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ELASTODYNAMICAL INTERACTIONS IN A FRACTIONAL ORDER MICROPOLAR THERMOELASTIC MASS DIFFUSION MEDIUM UNDER LASER PULSE HEATING

ELASTODINAMIČKE INTERAKCIJE KOD MIKROPOLARNE TERMOELASTIČNE MASENE DIFUZIONE SREDINE FRAKCIONOG REDA PRI ZAGREVANJU IMPULSNIM LASEROM

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- laser pulse
- micropolar
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- uniformly and linearly distributed source

Abstract

The present investigation deals with the deformation in a fractional order micropolar thermoelastic medium with mass diffusion subjected to thermomechanical loading due to input laser pulse. Laplace and Fourier transform technique is used to solve the problem. Concentrated normal force and thermal source are taken to illustrate the utility of approach. The compact form expressions for normal stress, tangential stress, tangential couple stress, mass concentration and temperature distribution are obtained in the transformed domain. Numerical inversion technique of Laplace transforms and Fourier transform has been applied to obtain the resulting quantities in the physical domain after developing a computer programme. The normal stress, tangential stress, tangential coupled stress, temperature distribution, and mass concentration are depicted graphically to show the effect of relaxation times. Some particular cases of interest are deduced from the present investigation.

Ključne reči
- impulsni laser
- mikropolarna sredina
- masena difuzija
- ravnomerno i linearno raspoređen izvor

Izvod

U radu se istražuju deformacije za slučaj mikropolarne termoelastične masene difuzione sredine frakcionog reda koja je opterećena termomehanički usled dejstva impulsnim laserom. Primenjene su metode transformacije Laplasa i Furijea za rešavanje problema. Radi ilustracije pristupa u rešavanju problema, razmatra se dejstvo skoncentrisane normalne sile i toplotnog izvora. Pri transformaciji domena, dobijaju se izrazi kompaktnog oblika za normalni napon, tangencijalni napon, tangencijalni spregnuti napon, koncentraciju mase i za raspodelu temperature. Numeričke inverzne metode Laplasove i Furijeove transformacije su primenjene za dobijanje rezultujućih veličina u fizičkom domenu nakon razvijanja programa za računar. Radi predstavljanja uticaja vremena relaksacije, grafički su predstavljeni normalni napon, tangencijalni napon, tangencijalni spregnuti napon, raspodela temperature, kao i koncentracija mase. Pojedinim slučajevima od interesa se posvećuje posebna pažnja.

INTRODUCTION

Eringen's micropolar theory of elasticity /1/ is a well known theory. The stepwise development of this theory of micropolar elasticity is given in a monograph by Eringen. In this theory, a load across a surface element is transmitted not only by a force stress vector. A micropolar elastic material can be considered as being composed of dumbbell-shaped molecules and these molecules in a volume element can undergo rotation about their centre of mass along with the linear displacement.

The dynamical interaction between thermal and mechanical fields in materials have numerous applications in aeronautics, nuclear reactors, and high energy particle accelerators. The micropolar theory was extended to include thermal effects by Nowacki /2/ and Eringen /3/. Maugin and Mild /4/ studied a solitary wave in micropolar elastic crystals. Shanker and Dhaliwal /5/ solved several dynamic thermoelastic problems in micropolar theory. Chirita /6/ proved the existence and uniqueness theorems for the equations of linear thermoelasticity with microstructures.

In recent years fractional calculus is very useful. This mathematical tool makes possible to obtain new challenging insights and surprising correlations between different branches of science and engineering. The application of fractional calculus is given by Abel in the solution of an integral equation that arises in the formulation of the tautochrone problem. The history of the development of fractional calculus is written by Ross /7/ and Miller /8/. Podlubny /9/ surveyed many applications of fractional calculus in the area of science and engineering. Most important advantage of fractional calculus in these applications is its non-local property. Non-local effects occur in space and time. The tools of fractional calculus are applicable to various fields of study. Povestenko /10/ has constructed a quasi-static uncoupled thermoelasticity model based on the heat conduction equation with fractional order time derivatives. Two general models of fractional heat conduction equation were derived by Ezzat and Karamany /11/. Sherief et al. /12/ developed another theory of fractional thermoelasticity and proved a uniqueness theorem and derived a reciprocity relation and a variational principle. Youssef and Al-Lehaibi /13/ introduced another model of fractional heat conduction equation and also presented a one-dimensional application. Ezzat /14/ formulated his model of fractional order generalised thermoelasticity. Youssef et al. /15/ derived a theory of generalised thermoelasticity with fractional order strain. A dynamical problem in fractional magneto-micropolar thermoelastic media with ramp type heating was studied by Kumar et al. /16/. The dynamical problem of fractional order micropolar thermoviscoelastic medium with diffusion under the effect of ramp type mechanical load was studied by Deswal et al. /17/. Lata and Kaur /18/ developed fractional order theory of thermal stresses for transversely isotropic thermoelastic materials. Yadav /19/ analysed the elastic wave propagation in a fractional micropolar diffusion porous medium.

Diffusion can be defined as a penetrative phenomenon from regions of high to regions of low concentration, derived from concentration differences of different regions of the materials. In recent past, several researchers have devoted their efforts to study this phenomenon inspired by its multifarious applications in geophysics, biology, and industry. For example, oil companies are interested in the process of thermodiffusion for more efficient extraction of oils from oil deposits. Thermodiffusion process also helps the investigation in the field associated with the advent of semiconductor devices and the advancement of microelectronics. Most of research associated with the presence of concentration and temperature gradients have been made with metals and alloys. Thermodiffusion in elastic solid is due to the coupling of temperatures, mass diffusion and strain in addition to the exchange of heat and mass with the environment. Nowacki /20-23/ presented the theory of thermoelastic diffusion in four research papers by using coupled thermoelastic model. Dudziak and Kowalski /24/ presented the theory of thermodiffusion, and Olesiak and Pyryev /25/ formulated coupled quasi-stationary problems of thermodiffusion for an elastic layer. Kumar /26/ studied a dynamical problem in laser irradiated microstretch thermoelastic medium with mass diffusion.

