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A REVIEW ON APPROACHES FOR STRUCTURAL DYNAMIC MODIFICATION

PREGLED METODA DINAMIČKE MODIFIKACIJE KONSTRUKCIJA

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- structural optimisation
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- FEM

Abstract

The paper aims to delve into the critical connection between optimal design and the reliability or safety of structures. The discussion primarily centres on structures whose reliability or failure probability can be reasonably assessed, especially in a redesigned context. A review of structural problems that have been optimised within a reliability framework is presented. Given the topic's focus on safety from a probabilistic perspective, it is necessary to address relevant issues of sensitivity, damage costs, limited empirical data, and safety philosophy. The paper primarily emphasizes the relationship between optimisation and reliability, considering the computational and philosophical questions that arise from failure analysis and reliability-based design. It highlights the drawbacks of current deterministic approaches and the potential benefits of a probabilistic approach to safety and design. It also notes that while most structural reliability analyses have been based on a static approach to loads and strength, a more holistic perspective should consider factors like stresses or fatigue strength that might be stochastic or time-dependent.

Ključne reči
- optimizacija konstrukcija
- pouzdanost
- sopstvene vrednosti
- MKE

Izvod

Cilj ovog rada uspostavljanje kritične veze između optimalnog dizajna i pouzdanosti i sigurnosti konstrukcija. Diskusija je mahom fokusirana na konstrukcije za koje se može odrediti verovatnoća otkaza sa dovoljno sigurnosti, posebno u kontekstu renoviranih konstrukcija. Pregled problema koji su optimizovani u okvirima pouzdanosti je takođe prikazan. Uzimajući u obzir fokus ove teme na pouzdanost sa tačke gledišta verovatnoće, neophodno je obratiti pažnju na probleme vezane za osetljivost, troškove usled oštećenja, ograničen broj empirijskih podataka, i filozofiju bezbednosti. Ovaj rad pre svega naglašava vezu između optimizacije i pouzdanosti, uzimajući u obzir računska i filozofska pitanja koja se javljaju pri analizi otkaza i dizajnu zasnovanom na pouzdanosti. Posebno su istaknuti nedostaci trenutnih determinističkih pristupa i potencijalne prednosti pristupa određivanju bezbednosti zasnovanog na verovatnoći. Takođe je napomenuto da, iako je većina analiza pouzdanosti konstrukcija zasnovana na statičkom opterećenju i čvrstoći, sveobuhvatniji pristup bi trebao da uzme u obzir i zamor, kao stohastičku veličinu, zavisnu od vremena.

INTRODUCTION

This paper offers a review of approximations in structural dynamics. Several key observations are outlined:
(i) by simultaneously altering configuration and implementing structural size modifications, designs can be significantly improved and made more cost-effective. However, additional shape/topology considerations are needed;
(ii) by incorporating stress and displacement constraints under various load conditions into the frequency constraint problem, designs can be made more realistic;
(iii) application of dynamics optimisation to smart structures for vibration control has yet to be thoroughly explored. Potential methodologies for investigation include neural networks and genetic algorithms;
(iv) Finite Element (FE) model validation has been significantly advanced through the study of primary procedures of the model validation process, as well as the development of model verification methods.

STRUCTURAL OPTIMISATION

The structural optimisation problem with frequency constraints is subjected in one of the following ways:
(i) Maximize the natural frequency or difference between two consecutive frequencies subject to a specified constraint

$$h(p) = g(p) - \tilde{g} = 0 , \tag{1}$$

and side constraints on the design values

$$p_i^l \le p_i \le p_i^u . \tag{2}$$

(ii) Minimize structural weight $g(v)$ subject to behaviour constraints

$$h_j(p) = \omega_j^2 - \tilde{\omega}_j^2 = 0 , \quad j = 1, 2, \ldots, k , \tag{3}$$

$$h_j(p) = \omega_j^2 - \tilde{\omega}_j^2 \ge 0 , \quad j = k+1, k+2, \ldots, m , \tag{4}$$

