

## MATHEMATICAL MODEL FOR FRACTIONAL MICROSTRETCH THERMOELASTIC MEDIUM MATEMATIČKI MODEL FRAKCIONE MIKROSTREGLJIVE TERMOELASTIČNE SREDINE

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### Keywords

- microstretch thermoelastic
- integral transform
- fractional order
- inclined forces
- thermal stresses

### Abstract

*The present investigation deals with thermal and mechanical interactions in a fractional order microstretch thermoelastic half-space subjected to inclined mechanical forces acting at the boundary of the surface of the half-space. Integral transform technique (Laplace and Fourier transform) has been applied to solve the basic equations mathematically. Mathematical expressions of mechanical stresses coupled tangential stress, microstress, and the temperature distribution are obtained numerically. Some particular results and special cases also have been derived from the present research.*

### INTRODUCTION

Eringen /1, 2/ developed the linear theory of micropolar thermoelasticity, theory of micropolar elastic solids with stretch and derived the basic equations of motion, constitutive relations and boundary conditions for these theories. The latter theory is very important for a class of materials which can stretch and contract. In this theory Eringen /2/ presented a new model which explained the motion of a certain class of materials, i.e. rigid chopped fibres, many categories of composites and granular materials.

Eringen /3/ developed the theory of thermo-microstretch elastic materials including microstructural expansion and contractions. The material points of microstretch thermoelastic material are able to stretch and contract independently of their translational and relational motion. Composite substances reinforced with chopped fibres, porous materials filled with asphalt and other insertions are also categorized as the microstretch thermoelastic materials. Here it is noted that the theory of microstretch thermoelastic solids is a particular case of micromorphic thermoelasticity and is the generalisation of micropolar theory of thermoelasticity.

The differential equations including higher-order fractional derivatives play a significant role in mathematical modelling of dynamical behaviour of some complex systems. The fractional calculus has also applications in the solution of problems of material engineering, physics, chemical engi-

### Ključne reči

- mikrorastegljiv, termoelastičan
- integralna transformacija
- sistemi frakcionog reda
- kose sile
- termički naponi

### Izvod

*Opisano je istraživanje u kojem se bavimo termičkim i mehaničkim interakcijama u sistemu frakcionog reda tipa mikrorastegljivog termoelastičnog poluprostora, u kojem deluju kose mehaničke sile na granici površine datog poluprostora. Metode integralne transformacije (Laplasova i Furijeova transformacija) su primenjene u matematičkom rešavanju osnovnih jednačina. Matematički izrazi za mehaničke napone, spregnuti tangencijalni napon, mikro-napon, i raspodelu temperature, su dobijeni numerički. Pojedini rezultati i specijalni slučajevi su takođe izvedeni u ovom istraživanju.*

neering and bio-medical dynamics. During the last decade many researchers worked on fractional calculus. First of all, Abel presented an application of fractional calculus in formulation of the tautochrone problem. Povstenko /4/ also developed a quasi-static thermoelastic model for uncoupled equations taking the fractional time derivative in the heat conduction equation. Later, Povstenko /5/ investigated the thermal stresses in an infinite medium including cylindrical holes by using the fractional heat conduction equation. Sherief et al. /6/ presented a mathematical model of fractional order theory of thermoelasticity by revising the Cattaneo law and deduced the basic equations, constitutive relations, uniqueness theorem, reciprocity theorem, and variational problems. Youssef /7/ constructed another mathematical model for fractional theory of thermoelasticity by taking different value of fractional parameter  $\alpha$ . He also discussed an application of this theory. Ezzat /8, 9/ proposed another theory of fractional order generalised thermoelasticity using Taylor's series expansion of time-fractional order. Later Ezzat and Fayik /10/ extended this fractional order generalised thermoelasticity by including the thermo-diffusion and presented a new theory named as fractional order thermoelasticity theory with diffusion in elastic medium. They also, derived the uniqueness theory, reciprocity theorem, and variational principle. Shaw and Mukhopadhyay /11/ extended the Sheriff's theory of fractional thermoelasticity to include the effect of two temperatures for micropolar thermoelastic