Laser technology has a vital range of application in non-destructive testing and evaluation of materials. If a material is heated with a laser pulse, it absorbs some energy which results in an increase in localized temperature. This causes thermal expansion and generation of ultrasonic waves in the material. There are generally two mechanisms for such wave generation, depending on the energy density deposited by the laser pulse. At high energy density, a thin surface layer of the solid material melts, followed by an ablation process, whereby particles fly off the surface, thus giving rise to forces that generate ultrasonic waves. At low energy density, the surface material does not melt, but it expands at a high rate and wave motion is generated due to thermoelastic processes. Very rapid thermal processes are interesting in the field of thermoelasticity, since they require a coupled analysis of temperature and deformation fields. A thermal shock induces very rapid movement in structural elements, giving rise to very significant inertial forces, and thereby, an increase in vibration. Rapidly oscillating contraction and expansion generates temperature changes in materials susceptible to diffusion of heat by conduction. Dubois et al. /27/ experimentally proved that penetration depth plays a very important role in the laser-ultrasound generation process. Ezzat et al. /28/ discussed the thermoelastic interaction in metal by fractional ultrafast laser. Al-Huniti and Al-Nimr /29/ discussed the thermoelastic changes of a composite slab under rapid dual-phase lag heating. The comparison of one- and two-dimensional axisymmetric approaches to the thermomechanical response caused by ultrashort laser heating was studied by Chen et al. /30/. Kim et al. /31/ investigated thermoelastic stresses in a bonded layer due to pulsed laser irradiation. Thermoelastic material response due to laser pulse heating with comparison in theories of thermoelasticity was presented by Youssef and Al-Bary /32/. Theoretical study of the effect on enamel parameters by laser induced surface acoustic waves in human incisor was studied by Yuan et al. /33/. A two-dimensional thermoelastic diffusion problem including laser pulse thermal heating was studied by Elhagary /34/. Othman et al. /35/ studied the influence of thermal loading due to laser pulse on generalised micropolar thermoelastic solid with comparison of different theories. The exact analysis of laser generated thermoelastic waves in an anisotropic infinite plate is mathematically done by Al-Qahtani and Datta /36/. Deswal et al. /37/ investigated a two-dimensional problem in magneto-thermoelasticity with laser pulse under different boundary conditions. Using normal mode analysis laser interactions in micropolar diffusive solid were studied by Kumar and Kumar /38/. Laser interactions in generalised microstretch thermoelastic medium were investigated by Kumar et al. /39/. A dynamical problem in piezo-microstretch thermoelastic solid was presented by Kumar and Ailawalia /40/. The effect of input pulsed laser heat source was discussed by Abo-Dahab et al. /41/.

This research includes a mass diffusion effect and radiation of ultra-short laser and establishes a model for fractional micropolar thermoelastic medium by using integral transform technique. Stress components and temperature distribution are computed numerically. Resulting quantities are presented graphically to show the effect of mass concentration and temperature.

BASIC EQUATIONS

Following Eringen /3/ and Al-Qahtani and Datta /36/ the basic equations for fractional micropolar generalised thermoelastic solid with mass diffusion in the absence of body forces and body couples are given by:

- stress equation of motion

$$(\lambda+\mu)\nabla(\nabla.\vec{u})+(\mu+k)\nabla^2\vec{u}+k\nabla\times\phi-\beta_1\nabla T-\beta_2\nabla C=\rho\ddot{\vec{u}}\,, \quad (1)$$

- couple stress equation of motion

$$(\gamma\nabla^2-2k)\vec{\phi}+(\alpha+\beta)\nabla(\nabla.\vec{\phi})+k\nabla\times\vec{u}=\rho j\ddot{\vec{\phi}}\,, \quad (2)$$

- fractional order equation of heat conduction

$$k^*\nabla^2 T=\left(\frac{\partial}{\partial t}+\frac{z_0^{\alpha'}}{(\alpha'+1)}\frac{\partial^{\alpha'+1}}{\partial t^{\alpha'+1}}\right)(\rho c^* T+\beta_1 T_0 e+aT_0 C)\,, \quad (3)$$

- equation of balance of stress moments

$$D\beta_2\nabla^2(\nabla.\vec{u})+Da\nabla^2T-Db\nabla^2C+\left(\frac{\partial}{\partial t}+\frac{z_0^{\alpha'}}{(\alpha'+1)}\frac{\partial^{\alpha'+1}}{\partial t^{\alpha'+1}}\right)C=0\,, \quad (4)$$

- the constitutive relations are

$$t_{ij}=\lambda\mu_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+k(u_{j,i}-\varepsilon_{ijk}\phi_k)-\beta_1\delta_{ij}T-$$
$$-\beta_2\delta_{ij}C\,, \quad (5)$$

$$m_{ij}=\alpha\phi_{k,k}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}\,, \quad (6)$$

$$p=-\beta_2e+bC-aT\,. \quad (7)$$

The plate surface is illuminated by laser pulse given by the heat input

$$Q=I_0f(t)g(x_1)h(x_3)\,. \quad (8)$$

Here, I_0 is the energy absorbed. The temporal profile $f(t)$ is represented as

$$f(t)=\frac{t}{t_0^2}e^{-(t/t_0)}\,. \quad (9)$$

Here, t_0 is the pulse rise time. The pulse is also assumed to have a Gaussian spatial profile in x_1

$$g(x)=\frac{1}{2\pi r^2}e^{-\left(x_1^2/r^2\right)}\,. \quad (10)$$

Here, r is the beam radius, and as a function of the depth x_3 the heat deposition due to the laser pulse is assumed to decay exponentially within the solid,

$$h(x_3)=\gamma^*e^{-\gamma^*x_3}\,. \quad (11)$$

Equation (8) with the aid of Eqs.(9)-(11) takes the form

$$Q=\frac{I_0\gamma^*}{2\pi r^2t_0^2}te^{-(t/t_0)}e^{-\left(x_1^2/r^2\right)}e^{-\gamma^*x_3}\,, \quad (12)$$

where: λ, μ, α, β, γ, K are material constants; ρ is mass density; $\boldsymbol{u}=(u_1, u_2, u3)$ is displacement vector; $\boldsymbol{\phi}=(\phi_1, \phi_2, \phi_3)$ is microrotation vector; T is temperature; T_0 is the reference temperature of the body chosen; k^* is coefficient of thermal conductivity; c^* is specific heat at constant strain; D is thermoelastic diffusion constant; a is the coefficient describing the measure of thermal diffusion; and b is the coefficient describing the measure of mass diffusion effects; j is microinertia; t_0 is pulse rise time;, I_0 is absorbed energy, t_{ij} are components of stress vector; m_{ij} are components of couple stress vector; δ_{ij} is Kronecker delta function; t_0, t_1 are thermal relaxation times with $t_0 \geq t_1 \geq 0$.

