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ELASTODYNAMICS OF MECHANICAL FORCES AND HALL CURRENT IN MAGNETO-MICROPOLAR THERMOELASTIC MASS DIFFUSION MEDIUM

ELASTODINAMIKA MEHANIČKIH SILA I HOLOVA STRUJA U MAGNETOMIKRO-POLARNOJ TERMOELASTIČNOJ MASENOJ DIFUZIONOJ SREDINI

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- micropolar thermoelastic
- normal mode analysis
- pulse laser
- concentrated normal force
- concentrated thermal source

Abstract

The present investigation deals with the deformation in micropolar thermoelastic diffusion medium due to inclined load subjected to thermal laser pulse. Normal mode analysis technique is used to solve the problem. The inclined load is assumed to be a linear combination of a normal load and a tangential load. The closed form expressions of normal stress, tangential stress, couple stress, temperature distribution and mass concentration are obtained. A computer programme has been developed to derive the physical quantities numerically. The variation of normal stress, tangential stress, coupled stress; temperature change and mass concentration are depicted graphically to show the effect of relaxation times and mass concentration. Some particular cases of interest are deduced from the present investigation.

Ključne reči
- mikropolarna termoelastičnost
- analiza u normalnom režimu
- impulsni laser
- skoncentrisana normalna sila
- skoncentrisan toplotni izvor

Izvod

U radu je prikazano istraživanje deformacije u mikropolarnoj termoelastičnoj difuzionoj sredini usled kosog opterećenja, pod dejstvom toplotnog laserskog impulsa. Za rešavanje problema se koristi analiza metodom normalnog režima. Koso opterećenje se pretpostavlja kao kombinacija normalnog i tangencijalnog opterećenja. Izrazi za normalni i tangencijalni napon, spregnuti napon, raspodelu temperature i koncentraciju mase, su dobijeni u zatvorenom obliku. Razvijen je računarski program za numeričko rešavanje mehaničkih veličina. Promene u normalnom, tangencijalnom i spregnutom naponu; temperaturi i koncentraciji mase su predstavljene grafički kako bi se prikazao efekat perioda relaksacije i koncentracije mase. Neki specifični slučajevi od interesa se izvode prema predstavljenom istraživanju.

INTRODUCTION

Modern engineering structures are often made up of materials possessing the internal structure. Polycrystalline materials, materials with fibrous or coarse grain structure come into this category. The classical theory of elasticity is not sufficient to explain the phenomenon of high frequency short wavelength and ultrasonic waves. So micropolar theory was developed to overcome the shortcomings of the classical theory of elasticity by considering the granular structure of the material of the medium. The micropolar theory of elasticity is applied to materials for problems where the classical theory of elasticity fails owing to the microstructure of the material. The linear theory of micropolar elasticity was developed by Eringen /1/. Under this theory, solids can undergo macro deformations and micro rotations. Also they can support couple stresses in addition to force stresses. Nowacki /2-4/ extended the micropolar theory of elasticity to include the thermal effects.

Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low-concentration region, and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal diffusion remains a practical process to separate isotopes of noble gases e.g., xenon and other light isotopes e.g., carbon for research purposes. In most of the applications, the concentration is calculated using Fick's law. This is a simple law which does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced, or the effect of temperature of this interaction.

In this research, taking into account the mass concentration effect and radiation of ultra short laser, we have established a model for micropolar thermoelastic medium with mass diffusion. The disturbance due to inclined loads has been studied in the proposed problem. The normal stress, tangential stress, coupled tangential stress, temperature distribution and mass concentration are obtained numerically.

