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INFLUENCE OF ELLIPTICAL OPENING POSITION ON THE DISTRIBUTION OF MINIMUM PRINCIPAL NORMAL STRESS

UTICAJ POLOŽAJA ELIPTIČNOG OTVORA NA RASPODELU MINIMALNOG GLAVNOG NORMALNOG NAPONA

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- minimum principal normal stress
- rectangular plate
- ANSYS© software

Ključne reči
- eliptični otvor
- minimalni glavni normalni napon
- pravougaona ploča
- računarski program ANSYS©

Abstract

Proper functioning of mechanical systems can only be achieved if machine parts are properly dimensioned. Since dimensioning of machine parts is impossible without previous stress state analysis, the aim of this paper is to obtain the distribution of minimum principal normal stress in the case of uniaxial tension of rectangular steel plates with an elliptical opening. For purposes of research, results showing minimum principal normal stress distribution are obtained using analytical and numerical methods, wherein the latter are performed using the finite element software package ANSYS©.

Izvod

Ispravno funkcionisanje mašinskih sistema moguće je jedino ako su mašinski delovi pravilno dimenzionisani. Kako dimenzionisanje mašinskih delova nije moguće obaviti bez prethodne analize naponskih stanja u delovima, cilj ovog rada je da se dođe do rezultata o uticaju položaja eliptičnog otvora u jednoosno zategnutim čeličnim pravougaonim pločama na raspodelu minimalnog glavnog normalnog napona. U radu su za dobijanje rezultata o raspodeli minimalnog glavnog normalnog napona korišćene analitičke i numeričke metode i računarski program ANSYS©.

INTRODUCTION

In engineering practice, parts with various discontinuities are commonly encountered. Openings and holes are one of the most frequent geometric discontinuities in machine parts and structures. One type of opening geometry is the elliptical shape. Although such openings are not too common in machine elements, investigating its effects can be of great use taking into account that some of analytical equations used for elliptical geometry can also be applied to circular opening, since it represents a special case of an elliptical opening, obtained by equating the length of elliptical axes.

Machine parts are subjected to a wide range of different loads during exploitation, hence engineering practice requires detailed insight into the effects of all these load cases on stress and strain distribution when designing mechanical structures and their elements. For the purpose of this paper, uniaxial tensile load effect on the distribution of minimum principal normal stress, $\sigma_{\min}$ is analysed.

Modern technological developments result in a wide variety of materials which can be used for the manufacture of mechanical structures. A significant position in this area is occupied by steels due to their isotropic nature. For this reason, the material used in obtaining the results for minimum principal normal stress $\sigma_{\min}$ is steel, as one of the most common materials.

