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THIN-WALLED OMEGA PROFILE EXPOSED TO CONSTRAINED TORSION - ANALYTICAL AND NUMERICAL STRESS AND STRAIN CALCULATION

TANKOZIDI OMEGA PROFIL PRI OGRANIČENOM UVIJANJU - ANALITIČKI I NUMERIČKI PRORAČUN NAPONA I DEFORMACIJA

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- finite element method
- equivalent stress
- deformation
- omega cross-section

Ključne reči
- tankozidi nosači
- metod konačnih elemenata
- ekvivalentni napon
- deformacija
- omega poprečni presek

Abstract

The paper presents analytical and numerical determination of the equivalent stress and strain of the open section thin-walled 'Ω' cantilever beams loaded with torsion. Analytical calculation was used to calculate the equivalent stress and strain values for the model encastred across the entire cross-section at one end, while the other end is free (model 1). The finite element method is applied for the calculation of the same 'Ω' cantilever beam model exposed to torsion, and the stress and strain values are obtained by numerical calculation. Numerical simulations have been performed using KOMIPS software. Analytical and numerical results are compared and discussed. In this paper, a numerical model for constrained torsion is also created in ABAQUS® and SolidWorks® software. A cantilever beam with an omega profile is encastred at one end, while at the other end there is a welded plate (model 2). Numerical models are created, and static calculation is done using the finite element method. The zones of stress concentration are identified and presented. Finally, the equivalent stress values of models 1 and 2 are compared. Conclusions obtained by examining this type of structures may be involved in the design process of new similar structures. The findings obtained during the implementation of this work can be directly applied to identify the behaviour of real structures in their working conditions.

Izvod

U radu je prikazano analitičko i numeričko određivanje ekvivalentnog napona i deformacije kod 'Ω' tankozidih konzola otvorenog poprečnog preseka opterećenih na uvijanje. Analitički proračun je korišćen za izračunavanje vrednosti ekvivalentnog napona i deformacija za model uklješten po celom poprečnom preseku na jednom kraju, dok je drugi kraj slobodan (model 1). Metoda konačnih elemenata je primenjena za proračun istog 'Ω' konzolnog modela koji je izložen uvijanju, i dobijene su vrednosti napona i deformacija numeričkim proračunom. Numerička simulacija je urađena korišćenjem softvera KOMIPS. Analitički i numerički rezultati su upoređeni i diskutovani. U ovom radu je takođe urađen numerički model pri ograničenom uvijanju u softverima ABAQUS® i SolidWorks®. Greda sa omega profilom je na jednom kraju uklještena dok se na drugom njenom kraju nalazi zavarena ploča (model 2). Napravljeni su računski modeli, a statički proračun je urađen metodom konačnih elemenata. Identifikovane su i prikazane zone koncentracije napona. Na kraju su upoređene vrednosti ekvivalentnih napona modela 1 i 2. Zaključci dobijeni ispitivanjem ove vrste konstrukcija mogu biti uključeni u proces projektovanja novih sličnih konstrukcija. Nalazi dobijeni tokom realizacije ovog rada mogu se direktno primeniti na identifikaciju ponašanja stvarnih objekata u uslovima njihovog rada.

INTRODUCTION

Thin-walled beams find a wide application in the construction and machinery industry as they enable obtaining any shape of the beam cross-section. Due to their low weight, thin-walled open section beams are widely applied in many structures. Many modern metal structures are manufactured using thin-walled elements (shells, plates, thin-walled beams) which are subjected to complex loads /1/. In most structures, such as automotive, railway vehicles, boats, and similar structures, they are installed in thin-walled elements. They are also often used as main structural elements, e.g., in making aircraft fuselages. Thin-walled elements can be different shapes, can have greater or lesser bending and torsional rigidity, but their common property is that they have a low weight compared to other possible structural shapes, /2-4/. The strength-to-weight ratio of such profiles has a key role for the material selection. The most common are metals (steel or aluminium), but for many years composite materials have been successfully used, such as GFRP or CFRP (glass or carbon fibre reinforced plastic), and currently the most popular laminates, /5/.