BASIC EQUATIONS

Following Eringen /29/, Sheriff /30/, Kumar /31/, and Al-Qahtani and Datta /32/, the basic equations for homogeneous, isotropic micropolar generalised thermoelastic solid with mass diffusion in the absence of body forces and body couples are given by:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T-\beta_2\cdot$$
$$\cdot\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C+\mu_0\varepsilon_{rji}J_rH_j=\rho\left(\ddot{\boldsymbol{u}}+\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u})+2\boldsymbol{\Omega}\times\frac{\partial u}{\partial t}\right), \quad (1)$$

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho\delta\left(\ddot{\boldsymbol{\phi}}+\boldsymbol{\Omega}\times\frac{\partial\phi}{\partial t}\right), \quad (2)$$

$$K^*\nabla^2T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)(\beta_1T_0\nabla.\dot{\boldsymbol{u}})+$$
$$+aT_0\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C, \quad (3)$$

$$D\beta_2\nabla^2(\nabla.\boldsymbol{u})+Da\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2T+\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-$$
$$-Db\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2C=0, \quad (4)$$

$$t_{ij}=\lambda u_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\varepsilon_{ijk}\phi_k)-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\cdot$$
$$\cdot\delta_{ij}T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\delta_{ij}C, \quad (5)$$

$$m_{ij}=\alpha\phi_{k,k}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}. \quad (6)$$

Here λ, μ, α, β, γ, K, are material constants, ρ is mass density, $\boldsymbol{u}=(u_1, u_2, u_3)$ is the displacement vector and $\boldsymbol{\phi}=(\phi_1, \phi_2, \phi_3)$ is the microrotation vector, T is temperature and T_0 is the reference temperature of the body chosen, C is the concentration of the diffusion material in the elastic body, K^* is the coefficient of the thermal conductivity, c^* is the specific heat at constant strain, D is the thermoelastic diffusion constant, a is the coefficient describing the measure of thermo diffusion and b is the coefficient describing the measure of mass diffusion effects, j is the microinertia, $\beta_1=(3\lambda+2\mu+K)\alpha_{t1}$, $\beta_2=(3\lambda+2\mu+K)\alpha_{c1}$, α_{t1} is the coefficient of linear thermal expansion and α_{c1} is the coefficient of linear diffusion expansion, t_{ij} are components of stress, m_{ij} are components of couple stress, e_{ij} are components of strain, e_{kk} is the dilatation, δ_{ij} is Kronecker delta function, τ^0, τ^1 are the diffusion relaxation times and τ_0, τ^1 are thermal relaxation times with $\tau_0\geq\tau_1\geq 0$. Here $\tau^0=\tau^1=\tau_0=\tau_1=\gamma_1=0$ for Coupled Thermoelastic theory (CT) model. $\tau_1=\tau^1=0$, $\varepsilon=1$, $\gamma_1=\tau_0$ for Lord-Shulman (LS) model, and $\varepsilon=0$, $\gamma_1=\tau^0$, where $\tau^0>0$ for Green-Lindsay (GL) model.

Let the micropolar thermoelastic mass diffusion medium rotate with angular velocity $\boldsymbol{\Omega}$. The equations of motion have two extra terms,

(i) the centripetal acceleration $\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{u})$ due to time varying motion;

(ii) the Coriolis acceleration $2(\boldsymbol{\Omega}\times\dot{\mathbf{u}})$.

The current density vector $\boldsymbol{J}$ can be expressed as:

$$J=\frac{\sigma_0}{1+m^2}\left[\boldsymbol{E}+\mu_0(\dot{\boldsymbol{u}}\times\boldsymbol{H})-\frac{\mu_0}{en_e}(\boldsymbol{J}\times\boldsymbol{H})\right]. \quad (7)$$

Here, $\boldsymbol{F}=\mu_0(\boldsymbol{J}\times\boldsymbol{H})$ is the Lorentz force, $\boldsymbol{H}$ is the magnetic field vector, $\boldsymbol{E}$ is the intensity of electric field, m is the Hall parameter, σ_0 is the electrical conductivity, e is the charge of an electron, n_e is the number density of electrons.

In the above equations symbol (',') followed by a suffix denotes differentiation with respect to spatial coordinates, and a superposed dot ('˙') denotes the derivative with respect to time.

FORMULATION OF THE PROBLEM

We consider a generalised micropolar thermoelastic with mass diffusion medium with rectangular Cartesian coordinate system $0X_1X_2X_3$ with x_3-axis pointing vertically downward the medium.

Suppose that an inclined line load F_0 per unit length is acting on the y-axis and its inclination to z-direction is θ (Fig. 1).