where: p_i is the design variable or updating parameter; p_i^l is the lower limit; p_i^u is the upper limit on the design variable; ω_j is the j-th natural frequency; $\tilde{\omega}_j$ is the specified value of the j-th natural frequency; $g(p)$ is the structural weight; $\tilde{g}$ is the specified weight; n is the number of design variables;

and m is the number of design constraints. The design variables are inherently dependent on the nature of the optimisation problem. When considering the design of structural components, such as stiffened panels and cylinders, design parameters typically encompass aspects such as stiffener spacing, size and shape, along with the thickness of the skin. In scenarios where the skin and/or stiffeners are composed of layered composites, fibre orientation and their proportion might become additional variables to consider.

In the case of a structural system with a fixed configuration, such as frames, trusses, wings, fuselages, etc., the sizes of the elements serve as design variables. These sizes can be represented by plate thickness, cross-sectional areas of bars, areas, moments of inertia, and torsional constants of beams.

When optimisation includes configuration, these parameters may be spatial. Furthermore, in dynamics problems, nonstructural mass locations and magnitudes may be viewed as variables. If the optimisation problem includes only frequency constraints, it is recommended to incorporate nonstructural masses into the structural model to represent aspects like fuel, payload, attachments, etc.

When executing a model updating procedure, every parameter in a Finite Element (FE) model can be seen as a potential updating parameter. In an FE model for a continuous structure, the number of independent parameters corresponds to the model's degrees of freedom.

EIGENSENSITIVITY ANALYSIS

Numerous techniques exist for dynamics approximations in mechanical structures with sensitivity analysis being among the most popular. This approach has been extensively developed and applied to the general eigenvalue problem /1-14/, and it's been specifically utilized in the context of structural dynamic modification analysis, /15-18/. Sensitivity derivatives are instrumental in examining the impact of parametric modifications, determining the search directions for an optimal design, building function approximations, and carrying out 'what-if' design trade-off studies. Recent reviews have shed light on the progress and applications of sensitivity analysis. This section focuses on the use of sensitivity analysis tools in frequency optimisation.

The eigenvalue problem is given as follows:

$$[K]\{\underline{Q}_i\} - \lambda_i [M]\{\underline{Q}_i\} = \{0\}, \tag{5}$$

where: $[K]$, $[M]$, $\lambda_i = \omega_i^2$, and $\{\underline{Q}_i\}$ are the stiffness matrix, mass matrix, eigenvalue, and eigenvector, respectively. Derivatives of the distinct eigenvalues with respect to the design variable p_j using the orthogonality conditions,

$\{\underline{Q}_i\}^T [M]\{\underline{Q}_i\} = 1$, are given as

$$\frac{\partial \lambda_i}{\partial p_j} = \{\underline{Q}_i\}^T \left(\frac{\partial [K]}{\partial p_j} - \lambda_i \frac{\partial [M]}{\partial p_j} \right) \{\underline{Q}_i\}. \tag{6}$$

Fox and Kapoor /2/ introduced methods for calculating the eigenvalue and eigenvector derivatives of symmetric matrices, offering two techniques for eigenvector gradients. The first method involved differentiating the algebraic eigenvalue problem in relation to the design variables, with derivatives calculated following algebraic manipulations. However, this approach disrupted the banded nature of the equations. In the second method, the derivative was expressed as a series of eigenvectors. Nelson /19/ later formulated an alternate approach for eigenvector derivatives that preserved the banded nature of matrices. Pritchard et al. /20/ derived an expression for the derivative of the nodal location of the mode shape with respect to the design variable for one-dimensional structures. Sutter and his team /21/ compared four methods for calculating the derivatives of vibration modes with respect to the design parameters and concluded that Nelson's method was superior for its accuracy and efficiency.