We have got the idea for the research in this work from paper /6/, where the analytical and numerical calculation of U and Z thin-walled cantilever beams loaded with torsion were done. Analytical calculation was made for the model which was encastred over the whole cross-section at one end, while the other end is free (model 1). The model is exposed to constrained torsion. Then, a numerical calculation was made for this cantilever beam model using the finite element method, and so the stress and strain values were obtained. Analytical and numerical results are compared and discussed. In this paper, a numerical model for constrained torsion is also done and stress and strain values are shown (model 2). This model 2 of the cantilever beam with an omega profile is encastred at one end, while at the other end there is a welded plate.

ANALYTICAL CALCULATION OF CONSTRAINED TORSION USING FEM - MODEL 1

First, the equivalent stress and strain are obtained analytically. The model 1 is encastred along the whole cross-section at one end, while the other end is free, during the action of constrained torsion. Properties of the cantilever beam material used in this paper are given in Table 1, /3/.

Table 1. Mechanical properties of steel S235JR.

Young's modulus (Pa)	Poisson's ratio	Yield stress (Pa)	Allowable stress (Pa)
2.1×10^{11}	0.3	235×10^{6}	160×10^{6}

The thin-walled cantilever beam has an open cross-section, the so-called Ω profile. Dimensions of the Ω profile are given in Fig. 1.

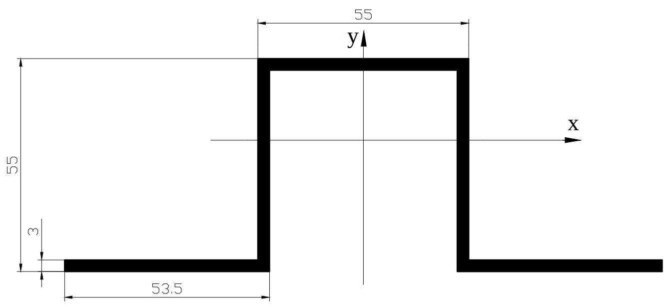

Figure 1. The Ω cross-section of the cantilever beam.

Cross-section area is calculated using Eq.(1), /7, 8/:

$$A=\sum_{i=1}^{3} b_i t_i \ . \tag{1}$$

Moments of inertia of the cross-sectional area about the centroidal axes x and y are given by expression, /8/:

$$I_x=\sum_{i=1}^{3} t_i \int y(s)y(s)ds \ , \tag{2}$$

$$I_y=\sum_{i=1}^{3} t_i \int x(s)x(s)ds \ . \tag{3}$$

The sectorial moment of inertia is given by, /8/:

$$I_\omega=\int_A \omega^2 dA=\sum_{i=1}^{3} t_i \int_S \omega(s)\omega(s)dS \ . \tag{4}$$

Torsional moment of inertia is given by expression /8/:

$$I_t=\frac{\eta}{3}\sum_{i=1}^{3} b_i t_i^3 \ , \tag{5}$$

where: η is coefficient of safety.

Torsional section modulus is given by:

$$W_t=\frac{I_t}{t_{\max}} \ . \tag{6}$$

The schematic representation, mechanical model of the cantilever beam exposed to the action of constrained torsion is given in Fig. 2.

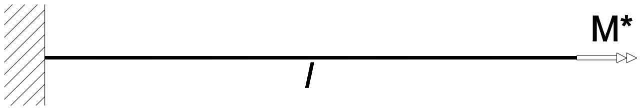

Figure 2. Constrained torsion of the cantilever beam.