Initial engineering tests during the design of structures rely exclusively on analytical methods, whereas nowadays numerical and experimental approaches are used. Mushelishvili /1/ used analytical methods in order to solve some basic problems related to mathematical elasticity theory, whereas Savin /2/ used such methods to determine the stress distribution around different shaped openings, in both isotropic and anisotropic fields. Kojić et al. /3/ covered the use of linear analysis in finite element methods (FEM), and Nikolić /4/ presented the mechanical analysis of toothed gears via FEM. The theoretical basis of FEM for solving linear static problems related to deformable body mechanics, including various finite element type formulations most commonly used in structural calculation of mechanical parts are given by Sorić /5/, whereas Josifović /6/ dealt with some of the experimental methods for testing mechanical structures. The magister thesis of Radojković /7/ provides a large number of examples with stress distribution in rectangular isotropic plates weakened with various opening geometries, subjected to tensile uni- and bi-axial loads. Ting et al. /8/ applied a mixed method in order to analyse changes in elements with multiple rhomboid-distributed openings. Strain concentration in thin composites with openings was the topic of research conducted by Pandit et al, /9/, wherein a mapping technique indicated that the highest stress concentration was around openings. Paper /10/ by Icardi involves experimental measuring of displacements in laminate composites using the electronic speckle photography method. Sanyal and Yadav /11/ suggest adding more openings to relieve stress concentration in axially loaded plates with individual circular openings. The influence of opening size and distance between them on stress concentration is examined using FEM in ANSYS software. Radojković et al. /12/ investigated the distribution of maximum principal normal stress and the effect of fillet radius in square openings of isotropic rectangular plates on stress distribution. Inverse methods are confirmed as a powerful tool for identification of properties of elastic-plastic metal materials by the work of Cooreman et al. /13/. The essence of inverse methods consists of comparing experimentally measured strain fields with the results obtained by using FEM. Use of parameterised geometric models by Wu /14/ determined the optimal shapes of openings which would decrease stress concentration. Bižić and Petrović /15/ identified the effect of circular opening on the stress state of a homogeneous isotropic plate subjected to uniaxial loading. Gunwant and Singh /16/ investigated stresses and displacements in a rectangular plate with a central elliptical opening. By using characteristic orthogonal polynomials, Okafor and Udeh /17/ provided a direct method for analysis of rectangular plates, whereas Srikanth and Kumar /18/ dealt with structural analysis of isotropic and orthotropic beams and plates using first order shear strain theory. Numerical analysis of thin plates is covered by Gokul et al. /19/, whose work involves the analysis of size and shape of an opening on stress distribution, using a well-known and previously mentioned ANSYS software. Jafari et al. /20/ determined optimal parameters for a plate with a square opening via optimisation algorithm. Gunwant /21/ also analysed stress concentration in flat plates with rectangular cutouts using FEM, and Konieczny et al. /22/ used the same approach to determine the stress distribution in plates with a diagonally distributed opening subjected to biaxial planar loads. By applying analytical methods, FEM, and artificial neural network techniques, Ozkan and Erdemir /23/ determined the theoretical stress concentration factor for reinforced circular and elliptical openings. Radojković et al. /24/ investigated the stress distribution in a biaxially loaded isotropic rectangular plate with rectangular opening. Stress concentration analysis in the vicinity of opening subjected to dynamic load was performed by Maksymovych et al. /25/, whereas Sieberer et al. /26/ investigated optical stress concentration while monitoring the stress gradient during testing which involved elastic-plastic fatigue via digital image correlation. The presence of openings is almost inevitable today with parts made of aluminium and its alloys /27/. Based on the above, it can be concluded that ANSYS© software is a reliable tool to obtain these and other similar results, /28/.

ANALYTICAL RESULTS

The following section briefly presents the procedure for determining analytical results for principal normal stress distribution in an isotropic field with an elliptical opening (with a half-axis ratio of b/a = 2/3 and b/a = 3/2), wherein a is the half-axis on the x axis, and b is the half-axis on the y axis. The whole procedure can be found in /2, 7/. The complex variable method is used to obtain analytical results.

According to /2, 7/, the function used to perform conform mapping γ of the unit circle interior onto the elliptical opening exterior with half-axes a and b has the following form:

$$z=\omega(\zeta)=R\left(\frac{1}{\zeta}+m\zeta\right), \quad (1)$$

where: ζ is independent variable; $R = (a + b)/2$; $m = (a - b)/(a + b) = (1 - k)/(1 + k)$; $k = a/b$.

Final forms of expressions for stress functions $\varphi(\zeta)$ and $\psi(\zeta)$ are:

$$\varphi(\zeta)=\frac{pR}{4}\left[\frac{1}{\zeta}+\left(2e^{2i\alpha}-m\right)\xi\right],$$

$$\psi(\zeta)=\frac{pR}{4}\left[\frac{e^{-2i\alpha}}{\zeta}+\frac{\zeta^3e^{2i\alpha}+\left(me^{2i\alpha}-m^2-1\right)\zeta}{m\zeta^2-1}\right], \quad (2)$$

whereas the expression used for calculating stress at contour points of an elliptical opening:

$$\sigma_\theta=p\frac{(1+k)^2\sin^2(\theta+\alpha)-\sin^2\alpha-k^2\cos^2\alpha}{\sin^2\theta+k^2\cos^2\theta}, \quad (3)$$

wherein: α is angle between directions of external forces and x axis; θ is angle between a normal through an opening contour point and x axis and is measured while following the contour in the positive mathematical direction.

