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RAYLEIGH WAVE PROPAGATION IN THERMOELASTIC DIFFUSION MEDIUM UNDER GREEN-NAGHDI MODELS

PROSTIRANJE REJLEJEVOG TALASA U TERMOELASTIČNOJ DIFUZIONOJ SREDINI SHODNO MODELIMA GRINA-NAGDIJA

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- Rayleigh wave
- diffusion
- thermoelastic
- phase velocity
- attenuation coefficient

Abstract

In this work, Rayleigh wave propagation in thermoelastic diffusion medium in the context of the Green-Naghdi theories of thermoelasticity (type II and type III) /1, 2/ is studied. We consider the effect of thermal and diffusion parameters on the propagation of Rayleigh waves. The propagation condition in complex irrational form after some algebraic manipulations is obtained in the form of polynomial equation of 15th degree at the stress free, isothermal, and isoconcentrated boundary. The roots of this polynomial equation are checked for satisfying the original dispersion equation and the property of decay with depth. The behaviour of particle motion is studied inside and at the surface of the medium. Furthermore, we use the numerical methods and computations to calculate the propagation characteristics like phase velocity, attenuation coefficient and particle path for the copper material. Some special cases are also discussed.

Ključne reči
- Rejlejev talas
- difuzija
- termoelastičnost
- fazna brzina
- koeficijent prigušenja

Izvod

U radu se proučava prostiranje Rejlejevog talasa u termoelastičnoj difuzionoj sredini shodno teorijama termoelastičnosti Grin-Nagdija (tip II i tip III), /1, 2/. Razmatra se uticaj toplotnih i difuzionih parametara na prostiranje Rejlejevih talasa. Posle nekih algebarskih izvođenja, uslov prostiranja koji je složenog iracionalnog oblika, dobija oblik polinoma 15-tog reda na izokoncentracionoj, izotermskoj granici, na kojoj nema napona. Izvršena je provera korena ovog polinoma, radi zadovoljavanja početne relacije disperzije, kao i osobine prigušenja sa dubinom. Ponašanje kretanja čestica se razmatra u kako u unutrašnjosti, tako i na površini date sredine. Zatim, primenjujemo numeričke metode i proračun za određivanje karakteristika prostiranja, na pr. fazne brzine, koeficijenta prigušenja i za putanju čestice bakarnog materijala. Takođe, data je diskusija još nekih specijalnih slučajeva.

INTRODUCTION

Biot /3/ formulated the coupled thermoelasticity theory to eliminate the paradox inherent in the classical uncoupled theory that elastic deformation has no effect on the temperature. The generalised theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity.

The theory of thermoelasticity introduced by Green and Naghdi /1/ is one of the generalised theories of thermoelasticity. They postulated a new concept of thermoelasticity which is called thermoelasticity without energy dissipation. In this theory, the classical Fourier law is replaced by a heat flux rate-temperature gradient relation. The general idea is postulated by Green and Naghdi /2/ in making use of the general entropy balance. Three types of the constitutive response functions are suggested. Type I, after linearization of the theory, is the same as the classical heat conduction theory (based on Fourier's law), while the types II and III permit propagation of thermoelastic disturbances with a finite speed, only type II without energy dissipation. Also GN model III of thermoelasticity theory involves a heat conduction law and one that involves the thermal displacement gradient among the constitutive variables.

Thermodiffusion in an elastic solid is due to the coupling of the fields of temperature, mass diffusion and that of strain. Nowacki /4-6/ developed the theory of thermoelastic diffusion using coupled thermoelastic model. Nowacki /7/ derived the basic equations for generalised thermoelastic diffusion. Sherief and Saleh /8/ developed the generalised theory of thermoelastic diffusion with one relaxation time which allows finite wave propagation speeds. When diffusion effects are considered, Kumar and Kansal /9/ derived the basic equations for generalised thermoelastic diffusion (GL model) and discussed the Lamb waves. Sharma /10/ and Sharma et al. /11/ investigated plane harmonic generalised thermoelastic diffusive waves and elasto-thermodiffusive surface waves in heat conducting solids.

The existence of Rayleigh waves on the free surface of an infinite homogeneous isotropic elastic solid was predicted by Rayleigh /12/. Lockett /13/ was the first who studied the problem of Rayleigh wave propagation in thermoelastic media. Buchwald /14/ studied the condition for the existence of Rayleigh waves in transversely isotropic media. Chadwick and Windle /15/ studied the propagation of Rayleigh waves for isothermal and insulated boundaries. Using the generalised theory of thermoelasticity, Sinha and Sinha /16/ studied the propagation of Rayleigh waves in thermoelastic media and obtained the asymptotic solutions.

Currie et al. /17/ solved the complex dispersion equation in viscoelastic medium for surface waves by rationalising this equation to the polynomial form. The effect of Rayleigh waves on a viscoelastic solid half space was studied by Romeo /18/. Stresses and displacements for some Rayleigh-type surface acoustic waves propagating on an anisotropic half space were demonstrated by Royer /19/. Kumar and Kansal /20-21/ discussed the propagation of Rayleigh waves in a homogeneous isotropic and transversely isotropic generalised thermoelastic diffusion half space, respectively. Rayleigh waves in rotating thermoelastic solid with voids was studied by Sharma and Kaur /22/. Abouelregal /23/ illustrated the effect of coupling parameter and phase-lags on Rayleigh waves in a thermoelastic solid half space. Singh et al. /24/ discussed the Rayleigh waves in a rotating magneto-thermo-elastic half-plane. Sharma /25, 26/ studied the propagation of Rayleigh waves in a partially saturated porous medium and dissipative poro-viscoelastic media, respectively. He calculated the polarization of Rayleigh waves at different depths in the considered medium. Sharma /27/ studied the propagation of Rayleigh waves in a generalised thermoelastic medium for isothermal or insulated surface.