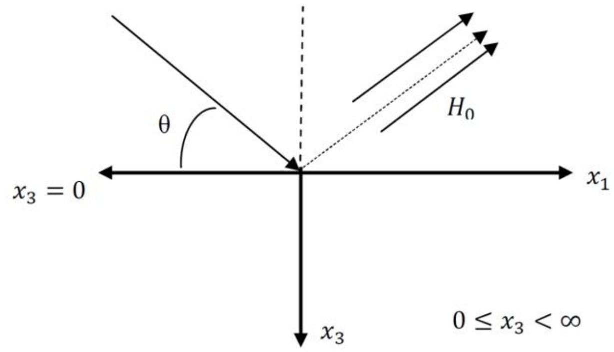

Figure 1. Inclined load and Hall current in a micropolar mass diffusion thermoelastic half-space.

For two-dimensional problems, we take the displacement vector and microrotation vector as:

$$\boldsymbol{u}=(u_1,0,u_3), \quad \boldsymbol{\phi}=(0,\phi_2,0). \quad (8)$$

Let us assume that the magnetic field $\boldsymbol{H}$ and the angular velocity $\boldsymbol{\Omega}$ act in the direction of x_2 axis as:

$$\boldsymbol{H}=(0,H_0,0), \quad (9)$$

$$\boldsymbol{\Omega}=(0,\Omega_0,0). \quad (10)$$

It is also assumed that $\boldsymbol{E}=0$, the generalised Ohm's law gives $\boldsymbol{J}_2=0$ in the medium of consideration. The current density components $\boldsymbol{J}_1$ and $\boldsymbol{J}_3$ are given by:

$$J_1=\frac{\sigma_0B_0}{1+m^2}\left(m\frac{\partial u_1}{\partial t}-\frac{\partial u_3}{\partial t}\right), \quad (11)$$

$$J_3=\frac{\sigma_0B_0}{1+m^2}\left(\frac{\partial u_1}{\partial t}+m\frac{\partial u_3}{\partial t}\right). \quad (12)$$

For further consideration it is convenient to introduce in Eqs.(1)-(5) the dimensionless quantities defined by:

$$u_i'=\frac{\rho\omega^*c_1}{\beta_1T_0}u_i, \quad x_i'=\frac{\omega^*}{c_1}x_i, \quad C'=\frac{\beta_2}{\rho c_1^2}C, \quad T'=\frac{T}{T_0}, \quad t'=\omega^*t,$$

$$\tau_1'=\omega^*\tau_1, \quad \tau_0'=\omega^*\tau_0, \quad t_{ij}'=\frac{1}{\beta_1T_0}t_{ij}, \quad \omega^*=\frac{\rho c^*c_1^2}{K^*},$$

$$\phi_i' = \frac{\rho c_1^2}{\beta_1 T_0}\phi_i\,,\ c_1^2 = \frac{\lambda + 2\mu + k}{\rho}\,,\ m_{ij}^* = \frac{\omega^*}{c\beta_1 T_0} m_{ij}\,,\ \Omega_0' = \frac{\Omega_0}{\omega^*}\,,$$

$$M = \frac{\sigma_0 B_0}{\rho c_1^2 \eta_0}\,,\ \eta_0 = \frac{\rho C_E}{K}\,,\ \varepsilon = \frac{\gamma^2 T_0}{\rho^2 c^* c_1}\,,$$

where: M is the Hartmann number or magnetic parameter.

Using relation Eq.(13) in Eqs.(1)-(4), the component-wise resulting equations are as the following:

$$a_1\frac{\partial e}{\partial x_1} + a_2\nabla^2 u_1 - a_3\frac{\partial \phi_2}{\partial x_3} + a_4\frac{\partial C}{\partial x_1} + \Omega_0^2 u_1 - 2\Omega_0\frac{\partial u_3}{\partial t} + \frac{M}{1+m^2}\cdot \left(\frac{\partial u_1}{\partial t} + \omega\frac{\partial u_3}{\partial t}\right) - \left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1} = \ddot{u}_1\,, \tag{14}$$