FORMULATION OF THE PROBLEM

We consider a fractional micropolar thermoelastic mass diffusion medium with rectangular Cartesian coordinate system $0x_1x_2x_3$ having origin on x_3-axis with x_3-axis pointing vertically inward the medium. We consider plane strain problem with all field variables depending on x_1, x_3 and t. For two-dimensional problems, we take

$$\boldsymbol{u}=(u_1,0,u_3)\,,\quad \boldsymbol{\phi}=(0,\phi_2,0)\,. \quad (13)$$

For further consideration, it is convenient to introduce in Eqs.(1)-(4) the dimensionless quantities defined as

$$(x_1^*,x_3^*,u_1^*,u_3^*)=w^*(x_1,x_3,u_1,u_3)\,,\quad (t,\tau_0^*,\tau^*)=w^*(t,\tau_0,\tau)\,,$$

$$T^*=\frac{\beta_1T}{\lambda+2\mu}\,,\ \tau_{ij}^*=\frac{\tau_{ij}}{\mu}\,,\ C^*=\frac{\beta_2C}{\lambda+2\mu}\,,\ \phi^*=\frac{p}{\beta_2}\,,\ \omega^*=\frac{\rho c^*C_0^2}{k^*}\,,$$

$$C_0^2=\frac{\lambda+2\mu+k}{\rho}\,,\ \phi_2^*=\frac{\rho C_0^2}{\beta_1T_0}\phi_2\,,\ m_{ij}^*=\frac{\omega^*}{\beta_1T_0C_0}m_{ij}\,,\ \tau_{ij}^*=\frac{\tau_{ij}}{\beta_1T_0}\,,$$

$$Q=\frac{k^*\omega^{*2}}{C^*}Q'\,. \quad (14)$$

Utilizing the expressions defined by Eq.(13) in Eqs.(1)-(4) and with the help of expressions defined in Eq.(14), we reach to the following equations,

$$(\lambda+\mu)\frac{\partial e}{\partial x_1}+(\mu+k)\nabla^2u_1-k\frac{\partial\phi_2}{\partial x_3}-\beta_1\frac{\partial T}{\partial x_1}-\beta_2\frac{\partial C}{\partial x_1}=\rho\ddot{u}_1\,, \quad (15)$$

$$(\lambda+\mu)\frac{\partial e}{\partial x_3}+(\mu+k)\nabla^2u_3+k\frac{\partial\phi_2}{\partial x_1}-\beta_1\frac{\partial T}{\partial x_3}-\beta_2\frac{\partial C}{\partial x_3}=\rho\ddot{u}_3\,, \quad (16)$$

$$(\gamma\nabla^2-2k)\phi_2+k\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)=\rho j\ddot{\phi}_2\,, \quad (17)$$

$$k^*\nabla^2T=\left(\frac{\partial}{\partial t}+\frac{z_0^{\alpha'}}{(\alpha'+1)}\frac{\partial^{\alpha'+1}}{\partial t^{\alpha'+1}}\right)(\rho c^*T+\beta_1T_0e+aT_0C)\,, \quad (18)$$

$$D\beta_2\nabla^2e+Da\nabla^2T-Db\nabla^2C+\left(\frac{\partial}{\partial t}+\frac{z_0^{\alpha'}}{(\alpha'+1)}\frac{\partial^{\alpha'+1}}{\partial t^{\alpha'+1}}\right)C=0\,, \quad (19)$$

$$a_1=\frac{kT_0}{\rho^2C_0^2}\,,\ a_2=\frac{\lambda+2\mu}{\rho C_0^2}\,,\ a_3=\frac{\mu+k}{\rho}\,,\ a_4=\frac{\beta_1T_0}{\rho C_0^2}\,,\ a_5=\frac{k}{\omega^2}\,,$$

$$a_6=\frac{j\beta_1T_0}{C_0^2}\,,\ a_7=\frac{k\omega(\lambda+2\mu)}{\beta_1^2T_0}\,,\ a_8=\frac{\rho(\lambda+2\mu)c^*}{\beta_1^2T_0}\,,\ a_9=\frac{a(\lambda+2\mu)}{\beta_1\beta_2}$$

$$a_{11}=\frac{b(\lambda+2\mu)}{\beta_2^2}\,,\ a_{12}=\frac{(\lambda+2\mu)}{\beta_2^2D\omega^*}\,,\ Q_0=\frac{a_{13}I_0\gamma^*}{2\pi r^2t_0^2}\,,\ \text{and}$$

$$f(x_1,t)=\left[t+\varepsilon\tau_0\left(1-\frac{t}{t_0}\right)\right]e^{-\left(\frac{x_1^2}{r^2}+\frac{t}{t_0}\right)}\,.$$

The displacement components u_1 and u_3 are related to non-dimensional potential functions ϕ and ψ as

$$u_1=\frac{\partial\phi}{\partial x_1}-\frac{\partial\psi}{\partial x_3}\,,\quad u_3=\frac{\partial\phi}{\partial x_3}+\frac{\partial\psi}{\partial x_1}\,. \quad (20)$$

Substituting the values of u_1 and u_3 from Eq.(20) in Eqs. (15)-(19) and with the aid of Eq.(13), we obtain

$$\nabla^2\phi-a_2T-a_2C=\frac{1}{C_0^2}\ddot{\phi}\,, \quad (21)$$

$$a_3\nabla^2\psi+a_1\phi_2=\ddot{\psi}\,, \quad (22)$$

$$\left[\left(\gamma\nabla^2-\frac{2k}{\omega^2}\right)a_4-a_6\frac{\partial^2}{\partial t^2}\right]\phi_2-a_5\nabla^2\psi=0\,, \quad (23)$$

$$\nabla^2\phi+\left[a_8\left(\frac{\partial}{\partial t}+\frac{\tau_0^{\alpha'}}{(\alpha'+1)}\frac{\partial^{\alpha'+1}}{\partial t^{\alpha'+1}}\right)-a_7\nabla^2\right]T+a_9C=0\,, \quad (24)$$

$$\nabla^4\phi+a_9\nabla^2T+\left[a_{12}\left(\frac{\partial}{\partial t}+\frac{\tau_0^{\alpha'}}{(\alpha'+1)}\frac{\partial^{\alpha'+1}}{\partial t^{\alpha'+1}}\right)-a_{11}\nabla^2\right]C=0\,. \quad (25)$$