For isothermal and isoconcentrated boundary, Kumar and Gupta /28/ studied the propagation of Rayleigh wave in thermoelastic medium with mass diffusion in the context of the Lord-Shulman /29/ and Green-Lindsay /30/ theories of thermoelasticity. Pathania and Kumar /31/ studied the thermo-mechanics of magneto-micropolar thermoelastic half space and obtained displacement, stress components and temperature distribution. Kumar and Ailawalia /32/ established a model of generalised magnetomicropolar thermoelasticity with two relaxation times.

In this present investigation, propagation of the Rayleigh wave is studied for Green-Naghdi theories of thermoelasticity (type II and type III) /1, 2/. The phase velocity, attenuation coefficient and path of surface particles of Rayleigh wave propagation are obtained from the secular equations. Using numerical examples, mathematical results are graphically illustrated.

BASIC EQUATIONS

The basic equations for a homogeneous isotropic generalised thermoelastic with mass diffusion (for Green-Naghdi theory of type III) in absence of body forces, heat sources, and mass diffusion sources are:

The constitutive relations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{kk} - \beta_1 T - \beta_2 C], \tag{1}$$

$$\rho T_0 S = \rho C_E T + \beta_1 T_0 e_{kk} + a T_0 C, \tag{2}$$

$$P = -\beta_2 e_{kk} - aT + bC. \tag{3}$$

The Fourier law (in the Green-Naghdi theory of type-III) is given by Chandrasekharaiah /17/ as

$$\mathbf{q} = -(K\nabla T + K^* \nabla \nu), \tag{4}$$

where: $\dot{\nu} = T$.

Fick's law

$$\boldsymbol{\eta} = -(D\nabla P + D^* \nabla \kappa), \tag{5}$$

where: $\dot{\kappa} = P$.

Mass concentration law

$$-\nabla \boldsymbol{\eta} = \dot{C}. \tag{6}$$

Equations of motion

$$(\lambda+\mu)\nabla(\nabla . u) + \mu \nabla^2 u - \beta_1 \nabla T - \beta_2 \nabla C = \rho \ddot{u}. \tag{7}$$

Equation of heat conduction

$$\left(K + K^* \frac{\partial}{\partial t}\right)\nabla^2 T = \rho C_E \ddot{T} + \beta_1 T_0 \ddot{e}_{kk} + a T_0 \ddot{C}. \tag{8}$$

Equation of mass diffusion

$$\left(D + D^* \frac{\partial}{\partial t}\right)\left[\beta_2 \nabla^2 \phi - b\nabla^2 C + a\nabla^2 T\right] + \ddot{C} = 0, \tag{9}$$

where: λ, μ are Lame's constants; ρ is density assumed to be independent of time; D is diffusivity; P is chemical potential per unit mass; C is concentration; u_i are components of displacement vector $\mathbf{u}$; K is coefficient of thermal conductivity; ν is thermal displacement; κ is chemical potential displacement; K^* and D^* are material thermal and diffusion constant characteristics, C_E is specific heat at constant strain; $T = \Theta - T_0$ is small temperature increment; Θ is absolute temperature of the medium; T_0 is reference temperature of the body chosen such that $|(T/T_0)| << 1$; a and b are in respect, the coefficients describing the measure of thermo-diffusion and mass diffusion effects; σ_{ij}, e_{ij} are components of stress and strain, respectively; e_{kk} is dilatation; S is entropy per unit mass; $\beta_1 = (3\lambda + 2\mu)\alpha_t$ and $\beta_2 = (3\lambda + 2\mu)\alpha_c$; α_t is coefficient of thermal linear expansion; α_c is coefficient of linear diffusion expansion. In the above equations, a comma followed by a suffix denotes spatial derivative, and a superposed dot denotes the derivative with respect to time.

SOLUTION OF THE FIELD EQUATIONS

Using Helmholtz resolution of the displacement vector $\mathbf{u}$, we have

$$u = \nabla\phi + \nabla \times \psi, \quad \nabla . \psi = 0. \tag{10}$$

Making use of Eq.(10) in Eqs.(7)-(9), we have

$$(\lambda + 2\mu)\nabla^2 \phi - \beta_1 T - \beta_2 C = \rho \ddot{\phi}, \tag{11}$$

$$\mu \nabla^2 \boldsymbol{\psi} - \rho \ddot{\boldsymbol{\psi}} = 0, \tag{12}$$

$$\left(K^* + K \frac{\partial}{\partial t}\right)\nabla^2 T = \rho C_E \ddot{T} + \beta_1 T_0 \nabla^2 \ddot{\phi} + a T_0 \ddot{C}, \tag{13}$$

$$\left(D^* + D \frac{\partial}{\partial t}\right)\left[\beta_2 \nabla^4 \phi - b\nabla^2 C + a\nabla^2 T\right] + \ddot{C} = 0. \tag{14}$$

To facilitate the solution, the following dimensionless quantities are introduced:

$$x_i' = \frac{x_i}{L}, \quad \phi' = \frac{\phi}{L^2}, \quad \psi' = \frac{\psi}{L^2}, \quad t' = \frac{c_1}{L}t, \quad T' = \frac{\beta_1}{\rho c_1^2}T,$$
$$C' = \frac{\beta_2}{\rho c_1^2}C, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad i = 1, 3, \tag{15}$$

where: L is arbitrary length.