materials. Sur and Kanoria /12/ investigated a one-dimensional problem in fractional thermoelasticity with two-temperatures in the context of LS-theory and GL-theory. Yu et al. /13/ discussed a problem in electromagnetic anisotropic medium using fractional order theory of thermoelasticity. Sumelka /14/ discussed some applications and qualitative aspects in fractional continuum mechanics. A one-dimensional fractional thermoelastic problem with diffusion in a half-space is discussed by Povstenko and Klekot /15/. Shaw and Mukhopadhyay /16/ developed a theory of fractional ordered thermoelastic diffusion. Recently Chirila and Marin /17/ worked on dipolar thermoelastic materials with the property of double porosity and presented a generalised theory of thermoelasticity with fractional order strain. Marin et al. /18/ recently presented a mathematical model of fractional order strain in dipolar thermoelasticity.

In this paper we have used the fractional theory of thermoelasticity developed by Ezzat and Fayik /10/ and analysed the thermo-mechanical interactions in a fractional order microstretch thermoelastic medium. The normal stress, tangential stress, coupled tangential stress, microstress, and temperature distribution are computed using the numerical method technique involving Laplace transform and Fourier transform. The computed physical quantities are also depicted graphically.

**BASIC EQUATIONS**

Following Eringen /3/, Ezzat and Fayik /10/, the basic equations for homogeneous, isotropic microstretch generalised thermoelastic solids in the absence of body forces, body couples and stretch forces are given by:

– stress equation of motion:  
 $(\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) + (\mu + K)\nabla^2 \mathbf{u} + K\nabla \times \boldsymbol{\phi} + \lambda_0 \nabla \phi^* - \beta_1 \nabla T = \rho \ddot{\mathbf{u}}, \quad (1)$

– couple stress equation of motion:  
 $(\gamma \nabla^2 - 2K)\boldsymbol{\phi} + (\alpha + \beta)\nabla(\nabla \cdot \boldsymbol{\phi}) + K\nabla \times \mathbf{u} = \rho \mathbf{j} \ddot{\boldsymbol{\phi}}, \quad (2)$

– equation of balance of stress moments:  
 $(\alpha_0 \nabla^2 - \lambda_1)\phi^* - \lambda_0 \nabla \cdot \mathbf{u} + v_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T = \frac{\rho j_0}{2} \ddot{\phi}^*, \quad (3)$

– fractional order equation of heat conduction:  
 $K^* \nabla^2 T = \left( \frac{\partial}{\partial t} + \frac{\tau_0^\alpha \partial^{\alpha+1}}{\Gamma(\alpha+1) \partial t^{\alpha+1}} \right) (\rho c^* T + v_1 T_0 \phi^*) + \left( 1 + \frac{\tau_0^\alpha \partial^\alpha}{\Gamma(\alpha+1) \partial t^\alpha} \right) (\beta_1 T_0 \nabla \cdot \dot{\mathbf{u}} - \rho Q), \quad (4)$

– the constitutive relations are:  
 $t_{ij} = (\lambda_0 \phi^* + \lambda u_{r,r}) \delta_{ij} + \mu (u_{i,j} + u_{j,i}) + K (u_{j,i} - \varepsilon_{ijk} \phi_k) - \beta_1 \delta_{ij} T, \quad (5)$

$$m_{ij} = \alpha \phi_{r,r} \delta_{ij} + \beta \phi_{i,j} + \gamma \phi_{j,i} + b_0 \varepsilon_{mji} \phi_{j,m}^*, \quad (6)$$

$$\lambda_i^* = \alpha_0 \phi_{,i}^* + b_0 \varepsilon_{ijm} \phi_{j,m}. \quad (7)$$