Now rearranging the Eqs.(21)-(25), we have

$$\left(\nabla^2-\frac{1}{C_0^2}\frac{\partial^2}{\partial t^2}\right)\phi-a_2T-a_2C=0\,, \quad (26)$$

$$\left(a_3\nabla^2-\frac{\partial^2}{\partial t^2}\right)\psi+a_1\phi_2=0\,, \quad (27)$$

$$\left[\left(\gamma\nabla^2-\frac{2k}{\omega^2}\right)a_4-a_6\frac{\partial^2}{\partial t^2}\right]\phi_2-a_5\nabla^2\psi=0\,, \qquad (28)$$

$$\nabla^2\phi+\left[a_8\left(\frac{\partial}{\partial t}+\frac{\tau_0^{\alpha'}}{(\alpha'+1)}\frac{\partial^{\alpha'+1}}{\partial t^{\alpha'+1}}\right)-a_7\nabla^2\right]T+a_9C=0\,, \qquad (29)$$

$$\nabla^4\phi+a_9\nabla^2T+\left[a_{12}\left(\frac{\partial}{\partial t}+\frac{\tau_0^{\alpha'}}{(\alpha'+1)}\frac{\partial^{\alpha'+1}}{\partial t^{\alpha'+1}}\right)-a_{11}\nabla^2\right]C=0\,. \qquad (30)$$

SOLUTION OF THE PROBLEM

We define Laplace- and Fourier transform respectively as

$$\bar{f}(s,x_1,x_3)=\int_0^\infty f(t,x_1x_3)e^{-st}dt\,, \qquad (31)$$

$$\hat{f}(x_3,\xi,s)=\int_{-\infty}^{\infty}\bar{f}(s,x_1x_3)e^{\iota\xi x_1}dx_1\,. \qquad (32)$$

Applying Laplace transform defined by Eq.(31) on Eqs. (26)-(30) and then applying Fourier transforms defined by Eq.(32) on the resulting quantities, we obtain

$$\left((D^2-\xi^2)-\frac{1}{C_0^2}s^2\right)\hat{\phi}-a_2\hat{T}-a_2\hat{C}=0\,. \qquad (33)$$

$$\left(a_3(D^2-\xi^2)-s^2\right)\hat{\psi}+a_1\hat{\phi}_2=0\,, \qquad (34)$$

$$\left[\left(\gamma(D^2-\xi^2)-\frac{2k}{\omega^2}\right)a_4-a_6s^2\right]\hat{\phi}_2-a_5(D^2-\xi^2)\hat{\psi}=0\,, \qquad (35)$$

$$(D^2-\xi^2)\hat{\phi}+\left[a_8\left(s+\frac{\tau_0^{\alpha'}}{(\alpha'+1)}s^{\alpha'+1}\right)-a_7(D^2-\xi^2)\right]\hat{T}+a_9\hat{C}=0\,, \qquad (36)$$

$$(D^2-\xi^2)^4\hat{\phi}+a_9(D^2-\xi^2)\hat{T}+\left[a_{12}\left(s+\frac{\tau_0^{\alpha'}}{(\alpha'+1)}s^{\alpha'+1}\right)-\right.$$
$$\left.-a_{11}(D^2-\xi^2)\right]\hat{C}=0\,. \qquad (37)$$

On rearranging the relations Eqs.(33)-(37), we have the following set of equations

$$(D^2-a_{13})\hat{\phi}-a_2\hat{T}-a_2\hat{C}=0\,, \qquad (38)$$

$$(D^2-\xi^2)\hat{\phi}-a_7D^2\hat{T}+a_9\hat{C}=0\,, \qquad (39)$$

$$(D^4-2\xi^2D^2+\xi^4)\hat{\phi}+a_9D^2\hat{T}+[-a_{11}D^2+a_{16}]\hat{C}=0\,, \qquad (40)$$

$$a_1\hat{\phi}_2+(a_3D^2-a_{17})\hat{\psi}=0\,, \qquad (41)$$

$$(a_{18}D^2-a_{19})\hat{\phi}_2+(-a_5D^2+a_{20})\hat{\psi}=0\,, \qquad (42)$$

where:

$$a_{13}=\xi^2+\frac{s^2}{C_0^2}\,;\; a_{14}=a_8\left(s+\frac{\tau_0^{\alpha'}}{(\alpha'+1)}s^{\alpha'+1}\right)+a_7\xi^2\,;\; a_{15}=a_9\xi^2\,;$$

$$a_{16}=a_{12}\left(s+\frac{\tau_0^{\alpha'}}{(\alpha'+1)}s^{\alpha'+1}\right)+a_{11}\xi^2\,;\; a_{17}=a_3\xi^2+s^2\,;\; a_{18}=a_4\gamma\,;$$

$$a_{19}=\left(\gamma\xi^2+\frac{2k}{\omega^2}\right)a_4+a_6s^2\,;\; a_{20}=a_5\xi^2\,.$$

Eliminating $\hat{C}$ and $\hat{T}$, $\hat{\phi}$ and $\hat{T}$, and $\hat{\phi}$ and $\hat{C}$ in respect from the resulting Eqs.(38)-(40), we obtain