Equations (11)-(14) with the aid of Eq.(15) yield

$$\nabla^2\phi - T - C = \ddot{\phi}, \tag{16}$$

$$\nabla^2\boldsymbol{\psi} = \frac{1}{\beta^2}\ddot{\boldsymbol{\psi}}, \tag{17}$$

$$\left(K^* + g_1\frac{\partial}{\partial t}\right)\nabla^2 T = g_2\ddot{T} + g_3\ddot{C} + g_4\nabla^2\ddot{\phi}, \tag{18}$$

$$\left(D^* + g_5\frac{\partial}{\partial t}\right)\left[\nabla^4\phi - g_6\nabla^2 C + g_7\nabla^2 T\right] + g_8\ddot{C} = 0, \tag{19}$$

where: $g_1 = \frac{Kc_1}{L}$; $g_2 = \rho C_E c_1^2$; $g_3 = \frac{aT_0 c_1^2\beta_1}{\beta_2}$; $g_4 = \frac{\beta_1^2 T_0 L^2}{\rho c_1^2}$;

$g_5 = \frac{Dc_1}{L}$; $g_6 = \frac{b\rho c_1^2}{\beta_2^2}$; $g_7 = \frac{a\rho c_1^2}{\beta_1\beta_2}$; $g_8 = \frac{\rho L c_1^3}{\beta_2^2}$.

Equation (17) represents the propagation of transverse wave with velocity $\beta = \sqrt{(\mu/\rho)}$. Real value of β shows that transverse wave travels without attenuation in thermoelastic diffusion medium.

For time harmonic vibrations ($\sim e^{-i\omega t}$) in $x_1 - x_3$ plane having ω as the angular frequency, Eqs.(16)-(19) yield

$$(\nabla^2 + \omega^2)\phi - T - C = 0, \tag{20}$$

$$\left(\nabla^2 + \frac{\omega^2}{\beta^2}\right)\boldsymbol{\psi} = \mathbf{0}, \tag{21}$$

$$g_{10}\nabla^2 T + \omega^2(g_2 T + g_3 C + g_4\nabla^2\phi) = 0, \tag{22}$$

$$g_9[\nabla^4\phi - g_6\nabla^2 C + g_7\nabla^2 T] - \omega^2 g_8 C = 0, \tag{23}$$

where: $g_9 = D^* - i\omega g_5$; $g_{10} = K^* - i\omega g_1$.

Equations (20), (22) and (23) after simplification give

$$[A_1\nabla^6 + A_2\nabla^4 + A_3\nabla^2 + A_4]\phi = 0, \tag{24}$$

where: $A_1 = \zeta_1 + \zeta_7$; $A_2 = \omega^2\zeta_1 + \zeta_2 + \zeta_4 + \zeta_6$; $A_3 = \omega^2\zeta_2 + \zeta_3 + \zeta_5$; $A_4 = \omega^2\zeta_3$; $\zeta_1 = g_6 g_9 g_{10}$; $\zeta_2 = \omega^2 g_8 g_{10} + \omega^2 g_9 (g_2 g_6 + g_3 g_7)$; $\zeta_3 = \omega^4 g_2 g_8$; $\zeta_4 = \omega^2 g_9 (g_4 g_6 + g_3)$; $\zeta_5 = \omega^4 g_4 g_8$; $\zeta_6 = \omega^2 g_9 (g_4 g_7 + g_2)$; $\zeta_7 = g_9 g_{10}$. The general solution of Eq.(24) can be written as

$$\phi = \sum_{i=1}^{3}\phi_i, \tag{25}$$

where: potentials ϕ_i, i = 1, 2, 3 are solution of wave equations, given by

$$\left[\nabla^2 + \frac{\omega^2}{V_i^2}\right]\phi_i = 0, \quad i = 1,2,3. \tag{26}$$

Here, V_1, V_2, V_3 are the velocities of three longitudinal waves, namely P, MD, and T wave, in descending order of their real part, and are derived from the roots of quadratic equation in V^2, given by

$$A_4 V^6 - A_3 V^4 + A_2 V^2 - A_1\omega^6 = 0. \tag{27}$$

The complex value of these velocities shows that these waves are attenuated. Equations (20), (22)-(23) with the aid of Eqs.(25) and (26) yield

$$\{T, C\} = \sum_{i=1}^{3}\{n_i, k_i\}\phi_i, \tag{28}$$

where: $n_i = \frac{\omega^2\left(\omega^2 V_i^2 g_3 - \omega^2 g_3 - \omega^2 g_4\right)}{g_{10}\omega^2 + V_i^2\left(\omega^2 g_3 - \omega^2 g_2\right)}$;

$k_i = \frac{\omega^4\left(g_{10} - V_i^2(g_2 + g_{10} + g_4) + \omega^2 g_2 V_i^4\right)}{\omega^2 V_i^4(g_2 - g_3) - \omega^2 V_i^2 g_{10}}$, i = 1,2,3.

FORMULATION OF THE PROBLEM

We consider the propagation of homogeneous plane wave in the x_1 direction and with attenuation in the half-space $x_3 \geq 0$ which is an isotropic elastic thermally conducting medium with mass diffusion at a uniform temperature T_0. The surface $x_3 = 0$ is subjected to stress free isothermal and isoconcentrated boundary.