Following Sherief /6/, the Caputo fractional derivative in the heat conduction Eq.(4) can be written as:

$$\frac{d^\alpha f(t)}{dt^\alpha} = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \frac{d^n f(\tau)}{d\tau^n} d\tau, & n-1 < \alpha < n \\ \frac{d^n f(\tau)}{d\tau^n}, & \alpha = n \end{cases} \quad (8)$$

in which  $n - 1 < \alpha < n$ ,  $m \in N = \{1, 2, \dots\}$ , where:  $\lambda, \mu, \alpha, \beta, \gamma, K, \lambda_0, \lambda_1, \alpha_0, b_0$  are material constants;  $\rho$  is mass density;  $\mathbf{u} = (u_1, u_2, u_3)$  is displacement vector; and  $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$  is microrotation vector;  $\phi^*$  is scalar microstretch function;  $T$  is temperature; and  $T_0$  is reference temperature of the body chosen;  $K^*$  is coefficient of thermal conductivity;  $c^*$  is specific heat at constant strain;  $j$  is microinertia;  $t_{ij}$  are components of stress;  $m_{ij}$  are components of coupled stress;  $\lambda_i^*$  is microstress tensor;  $\delta_{ij}$  is Kronecker's delta function.

**FORMULATION OF THE PROBLEM**

We consider an isotropic homogeneous fractional microstretch thermoelastic half-space in an intact form at uniform temperature  $T_0$ . The origin of rectangular Cartesian coordinate system is taken on the  $x_3$  axis with  $x_3$ -axis pointing vertically downward the medium.

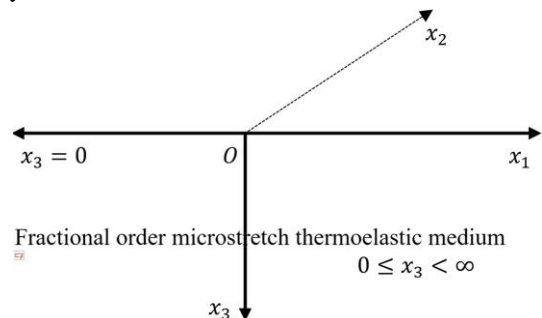


Figure 1. Geometry of the problem.

We consider plane strain problem with all field variables depending on  $(x_1, x_3, t)$ . For two-dimensional problems, we take

$$\mathbf{u} = (u_1, 0, u_3), \quad \boldsymbol{\phi} = (0, \phi_2, 0). \quad (9)$$

For further consideration, it is convenient to introduce in Eqs.(1)-(4) the dimensionless quantities defined as:

$$\begin{aligned} x'_i &= \frac{\omega^*}{c_1} x_i, \quad u'_i = \frac{\rho \omega^* c_1}{\beta_1 T_0} u_i, \quad \phi'_i = \frac{\rho c_1^2}{\beta_1 T_0} \phi_i, \quad \phi'^* = \frac{\rho c_1^2}{\beta_1 T_0} \phi^*, \\ T' &= \frac{T}{T_0}, \quad t' = \omega^* t, \quad \tau'_1 = \omega^* \tau_1, \quad \tau'_0 = \omega^* \tau_0, \quad t'_{ij} = \frac{1}{\beta_1 T_0} t_{ij}, \\ \omega^* &= \frac{\rho c_1^2}{K^*}, \quad c_1^2 = \frac{\lambda + 2\mu + k}{\rho}, \quad m'_{ij} = \frac{\omega^*}{c \beta_1 T_0} m_{ij}. \end{aligned} \quad (10)$$

Utilizing the expressions defined by Eq.(10) in Eqs.(1)-(4) and with the help of Eq.(9), we reach to the following equations:

$$a_1 \frac{\partial e}{\partial x_1} + a_2 \nabla^2 u_1 - a_3 \frac{\partial \phi_2}{\partial x_3} + a_4 \frac{\partial \phi^*}{\partial x_1} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial x_1} = \ddot{u}_1, \quad (11)$$

$$a_1 \frac{\partial e}{\partial x_3} + a_2 \nabla^2 u_3 + a_3 \frac{\partial \phi_2}{\partial x_1} + a_4 \frac{\partial \phi^*}{\partial x_3} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial x_3} = \ddot{u}_3, \quad (12)$$

$$\nabla^2 \phi_2 - 2a_6 \phi_2 + a_6 \left( \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \right) = a_7 \ddot{\phi}_2, \quad (13)$$

$$\nabla^2 \phi^* - a_8 \phi^* - a_9 e + a_{10} \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) T = a_{12} \ddot{\phi}^*, \quad (14)$$

$$\nabla^2 T - \left( \frac{\partial}{\partial t} + \frac{\tau_0 \omega^{*\alpha-1} \partial^{\alpha+1}}{\partial t^{\alpha+1}} \right) (T - a_{13} \nabla^2 \phi + a_{14} \phi^*) = Q_0, \quad (15)$$