The cantilever beam is loaded with a torsional moment according to the expression:

$$M^*=15700 \text{ Nmm} \ . \tag{7}$$

The reduced Young's modulus is given by expression:

$$\bar{E}=\frac{E}{1-\nu} \ . \tag{8}$$

The bending-torsional characteristic is given by, /6/:

$$k=\sqrt{\frac{GI_t}{\bar{E}I_\omega}} \ . \tag{9}$$

The bimoment and the maximum normal stress are given by Eqs.(10) and (11), respectively, /1, 2/:

$$B_{\max}=-\frac{M^*}{k}\tanh(kl) \ , \tag{10}$$

$$\sigma_{\max}=\frac{B_{\max}}{I_\omega}\omega_{\max} \ . \tag{11}$$

In case of loads by concentrated torsional moment on the free end of the cantilever beam, the moment of pure torsion on the free end is given by expression, /3/:

$$M_{t\max}=M^*\left(1-\frac{1}{\cosh(kl)}\right) . \tag{12}$$

The shear stress is given by:

$$\tau_{\max}=\frac{M_{t\max}}{W_t} \ . \tag{13}$$

In the case of a complex load (normal stress and shear stress are taken together in the calculation), the equivalent stress is defined, calculated by the Hencky-Mises hypothesis, /8/:

$$\sigma_e=\sqrt{\sigma_{\max}^2+3\tau_{\max}^2} \ . \tag{14}$$

Based on the previous equations and equations presented in literature /1, 2/, the geometrical characteristics of the cross-section of the given cantilever beam (Fig. 1) are obtained and presented in Table 2.

Table 2. Geometrical characteristics of the Ω cross-section.

Profile	Ω
A (cm^2)	7.8
I_x (cm^4)	36.68
I_y (cm^4)	116.3
W_x (cm^3)	11.22
W_y (cm^3)	14.91
I_t (cm^4)	0.398
W_t (cm^3)	1.326
I_ω (cm^6)	377.29

According to Eqs.(1)-(14) and the equations given in the literature /9, 10/, normal, tangential, equivalent stresses and deformations are obtained. The obtained values are shown in Table 3. The models are designed to have the same cross-sectional area and are loaded with the same intensity of the torsional moment. Cantilever beam lengths are l = 1000 mm.

Table 3. Stress and strain.

Profile	Ω
M* (Nmm)	15700
B_{max} (Nmm^2)	7825500
σ_{max} (MPa)	30.68
τ_{max} (MPa)	8.44
σ_e (MPa)	33.9
θ_{max} (°)	1.404

NUMERICAL ANALYSIS OF CONSTRAINED TORSION USING FEM - MODEL 1

The finite element method (FEM) was applied to the same Ω cantilever beam model exposed to constrained torsion, as model 1. Stress and strain values are obtained by numerical calculation. Numerical simulations /11-14/ are performed using KOMIPS software. Units for strains are (cm) and for the loads are (kN).

In Fig. 3 load and boundary conditions are shown, /11/. Shell elements were used (shell model), /11/. The torsional moment is introduced through the coupling of forces F = 302 N through the centre of gravity of the cross-section, and the moment they cause is M^* = 15700 Nmm.

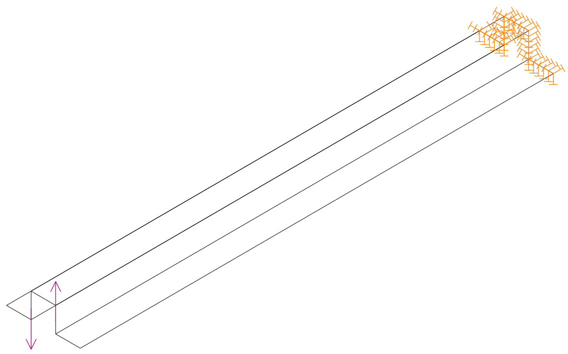

Figure 3. Load and boundary conditions - KOMIPS software.

DISCUSSION OF ANALYTICAL AND NUMERICAL RESULTS - MODEL 1

The model displacement is shown in Fig. 4, and maximal displacement f_{max} is given in (mm).

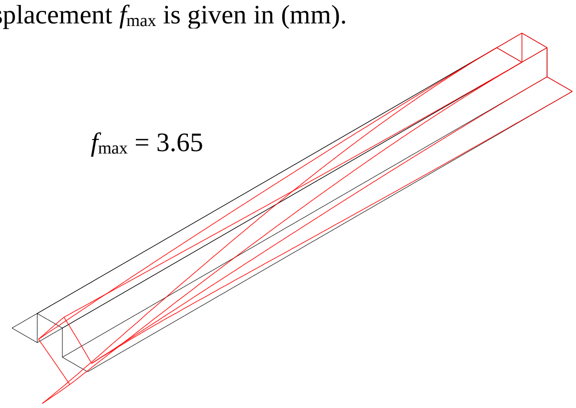

Figure 4. Deformed model 1 with maximal displacement.