Stress magnitudes σ_θ at the elliptic opening contour with half-axis ratio of $k = b/a$ = 2/3, under uniaxial load, can be calculated according to /2, 7/. These values determined by analysis presented in this paper are given in Table 1, whereas Fig. 1 shows maximum and minimum principal normal stresses, σ_{max} (upper half), and σ_{min} (lower half), in respect.

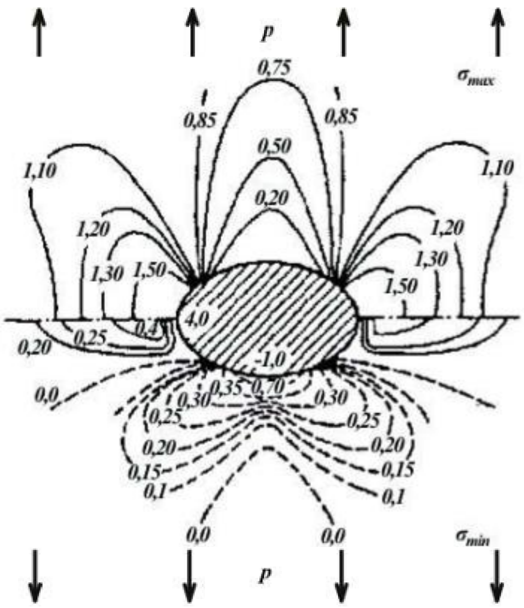

Figure 1. Maximum (σ_{max}, upper half) and minimum (σ_{min}, lower half) principal normal stresses under uniaxial tensile load acting on an isotropic field with elliptical opening of ratio $k = b/a$ = 2/3.

In addition, the procedure shown in /2, 7/ can be used to determine stress values at the opening contour for a case of uniaxial tension acting on an isotropic field with $k = b/a$ =

3/2. Stress values for this case are given in Table 1, whereas Fig. 2 shows maximum (upper half) and minimum (lower half) principal normal stress lines.

Table 1. Stress values σ_θ at elliptical opening contour.

σ_θ (N/m^2)					
θ°	b/a = 2/3	b/a = 3/2	θ°	b/a = 2/3	b/a = 3/2
0	-1.00	-1.00	50	0.93	1.54
10	-0.93	-0.78	60	1.86	1.90
20	-0.72	-0.23	70	2.85	2.15
30	-0.35	0.43	80	3.67	2.29
40	0.19	1.04	90	4.00	2.33

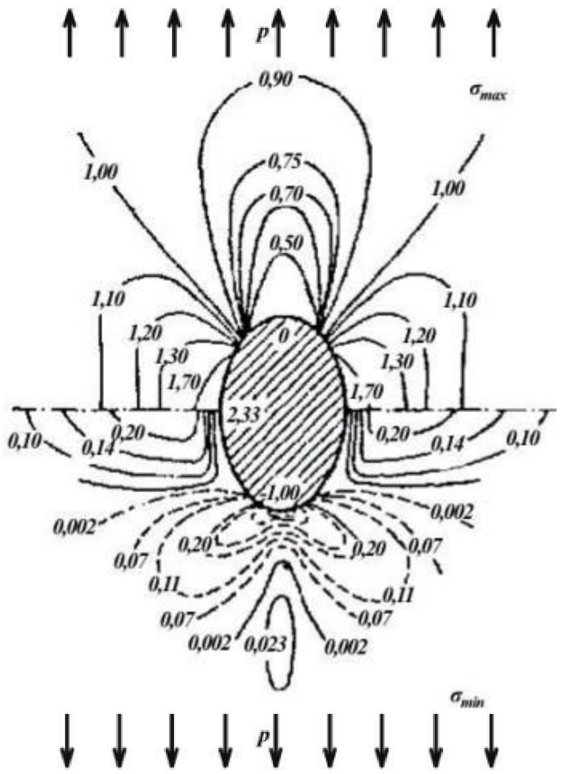

Figure 2. Maximum (σ_{max}, upper half) and minimum (σ_{min}, lower half) principal normal stresses under uniaxial tensile load acting on an isotropic field with elliptical opening of ratio $k = b/a$ = 3/2.

