoparticle Rayleigh number have destabilizing effects for both stationary and oscillatory convection.

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