The displacement vector, temperature change, and mass concentration in x_1–x_3 plane with taken as:

$$\mathbf{u} = (u_1, 0, u_3), \quad \mathbf{T}(x_1, x_3, t), \quad \mathbf{C}(x_1, x_3, t). \tag{29}$$

From Eqs.(10), (25) and (29), we have

$$u_1 = \sum_{i=1}^{3}\frac{\partial\phi_i}{\partial x_1} + \frac{\partial\phi_4}{\partial x_3}, \quad u_3 = \sum_{i=1}^{3}\frac{\partial\phi_i}{\partial x_3} - \frac{\partial\phi_4}{\partial x_1}, \tag{30}$$

where: $\phi_4 = (-\boldsymbol{\Psi})_y$. For the propagation of plane harmonic waves with exponential decay in x_1-x_3 plane, we use the displacement potentials as

$$\phi_i = A_j e^{i\omega\left(\frac{x_1}{c} - t\right) - \frac{q_j\omega x_3}{c}}, \quad j = 1,2,3,4, \tag{31}$$

where: $q_j = \sqrt{\frac{c^2}{V_j^2} - 1}$, j = 1, 2, 3; $q_4 = \sqrt{\frac{c^2}{\beta^2} - 1}$; and c is the apparent phase velocity. Also it is required that imaginary parts of the vertical slowness q_j/c, j = 1, 2, 3, 4 should be positive.

BOUNDARY CONDITIONS

Boundary conditions to be satisfied at the interface $x_3 = 0$ are the vanishing of normal and tangential component of stress, isothermal, isoconcentrated boundary surface. Mathematically, these can be written as

(i) $$\sigma_{33} = 0, \tag{32}$$

(ii) $$\sigma_{31} = 0, \tag{33}$$

(iii) $$T = 0, \tag{34}$$

(iv) $$C = 0. \tag{35}$$

Making use the value of ϕ_I from Eq.(31) in the boundary conditions Eqs.(32)-(35) and with the aid of Eqs.(1), (28) and Eqs.(29)-(31), we get a system of four homogeneous equations which can be written as

$$\sum_{j=1}^{4} d_{ij}A_j = 0, \tag{36}$$

where: $d_{1j} = -\omega^2\Pi_j + 2\frac{\omega^2}{h} - \beta_1\frac{n_j}{\rho} - \beta_2\frac{k_j}{\rho}$; $d_{2j} = 2q_j$; $d_{3j} = n_j$; $d_{4j} = k_j$; $d_{14} = 2(\omega^2 q_4)/h$; $d_{24} = q_4^2 - 1$; $d_{34} = d_{44} = 0$, j = 1, 2, 3.

The system of Eqs.(36) have a non-trivial solution if the determinant of the coefficients of this system vanishes, which yield the dispersion equation for propagation of Rayleigh waves as

$$(2-h)\left[(2-\Pi_1 h)+p_1(2-\Pi_2 h)+p_2(2-\Pi_3 h)\right]= \\ =-4(q_1+p_1q_2+p_2q_3)q_4\,, \quad (37)$$

where: $h=\dfrac{c^2}{\beta^2}$; $p_1=\dfrac{n_3k_1-n_1k_3}{n_2k_3-n_3k_2}$; $p_2=\dfrac{n_1k_2-n_2k_1}{n_2k_3-n_3k_2}$; and $\Pi_j=\dfrac{(\lambda+2\mu)}{\rho V_j^2}$, j = 1, 2, 3.

As Eq.(37) is an irrational, so it cannot be solved through algebraic methods. After three squaring and some algebraic manipulations, the equation is reduced to a polynomial form of degree 15, which can be written as follows:

$$\sum_{j=0}^{15} C_j h_j = 0\,, \quad (38)$$

where: coefficients C_j are given in the Appendix A.

In the fifteen roots of algebraic Eq.(38), some roots are those which are added while converting irrational Eq.(37) to the polynomial Eq.(38). These roots are identified for not satisfying the original dispersion Eq.(37). The remaining roots, which satisfy Eq.(37) are again verified for the decay of the wavefield with increase in x_3, i.e. as we move away from the surface. The roots which satisfy both, the checks represent the existence and propagation of Rayleigh waves in the generalised thermoelastic with mass diffusion. In this case only one root is obtained which satisfies both checks. The value of phase velocity calculated from the root of Eq.(38) depends upon the frequency ω ensuring that Rayleigh wave is dispersive in generalised thermoelastic with mass diffusion. The complex value of c shows that Rayleigh waves are attenuated. Also it shows that these waves are inhomogeneous waves, which decay as we move away from the surface. For the complex c, the positive imaginary parts of the vertical slowness q_j/c, j = 1,2,3,4 in Eq.(31) ensures the decay of these waves in the region $x_3 > 0$.

