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STRUCTURAL BEHAVIOUR OF ANNULAR ISOTROPIC DISK MADE OF STEEL/COPPER MATERIAL WITH GRADUALLY VARYING THICKNESS SUBJECTED TO INTERNAL PRESSURE

PONAŠANJE KONSTRUKCIJE PRSTENASTOG IZOTROPNOG DISKA OD ČELIKA/BAKRA SA POSTEPENOM PROMENOM DEBLJINE OPTEREĆEN UNUTRAŠNJIM PRITISKOM

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Ključne reči
- napon
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Abstract

This article deals with the study of stress analysis in an annular isotropic disk made of steel/copper material with gradually varying thickness and subjected to internal pressure. Analysis is based on Seth's transition theory, generalised strain measure and Hooke's law. Convergent and divergent disks are considered. Analytical solutions for stress distribution and critical pressure are obtained. The effects of different pertinent parameters (i.e. thickness and pressure) are considered for the annular isotropic disk made of steel and copper material. From the obtained results it is noticed that disk made of copper material requires a maximal hoop stress at the outer surface of the convergent case, but reverse results are obtained in the case of uniform/divergent case.

Izvod

U radu se istražuje analiza napona kod prstenastog izotropnog diska od čelika/bakra, sa postepenom promenom debljine, a koji je opterećen unutrašnjim pritiskom. Analiza se zasniva na teoriji prelaznih napona Seta, generalisanom merom deformacija i na Hukovom zakonu. Razmatraju se konvergentni i divergentni diskovi. Dobijena su analitička rešenja za raspodelu napona i kritični pritisak. Razmatraju se i uticaji različitih relevantnih parametara (na pr. debljine i pritiska) kod prstenastog izotropnog diska od čelika i bakra. Prema dobijenim rezultatima primećuje se da je u slučaju diska od bakra potreban maksimalan napon u obimskom pravcu na spoljnoj površini kod konvergentnog diska, dok su rezultati suprotni za uniformni/divergentni disk.

INTRODUCTION

The study of disks has become important in the last few years. Particularly found in various engineering applications in chemical processing, aerospace industries, and mechanical engineering such as compressors, computer disks, high speed gear engines, steam turbine, flywheels, sink fits, pumps, and turbo generators. etc. The analytical studies of elastoplastic deformation in a rotating disk can be found in various textbooks, /1-4/. Güven /5/ discussed the effects of stress distribution in a hyperbolic disk fixed with rigid shaft by taking assumptions of plane stress and Tresca's yield condition. Vivio et al. /6/ analyse the stress distribution in a rotating conical disk, which is either annular or solid subjected to thermal loads and having fictitious density variation. Calderale et al. /7/ discussed the thermal stresses in a hyperbolic disk under thermal load condition as a particular case of nonlinearly variable disk thickness. Further, Vivio et al. /8/ studied the stresses and strains in a hyperbolic disk subjected to thermal load and with omission of singularities. Deepak et al. /9/ presented steady state creep in a rotating functionally graded Al-SiC$_p$. Woncheol et al. /10/ examined the stresses in a rotating annular hyperbolic disk by using Drucker-Prager yield criterion. Yıldırım /11/ studied the analytic solution of a power-law graded hyperbolic rotating disk made of a power-law graded material. Ersin et al. /12/ investigated the elastoplastic stresses of functionally graded hyperbolic disk under steady-state temperature. Furthermore, Yıldırım /13/ explored the parametric study of displacements and centrifugal force-induced stress in power-law graded hyperbolic disk. In addition, Yıldırım /14/ concluded the closed-form formulas of hyperbolic rotating disk under combined mechanical and thermal loads. Singh et al. /15/ investigated the creep deformation in a rotating composite hyperbolic disk. The creep behaviour of the disk was described by threshold stress-based law and yielding is assumed to follow Tresca criterion. Thakur et al. /16/ studied the analytical solution of hyperbolic deformable disk having variable density. The effects of pressure and stress distribution in an annular isotropic disk with variable thickness and subjected to uniform pressure have been discussed by using transition theory and generalised strain measure.

PROBLEM FORMULATION

Figure 1 displays the configuration of an annular thin isotropic disk made of copper and steel materials with inner radius a, and outer radius b, subjected to internal pressure $p_{int.}$. The thickness of the disk varies along the radius and is given by /16/:

$$h = h_i (r/a)^k , \tag{1}$$

where: k is thickness parameter, and for convergent $k < 0$ and divergent $k > 0$.