NUMERICAL RESULTS

The FEM is used to obtain numerical results, and all calculations are performed in ANSYS software. Results in question are related to the distribution of minimum principal normal stress σ_{min} in the case of plate elements subjected to uniaxial tensile load, wherein the dimensions of plates in all examples are 2×5×0.1 m. The plate is positioned in such a way that its shorter side is parallel to x axis in both cases considered. As for the first case, the rectangular plate was weakened by a central elliptical opening, with half-axis lengths of a = 100 mm and b = 66.67 mm (b/a = 2/3) and opening positioned so that its longer half-axis coincides with the x axis, and the tensile load is transferred via surface forces with an intensity of p = 1 N/m^2 along the y axis. Plates were made of steel of elasticity module E = 2.1×10^{10} N/m^2, and Poisson's ratio of μ = 0.33. Model discretisation in both cases is done using 2D tetrahedral solid finite elements with six nodes.

Figure 3 shows a detail from the finite element mesh in the vicinity of an elliptical opening made in ANSYS, and Fig. 4 shows the distribution of minimum principal normal stress σ_{min} in a steel plate with central elliptical opening (a = 100 mm, b = 66.7 mm), subjected to uniaxial tensile loading. The longer half-axis coincides with the x-axis and tensile load is applied via forces with an intensity of p = 1 N/m^2 along y-axis. It can be seen from Fig. 4 that the highest stresses are obtained at the intersection of the opening contour and shorter half-axis (y axis), and in this case the value is σ_{min} = -0.996995 N/m^2.

Figure 5 shows a detail from the finite element mesh in the vicinity of an elliptical opening made in ANSYS, and Fig. 6 shows the distribution of minimum principal normal stress σ_{min} in a steel plate with a central elliptical opening (a = 100 mm, b = 150 mm), subjected to uniaxial tensile loading. The longer half-axis coincides with the x-axis and tensile load is applied via forces with an intensity of p = 1 N/m^2 along the y-axis. It can be seen from Fig. 6 that highest stress values are obtained at the intersection of the opening contour and shorter half-axis (y axis), and in this case the value is σ_{min} = -0.981429 N/m^2.

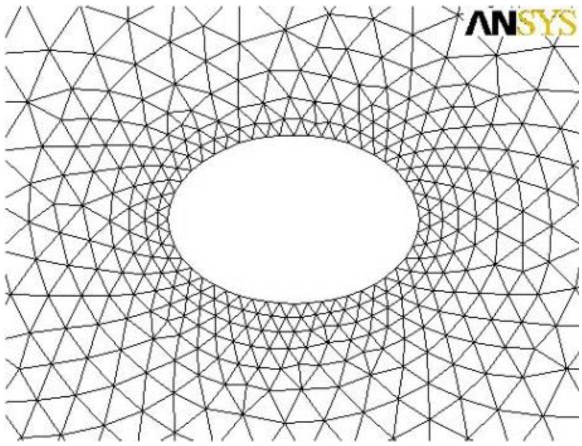

Figure 3. A detailed view of the finite element mesh of a plate with elliptical opening subjected to tension along the b half-axis, with half-axes lengths a = 100 mm and b = 66.67 mm.

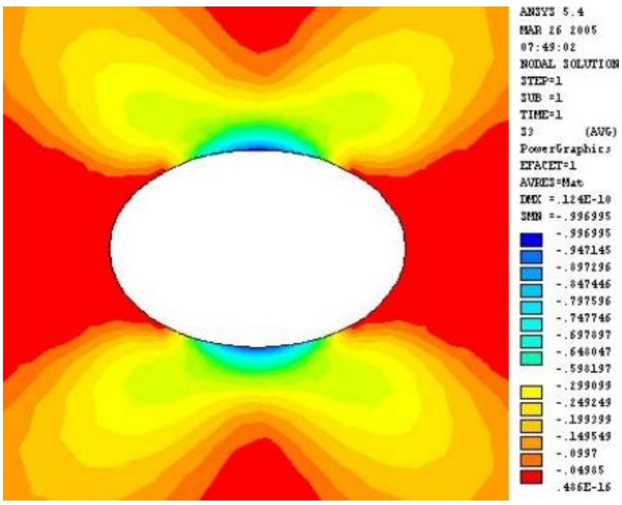

Figure 4. Distribution of σ_{min} in a plate with an elliptical opening subjected to tension along the b half-axis, with half-axes lengths a = 100 mm and b = 66.67 mm.