The phase velocity and attenuation coefficient of the Rayleigh wave is calculated by using the expressions:

$$V=\frac{|c^2|}{\mathrm{Re}(c)}=\frac{\beta(h)}{\mathrm{Re}(\sqrt{h})}\,, \quad (39)$$

$$Q^{-1}=\frac{\mathrm{Im}\left(\dfrac{1}{c^2}\right)}{\mathrm{Re}\left(\dfrac{1}{c^2}\right)}=-\frac{\mathrm{Im}(h)}{\mathrm{Re}(h)}\,. \quad (40)$$

POLARIZATION

Displacement potentials Eq.(31) can be rewritten as

$$\phi_i = A_1\gamma_j e^{i(kx_1-\omega t)+ikx_3q_j}\,,\; j = 1,2,3,4\,, \quad (41)$$

where: $k = \omega/c$ is complex wave number; and $\gamma_j = A_j/A_1$, j = 1,2,3,4 are the solutions of system of Eqs.(36):

$$\gamma_1=1,\; \gamma_2=\frac{n_3(n_1k_2-n_2k_1)}{n_2(n_2k_3-n_3k_2)}-\frac{n_1}{n_2}\,,\; \gamma_3=\frac{n_1k_2-n_2k_1}{n_2k_3-n_3k_2}\,,$$

$$\gamma_4=\frac{2q_1+2q_2\left(\dfrac{n_3}{n_2}p_2-\dfrac{n_1}{n_2}\right)+2q_3p_2}{1-q_4^2}\,.$$

Substituting the value of ϕ_i, i = 1,2,3,4 from Eq.(41) in Eq.(30), we have

$$(u_1,u_3)=(|U_0|e^{i\theta_1},|W_0|e^{i\theta_2})e^{i(kx_1-\omega t)}\,, \quad (42)$$

$$U_0=ikA_1(e^{ik_Rx_3\delta_1}+\gamma_2e^{ik_Rx_3\delta_2}+\gamma_3e^{ik_Rx_3\delta_3}+ \\ +\gamma_4q_4e^{ik_Rx_3\delta_4})e^{i(kx_1-\omega t)}\,, \quad (43)$$

$$W_0=kA_1(q_1e^{ik_Rx_3\delta_1}+\gamma_2q_2e^{ik_Rx_3\delta_2}+\gamma_3q_3e^{ik_Rx_3\delta_3}- \\ -\gamma_4e^{ik_Rx_3\delta_4})e^{i(kx-\omega t)}\,, \quad (44)$$

where: $\theta_1 = \arg U_0$; $\theta_2 = \arg W_0$; and $\delta_j=\left(1-i\dfrac{c_I}{c_R}\right)q_j$, j = 1, 2, 3, 4. R and I denotes the real and imaginary part of the corresponding complex quantity.

Similarly, from Eqs.(28) and (41), we have

$$(T,C)=(|\theta_0|e^{i\theta_3},|C_0|e^{i\theta_4})e^{i(kx_1-\omega t)}\,, \quad (45)$$

$$\theta_0=A_1(n_1e^{ik_Rx_3q_1}+n_2\gamma_2e^{ik_Rx_3q_2}+n_3\gamma_3e^{ik_Rx_3\delta_3}+n_4\gamma_4e^{ik_Rx_3\delta_4})\,, \quad (46)$$

$$C_0=A_1(k_1e^{ik_Rx_3q_1}+k_2\gamma_2e^{ik_Rx_3q_2}+k_3\gamma_3e^{ik_Rx_3\delta_3}+k_4\gamma_4e^{ik_Rx_3\delta_4})\,, \quad (47)$$

where: $\theta_3 = \arg T_0$; and $\theta_4 = \arg C_0$.

On the surface x_3 = 0, the Eq.(42) on retaining real parts leads to

$$U_S=|U_0|e^{-k_Ix_1}\cos(\theta_1+\Phi), \\ W_S=-|W_0|e^{-k_Ix_1}\sin(\theta_2+\Phi), \quad (48)$$

where: $k_R(k_I)$ denotes real (imaginary) parts of the complex wavenumber k. Parameter Φ (= $k_Rx - \omega t$) is varied in [0, 2π) to show the path traced at depth x_3. The parametric representation of the curve shows that the surface particles trace elliptical path.

PARTICULAR CASE

(i) When $K^* = D^* = 0$ in Eq.(37), we obtain the resulting equation for the Green-Naghdi (Type-II) theory of thermoelasticity. In this case the roots of this dispersion equation (Green-Naghdi (Type-II)) are real, indicating that purely thermoelastic diffusive waves in thermoelasticity of Green-Naghdi (Type-II) are unattenuated and nondispersive (without energy dissipation).

(ii) In the absence of diffusion effect, the dispersion Eq.(37) reduces for the propagation of Rayleigh waves at the stress free isothermal boundary of the thermoelastic solid with, and without, energy dissipation.

(iii) In the absence of thermal and diffusion effects, the dispersion Eq.(37) reduces to the propagation of Rayleigh waves at the stress free boundary of the isotropic elastic half-space.

(iv) If we take $\Pi_1 = 1$, $\Pi_2 = \Pi_3 = 0$, $p_1 = 0$, $p_2 = 0$ in the dispersion Eq.(37), we obtain the corresponding equation for the propagation of Rayleigh wave in perfectly elastic solid as

$$(2-h)^2=-4q_1q_4\,,$$

which is similar to that given in Ewing et al., /33/.

NUMERICAL RESULTS AND DISCUSSION

We now represent some numerical results for copper material /8/, the physical data which are given below: $\lambda = 7.76\times10^{10}$ kgm^{-1}s^{-2}, $\mu = 3.86\times10^{10}$ kg m^{-1}s^{-2}, $T_0 = 0.293\times10^3$ K, $C_E = 0.3831\times10^3$ Jkg^{-1}K^{-1}, $\alpha_t = 1.78\times10^{-5}$ K^{-1}, $\rho = 8.954\times10^3$ kgm^{-3}, $K = 0.383\times10^3$ Wm^{-1}K^{-1}, $K^* = 1.483\times10^{13}$ Wm^{-1}K^{-1}s^{-1}.