The computation of sensitivity for repeated eigenvalues has also been explored by various researchers. Given that repeated eigenvalues are not differentiable, only directional derivatives can be obtained. Studies /22-25/ have addressed structural optimisation problems with repeated eigenvalues using directional derivatives. In the case of real symmetric matrices, a generalised version of Nelson's method was presented in references /26-28/, maintaining the bandedness of the matrix. The complexity in sensitivity computation arises from the non-uniqueness of the eigenvectors of the repeated eigenvalues. The eigenvalue derivatives for repeated roots can be obtained by solving a sub-eigenvalue problem,

$$\left[\{Q_i\}^T \left(\frac{\partial [K]}{\partial p_j} - \lambda_i \frac{\partial [M]}{\partial p_j} \right) \{Q_i\} - \frac{\partial \lambda_i}{\partial p_j} [I] \right] \{A_i\} = 0, \tag{7}$$

where: $\{Q_i\}$ consists of eigenvectors corresponding to repeated roots; $\{A_i\}$ is a coefficient vector; and $[I]$ is an identity matrix. The eigenvalues of Eq.(7) represent the $\partial \lambda_i / \partial p_j$ vector.

Reference /29/ made use of reduced-order models for the computation of sensitivities for both repeated and non-repeated frequencies. Hou and Chuang /30/ formulated equations for eigenvalue and eigenvector sensitivities in continuous beams subject to variations in support locations, employing both domain and boundary methods in their derivations.

APPROXIMATIONS OF FREQUENCY CONSTRAINTS

Barthelemy and Haftka /31/ have categorised function approximations used in structural optimisation into local, medium-range, and global types in their paper. While most conventional approximation techniques are suitable for frequency functions, a few researchers have developed premium approximations specifically for frequency problems /32-36/, to achieve stable convergence with less restrictive move limits. Owing to the inherent nonlinear attributes of natural frequency constraints, Miura and Schmit /33/ introduced a second-order Taylor series approximation for each eigenvalue to enhance stability and overall efficiency of the synthesis process. Their research indicates the high nonlinearity of eigenvalues in both direct and reciprocal design variable space, necessitating strict move limits. They reported that while the second-order approximation yielded stable convergence without stringent move limits, the total computation time was similar to that required with first-order approximation with move limits. Starnes and Haftka /34/

proposed that a hybrid constraint using mixed variables (a blend of direct and reciprocal variables) delivers a more conservative approximation. Woo /36/ extended this concept in his generalised hybrid constraint approximation, where a variable exponent determines the conservativeness of the convex approximation, with the concepts demonstrated on a space frame structures design. The presentation of a second-order approximation employing a half-quadratic scheme, a generalised power approach, a generalised method of moving asymptotes, and a full second-order Taylor approximation can be found in /37/. Considering the computational cost, the authors suggest that second-order approximations are suitable for challenging problems, whereas for less sensitive problems, approximate second-order information is recommended. Pritchard and Adelman /38/ presented an innovative approach by interpreting sensitivity expressions as differential equations, thereby obtaining closed-form exponential approximations for eigenvalues and eigenvectors, which proved superior to linear models. Despite the evident nonlinearity of frequencies via the appearance of cross-sectional variables in both the numerator and denominator of Rayleigh's quotient, Venkayya and Tischler /39/, and Maneski /40/ argue that in practical structures, the denominator (kinetic energy) is predominantly influenced by the nonstructural mass. In this scenario, the eigenvalues are nearly linear in the cross-sectional property (direct design variable space). Vanderplaats and Salajegheh /35/ illustrated better quality using a linear approximation of the eigenvalues concerning the member section properties of frame elements, given the optimisation design variables were cross-sectional dimensions. However, no effort was made to create a convex or separable form of the optimisation problem. The optimality criterion approach proposed by Venkayya and Tischler /39/, and Grandhi and Venkayya /41, 42/ indicate that the modal strain and kinetic energies could be more appropriate quantities for approximation than the eigenvalue. Canfield /32/ constructed the Rayleigh quotient approximation (RQA) by creating first-order approximations to the modal strain and kinetic energies, independently,