$$a_1\frac{\partial e}{\partial x_3} + a_2\nabla^2 u_3 + a_3\frac{\partial \phi_2}{\partial x_1} + a_4\frac{\partial C}{\partial x_3} + \Omega_0^2 u_3 + 2\Omega_0\frac{\partial u_1}{\partial t} - \frac{M}{1+m^2}\cdot \left(\omega\frac{\partial u_1}{\partial t} - \frac{\partial u_3}{\partial t}\right) - \left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3} = \ddot{u}_3\,, \tag{15}$$

$$\nabla^2\phi_2 - 2a_6\phi_2 + a_6\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) = a_7\ddot{\phi}_2\,, \tag{16}$$

$$-\nabla^2 T + \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + a_{13}\left(\frac{\partial}{\partial t} + \varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)e + a_{14}\left(\frac{\partial}{\partial t} + \gamma_1\frac{\partial^2}{\partial t^2}\right)C = 0\,, \tag{17}$$

$$\nabla^2 e + a_9\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2 T + a_{10}\left(1+\varepsilon\tau^0\frac{\partial}{\partial t}\right)\dot{C} - a_{11}\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2 C = 0\,. \tag{18}$$

Here,

$$M_1 = 2\Omega_0 + \frac{M}{1+m^2}\,,\ a_1 = \frac{\lambda+\mu}{\rho c_1^2}\,,\ a_2 = \frac{\mu+K}{\rho c_1^2}\,,\ a_3 = \frac{K}{\rho c_1^2}\,,$$

$$a_4 = \frac{\lambda_0}{\rho c_1^2}\,,\ a_6 = \frac{K c_1^2}{\gamma\omega^{*2}}\,,\ a_7 = \frac{\rho j c_1^2}{\gamma}\,,\ a_8 = \frac{\lambda_1 c_1^2}{\alpha_0\omega^{*2}}\,,\ a_9 = \frac{\lambda_0 c_1^2}{\alpha_0\omega^{*2}}\,,$$

$$a_{10} = \frac{\upsilon_1\rho c_1^4}{\beta_1\alpha_0\omega^{*2}}\,,\ a_{12} = \frac{\rho c_1^2 j_0}{2\alpha_0}\,,\ a_{13} = \frac{\beta_1 T_0^2}{\rho\omega^* K^*}\,,\ a_{14} = \frac{\upsilon_1\beta_1 T_0}{\rho\omega^* K^*}\,,$$

and $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$ is the Laplacian operator.

The displacement components u_1 and u_3 are related to the non-dimensional potential functions ϕ and ψ as:

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}\,,\quad u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}\,. \tag{19}$$

Substituting the values of u_1 u_3 from Eq.(19) in Eqs.(14)-(18), and with the aid of Eq.(13), we obtain:

$$\left(\nabla^2\phi + \Omega_0^2 - \frac{M}{1+m^2}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right)\phi - M_1\frac{\partial\psi}{\partial t} + a_4 C - \left(1+\tau_1\frac{\partial}{\partial t}\right)T = 0\,, \tag{20}$$

$$-\nabla^2 T + \left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T} + a_{13}\left(\frac{\partial}{\partial t} + \varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\phi + a_{14}\left(\frac{\partial}{\partial t} + \gamma_1\frac{\partial^2}{\partial t^2}\right)C = 0\,, \tag{21}$$

$$\nabla^4\phi + a_9\left(1+\tau_0\frac{\partial}{\partial t}\right)\nabla^2 T + a_{10}\left(1+\varepsilon\tau^0\frac{\partial}{\partial t}\right)\dot{C} - a_{11}\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2 C = 0\,, \tag{22}$$

$$\left(\Omega_0^2 a_2\nabla^2 - \frac{M}{1+m^2}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right)\psi + M_1\frac{\partial\phi}{\partial t} + a_3\phi_2 = 0\,, \tag{23}$$

$$\nabla^2\phi_2 - 2a_6\phi_2 - a_6\nabla^2\psi = a_7\ddot{\phi}_2\,, \tag{24}$$

Here, $a_1 = \frac{\lambda+\mu}{\rho c_1^2}$, $a_2 = \frac{\mu+K}{\rho c_1^2}$, $a_3 = \frac{K}{\rho c_1^2}$, $a_4 = \frac{\rho c_1^2}{\beta_1 T_0}$,

$a_6 = \frac{K c_1^2}{\gamma\omega^{*2}}$, $a_7 = \frac{\rho j c_1^2}{\gamma}$, $a_5 = \frac{\beta_1^2 T_0}{\rho K^*\omega^*}$, $a_8 = \frac{a\rho c_1^4}{\omega^*\beta_2 K^*}$,

$a_9 = \frac{a\rho c_1^2}{\beta_1\beta_2}$, $a_{10} = \frac{\rho c_1^4}{D\omega^*\beta_2^2}$, $a_{11} = \frac{b\rho c_1^2}{\beta_2^2}$, and here

$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$ is the Laplacian operator.