The diffusion parameters are taken as:
$\alpha_c = 1.98\times10^{-4}$ m^{3}kg^{-1}, $a = 1.2\times10^4$ m^{2}s^{-2}K^{-1},
$b = 9\times10^5$ kg^{-1}m^{5}s^{-2}, $D = 0.85\times10^{-8}$ kg·s·m^{-3},
$D^* = 0.65\times10^{-8}$ kg·m^{-3}, and $L = 1$.

The software Matlab 7.0.4 has been used to determine the values of phase velocity V and attenuation coefficient Q^{-1} defined in the previous section for different values of frequency ω, ranging from $2\times\pi$ Hz to $2\times\pi\times10$ Hz.

In Figs. 1-8, the numerical data is varied around their reference values, to depict their effect on the phase velocity, attenuation coefficient, and particle motion of Rayleigh wave propagation in thermoelastic diffusion half space.

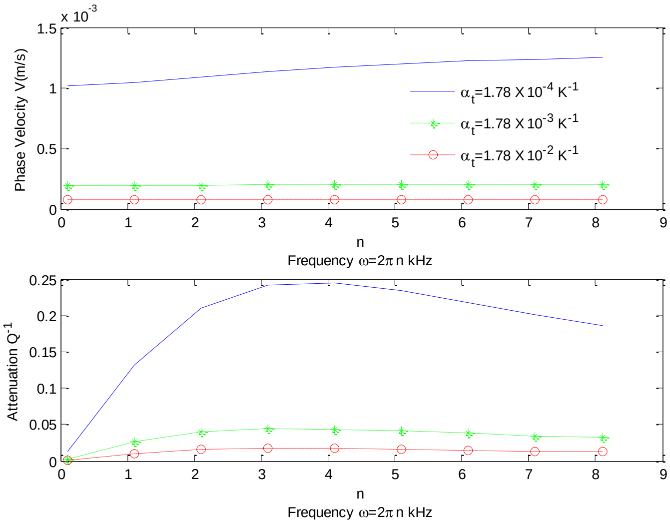

Figure 1. Variation of phase velocity V (m/s) and attenuation Q^{-1} with frequency ω (Hz) for different values of α_t.

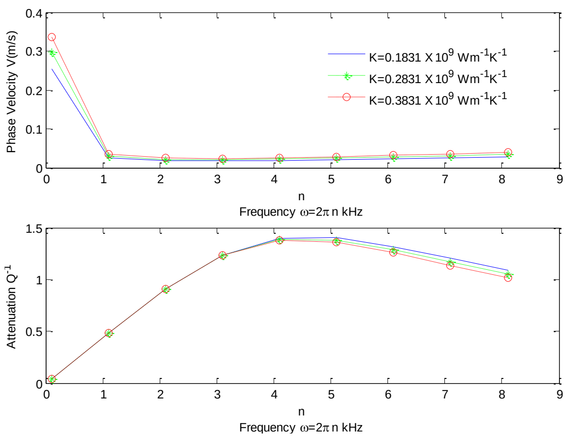

Figure 2. Variation of phase velocity V (m/s) and attenuation Q^{-1} with frequency ω (Hz) for different values of K.

Figures 1-4 show respectively the effect of α_t, K, a, D on the variation of phase velocity V and attenuation coefficient Q^{-1} with frequency ω.

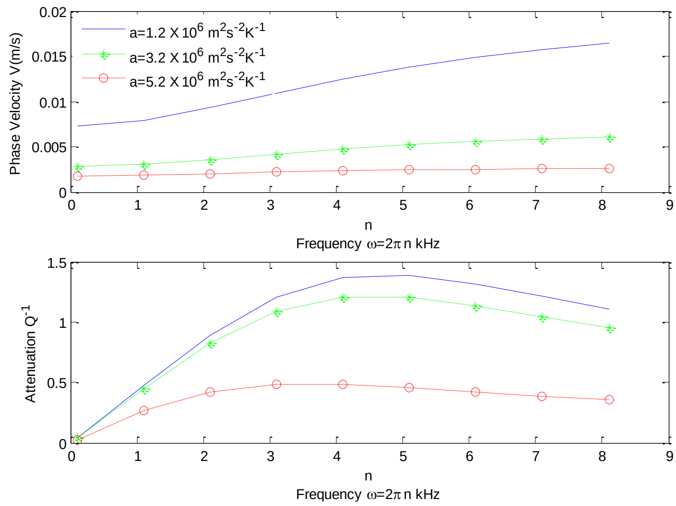

Figure 3. Variation of phase velocity V (m/s) and attenuation Q^{-1} with frequency ω (Hz) for different values of a.

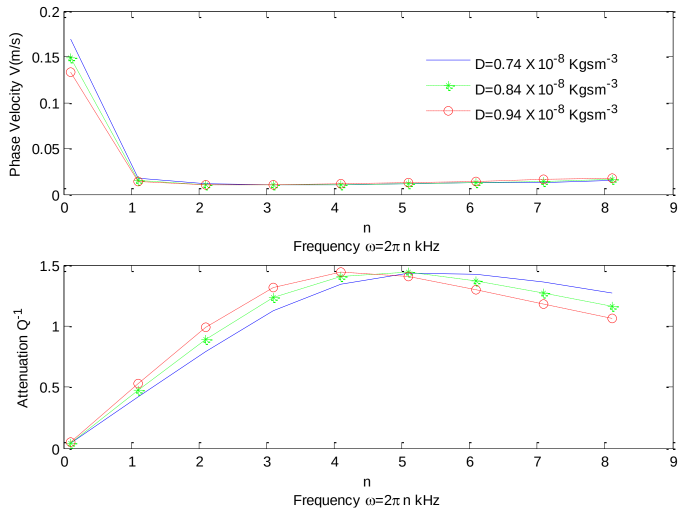

Figure 4. Variation of phase velocity V (m/s) and attenuation Q^{-1} with frequency ω (Hz) for different values of D.

