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COMPUTER PROGRAMME FOR MATHEMATICAL MODELLING AND OPTIMISATION OF HYDRODYNAMIC PROCESSES OF A RADIAL PISTON PUMP

RAČUNARSKI PROGRAM ZA MATEMATIČKO MODELIRANJE I OPTIMIZACIJU HIDRODINAMIČKIH PROCESA RADIJALNE KLIPNE PUMPE

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Keywords
- radial piston pump
- mathematical modelling
- experimental research

Abstract

Fundamental basis in developing the radial piston pump is presented with experimental research and mathematical modelling of non-stationary high-dynamic hydraulic processes in the pump cylinder, discharge space, and the intake and discharge pipeline as a function of the shaft angle of action. Based on experimental research results and the results of mathematical modelling, the development and application of the method for identifying unknown parameters in the mathematical model, a computer programme is developed that enables sufficiently accurate determination of some process parameters of radial piston pumps.

Ključne reči
- klipno radijalna pumpa
- matematičko modeliranje
- eksperimentalno istraživanje

Izvod

Osnovu razvoja radijalne klipne pumpe predstavlja eksperimentalno istraživanje i matematičko modeliranje nestacionarnih visokodinamičkih hidrauličnih procesa u cilindru pumpe, potisnom prostoru i usisnom i ispusnom cevovodu u funkciji ugla dejstva osovine. Na osnovu rezultata eksperimentalnih istraživanja i rezultata matematičkog modeliranja, razvoja i primene metode identifikacije nepoznatih parametara matematičkog modela, razvijen je računarski program koji omogućava dovoljno precizno određivanje nekih parametara radnih procesa radijalnih klipnih pumpi.

INTRODUCTION

Modern design of radial piston pump, based on computer aided design (CAD), requires description of all processes and parameters in the pump. Complexity of hydrodynamic and dynamic processes in a radial piston pump (cylinder, intake and discharge space, discharge valve, and high pressure pipeline) requires physical and mathematical analysis. Based on experimental results and results of mathematical modelling, a method for identifying unknown parameters of non-stationary high-dynamic processes and an optimisation technique is developed and applied. This way of solving the problem is only possible with a special computer programme.

MATHEMATICAL MODEL

The following general suppositions have been adapted for mathematical modelling of hydrodynamic and dynamic processes in a radial piston pump (pump cylinder, intake and discharge space, discharge valve and high pressure pipeline), Fig. 1:

(a) changes of the fluid state are pseudo-stationary, except in the discharge pipeline;
(b) kinetic energy of fluid in each control space, except in the discharge pipeline, is neglected;
(c) fluid flow through clearances (crevices between piston and cylinder, flow through a split panel and discharge valve) is pseudo-stationary;
(d) processes in control spaces are isothermal or isentropic.

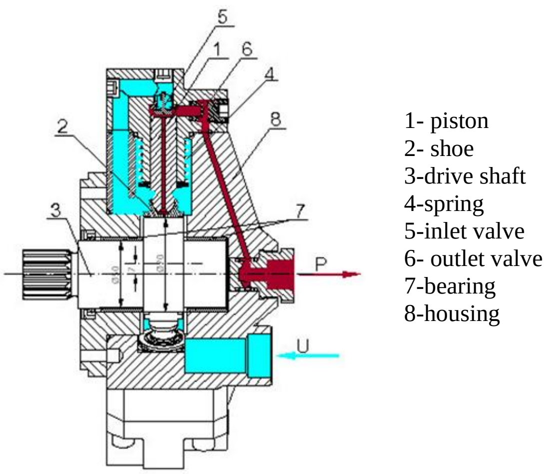

Figure 1. Structure and control spaces of a radial piston pump.

Mathematical model of pump process

The mathematical model is given for each element. This makes the programming module, further improvements and monitoring much easier.