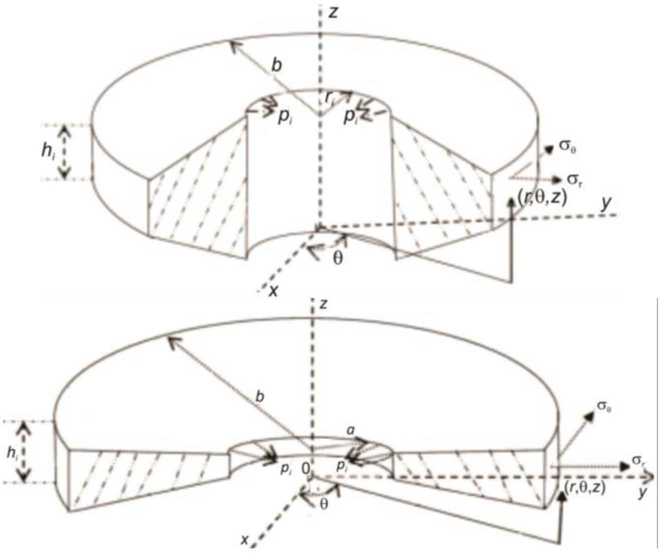

Figure 1. Geometry of the disk (convergent/divergent).

Boundary conditions: the boundary conditions of annular disk under internal pressure are given as:

$$\tau_{rr} = -p_{\text{int.}} \text{ at } r = a \text{ and } \tau_{rr} = 0 \text{ at } r = b \tag{2}$$

where: $p_{\text{int.}}$ is pressure applied at the disk inner surface.

BASIC GOVERNING EQUATION

The components of displacement in (r,θ,z) are given by /17, 18/:

$$u = r(1-\beta) , \quad v = 0 , \quad w = dz . \tag{3}$$

The stress components are given by /19/:

$$\tau_{rr} = \frac{2\mu}{n}\left[3-2c-(2-c)\left\{2\beta(\eta+1)-1\right\}^{n/2} - (1-c)(2\beta-1)^{n/2}\right],$$

$$\tau_{\theta\theta} = \frac{2\mu}{n}\left[3-2c-(1-c)\left\{2\beta(\eta+1)-1\right\}^{n/2} - (2-c)(2\beta-1)^{n/2}\right],$$

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = \tau_{zz} = 0 , \tag{4}$$

where: $r\beta' = \beta\eta$ (η is function of β, and β is function of r); and $c = 2\mu/(\lambda + 2\mu)$. For an isotropic disk centrifugal forces are zero, therefore equations of equilibrium become:

$$\frac{d}{dr}(rh\tau_{rr}) - h\tau_{\theta\theta} = 0 , \tag{5}$$

where: h is the disk thickness. Using Eq.(4) into Eq.(5), we get the following nonlinear differential equation in β as:

$$(2-c)n\beta^2 \left\{2\beta(P+1)-1\right\}^{\frac{n}{2}-1}\frac{d\eta}{d\beta} = \Big[(2\beta-1)^{n/2} -$$

$$\left\{2\beta(\eta+1)-1\right\}^{n/2} - (1-c)n\eta\beta(2\beta-1)^{\frac{n}{2}-1} + \frac{rh'}{h}\times$$

$$\times\left\{3-2c-\left\{2\beta(\eta+1)-1\right\}^{n/2} - (2\beta-1)^{n/2}(1-c)\right\}\Big] , \tag{6}$$

where: $\beta' = d\beta/dr$.

ELASTIC TO PLASTIC SOLUTIONS

The principal stress /16-31/ leads from elastic to plastic state at the transition point $\eta \to \pm\infty$, we define the transition function Ψ as:

$$\Psi = \frac{n}{2\mu}\tau_{\theta\theta} = 3-2c-(1-c)\{2\beta(P+1)-1\}^{n/2} - (2-c)(2\beta-1)^{n/2} . \tag{7}$$

By taking logarithmic differentiation of Eq.(7) with respect to r and using Eq.(6), after that taking $\eta \to \pm\infty$ and then integrating, we get

$$\Psi = \left(\frac{A}{h}\right) r^{\nu-1} , \tag{8}$$

where: $\nu = (1-c)/(2-c)$. Therefore, Eqs.(7) and (8) become:

$$\tau_{\theta\theta} = \left(\frac{2\mu}{n}\right)\frac{Ar^{\nu-1}}{h} . \tag{9}$$