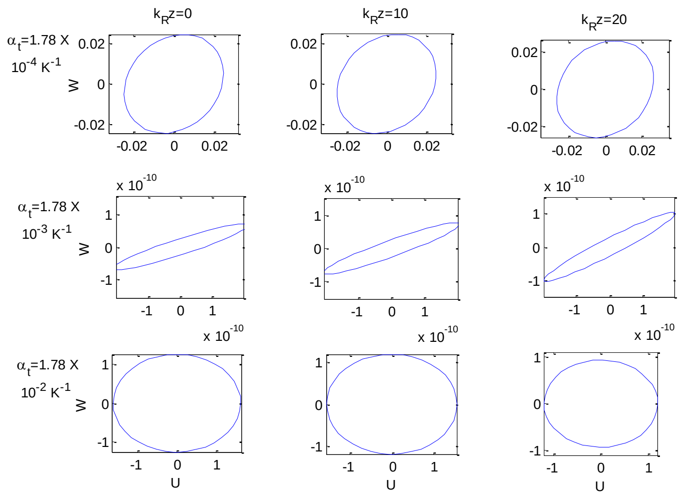

Figure 5. Variation of particle motion U, V with depth k_Rz for different values of α_t.

Figure 1 shows that phase velocity and attenuation coefficient increase continuously with increase in ω and appear stationary for higher values of ω. Increase in the value of α_t decreases phase velocity and attenuation.

Figure 2 shows that phase velocity decreases smoothly for $0 \leq \omega \leq 1$ and shows stationary behaviour for other values of frequency. Attenuation coefficient increases smoothly, then decreases continuously with frequency. Increase in the value of K, increases phase velocity while decreases attenuation.

Figure 3 shows that phase velocity and attenuation coefficient decrease smoothly with increase in ω and shows stationary behaviour for $\omega \geq 12$. Also phase velocity increases with increase in the value of α, increasing sharply for $\omega \leq 3$ and decreasing smoothly for other values of frequency. Increase in a decreases phase velocity and attenuation.

Figure 4 shows that phase velocity decreases sharply for $0 \leq \omega \leq 1$ while attenuation coefficient increases smoothly until it becomes stationary with increase in ω. Increase in T_0 increases phase velocity while it decreases the attenuation. Increase in D, decreases phase velocity while it increases attenuation for $0 \leq \omega \leq 5$ and decreases for $\omega > 5$.

Figure 5 depicts the particle motion U, W computed at different depths, i.e. $k_R z = 1$, 10, 20. The effect of α_t and depth are observed on polarizations of the Rayleigh wave. It shows that increase in α_t, tilts the particle motion of the Rayleigh wave. Increase in α_t initially weakens and then again broadens particle motion. Polarization of Rayleigh wave is not affected to a large extent with the increase in depth.

CONCLUSION

Rayleigh wave propagation at the stress free, isothermal, and isoconcentrated boundary in thermoelastic diffusion medium under Green-Naghdi theories is a significant problem of continuum mechanics. After developing mathematical formulation, the dispersion equation is obtained, which happens to be complex and irrational. This resulting equation is resolved into a polynomial form to find the roots, and therefore, to find the existence and propagation of Rayleigh wave. The dispersive character of Rayleigh waves and their inhomogeneous nature is ensured. The effect of depth and thermal, diffusion parameters is significant on the values of phase velocity, attenuation coefficient, and polarization of the Rayleigh wave.

The problem studied is applicable to a wide range of problems in geophysics and rock mechanics. The results obtained as a result of this study may provide a useful information for experimental scientists, researchers, and seismologists working on thermoelastic medium with mass diffusion. Also, the present model represents a more realistic earth model for further study.