$$[D^6+AD^4+BD^2+C]\hat{\phi}=f_1e^{-\gamma^*x_3}\,, \qquad (43)$$

$$[D^6+AD^4+BD^2+C]\hat{T}=f_2e^{-\gamma^*x_3}\,, \qquad (44)$$

$$[D^6+AD^4+BD^2+C]\hat{C}=f_3e^{-\gamma^*x_3}\,. \qquad (45)$$

Eliminating $\hat{\phi}_2$ respectively from resulting Eqs.(41)-(42), we obtain

$$[D^4+ED^2+F]\hat{\psi}=0\,, \qquad (46)$$

where: $f_2=Q_1(a_{39}\gamma^{*4}-a_{40}\gamma^{*2}+a_{41})/a_{39}$; $f_3=Q_1(a_{42}\gamma^{*4}+a_{43}\gamma^{*2}+a_{44})/a_{39}$; $f_4=(\gamma^{*6}+A\gamma^{*4}+B\gamma^{*2}+C)$; $k_1=a_7a_{11}-a_2a_7$; $k_2=a_7a_{16}-a_{11}a_{14}-a_9a_{10}-a_7a_{11}a_{13}-a_2a_{11}-2a_2a_9+2a_2a_7+a_2a_{14}$; $k_3=a_{14}a_{16}+a_9a_{15}+a_7a_{13}a_{16}+a_{11}a_{13}a_{14}+a_9a_9a_{13}+a_2a_{16}+a_2a_{11}\xi^2+2a_2a_9-2a_2a_7\xi^4+a_2a_{15}+a_2a_9\xi^2-2a_2a_{14}\xi^2$; $k_4=-a_{13}a_{14}a_{16}-a_9a_{13}a_{15}-a_2a_{16}\xi^2-a_2a_9\xi^4-a_2a_{15}\xi^2+a_2a_{14}\xi^4$; $k_5=-a_3a_{18}$; $k_6=-a_1a_5+a_{17}a_{18}+a_3a_{19}$; $k_7=-a_{17}a_{19}$; $A=k_2/k_1$; $B=k_3/k_1$; $C=k_4/k_1$; $E=k_6/k_5$; $F=k_7/k_5$.

Solutions of Eqs.(43)-(46) satisfying the radiation conditions that ($\hat{\phi}$, ϕ^*, $\hat{T}$, $\hat{\phi}_2$, $\hat{\psi}$) $\to 0$ as $x_3\to\infty$ are given by

$$\hat{\phi}=B_1e^{-m_1x_3}+B_2e^{-m_2x_3}+B_3e^{-m_3x_3}+L_1e^{-\gamma^*x_3}\,, \qquad (47)$$

$$\hat{T}=d_1B_1e^{-m_1x_3}+d_2B_2e^{-m_2x_3}+d_3B_3e^{-m_3x_3}+L_2e^{-\gamma^*x_3}\,, \qquad (48)$$

$$\hat{C}=e_1B_1e^{-m_1x_3}+e_2B_2e^{-m_2x_3}+e_3B_3e^{-m_3x_3}+L_3e^{-\gamma^*x_3}\,, \qquad (49)$$

$$\hat{\psi}=B_4e^{-m_4x_3}+B_5e^{-m_5x_3}\,, \qquad (50)$$

$$\hat{\phi}_2=h_4B_4e^{-m_4x_3}+h_5B_5e^{-m_5x_3}\,, \qquad (51)$$

where: $d_i=\dfrac{a_{39}m_i^4-a_{40}m_i^2+a_{41}}{a_{37}m_i^2+a_{38}}$, $e_i=\dfrac{a_{42}m_i^4+a_{48}m_i^2+a_{44}}{a_{37}m_i^2+a_{38}}$,

$L_i=\dfrac{f_i}{m_i^6+Am_i^4+Bm_i^2+C}$, $i=1, 2, 3$; $h_i=\dfrac{a_2(m_i^2-\xi_1)}{a_3}$, $i=5, 6$;

and m_i^2 ($i=1, 2, 3$) are the roots of the characteristic equation of Eq.(27), and m_i^2 ($i=4, 5$) are the roots of the characteristic equation of Eq.(30).

BOUNDARY CONDITIONS

We consider concentrated normal force and concentrated thermal source at the boundary surface $x_3=0$, mathematically, these can be written as

$$t_{33}=-F_1\psi_1(x_1)\delta(t)\,,\; t_{31}=0\,,\; m_{32}=0\,,\; T=F_2\psi_1(x_1)\delta(t)\,,$$
$$C=F_3\psi_1(x_1)\delta(t)\,, \qquad (52)$$

where: F_1 is the magnitude of the applied force; and F_2 is the constant temperature applied on the boundary. Also, substituting the values of $\hat{\phi}$, ϕ^*, $\hat{T}$, $\hat{\psi}$, $\hat{\phi}_2$ from Eqs.(47)-(51) in the boundary condition Eq.(52), and using Eqs.(5)-(11), (13)-(14), (31)-(32) and solving the resulting equations, we obtain:

$$\hat{t}_{32}=\sum_{i=1}^5G_{1i}e^{-m_ix_3}-M_1e^{-\gamma^*x_3}\,,\; i=1,2,\ldots,5\,, \qquad (53)$$

$$\hat{t}_{31}=\sum_{i=1}^5G_{2i}e^{-m_ix_3}-M_2e^{-\gamma^*x_3}\,,\; i=1,2,\ldots,5\,, \qquad (54)$$

$$\hat{m}_{32}=\sum_{i=1}^5G_{3i}e^{-m_ix_3}-M_3e^{-\gamma^*x_3}\,,\; i=1,2,\ldots,5\,, \qquad (55)$$

$$\hat{T}=\sum_{i=1}^5G_{4i}e^{-m_ix_3}-M_4e^{-\gamma^*x_3}\,,\; i=1,2,\ldots,5\,, \qquad (56)$$

$$\hat{C}=\sum_{i=1}^5G_{5i}e^{-m_ix_3}-M_5e^{-\gamma^*x_3}\,,\; i=1,2,\ldots,5\,, \qquad (57)$$

where: $G_{mi}=g_{mi}C_i$, $C_i=\Delta_i/\Delta_0$, $i=1,2,\ldots,5$; $g_{1i}=(m_i^2-b_2\xi^2)-(1+\tau_1s)\alpha_{1i}-b_{11}\alpha_{2i}(1+\tau^1s)$, $g_{2i}=(b_5+b_6)\iota\xi$, $g_{3i}=0$, $g_{4i}=\alpha_i$, $g_{5i}=m_i\beta_i$, $i=1,2,3$; $g_{1l}=-\iota\xi b_3m_l$, $g_{2l}=(b_6m_l^2+b_5\xi^2)-b_7\alpha_{3l}$, $g_{3l}=-b_8m_l\alpha_{3l}$, $g_{4l}=0$, $g_{5l}=0$, $l=4,5$;

$$\Delta_0 = \begin{vmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35} \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{45} \\ g_{51} & g_{51} & g_{51} & g_{51} & g_{51} \end{vmatrix}$$