$$\lambda_i = \frac{\{\underline{Q}_i\}^T [K] \{\underline{Q}_i\}}{\{\underline{Q}_i\}^T [M] \{\underline{Q}_i\}} = \frac{U_{i,A}}{T_{i,A}}, \tag{8}$$

where: $U_{i,A}$ and $T_{i,A}$ are first-order approximations for modal strain (potential energies) and kinetic energies, respectively. He achieved fast and stable convergence with generous move limits. Interestingly, this concept is akin to an alternative approximation proposed by Fox and Kapoor /2/, excluding the use of the eigenvector's first-order estimate here. Methods for approximate eigenvalue reanalysis of locally modified structures are developed, drawing on the generalised Rayleigh's quotients as presented by Wang and Pilkey /43/. For straightforward modifications such as the addition of springs and masses or alterations to the truss member's cross-sectional area, closed-form formulas are provided. Two effective approaches for achieving excellent results based on one-term approximations via Rayleigh's quotients are detailed, including their practical applications as per Hodges /44/. As demonstrated therein, the utilisation of the improved functions in Rayleigh's quotient allows for an upper frequency bound, boasting better accuracy than the lower bound directly derived from the method itself. An enhanced first-order approximation procedure for the reanalysis of eigenvalues and eigenvectors of modified structural dynamic systems has been proposed by Nair, Keane, and Langley /45/. The current methodology, as shown, can be employed to acquire reliable estimates of the natural frequencies during simultaneous structural parameter perturbations. Furthermore, this method can be utilised without significant accuracy loss when using approximate eigenvector derivatives/perturbations to calculate basis vectors.

CONCLUSION

The following key points are noted:

- Significant improvements and cost-effective designs can be achieved through simultaneous alterations in configuration (beam lengths, boundary, or support conditions) in conjunction with modifications to the structural size.
- More authentic designs can be achieved by incorporating stress and displacement constraints under various load conditions within the frequency constraint problem.
- Recent studies /46, 47, 48/ focus on the experimental validation of optimised designs that meet frequency requirements. This is a promising area of exploration, given the increased sensitivity of optimised designs to the parametric uncertainties inherent in the physical system.

Future research efforts should also include the development of formal methodologies for the observation of mode-switching phenomena during optimisation.

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REFERENCES

1. Faddeev, D.K., Faddeeva, V.N., Computational Methods of Linear Algebra, W.H. Freeman and Company, San Francisco, London, 1963.
2. Fox, R.L., Kapoor, M.P. (1968), *Rates of change of eigenvalues and eigenvectors*, AIAA J, 6(12): 2426-2429. doi: 10.25 14/3.5008
3. Jacobi, C.G.J. (1971), *Uber ein leichtes Verfahren die in der Theorie der Säcularstörungen vorkommenden Gleichungen numerisch aufzulösen*, Zeitschrift für Reine und Angewandte Mathematik, 30(1846): 51-95.
4. Jahn, H.A. (1948), *Improvement of an approximate set of latent roots and modal columns of a matrix by methods akin to those of classical perturbation theory*, Quart. J Mech. Appl. Math. 1: 132-144.
5. Lancaster, P. (1964), *On eigenvalues of matrices dependent on a parameter*, Numer. Math. 6(5): 377-387. doi: 10.1007/BF013 86087
6. Wilkinson, J.H., The Algebraic Eigenvalue Problem, Oxford University Press, London, 1963, pp.62-109.
7. Rosenbrock, H.H. (1965), *Sensitivity of an eigenvalue to changes in the matrix*, Electr. Lett. 1(10): 278-279. doi: 10.1049/el:1965 0252