SOLUTION OF THE PROBLEM

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:

$$\{\phi,\psi,T,\phi_2,C\}(x_1,x_3,t) = \{\bar{\phi},\bar{\psi},\bar{T},\bar{\phi}_2,\bar{C}\}(x_3)e^{i(kx_1-\omega t)}\,. \tag{25}$$

Here ω is the angular velocity and k is wave number.

Making use of Eq.(25) and Eqs.(20)-(24), after some simplifications, we obtain:

$$[A\boldsymbol{D}^{10} + B\boldsymbol{D}^8 + C\boldsymbol{D}^6 + E\boldsymbol{D}^4 + F\boldsymbol{D}^2 + G](\bar{\phi},\overline{\phi^*},\bar{T},\bar{\phi}_2,\bar{\psi}) = 0\,. \tag{26}$$

The solution of the above Eq.(26) satisfying the radiation conditions that ($\bar{\phi},\bar{\psi},\bar{T},\bar{\phi}_2,\bar{C}$) $\to 0$ as $x_3 \to \infty$, are given as following:

$$(\bar{\phi}) = \textstyle\sum_{i=1}^{3} c_i e^{-m_i x_3}\,, \tag{27}$$

$$\bar{T} = \textstyle\sum_{i=1}^{3} \alpha_i c_i e^{-m_i x_3}\,, \tag{28}$$

$$\bar{C} = \textstyle\sum_{i=1}^{3} \beta_i c_i e^{-m_i x_3}\,, \tag{29}$$

$$(\bar{\psi},\overline{\phi_2}) = \textstyle\sum_{i=4}^{5} (1,\delta_i) c_i e^{-m_i x_3}\,, \tag{30}$$

where: m_i^2 (i = 1,2,…,5) are the roots of Eq.(26); and $\alpha_i = -\Delta_{2i}/\Delta_{1i}$, $\beta_i = -\Delta_{3i}/\Delta_{1i}$, i = 1,2,3, and $\delta_i = -\Delta_{5i}/\Delta_{4i}$, i = 4,5. Here, Δ_{1i}, Δ_{2i}, … are given in Appendix B.

Substituting the values of $\bar{\phi},\bar{\psi},\bar{T},\bar{\phi}_2,\bar{C}$ from Eqs.(27)-(30) in Eqs.(5)-(6), and after that solving the resulting equations and simplifying, we obtain:

$$t_{33} = \textstyle\sum_{i=1}^{5} G_{1i} e^{-m_i x_3} e^{i(kx_1-\omega t)}\,, \tag{31}$$

$$t_{31} = \textstyle\sum_{i=1}^{5} G_{2i} e^{-m_i x_3} e^{i(kx_1-\omega t)}\,, \tag{32}$$

$$m_{32} = \textstyle\sum_{i=1}^{5} G_{3i} e^{-m_i x_3} e^{i(kx_1-\omega t)}\,, \tag{33}$$

$$T = \textstyle\sum_{i=1}^{5} G_{4i} e^{-m_i x_3} e^{i(kx_1-\omega t)}\,, \tag{34}$$

$$C=\sum_{i=1}^{5} G_{5i} e^{-m_i x_3} e^{i(kx_1-\omega t)}\,, \tag{35}$$

where: $G_{mi} = g_{mi}C_i$, $i = 1,2,\ldots,5$.