Substituting Eq.(9) into Eq.(5) and using Eq.(1) after that integrating, we get:

$$\tau_{rr} = \left(\frac{2\mu}{n\nu}\right)\frac{Aa^k r^{\nu-k-1}}{h_i} + \frac{Ba^k}{h_i r^{k+1}} . \tag{10}$$

By applying boundary conditions Eq.(2) into Eq.(10), we get:

$$A = \left(\frac{n}{2\mu}\right)\frac{p_{\text{int.}} h_0 a\nu}{(b^\nu - a^\nu)} \quad \text{and} \quad B = -\frac{p_{\text{int.}} ab^\nu h_0}{(b^\nu - a^\nu)} .$$

Substituting the values of A and B into Eqs.(9) and (10), we get the stresses:

$$\tau_{rr} = \frac{p_{\text{int.}} R_0^{k+1}(R^\nu - 1)}{R^{k+1}(1-R_0^\nu)} , \quad \tau_{\theta\theta} = \frac{p_{\text{int.}} \nu R_0^{k+1} R^{\nu-k-1}}{(1-R_0^\nu)} . \tag{11}$$

From Eq.(11), we get:

$$\tau_{\theta\theta} - \tau_{rr} = \frac{p_{\text{int.}} R_0^{k+1}}{R^{(k+1)}(1-R_0^\nu)}\left[(\nu-1)R^\nu + 1\right] , \tag{12}$$

where: $R = r/b$ and $R_0 = a/b$. The thickness parameter k varies: $k < 0$ and $k > 0$.

Convergent disk for $\frac{(\nu-1)^2 - 1}{(\nu-1)+1} < k < \frac{R_0^\nu(\nu-1)^2 - 1}{R_0^\nu(\nu-1)+1}$, the maximal value of shear stress $|\tau_{\theta\theta} - \tau_{rr}|$ is localized inside the disk at $R = R_i = \left[\frac{k+1}{(\nu-1)(\nu-k-1)}\right]^{1/\nu}$ for $R_0 = R_i < 1$. Different values of Poisson's ratio take place in the range: $\nu = 0.27$, $R_0 = 0.5$: $-1.73 < k < -1.40$; $\nu = 0.4$, $R_0 = 0.5$: $-1.6 < k < -1.32$; $\nu = 0.5$, $R_0 = 0.5$: $-1.5 < k < -1.27$. Eq.(12) becomes:

$$|\tau_{\theta\theta} - \tau_{rr}|_{R \cong R_i} = \left|\frac{p_{\text{int.}} R_0^{k+1}}{(1-R_0^\nu)}\left(\frac{\nu}{\nu-k-1}\right)\left[\frac{k+1}{(\nu-1)(\nu-k-1)}\right]^{-\frac{(k+1)}{\nu}}\right| \cong Y ,$$

where Y is the yielding stress. The pressure required for initial yielding is given by:

$$p_i = \frac{p_{\text{int.}}}{Y} = \left|\frac{(1-R_0^\nu)}{R_0^{k+1}}\left(\frac{\nu-k-1}{\nu}\right)\left[\frac{(\nu-1)(\nu-k-1)}{k+1}\right]^{-\frac{(k+1)}{\nu}}\right| . \tag{13}$$

Substituting Eq.(13) into Eq.(11), we get:

$$\sigma_r = \frac{p_i R_0^{k+1}(R^{\nu}-1)}{R^{k+1}(1-R_0^{\nu})}, \quad \sigma_\theta = \frac{p_i \nu R_0^{k+1} R^{\nu-k-1}}{(1-R_0^{\nu})}, \qquad (14)$$

where: $\sigma_r = \tau_{rr}/Y$ and $\sigma_\theta = \tau_{\theta\theta}/Y$. Eqs.(13)-(14), the pressure and stress distribution for fully-plastic stage (i.e. $\nu \to 1/2$), becomes:

$$p_f = \frac{p_{\text{int.}}}{Y} = \left| \frac{(1-\sqrt{R_0})(2k+1)}{R_0^{k+1}} \left[\frac{2k+1}{4(k+1)} \right]^{-2(k+1)} \right|, \qquad (15)$$

$$\sigma_r = \frac{p_f R_0^{k+1}(\sqrt{R}-1)}{R^{k+1}(1-\sqrt{R_0})}, \quad \sigma_\theta = \frac{p_f R_0^{k+1}}{2R^{k+\frac{1}{2}}(1-\sqrt{R_0})}, \qquad (16)$$

where: p_f is pressure required for the fully-plastic stage.