, Δ_1, Δ_2, Δ_3, Δ_4, and Δ_5 are obtained by replacing the 1st, 2nd, 3rd, 4th, 5th columns by $\left[(M_1+F_1\hat{\psi}_1(s)), M_2, M_3, (M_4-F_1\hat{\psi}_1(s)), M_5\right]$ in Δ_0, and

$$M_1 = -\frac{(\gamma^{*2}-b_2\xi^2)f_1-(1+\tau_1 s)f_2-b_{11}(1+\tau^1 s)f_3}{f_4},$$

$$M_2 = -\frac{(b_5+b_6)\iota\xi\gamma^* f_1}{f_4},\quad M_3=0,\quad M_4=-\frac{\gamma^* f_2}{f_4},\quad M_5=-\frac{\gamma^* f_3}{f_4}.$$

Case 1: for the thermal source: $F_1 = 0$.
Case 2: for the normal source: $F_2 = 0$.

APPLICATIONS

(a) Uniformly distributed source

The solution due to uniformly distributed force applied on the half-space is obtained by setting

$$\psi_1(x_1) = \begin{cases} 1, & |x_1| \le d \\ 0, & |x_1| > d \end{cases} \tag{58}$$

Applying Laplace and Fourier transforms on Eq.(58), gives

$$\hat{\psi}_1(\xi) = \frac{2\sin(\xi d)}{\xi},\quad \xi \ne 0. \tag{59}$$

(b) Linearly distributed source

The solution due to linearly distributed force over a strip of non-dimensional width $2d$, applied on the half-space is obtained by setting

$$\psi_1(x_1) = \begin{cases} 1-\dfrac{|x_1|}{d}, & |x_1| \le d \\ 0, & |x_1| > d \end{cases} \tag{60}$$

Applying Laplace and Fourier transforms on (60), gives

$$\hat{\psi}_1(\xi) = \frac{2(1-\cos(\xi d))}{\xi^2 d},\quad \xi \ne 0. \tag{61}$$

SPECIAL CASE

Micropolar thermoelastic solid: in absence of mass diffusion effect in Eqs.(53)-(57), we obtain the corresponding expressions of stresses, displacements, and temperature for micropolar generalised thermoelastic half space.

INVERSION OF THE TRANSFORMS

The transformed components of displacements, stresses, temperature, chemical potential, and concentration deviation are functions of z and the parameters of Laplace and Fourier transforms s and ξ respectively, and hence are of the form $f(s, \xi, z)$. We invert the Laplace and Fourier transform by using the methodology of Rakshit and Mukhopadhyay /44/ to find the solution of the problem in physical domain.

NUMERICAL RESULTS AND DISCUSSIONS

The analysis is conducted for a magnesium crystal-like material. For numerical computations, following Eringen /42/, values of physical constants are: $\lambda = 9.4\times10^{10}$ Nm^{-2}; $\mu = 4.0\times10^{10}$ Nm^{-2}; $K = 1.0\times10^{16}$ Nm^{-2}; $\rho = 1.74\times10^3$ kgm^{-3}; $j = 0.2\times10^{-19}$ m^2; $\gamma = 0.779\times10^{-9}$ N.

Following Dhaliwal and Singh /43/, thermal and diffusion parameters are given by: $c^* = 1.04\times10^3$ Jkg^{-1}K^{-1}; $K^* = 1.7\times10^6$ Jm^{-1}s^{-1}K^{-1}; $\alpha_{t1} = 2.33\times10^{-5}$ K^{-1}; $\alpha_{c1} = 2.48\times10^{10}$ K^{-1}; $T_0 = 298$ K; $\tau_0 = 0.02$; $\tau_1 = 0.01$; $\alpha_{c1} = 2.65\times10^{-4}$ m^{3}kg^{-1}; $a = 2.9\times10^4$ m^{2}s^{-2}K^{-1}; $b = 32\times10^5$ kg^{-1}m^{5}s^{-2}; $\tau_1 = 0.04$; $\tau_0 = 0.03$; $D = 0.85\times10^{-8}$ kgm^{-3}s.

Effect of laser pulse on thermal stresses: a comparison of the dimensionless form of the field variables for the cases of fractional micropolar mass diffusion thermoelastic medium with laser heat source (FMPMDLSR) and fractional micropolar mass diffusion thermoelastic medium (FMPMD) for two different values of laser parameter I, i.e., $I = 10^5$ and $I = 0$, subjected to mechanical forces are shown in Figs. 1-5.

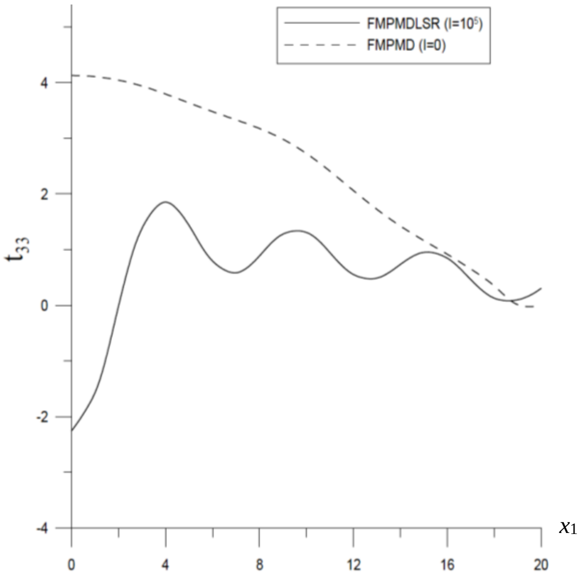

Figure 1. Variation of normal stress vs. distance.