Boundary conditions

We consider normal and tangential force acting at the surface $x_3 = 0$ along with vanishing of couple stress in addition to thermal and mass concentration boundaries considered at $x_3 = 0$. Mathematically this can be written as:

$$t_{33}=-F_1 e^{-(kx_1-\omega t)}\,,\ t_{31}=-F_2 e^{-(kx_1-\omega t)}\,,\ m_{32}=0\,,$$

$$h_1\frac{\partial T}{\partial x_3}+h_2 T=0\,,\ h_3\frac{\partial C}{\partial x_3}+h_4 C=0\,, \tag{36}$$

where: F_1 and F_2 are the magnitude of the applied force.

Here, h_2, $h_4 \to 0$ corresponds to insulated impermeable boundaries. Similarly, h_1, $h_3 \to 0$ corresponds to isothermal and isoconcentrated boundaries.

Substituting the expression of the variables considered into these boundary conditions Eq.(36), we can obtain the following equations:

$$\sum_{i=1}^{5} g_{1i}c_i=-F_1\,, \tag{37}$$

$$\sum_{i=1}^{5} g_{2i}c_i=-F_2\,, \tag{38}$$

$$\sum_{i=1}^{5} g_{3i}c_i=0\,, \tag{39}$$

$$\sum_{i=1}^{5} (h_2-m_i h_1) g_{1i}c_i=0\,, \tag{40}$$

$$\sum_{i=1}^{5} (h_4-m_i h_3) g_{1i}c_i=0\,. \tag{41}$$

The system of Eqs.(37)-(41) are solved by using the matrix method as follows:

$$\begin{bmatrix} c_1\\ c_2\\ c_3\\ c_4\\ c_5 \end{bmatrix}=\begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15}\\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25}\\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35}\\ (h_2-m_1h_1)g_{41} & (h_2-m_2h_1)g_{42} & (h_2-m_3h_1)g_{43} & (h_2-m_4h_1)g_{44} & (h_2-m_5h_1)g_{45}\\ (h_4-m_1h_3)g_{51} & (h_4-m_2h_3)g_{52} & (h_4-m_3h_3)g_{53} & (h_4-m_4h_3)g_{54} & (h_4-m_5h_3)g_{55} \end{bmatrix}^{-1}\begin{bmatrix} -F_1\\ -F_2\\ 0\\ 0\\ 0 \end{bmatrix}, \tag{42}$$

Special cases

(a) *Micropolar thermoelastic mass diffusion solid*: in absence of Hall effect and rotation in Eqs.(37)-(41), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic mass diffusion solid.

(b) *Magneto-micropolar thermoelastic solid*: in absence of mass concentration effect in Eqs.(37)-(41), we obtain the corresponding expressions of stresses, displacements, and temperature for magneto-micropolar generalised thermoelastic solid with rotation.

(c) *Micropolar thermoelastic solid*: in absence of Hall effect and mass-diffusion effect in Eqs.(37)-(41), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar generalised thermoelastic solid.

Applications

Inclined line load

For an inclined line load F_0 we have:

$$F_1=F_0\cos\theta\,,\quad F_2=F_0\sin\theta\,. \tag{43}$$

Making use of Eq.(43) in Eqs.(37)-(41), we obtain the corresponding expressions of normal and tangential stress, couple stress, temperature distribution, and mass concentration due to inclined load.

Particular cases

(i) If we take $\tau_1 = \tau^1 = 0$, $\varepsilon = 1$, in Eqs.(37)-(41) and with the aid of Eq.(36), we obtain corresponding expressions of stresses, displacements, and temperature distribution for micropolar mass diffusion thermoelastic half space with one relaxation time.

(ii) If we take $\varepsilon = 0$ in Eqs.(37)-(41) and with the aid of Eq. (36), the corresponding expressions of stresses, displacements, and temperature distribution are obtained for micropolar mass diffusion thermoelastic half space with two relaxation times.

(iii) Taking $\tau^0 = \tau^1 = \tau_0 = \tau_1 = 0$ in Eqs.(37)-(41) and with the aid of Eq.(36) yield the corresponding expressions of stresses, displacements, and temperature distribution for micropolar mass diffusion coupled thermoelastic half space.

(iv) If $\theta = 0$ corresponds to normal load.

(v) If $\theta = \pi/2$ corresponds to tangential load.