Divergent disk: the initial yielding when $k > 0$, from Eq.(11) $|\tau_{\theta\theta}|$ is maximal at the internal surface, therefore, yielding will take place at the internal surface of the disk. Eq.(11) becomes: $|T_{\theta\theta}|_{R=R_0} = \left| \frac{p\nu R_0^{\nu}}{(1-R_0^{\nu})} \right| \cong Y_1$; where Y_1 is the yielding stress. Therefore, the required pressure for yielding is given:

$$p_i^* = \frac{p_{\text{int.}}}{Y_1} = \left| \frac{(1-R_0^{\nu})}{\nu R_0^{\nu}} \right|. \qquad (17)$$

Substituting Eq.(17) into Eqs.(11), we get:

$$\sigma_r = \frac{p_i^* R_0^{k+1}(R^{\nu}-1)}{R^{k+1}(1-R_0^{\nu})}, \quad \sigma_\theta = \frac{p_i^* \nu R_0^{k+1} R^{\nu-k-1}}{(1-R_0^{\nu})}, \qquad (18)$$

where: $\sigma_r = \tau_{rr}/Y_1$ and $\sigma_\theta = \tau_{\theta\theta}/Y_1$. Eqs.(17)-(18), stress distribution and pressure for fully-plastic stage (i.e. $\nu \to 0.5$) is:

$$\sigma_r = \frac{p_f^* R_0^{k+1}(\sqrt{R}-1)}{R^{k+1}(1-\sqrt{R_0})}, \quad \sigma_\theta = \frac{p_f^* R_0^{k+1}}{2R^{k+\frac{1}{2}}(1-\sqrt{R_0})}, \qquad (19)$$

and

$$p_f^* = \frac{p_{\text{int.}}}{Y_1} = \left| \frac{2(1-\sqrt{R_0})}{\sqrt{R_0}} \right|, \qquad (20)$$

where: p_f^* pressure required for the fully-plastic stage.

VALIDATION OF RESULTS

By taking $k \to 0$ from Eqs.(14), (16), (18), and (19) for the initial/fully-plastic stage is same as given by Thakur /31/ for the case of uniform disk. Therefore, the present results are correct and authenticate the validity of the derived solutions.

RESULTS AND DISCUSSION

To investigate the effect of stress distribution and pressure for annular isotropic disk made of copper and steel material with a Poisson's ratio $\nu = 0.33$ and 0.27, /1/, the following numerical values have been taken: $a = 1$, $b = 2$, $a \le r \le b$, $k > 0$ for divergent case, and $k < 0$ for convergent case.

Table 1. Perc. in pressure required for initial/fully plastic stage.

Symmetry of disk	Mater.	Yielding starts at $R = R_0$			Perc. increase in pressure from initial yielding to fully-plastic stage P (%)
		thickness (k)	pressure p_i	pressure p_f	$P = \left(\frac{p_f - p_i}{p_i}\right)100$
converg. ($k < 0$)	copper	-1.32	0.444	0.48	9.4
	steel	-1.40	0.432	0.48	12.3
diverg. ($k > 0$)	copper	0.1	0.798	1.25	57.2
	steel	0.1	0.762	1.25	64.7