Here,  $a_2 = \frac{\mu+k}{\rho c_1^2}$ ,  $a_3 = \frac{k}{\rho c_1^2}$ ,  $a_4 = \frac{\lambda_0}{\delta c_1^2}$ ,  $a_6 = \frac{2kc_1^2}{\gamma \omega^{*2}}$ ,

$$a_7 = \frac{\delta j c_1^2}{\gamma}, \quad a_8 = \frac{\lambda_1 c_1^2}{\alpha_0 \omega^{*2}}, \quad a_9 = \frac{\lambda_0 c_1^2}{\alpha_0 \omega^{*2}}, \quad a_{10} = \frac{v_1 \delta c_1^4}{\beta_1 \alpha_0 \omega^{*2}},$$

$$a_{12} = \frac{\delta c_1^4 j_0}{2\alpha_0}, \quad a_{13} = -\frac{\beta_1^2 T_0}{k^* \omega^*}, \quad a_{14} = \frac{v_1 \beta_1 T_0}{\rho \omega^* k^*},$$

$$a_{15} = s + \tau_0 \omega^{*\alpha-1} s^{\alpha+1}, \text{ and } \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}, \text{ is the Lapla-}$$

cian operator.

Making use of Helmholtz's decomposition theorem i.e. representation of a vector into scalar and vector potentials, the displacement components  $u_1$  and  $u_3$  are related to non-dimensional potential functions  $\phi$  and  $\psi$  as:

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1}. \quad (16)$$

Substituting the values of  $u_1$  and  $u_3$  from Eq.(16) in Eqs. (11)-(15), we obtain:

$$\nabla^2 \phi - \ddot{\phi} + a_4 \phi^* - \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) T = 0, \quad (17)$$

$$\left( \nabla^2 - a_8 - a_{12} \frac{\partial^2}{\partial t^2} \right) \phi^* - a_9 \nabla^2 \phi + a_{10} \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) T = 0, \quad (18)$$

$$\nabla^2 T - \left( \frac{\partial}{\partial t} + \frac{\tau_0 \omega^{*\alpha-1} \partial^{\alpha+1}}{\partial t^{\alpha+1}} \right) (T - a_{13} \nabla^2 \phi + a_{14} \phi^*) = Q_0, \quad (19)$$

$$a_2 \nabla^2 \psi - \ddot{\psi} + a_3 \phi_2 = 0, \quad (20)$$

$$\nabla^2 \phi_2 - 2a_6 \phi_2 - a_6 \nabla^2 \psi = a_7 \ddot{\phi}_2. \quad (21)$$

### SOLUTION OF THE PROBLEM

We define the Laplace transform and Fourier transform respectively as:

$$\bar{f}(s, x_1, x_3) = \int_0^\infty f(t, x_1, x_3) e^{-st} dt, \quad (22)$$

$$\hat{f}(x_3, \xi, s) = \int_{-\infty}^\infty \bar{f}(s, x_1, x_3) e^{i\xi x_1} dx_1. \quad (23)$$

Applying Laplace transform defined by Eq.(22) on Eqs. (17)-(21), and then applying Fourier transforms defined by Eq.(23) on the resulting quantities, we obtain:

$$\left( \frac{d^2}{dx^2} - \xi_1 \right) \hat{\phi} + a_4 \phi^* - \tau_{11} \hat{T} = 0, \quad (24)$$

$$-a_9 \left( \frac{d^2}{dx^2} - \xi^2 \right) \hat{\phi} + \left( \frac{d^2}{dx^2} - a_{20} \right) \phi^* + a_{21} \hat{T} = 0, \quad (25)$$

$$(D^2 - \xi^2) \hat{T} - (s + z_0 \omega^{*\alpha-1} s^{\alpha+1}) (\hat{T} - a_{13} (D^2 - \xi^2) \hat{\phi} + a_{14} \phi^*) = Q_1, \quad (26)$$

$$\left( \frac{d^2}{dx_3^2} - \xi^2 - s^2 \right) \hat{\psi} - a_3 \phi_2 = 0, \quad (27)$$

$$\left( \frac{d^2}{dx_3^2} - a_{30} \right) \phi_2 + a_6 \left( \frac{d^2}{dx_3^2} - \xi^2 \right) \hat{\psi} = 0. \quad (28)$$