8. Reddy, D.C. (1966), *Sensitivity of an eigenvalue of a multivariable control system*, Electr. Lett. 2(12): 446. doi: 10.1049/el:19660374
9. Rogers, L.C. (1970), *Derivatives of eigenvalues and eigenvectors*, AIAAJ, 8(5): 943-944. doi: 10.2514/3.5795
10. Vanhonacker, P. (1980), *Differential and difference sensitivities of natural frequencies and mode shapes of mechanical structures*, AIAAJ, 18(12): 1511-1514. doi: 10.2514/3.7738
11. Rudisill, C.S., Bhatia, K.G. (1972), *Second derivatives of the flutter velocity and the optimization of aircraft structures*, AIAAJ, 10: 1511-1514.
12. Plaut, R.H., Huseyin, K. (1973), *Derivatives of eigenvalues and eigenvectors in non-self-adjoint systems*, AIAAJ, 11(2): 250-251. doi: 10.2514/3.6740
13. Rudisill, C.S. (1974), *Derivatives of eigenvalues and eigenvectors for a general matrix*, AIAAJ, 12(5): 721-722. doi: 10.2514/3.49330
14. Belle, H.V. (1982), *Higher order sensitivities in structural systems*, AIAAJ, 20(2): 286-288. doi: 10.2514/3.7911
15. Skingle, G.W., Ewins, D.J. (1989), *Sensitivity analysis using resonance and anti-resonance frequencies - A guide to structural modification*, Proc. 7th Int. Modal Analysis Conf., Las Vegas, Nevada, Union College, 1989.
16. Lim, K.B., Junkins, J.L., Wang, B.P. (1987), *Re-examination of eigenvector derivatives*, J Guid. Control Dyn. 10(6): 581-587. doi: 10.2514/3.20259
17. Wang, J., Heylen., W., Sas, P. (1987), *Accuracy of structural modification techniques*, Proc. 5th Int. Modal Analysis Conf., pp.65-71.
18. Noor, A.K., Whitworth, S.L. (1988), *Reanalysis procedure for large structural systems*, Int. J Numer. Methods Eng. 26(8): 1729-1748.
19. Nelson, R.B. (1976), *Simplified calculation of eigenvectors derivatives*, AAIAJ, 14(9): 1201-1225. doi: 10.2514/3.7211
20. Pritchard, J.I., Adelman, H.M., Haftka, R.T. (1987), *Sensitivity analysis and optimization of nodal point placement for vibration reduction*, J Sound Vibr. 119(2): 277-289. doi: 10.1016/0022-460X(87)90455-X
21. Sutter, T.R., Camarda, C.J., Walsh, J.L., Adelman, H.M. (1988), *Comparison of several methods for calculating vibration mode shape derivatives*, AIAAJ, 26(12): 1506-1511. doi: 10.2514/3.10070
22. Bartholomew, P., Pitcher, N. (1984), *Optimization of structures with repeated normal-mode frequencies*, Eng. Optim. 7(3): 195-208. doi: 10.1080/03052158408960639
23. Choi, K.K., Haung, E.J., Seong, H.G. (1983), *An iterative method for finite dimensional structural optimization problems with repeated eigenvalues*, Int. J Numer. Meth. Eng. 19(1): 93-112. doi: 10.1002/nme.1620190110
24. Haug, E.J., Choi, K.K. (1982), *Systematic occurrence of repeated eigenvalues in structural optimization*, J Optim. Theory Appl. 38(2): 251-274. doi: 10.1007/BF00934087
25. Myslinski, A. (1985), *Bimodal optimal design of vibrating plates using theory and methods of nondifferentiable optimization*, J Optim. Theory Appl. 46: 187-203. doi: 10.1007/BF00938423
26. Dailey, R.L. (1989), *Eigenvector derivatives with repeated eigenvalues*, AAIAJ, 27(4): 486-491. doi: 10.2514/3.10137
27. Mills-Curran, W.C. (1988), *Calculation of eigenvector derivatives for structures with repeated eigenvalues*, AAIAJ, 26(7): 867-871. doi: 10.2514/6.1989-1333
28. Ojalvo, I.U. (1987), *Efficient computation of mode-shape derivatives for large dynamic systems*, AAIAJ, 25(10): 1386-1390. doi: 10.2514/3.9793