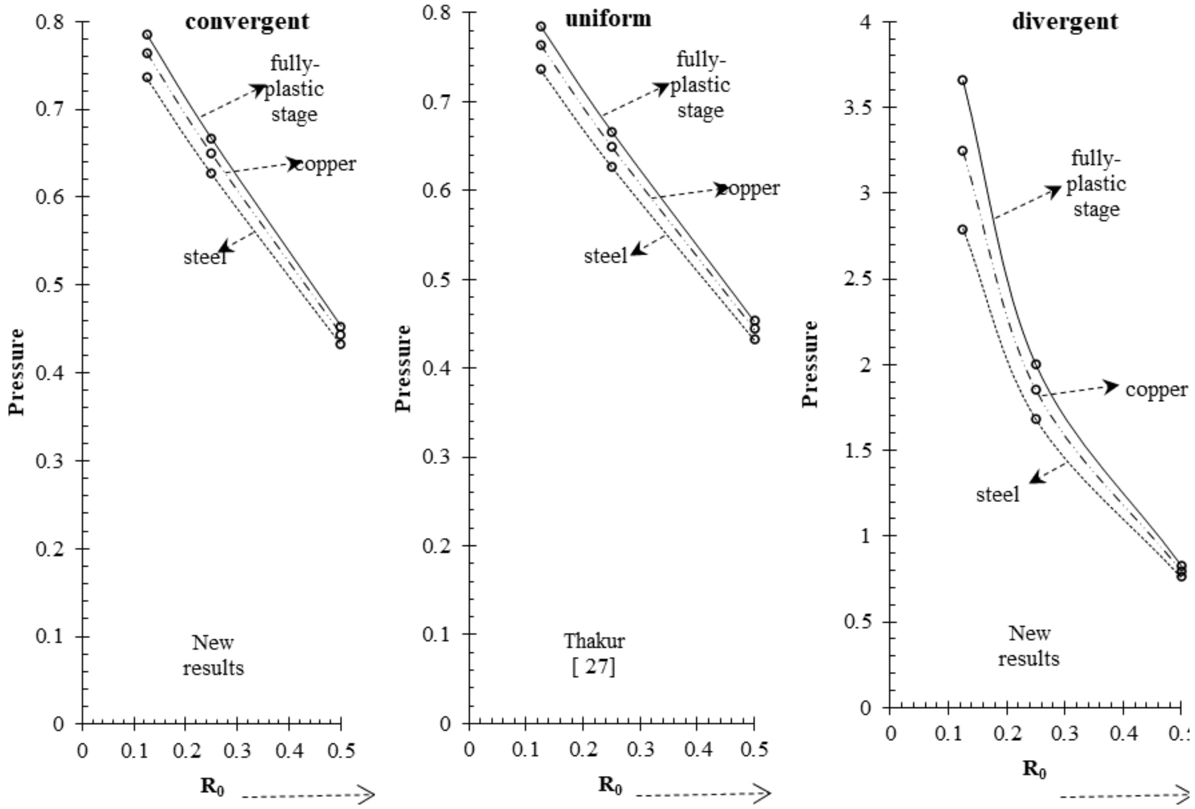

Figure 2. Graphical representation of pressure for initial yielding/fully-plastic stage vs. radii ratio $R_0 = a/b$.