Eliminating  $\phi^*$  and  $T$ ,  $\phi$  and  $T$ , and  $\phi$  and  $T$ , in respect from Eqs.(24)-(26), we obtain:

$$[D^6 - AD^4 - BD^2 + C] \phi = f_1, \quad (29a)$$

$$[D^6 - AD^4 - BD^2 + C] \phi^* = f_2, \quad (29b)$$

$$[D^6 - AD^4 - BD^2 + C] T = f_3. \quad (29c)$$

Also eliminating  $\phi_2$  in Eqs.(27)-(28) yields:

$$[D^4 + ED^2 + F] \hat{\psi} = 0, \quad (30)$$

where:  $\xi_{11} = \xi^2 + s^2$ ;  $a_{16} = \xi^2 + a_8 + a_{12}s^2$ ;  $a^{17} = \xi^2 + a_{15}$ ;  $a_{18} = a_{13}a_{15}$ ;  $a_{19} = a_{14}a_{15}$ ;  $a_{20} = a_2\xi^2 - s^2$ ;  $a_{21} = \xi^2 + 2a_6 + a_7s^2$ ;  $f_1 = -Q_1(\tau_{11}\gamma^{*2} + a_{25})$ ;  $f_2 = -Q_1(a_{26}\gamma^{*2} + a_{27})$ ;  $f_3 = -Q_1(\gamma^{*4} - a_{28}\gamma^{*2} + a_{29})$ ;  $f_4 = [\gamma^{*6} - A\gamma^{*4} - B\gamma^{*2} + C]$ ;  $A = -a_{16} - a_{17} - \xi_{11} + a_4a_9 + a_{18}$ ;  $B = a_{10}a_{19} + \xi_{11}a_{16} + \xi_{11}a_{17} - a_4a_9\xi^2 - a_4a_9a_{17} - a_{10}a_{18} - a_9a_{19} - a_{18}\xi^2 - a_{16}a_{18}$ ;  $C = -\xi_{11}a_{16}a_{17} - \xi_{11}a_{10}a_{19} - a_4a_{17}\xi^2 + a_{10}a_{18}\xi^2 + a_{16}a_{18}\xi^2$ ;  $B_2 = (a_3a_6 - a_2a_{21} - a_{20})/a_2$ ;  $B_3 = (a_{20}a_{21} - a_3a_6\xi^2)/a_2$ .

Mathematical solutions of Eqs.(29)-(30) satisfying the radiation conditions that  $(\phi, \phi^*, T, \phi_2, \hat{\psi}) \rightarrow 0$  as  $x_3 \rightarrow \infty$  are given by:

$$\hat{\phi} = B_1 e^{-m_1 x_3} + B_2 e^{-m_2 x_3} + B_3 e^{-m_3 x_3} + L_1, \quad (31)$$

$$\phi^* = d_1 B_1 e^{-m_1 x_3} + d_2 B_2 e^{-m_2 x_3} + d_3 B_3 e^{-m_3 x_3} + L_2, \quad (32)$$

$$T = e_1 B_1 e^{-m_1 x_3} + e_2 B_2 e^{-m_2 x_3} + e_3 B_3 e^{-m_3 x_3} + L_3, \quad (33)$$

$$\hat{\psi} = B_4 e^{-m_4 x_3} + B_5 e^{-m_5 x_3}, \quad (34)$$

$$\phi_2 = h_4 B_4 e^{-m_4 x_3} + h_5 B_5 e^{-m_5 x_3}, \quad (35)$$

where:

$$d_i = \frac{a_9 m_i^4 - (a_9 a_{17} + a_9 \xi^2 - a_{10} a_{18}) m_i^2 + a_9 \xi^2 a_{17} - a_{10} \xi^2 a_{18}}{m_i^4 - (a_{16} + a_{17}) m_i^2 + a_{16} a_{17} + a_{10} a_{19}}$$

$$e_i = \frac{-a_{18} m_i^4 - (a_9 a_{19} + a_{18} \xi^2 + a_{16} a_{18}) m_i^2 - a_9 \xi^2 a_{19} - a_{16} \xi^2 a_{18}}{m_i^4 - (a_{16} + a_{17}) m_i^2 + a_{16} a_{17} + a_{10} a_{19}}$$

$$i = 1, 2, 3; \text{ and } h_l = \frac{a_2 (m_l^2 - \xi_1)}{a_3} \quad (l = 4, 5) \text{ and } m_i^2 \quad (i = 1, 2, 3)$$

are the roots of the characteristic equation given Eq.(29a); and  $m_l^2$  ( $l = 4, 5$ ) are the roots of the characteristic equation of Eq.(30).

### Boundary conditions

We consider concentrated normal force and concentrated thermal source at the boundary surface  $x_3 = 0$ , mathematically, these can be written as:

$$t_{33} = -F_1 \delta(x_1) \delta(t), \quad t_{31} = -F_2 \delta(x_1) \delta(t), \quad m_{32} = 0, \\ \lambda_3^* = 0, \quad \frac{\partial T}{\partial x_3} = 0, \quad (36)$$

where:  $F_1, F_2$  are the magnitudes of applied forces.

Substituting the values of  $\phi$ ,  $\phi^*$ ,  $T$ ,  $\hat{\psi}$ ,  $\phi_2$  from Eqs. (31)-(35) in the boundary condition Eq.(36) and using Eqs. (5)-(7), (9)-(10), (22)-(23), and solving the resulting equations, we obtain:

$$t_{33} = \sum_{i=1}^5 G_{1i} e^{-m_i x_3} + M_1 e^{-\gamma^* x_3}, \quad (37)$$

$$t_{31} = \sum_{i=1}^5 G_{2i} e^{-m_i x_3} + M_2 e^{-\gamma^* x_3}, \quad (38)$$

$$m_{32} = \sum_{i=1}^5 G_{3i} e^{-m_i x_3} + M_3 e^{-\gamma^* x_3}, \quad (39)$$

$$\lambda_3^* = \sum_{i=1}^5 G_{4i} e^{-m_i x_3} + M_4 e^{-\gamma^* x_3}, \quad (40)$$

$$\hat{T} = \sum_{i=1}^5 G_{5i} e^{-m_i x_3} + M_5 e^{-\gamma^* x_3}. \quad (41)$$

Here,

$$b_1 = \frac{\lambda_0}{\rho c_1^2}, \quad b_2 = \frac{\lambda}{\rho c_1^2}, \quad b_3 = \frac{2\mu + K}{\rho c_1^2}, \quad b_5 = \frac{\mu + K}{\rho c_1^2}, \quad b_6 = \frac{\mu}{\rho c_1^2},$$

$$b_7 = \frac{K}{\rho c_1^2}, \quad b_8 = \frac{\omega^* \gamma}{\rho c_1^4}, \quad b_9 = \frac{\omega^* b_0}{\rho c_1^4}, \quad b_{10} = \frac{\omega^*}{\rho c_1^4}, \quad G_{mi} = g_{mi} C_i,$$

$$C_i = \frac{\Delta_i}{\Delta_0}, \quad i = 1, 2, \dots, 5.$$

*Special case*

*Micropolar thermoelastic solid:*

In the absence of microstretch effect in Eqs.(37)-(41), we obtain the corresponding expressions of stresses, displacements, and temperature for micropolar generalised thermoelastic half-space.

*Inversion of the transform*

The transformed displacements, stresses and temperature changes are functions of the parameters of Laplace and Fourier transforms  $s$  and  $\xi$ , respectively, and hence, these are of the form  $f(s, \xi, z)$ . To obtain the solution of the problem in the physical domain, we must invert the Laplace and Fourier transform by using the method applied by Kumar, /19/.

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