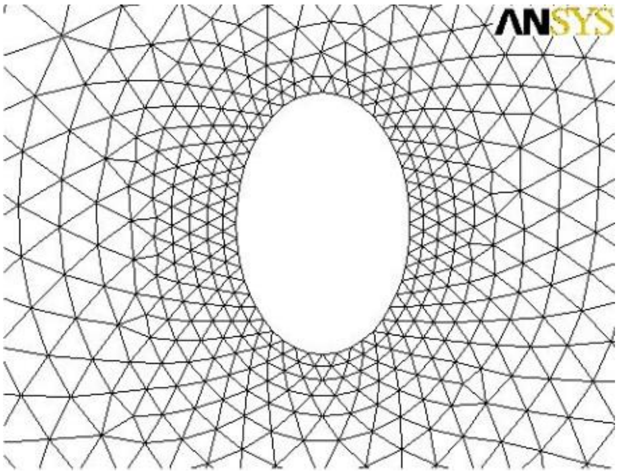

Figure 5. A detailed view of the finite element mesh of a plate with elliptical opening subjected to tension along the *b* half-axis, with half-axes lengths a = 100 mm and b = 150 mm.

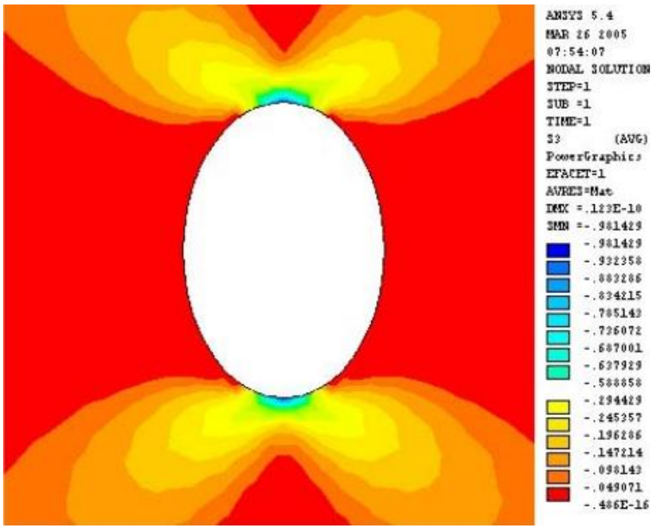

Figure 6. Distribution of σ_{min} in a plate with elliptical opening subjected to tension along the *b* half-axis, with half-axes lengths a = 100 mm and b = 150 mm.

MINIMUM PRINCIPAL NORMAL STRESS DISTRIBUTION

In order to obtain numerical results for the distribution of minimum principal normal stress, FEM was used and calculations are made in ANSYS software. The validity of results is checked by comparing them to analytically obtained results (via complex variable function methods), in accordance with the theory given in /2, 7/. Table 2 shows the values of minimum principal normal stress obtained in both ways (analytically and FEM) and are related to uniaxial tension in steel plates with elliptical openings with half-axes a = 100 mm and b = 66.67 mm (b/a = 2/3) in the first case, and a = 100 mm and b = 150 mm (b/a = 3/2) in the second case.

Table 2. Values of minimum principal normal stress σ_{min}.

Method	σ_{min} (N/mm^2)	
	b/a = 2/3	b/a = 3/2
Analytical	-1.000	-1.000
FEM	-0.997	-0.981

Based on results shown in Table 2 it can be seen that the analytical method provides the same results in both cases, hence it can be concluded that the position of the elliptical opening relative to the direction of acting tensile forces does not affect the minimum principal normal stress value. In addition, these results have shown that values obtained using FEM (ANSYS) are almost identical and very close to the analytical ones.