Figure 5 shows the equivalent stress distribution according to von Mises hypothesis. The equivalent stress value is given in (MPa), and the maximal value is 40 MPa.

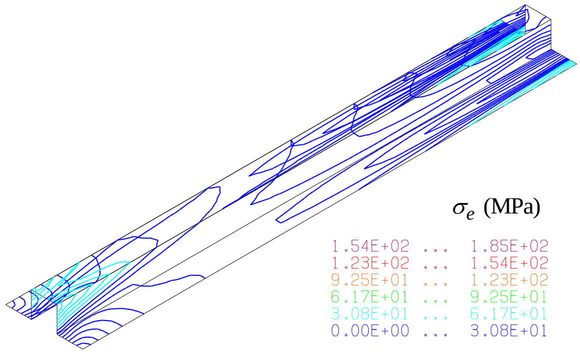

Figure 5. Equivalent stress according to von Mises and corresponding scale - KOMIPS software.

If we compare equivalent stress values from Table 3 with values from Fig. 5, we see that the stress obtained by numerical simulation is 17 % higher than the stress obtained analytically. This relatively large deviation is obtained primarily due to the complex geometry of the Ω profile.

NUMERICAL ANALYSIS OF CONSTRAINED TORSION USING FEM - MODEL 2

Numerical simulations /3/ are performed in ABAQUS® and SolidWorks®. These two programmes were used to confirm the accuracy of the obtained results, as well as to be able to trust both programmes. A cantilever beam with an Ω profile is encastred at one end, while at the other end there is a welded plate.

Numerical simulation is first done in ABAQUS®. The load is applied as two concentrated forces to the corners of the rectangular plate in order to prevent torsion, with a magnitude of 92.4 N for each force. Load and boundary conditions can be seen in Fig. 6. Plate sizes are 170 × 170 × 3 mm. The cantilever beam had an Ω profile cross-section (Fig. 1). The torsional moment value is the same for both models, M^* = 15700 Nmm.

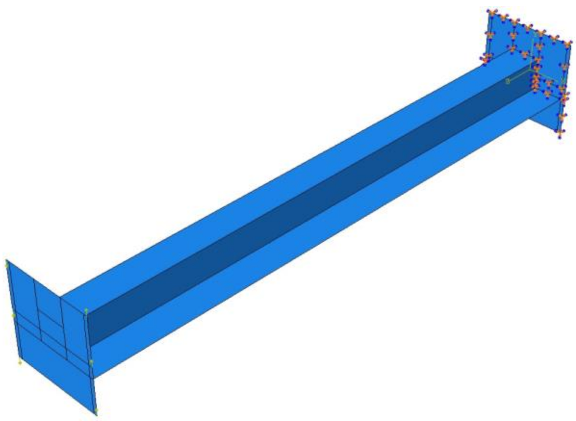

Figure 6. Load and boundary conditions - ABAQUS® software.

The finite element mesh is shown in Fig. 7. The equivalent stress distribution field according to von Mises is given in Fig. 8. Stress values are given in (MPa), σ_e (MPa). It can be seen that the maximal equivalent stress is 57.64 MPa.

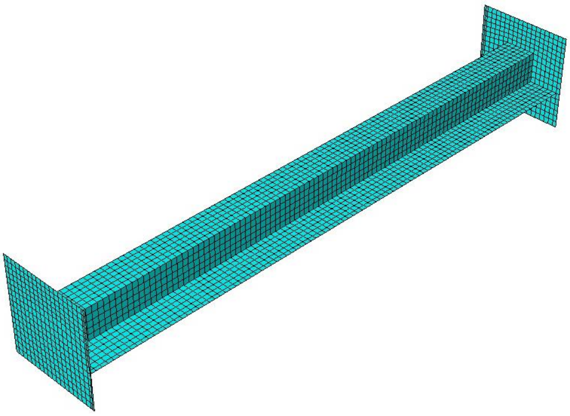

Figure 7. Finite element mesh - ABAQUS® software.