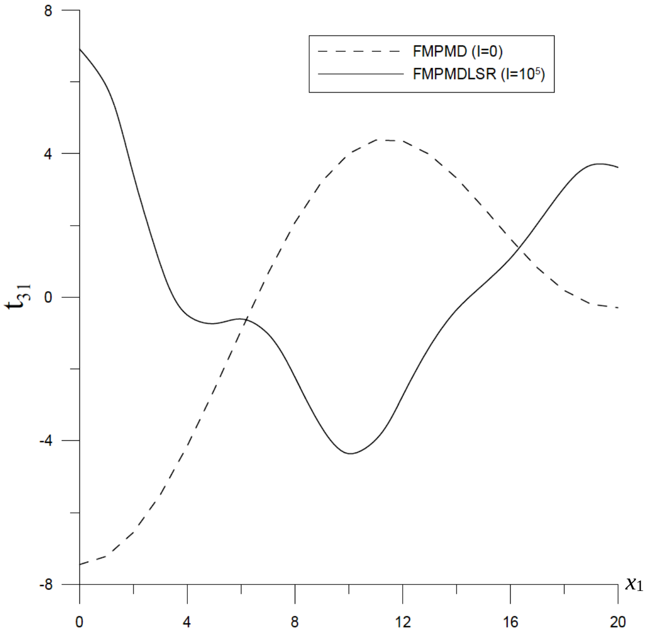

Figure 2. Variation of tangential stress vs. distance.

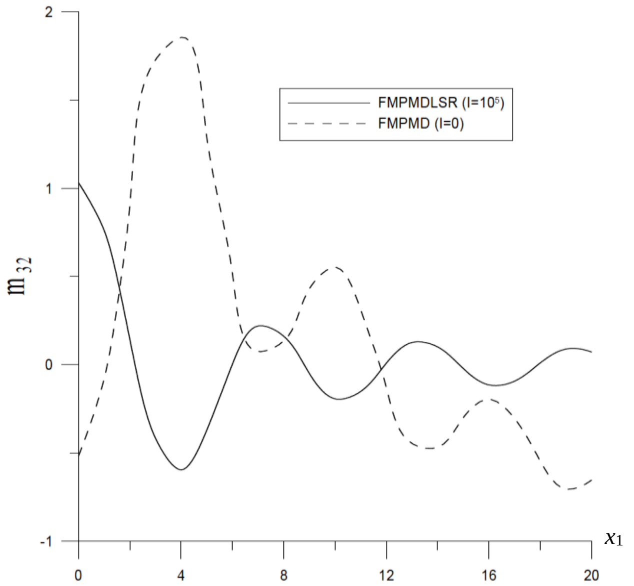

Figure 3. Variation of couple tangential stress vs. x_1.

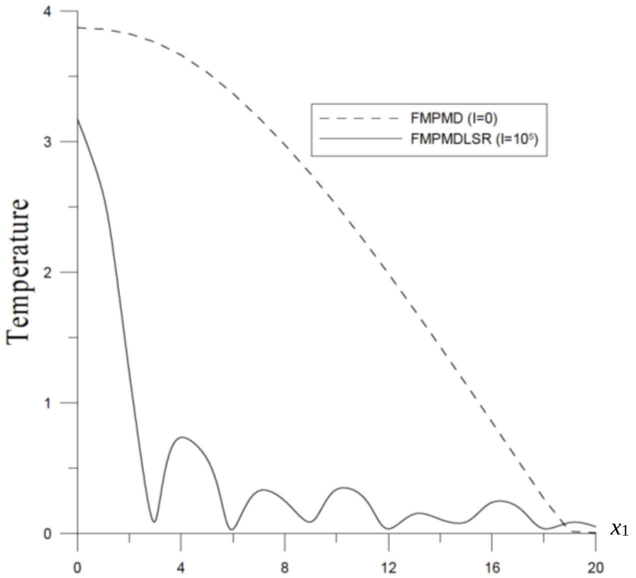

Figure 4. Variation of temperature vs. x_1.

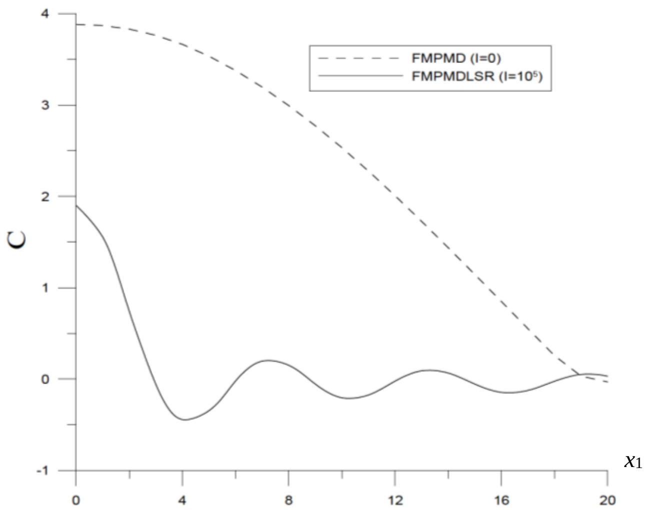

Figure 5. Variation of mass diffusion vs. x_1.

The values of all physical quantities for all cases are shown in the range $0 \leq x_1 \leq 20$.

Solid and dashed lines correspond to fractional micropolar thermoelastic mass diffusion medium including input laser heating (FMPMDLSR) and fractional micropolar thermoelastic mass diffusion medium (FMPMD) without laser, in respect.

Effect of fractional parameter on various physical quantities: a comparison of the dimensionless form of field variables for the cases of fractional micropolar mass diffusion thermoelastic medium with laser heat source (FMPMDL SR) for two different values of fractional parameter α', i.e., $\alpha' = 1$ and $\alpha' = 0.5$, subjected to mechanical forces are shown in Figs. 6-10. The values of all physical quantities for all cases are shown in the range $0 \leq x_1 \leq 20$.

Solid and dashed lines correspond to fractional micropolar thermoelastic mass diffusion medium including input laser heating (FMPMDL $\alpha' = 1$) and fractional micropolar thermoelastic mass diffusion medium (FMPMDL $\alpha' = 0.5$) with laser, respectively.

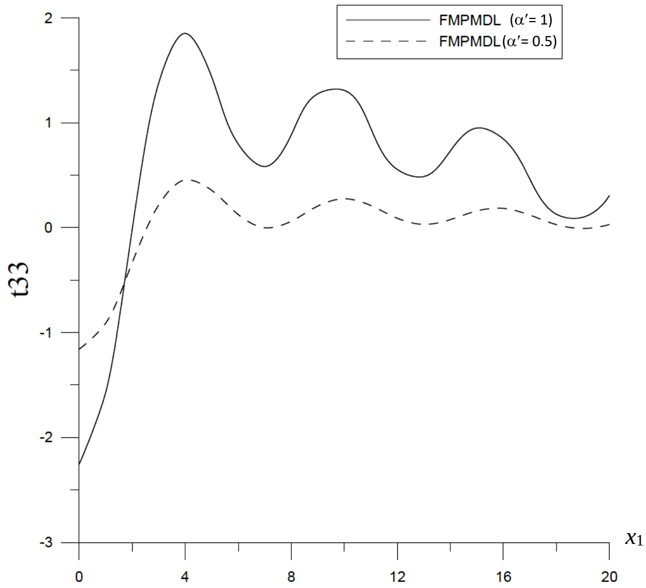

Figure 6. Variation of normal stress vs. distance.