CONCLUSION

The aim of this research is to determine the distribution of minimum principal normal stress σ_{min} and to analyse the effect of opening position in rectangular plate elements subjected to uniaxial tension. It is known that a circular opening represents a special case of an elliptical opening, hence all of the output obtained analytically for uniaxial tension in the case of elliptical openings also applies to a rectangular plate with a circular opening. In other words, when half-axes lengths of the ellipse are equated in expressions used for this analysis (a = b), results for a circular opening are obtained. Results shown here are related to plates made of isotropic materials, and in addition to analytical methods, they are obtained using FEM software ANSYS.

This investigation has shown that rectangular steel plates with a central elliptical opening under uniaxial tension resulted in the same values of minimum principal normal stresses for both considered cases, which implies that the orientation of the opening relative to the acting forces does not affect the stress distribution. Comparison of analytical and numerical results proved excellent agreement, with a difference of 1.9 %.

Further research on this topic will be related to the distribution and concentration of stresses in machine parts which contain elliptical openings, considering different orientations, opening size and half-axis ratios.

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28. https://www.ansys.com

APPENDIX - Homage to Stojan Sedmak (1929-2014)

Extracts from his Magister Thesis, mentor Nikola Hajdin, University of Belgrade, Faculty of Sciences and Mathematics, 1966

Photo-elasticity was a standard method to solve stress concentration problems, but it was not so often applied to solve fracture mechanics problems back in sixties. On the contrary, it was quite difficult to apply this method to solve the Bowie's problem, Fig. A1, requiring long and tedious work, for more than a year. Actually, the first results (Figs. A2, A3) indicated SCF = 3, which was not correct for notches emanating from the circular opening. By repeating experiments to get more readable photos, Fig. A2, Prof. Stojan was capable to get more realistic results. Also, one should notice the extrapolation technique, to get more precise number of isochromes, as shown in Fig. A3.

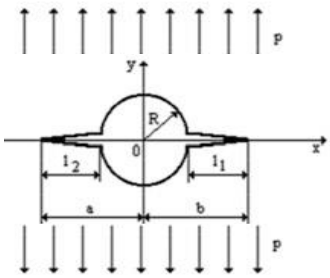

Figure A1. Bowie's problem.

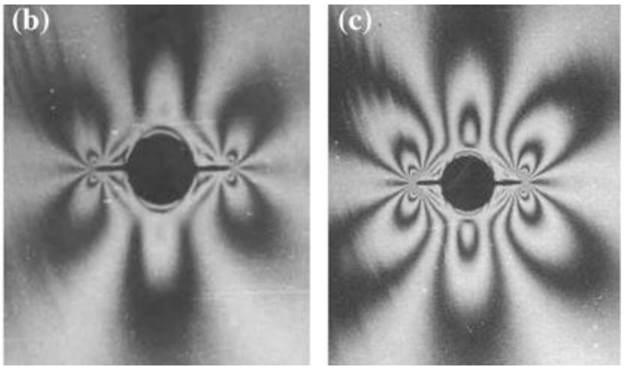

Figure A2. Isochromes - initial results.

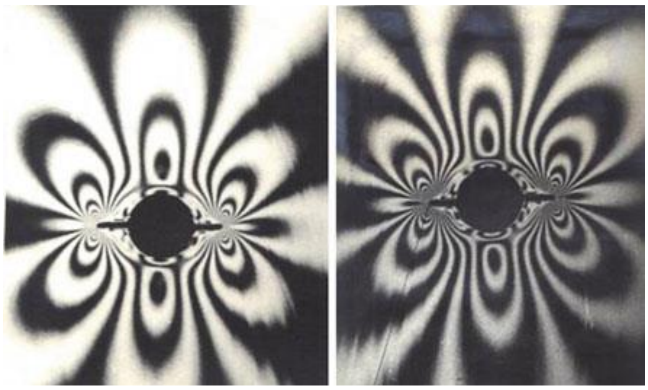

Figure A3. Isochromes - final results.