29. Chen, T.-Y. (1992), *Optimum design of structures with both natural frequency response constraints*, Int. J Numer. Meth. Eng. 33(9): 1927-1940. doi: 10.1002/nme.1620330910
30. Haug, E.J., Choi, K.K. (1990), *Design sensitivity analysis and optimization of vibrating beams with variable support locations*, 16th Design Automation Conf., DE-Vol.23-2, ASME, Chicago, IL, 1990, pp.281-290.
31. Barthelemy, J.-F.M., Haftka, R.T. (1991), *Recent advances in approximation concepts for optimum structural design*, NASA Tech. Memorandum, 104032, p.24.
32. Canfield, R.A. (1990), *High-quality approximation of eigenvalues in structural optimization*, AIAAJ, 28(6): 1116-1122. doi: 10.2514/3.25175
33. Miura, H., Schmit, L.A. Jr. (1978), *Second order approximation of natural frequency constraints in structural synthesis*, Int. J Numer. Meth. Eng. 13(2): 337-351. doi: 10.1002/nme.1620130209
34. Starnes, J.H. Jr., Haftka, R.T. (1979), *Preliminary design of composite wings for buckling, strength, and displacement constraints*, J Aircraft, 16(8): 564-570. doi: 10.2514/3.58565
35. Vanderplaats, G.N., Salajegheh, E. (1988), *An efficient approximation technique for frequency constraints in frame optimization*, Int. J Numer. Meth. Eng. 26(5): 1057-1069. doi: 10.1002/nme.1620260505
36. Woo, T.H. (1987), *Space frame optimization subject to frequency constraints*, AIAAJ, 25(10): 1396-1404. doi: 10.2514/3.9795
37. Mlejnek, H.P., Jehle, U., Schirrmacher, R. (1982), *Second order approximations in structural genesis and shape finding*, Int. J Numer. Meth. Eng. 34(3): 853-872. doi: 10.1002/nme.1620340311
38. Pritchard, J.I., Adelman, H.M. (1991), *Differential equation based method for accurate modal approximations*, AIAAJ, 29 (3): 484-486. doi: 10.2514/3.10609
39. Vekayya, V.B., Tischler, V.A. (1983), *Optimization of structures with frequency constraints*, Comp. Meth. Nonlinear Solids Struct. Mech., ASME, AMD-54: 239-251.
40. Maneski, T., *Contribution to development of design via computational modeling of supporting structure of machine tools*, Ph.D. Thesis, University of Belgrade, Faculty of Mechanical Engineering, Belgrade, 1992. (in Serbian)
41. Grandhi, R.V., Venkayya, V.B. (1988), *Structural optimization with frequency constraints*, AIAAJ, 26(7): 858-866. doi: 10.2514/3.9979
42. Grandhi, R.V., Venkayya, V.B. (1989), *Optimum design of wing structures with multiple frequency constraints*, Finite Elem. Anal. Des. 4(4): 303-313. doi: 10.1016/0168-874X(89)90025-5
43. Wang, B.P., Pilkey, W.D. (1986), *Eigenvalue reanalysis of locally modified structures using a generalized Rayleigh's method*, AIAAJ, 24(6): 983-990. doi: 10.2514/3.9374
44. Hodges, D.H. (1997), *Improved approximations via Rayleigh's quotient*, J Sound Vibr. 199(1): 155-164. doi: 10.1006/jsvi.1996.0641
45. Nair, P.B., Keane, A.J., Langley, R.S. (1998), *Improved first-order approximation of eigenvalues and eigenvectors*, AIAAJ, 36(9): 1722-1727. doi: 10.2514/2.578
46. Adelman, H.M. (1992), *Experimental validation of the utility of structural optimization*, Struct. Optim. 5: 3-11. doi: 10.1007/BF01744689
47. Imamovic, N., *Validation of large structural dynamics models using modal test data*, Ph.D. Thesis, Imperial College, Univ. of London, 1998.
48. Fotsch, D.W., *Development of valid FE models for structural dynamic design*, Ph.D. Thesis, Imperial College, Univ. of London, 2001