Appendix A

$C_{15} = Y_1$, $C_{14} = Y_2$, $C_{13} = Y_3$, $C_{12} = Y_4$, $C_{11} = Y_5$, $C_{10} = Y_6$, $C_9 = (Y_7 - F_1)$, $C_8 = (Y_8 + F_1 - F_2)$, $C_7 = (Y_9 + F_2 - F_3)$, $C_6 = (Y_{10} + F_3 - F_4)$, $C_5 = (Y_{11} + F_4 - F_5)$, $C_4 = (Y_{12} + F_5 - F_6)$, $C_3 = (Y_{13} + F_6 - F_7)$, $C_2 = (Y_{14} + F_7 - F_8)$, $C_1 = (Y_{15} + F_8 - F_9)$, $C_0 = (Y_{16} + F_9 - F_{10})$, $F_0 = 4096 p_1^2 p_2^2$, $F_1 = F_0 D_1 B_1$, $F_2 = F_0(E_1 B_1 + D_1 B_2)$, $F_3 = F_0(E_2 B_1 + E_1 B_2 + D_1 B_3)$, $F_4 = F_0(E_3 B_1 + E_2 B_2 + E_1 B_3 + D_1 B_4)$, $F_5 = F_0(E_4 B_1 + E_3 B_2 + E_2 B_3 + E_1 B_4)$, $F_6 = F_0(E_4 B_2 + E_3 B_3 + E_2 B_4 + D_1 B_5 + E_1 B_5 - B_1)$, $F_7 = F_0(E_4 B_3 + E_3 B_4 + E_2 B_5 - B_2)$, $F_8 = F_0(E_4 B_4 + E_3 B_5 - B_3)$, $F_9 = F_0(E_4 B_5 - B_4)$, $F_{10} = -B_5$, $E_1 = D_2 - 2D_1$, $E_2 = D_3 + D_1 - 2D_1$, $E_3 = D_2 - 1 - 2D_3$, $E_4 = D_3 + 2$, $D_1 = \varepsilon_1 \varepsilon_2 \varepsilon_3$, $D_2 = -\varepsilon_1 \varepsilon_2 - \varepsilon_2 \varepsilon_3 - \varepsilon_3 \varepsilon_1$, $D_3 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$, $B_1 = N_1^2$, $B_2 = 2N_1 N_2$, $B_3 = N_2^2 + 2N_1 N_3$, $B_4 = 2N_2 N_3$, $B_5 = N_3^2$, $Y_1 = S^8$, $Y_2 = 2R_1 S^4$, $Y_3 = R_1^2 + 2W_0 S^4$, $Y_4 = 2W_0 R_1 + 2W_1 S^4$, $Y_5 = W_0^2 + 2W_1 R_1 + 2W_2 S^4$, $Y_6 = 2W_0 W_1 + 2W_2 R_1 + 2W_3 S^4$, $Y_7 = W_1^2 + 2W_0 W_2 + 2W_3 R_1 + 2W_4 S^4$, $Y_8 = 2W_1 W_2 + 2W_0 W_3 + 2W_4 R_1 + 2W_5 S^4$, $Y_9 = W_2^2 + 2W_1 W_3 + 2W_0 W_4 + 2W_5 R_1 + 2W_6 S^4$, $Y_{10} = 2W_1 W_4 + 2W_0 W_5 + 2W_6 R_1 + 2W_2 W_3$, $Y_{11} = W_3^2 + 2W_0 W_6 + 2W_1 W_5 + 2W_2 W_4$, $Y_{12} = 2W_1 W_6 + 2W_2 W_5 + 2W_3 W_4$, $Y_{13} = W_4^2 + 2W_2 W_6 + 2W_3 W_5$, $Y_{14} = 2W_3 W_6 + 2W_4 W_5$, $Y_{15} = W_5^2 + 2W_4 W_6$, $Y_{16} = 2W_5 W_6$, $Y_{17} = W_6^2$, $U_0 = 1024 p_1^2 p_2^2$, $W_0 = R_2 - H_1$, $W_1 = R_3 - H_2$, $W_2 = R_4 - U_0 \varepsilon_2 \varepsilon_3 - H_3$, $W_3 = R_5 - U_0 U_1 - H_4$, $W_4 = R_6 - U_0 U_2 - H_5$, $W_5 = R_7 - U_0 U_3 - H_6$, $W_6 = Q_3^2 - U_0 - N_3^2$, $U_1 = -(\varepsilon_2 + \varepsilon_3)^2$, $U_2 = 1 + 2\varepsilon_2 + 2\varepsilon_3 + \varepsilon_2 \varepsilon_3$, $U_3 = -(\varepsilon_2 + \varepsilon_3 + 2)$, $V_1 = T_1 - N_1^2$, $V_2 = T_2 - T_1$, $V_3 = T_3 - T_2$, $V_4 = N_3^2 - T_3$, $H_1 = \varepsilon_1 N_1^2$, $H_2 = \varepsilon_1 V_1 - N_1^2$, $H_3 = \varepsilon_1 V_2 - V_1$, $H_4 = \varepsilon_1 V_3 - V_2$, $H_5 = \varepsilon_1 V_4 - V_3$, $H_6 = -(\varepsilon_1 N_3^2 + V_4)$, $T_1 = 2N_1 N_2$, $T_2 = N_2^2 + 2N_1 N_3$, $T_3 = 2N_2 N_3$, $R_1 = 2M_1 S^2$, $R_2 = M_1^2 + 2Q_1 S^2$, $R_3 = 2M_1 Q_1 + 2Q_2 S^2$, $R_4 = Q_1^2 + 2Q_3 S^2 + 2M_1 Q_2$, $R_5 = 2Q_1 Q_2 + 2M_1 Q_3$, $R_6 = 2Q_1 Q_3 + Q_2^2$, $R_7 = 2Q_2 Q_3$, $N_1 = -8S$, $N_2 = 8(2S+R)$, $N_3 = -16R$, $Q_1 = M_2 - 16G_1 + 16\varepsilon_1$, $Q_2 = M_3 - 16(\varepsilon_1 + 1) + 16(G_1 + G_2)$, $Q_3 = 4R^2 + 16 - 16G_2$, $M_1 = -2S(2S+R)$, $M_2 = (2S+R)^2 + 4SR$, $M_3 = -4R(2S+R)$, $G_1 = \varepsilon_2 p_1^2 + \varepsilon_3 p_2^2$, $G_2 = p_1^2 + p_2^2$, $R = 2(1 + p_1 + p_2)$, $S = \Pi_1 + p_1 \Pi_2 + p_2 \Pi_3$, $\varepsilon_j = (\beta / \alpha_j^2)$, $j = 1,2,3$.

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Plenary speakers
Stavros Kourkoulis, *Quantifying elastic contact stresses on the lips of 'mathematical' cracks*
Aleksandar Sedmak, *Numerical simulation of fatigue crack growth*

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