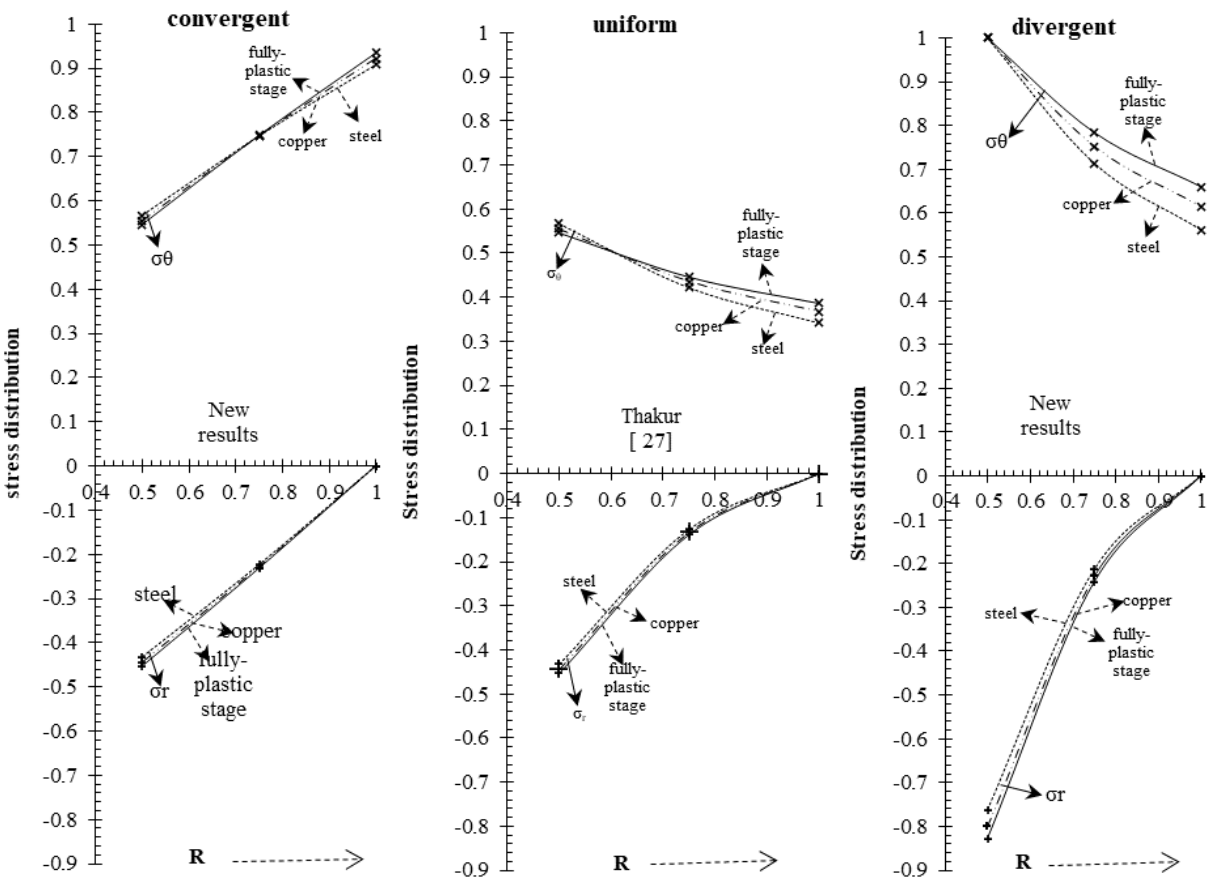

Figure 3. Graphical representations of stress distributions for initial/fully-plastic stage vs. radii ratio $R = r/b$.

From Table 1, a percentage increase in pressure required for initial yielding to become fully-plastic stage is discussed. It can also be seen from Table 1 that annular isotropic disk made of steel material requires higher percentage values in pressure (i.e., P = 12.3 %, 64.7 %) in convergent/divergent case with variable thickness (i.e., k = -1.40 and 0.1) to become fully plastic as compared to the copper disk (i.e., P = 9.4 %, 57.2 %) in convergent/divergent case with variable thickness (i.e. k = -1.32 and 0.1), respectively. Further, in the case of divergent, the disk made of copper and steel material acquires higher percentage ratio (i.e., P %) for the initial yielding stage to become fully plastic in comparison to the convergent. Moreover the convergent disk is more convenient than the divergent disk.

Figures 2 and 3 show a graphical comparison between the new and previously published results, pressure versus radii ratio $R_0 = a/b$ and stress distribution versus radius $R = r/b$. By taking $k \to 0$ in Eqs.(14), (16), (18), and (19) for the initial and fully-plastic stage, the present results reduce to previously published outcomes done by Thakur /31/. From this comparison, it is found that the present results are correct and make sure of the validity of present solutions. The shortcomings of this study /31/ are that the author has discussed only the uniform disk, but in the present study we have discussed a new addition of convergent and divergent case of the annular isotropic disk and the comparison of published and new results and found a more convenient disk.

In Figure 2, curves are drawn between pressure required for initial yielding/fully-plastic stage versus radii ratio $R_0 = a/b$ for the convergent/uniform/divergent case. It is observed that the copper disk requires higher pressure to yield at the internal surface as compared to steel material. Moreover, the divergent disk needs high pressure for the initial yielding stage in comparison to the convergent/ uniform disk.

Figure 3 are portrayed in order to demonstrate the behaviour of stress distribution for the initial/fully-plastic stage versus radii ratio $R = r/b$ in the convergent/uniform and divergent case and having thickness $k < 0$, $k = 0$, and $k > 0$, respectively. It is observed that disk made of copper material requires a maximum hoop stress at the outer surface of the convergent case, but reverse results are obtained in the case of uniform/divergent. Moreover divergent disk attains maximum radial as well as hoop stress at the inner/outer surface in comparison to the uniform/convergent case. Furthermore, the disk made of copper material is more convenient than that of steel material.

CONCLUSION

The main finding follows:

- the steel disk requires less pressure to yield at the internal surface as compared to copper material;
- the divergent disk needed high pressure for the initial yielding stage in comparisons to convergent disk;
- the disk made of copper material requires a maximum hoop stress at the outer surface of the convergent case, but reverse results are obtained in the case of uniform/ divergent case;
- the results of Thakur /31/ can be obtained by taking $k \to 0$ in the resulting equations.

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Abbreviations

λ, μ - Lame's constants,
C - compressibility factor,
a, b - internal and external radius,
u, v, w - displacement components,
ν - Poisson's ratio,
τ_{ij}, ε_{ij} - stress and strain tensors,
Y - yield stress,
p_i, p_f - pressure required for initial and fully-plastic stage,
A, B - constants of integration,
k - thickness parameters,
δ_{ij} - Kronecker's delta,
r - function of x and y,
β - function of r only,
Ψ - transition function,
d - constant

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