CONCLUSION

The results obtained here are useful in engineering problems, particularly in the determination state of stresses in a micropolar thermoelastic mass diffusion medium. Also, any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the problem.

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Appendix A:

$A = K_{13}L_{10}$; $B = L_2L_3 + K_{12}L_{10} - K_{15}K_{13}L_{10}$; $C = L_1L_3 + L_2L_4 + L_7 + K_{13}L_{12} + K_{12}L_{11} - K_{13}L_{11} - K_{15}K_{12}L_{10}$; $E = L_1L_4 + L_2L_5 + L_8 + K_{12}L_{12} + K_{13}L_{13} - K_{15}(L_7 + K_{13}L_{12} + K_{12}L_{11} - K_{13}L_{11})$; $F = L_1L_5 + L_2L_6 + L_9 + K_{12}L_{13} - K_{15}(L_8 + K_{12}L_{12} + K_{13}L_{13})$; $G = L_1L_5 - K_{15}(L_9 + K_{12}L_{13})$; $L_1 = -a_3K_{14}$; $L_2 = a_3a_6$; $L_3 = K_3 - K_{10}$; $L_4 = K_6K_{10} - K_9 - K_1K_{10} + K_2K_7 - 2k^2K_3 - K_3K_6$; $L_5 = K_1K_6K_{10} - K_1K_9 + K_6K_9 - K_7K_8 - K_2K_5K_{10} - 2k^2K_2K_7 + k^4K_3 + 2k^2K_3K_6$; $L_6 = K_1K_6K_9 - K_1K_7K_8 - K_2K_5K_9 + K_3K_5K_8 - k^4K_3K_6$; $L_7 = K_{11}K_4K_{10}$; $L_8 = -K_4K_{11}(K_6K_{10} - K_9)$; $L_9 = -K_4K_{11}(K_6K_9 - K_7K_8)$; $L_{10} = -K_{10} - K_3$; $L_{11} = K_6K_{10} - K_9 + K_2K_7 + K_3K_6 + 2k^2K_3$; $L_{12} = K_1K_6K_{10} - K_2K_5K_{10} - 2k^2(K_2K_7 + K_3K_6) - k^4K_3$; $L_{13} = K_1K_6K_9 - K_1K_7K_8 - K_2K_5K_9 + k^4K_2K_7 - K_3K_5K_8 + k^4K_3K_6$; $K_1 = \Omega_0^2 - k^2 - \omega^2 + (i\omega M/(1+m^2))$; $K_2 = -1 + i\omega\tau_1$; $K_3 = a_4$; $K_4 = i\omega M_1$; $K_5 = -a_{13}(i\omega + \omega^2\varepsilon\tau_0)$; $K_6 = k^2 - i\omega - \omega^2\tau_0$; $K_7 = -a_{14}(i\omega + \gamma_1\omega^2)$; $K_8 = a_9(1 - i\omega\tau_1)$; $K_9 = -a_{10}(i\omega + \varepsilon\tau^0\omega^2) + a_{11}k^2(1 - i\omega\tau^1)$; $K_{10} = -a_{11}(1 - i\omega\tau^1)$; $K_{11} = - i\omega M$; $K_{12} = -a_2\Omega_0^2k^2 + \omega^2 + (i\omega M/(1+m^2))$; $K_{13} = a_2\Omega_0^2$; $K_{14} = a_6k^2$; $K_{15} = a_7\omega^2 - k^2 - 2a_6$.

Appendix B:

$\Delta_{1i} = -b_1b_4 + b_1b_5(m_i^2 - k^2) + b_2b_3(m_i^2 - k^2)$; $\Delta_{2i} = b_4(m_i^2 - k_1) - b_5(m_i^2 - k_1)(m_i^2 - k^2) + b_2(m_i^4 + k^4 - 2m_i^2k^2)$; $\Delta_{3i} = b_3(m_i^2 - k_1)(m_i^2 - k^2) + b_1(m_i^4 + k^4 - 2m_i^2k^2)$, $i = 1,2,3$; $\Delta_{4i} = a_6(m_i^2 - k^2)$; $\Delta_{5i} = (m_i^2 - k^2 - 2a_6 - a_7\omega^2)$, $i = 4,5$.