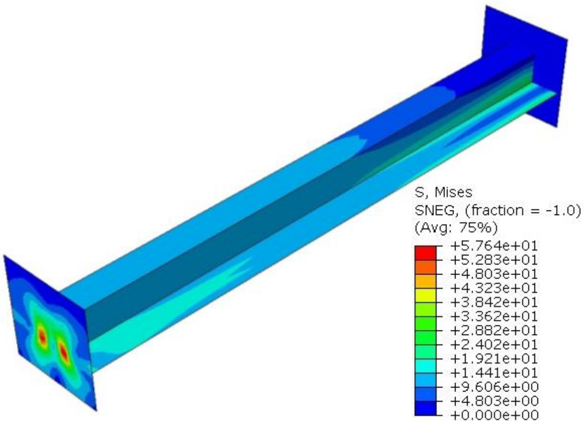

Figure 8. Distribution of equivalent stress according to von Mises - ABAQUS® software.

Figure 9 shows the displacement distribution. Displacement is calculated in (mm), and the corresponding scale is given in Fig. 9.

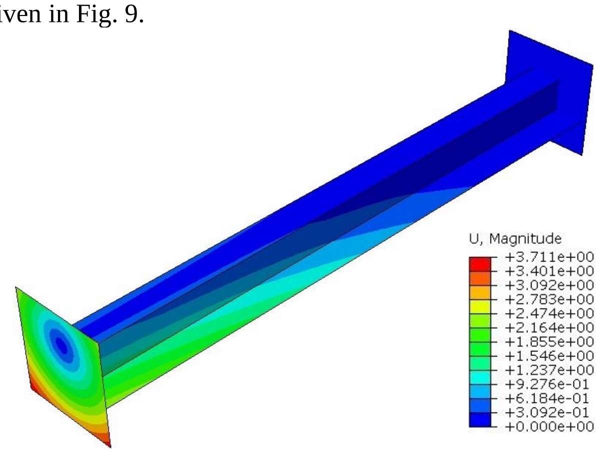

Figure 9. Displacement distribution and corresponding scale in (mm) - ABAQUS® software.

Numerical simulation is performed in SolidWorks® for the Ω profile model, with the same dimensions of the model as ABAQUS®. The load values and the method of loading were also the same. The mesh model is shown in Fig. 10, also with the loading and boundary conditions.

The equivalent stress distribution field according to von Mises is shown in Fig. 11. Stress values are given as σ_e (Nm^{-2}) and the corresponding scale is shown in Fig. 13a. It can be seen that maximal equivalent stress is 58.06 MPa.

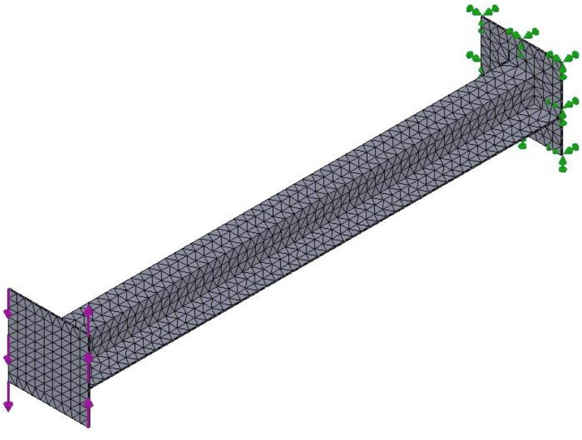

Figure 10. Mesh model, loading and boundary conditions - SolidWorks® software.

By comparing stress values obtained in the two programmes, we conclude that the difference is about 0.7 %. Therefore, both ABAQUS® and SolidWorks® provide reliable results for the numerical simulation of the Ω profile cantilever beam exposed to constrained torsion.