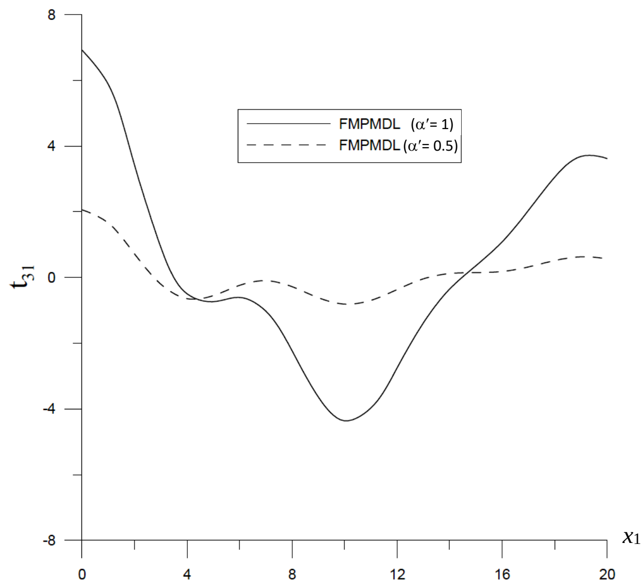

Figure 7. Variation of tangential stress vs. distance.

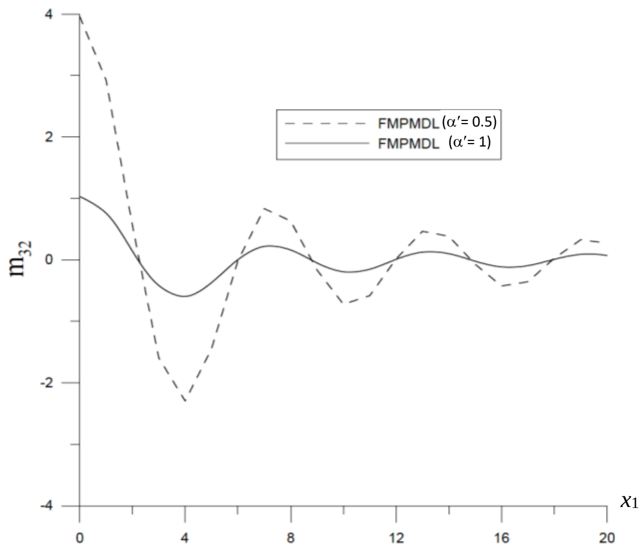

Figure 8. Variation of couple tangential stress vs. x_1.

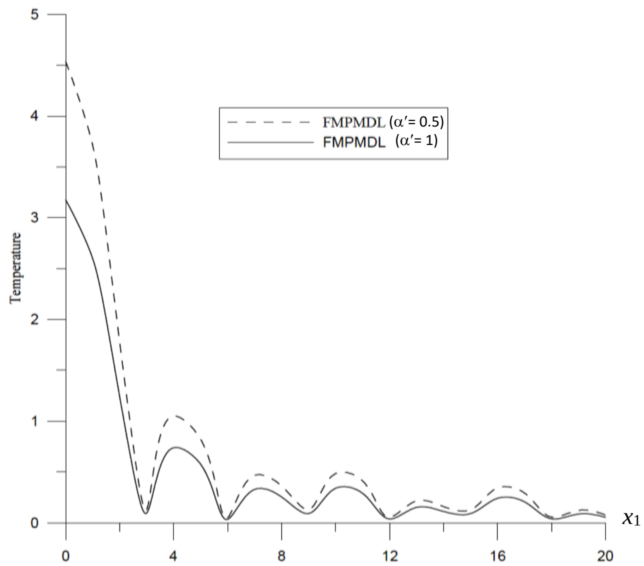

Figure 9. Variation of temperature vs. x_1.

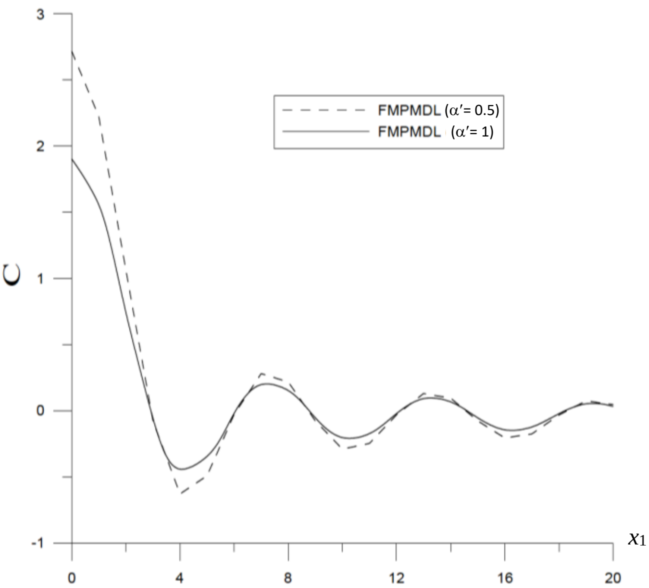

Figure 10. Variation of mass diffusion vs. x_1.

CONCLUSIONS

The problem of laser irradiation on micropolar thermoelastic mass diffusion medium is a significant problem in continuum mechanics. It is observed that the physical quantities are also affected by the different non-classical theories of thermoelasticity with mass diffusion. It is observed from Figs. 1-10 that laser and fractional parameters have effect on stress components, temperature change, and mass concentration depending on the distance x_1.

The present problem has a significant application in geophysics and electronics engineering. The effect of diffusion is used to improve the conditions of oil extractions. Nowadays, there is a great deal of interest in the study of this phenomenon due to its application in geophysics and the electronic industry.

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