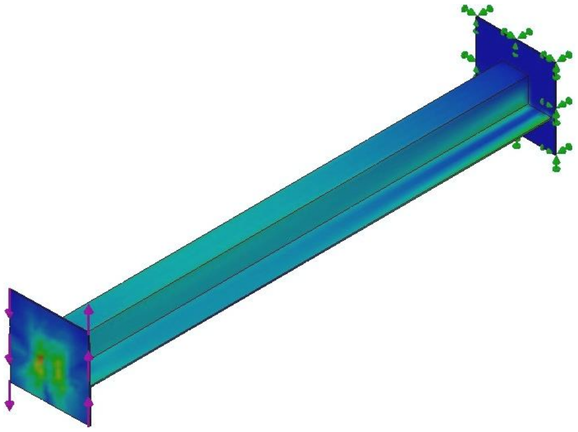

Figure 11. Equivalent stress according to von Mises - SolidWorks®.

Displacement distribution is given in Fig. 12. The displacement is calculated in (mm) and the corresponding scale is shown in Fig. 13b. If we look at Figs. 9 and 13b we see maximal displacement values of model 2 are approximately the same in both software codes (about 3.5 mm).

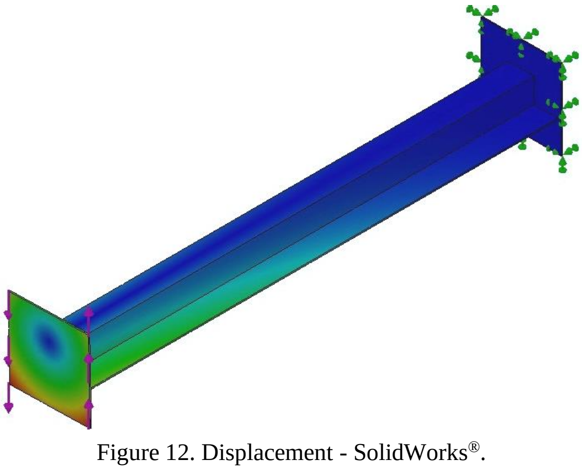

Figure 12. Displacement - SolidWorks®.

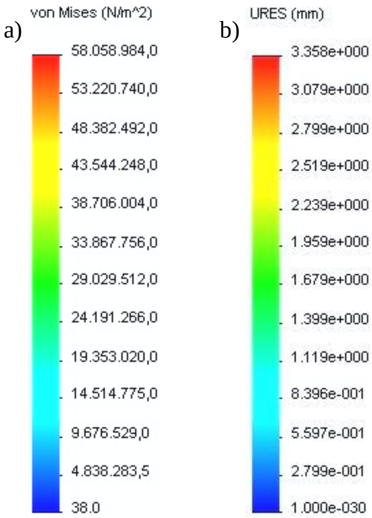

Figure 13. a) Equivalent von Mises stress scale (Pa) - SolidWorks®; b) displacement scale in (mm) - SolidWorks®.

CONCLUSIONS

The paper provides initial considerations that include an overview of current studies of the stress and deformation states in thin-walled omega profile, as well as a review of available literature. Numerical models are made in three different numerical software codes. Static calculation is carried out analytically and using FEM. The zones of stress concentration are identified.

In case of model 1, by comparing stress values obtained analytically and numerically, a difference of about 17 % is obtained. This relatively large deviation value is obtained primarily due to the complex geometry of the omega profile.

For model 2, the values of equivalent stresses and displacements are obtained in two software codes. Stress values differ by only 0.7 %. Therefore, both ABAQUS® and SolidWorks® codes provide reliable results for numerical simulation of a profile cantilever beam exposed to constrained torsion.

Stress values obtained by numerical calculation at constrained torsion in model 1 and in model 2 are compared. The maximal stress at constrained torsion in model 1 is 45 % less than in model 2. Such a difference in stress values is expected because the cantilever beam in model 2 has a welded plate at the other end. The plate increases the stiffness of the cantilever, and thus leads to an increase in equivalent stress.

Conclusions obtained by examining this type of structure may be involved in the design process of new similar structures. The findings obtained in the implementation of this work can be directly applied to identify the behaviour of real structures in their working conditions, i.e., in exploitation.

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