- Mass flow through opening 1, at the entrance into the intake space of the pump, Fig. 1:

$$\frac{dm_1}{dt} = \sigma_1 \mu_1 A_1 \sqrt{2\rho_s |p_u - p_s|} \tag{1}$$

where: $\sigma_1 = 1$ for $p_u \geq \Delta p_s$, $\sigma_1 = -1$ for $p_u < \Delta p_s$; A_1-geometrical flow section of the intake pipe.

- Fluid mass flow through the split pump organ during the filling of one of the pump cylinders:

$$\frac{dm_u}{dt} = \sigma_u \mu_u A_u \sqrt{2\rho_s |p_s - p_c|}\,, \qquad (2)$$

where: $\sigma_u = 1$ for $p_s \geq \Delta p_c$, $\sigma_u = -1$ for $p_s < \Delta p_c$; A_u-geometrical flow section of the intake split organ; μ_u-flow coefficient.

- Mass balance of the intake space is:

$$\frac{dm_s}{dt} = \frac{dm_1}{dt} - \sum_{j=1}^{z_c} \frac{dm_{u,j}}{dt}\,, \qquad (3)$$

where: $j = 1, 2, ..$, order number of z_c cylinder.

- Differential pressure equation in the intake pump space:

$$\frac{dp_s}{d\phi} = \frac{E}{V_s \rho_s}\left(\frac{dm_1}{d\phi} - \sum_{j=1}^{z_c} \frac{dm_{u,j}}{d\phi}\right), \qquad (4)$$

where: E - modulus of elasticity.

- Differential pressure equation in the pump cylinder:

$$\frac{dp_c}{d\phi} = \frac{E}{V_c}\left[\frac{A_c v_k}{\omega} + \frac{1}{\rho_c}\left(\frac{dm_u}{d\phi} - \frac{dm_i}{d\phi}\right)\right], \qquad (5)$$

where: $V_c = V_{c\min} + V_{cx}$; $V_{cx} = A_c x_k$ is immediate volume of cylinder; change of volume of the pump cylinder caused by piston movement, $dV_c/dt = -A_c v_k$; x_k - immediate displacement of the piston.

- Mass balance of the discharge space is:

$$\frac{dm_v}{dt} = \sum_{j=1}^{z_c} \frac{dm_{i,j}}{dt} - \frac{dm_2}{dt}\,, \qquad (6)$$

where: $j = 1, 2, ...$, order number of z_c cylinders.

- Mass flow out of the discharge space into the discharge pipe:

$$\frac{dm_2}{dt} = \sigma_2 \mu_2 A_2 \sqrt{2\rho_t |p_v - p_n|}\,, \qquad (7)$$

where: $\sigma_2 = 1$ for $p_v \geq p_n$, $\sigma_2 = -1$ for $p_v < p_n$; A_2- geometrical flow section of the discharge pipeline.

- Differential pressure equation in discharge pump space:

$$\frac{dp_v}{d\phi} = \frac{E}{V_v \rho_v}\left(\sum_{j=1}^{z_c} \frac{dm_{i,j}}{d\phi} - \frac{dm_2}{d\phi}\right). \qquad (8)$$

- Mass flow through a concentric clearance between cylinder and piston:

$$\frac{dm_z}{dt} = \frac{\pi D_c \Delta r^3}{12\eta x_k(\phi)}(p_c - p_s)\rho_c\,, \qquad (9)$$

where: D_c- diameter of cylinder; Δr- radial clearance between piston and cylinder; η- dynamic viscosity; $x_k(\varphi)$ - immediate displacement of piston; p_c - pressure in cylinder; p_s - pressure in intake space; ρ_c - density of fluid in the cylinder.

STRUCTURE OF COMPUTER PROGRAMME

Simultaneous integration of previous nonlinear differential equations of boundary conditions and partial differential equations of streaming in a discharge pipeline have required the application of corresponding software, /1-4/.

The programme connecting and solving simultaneously all listed differential equations, equations of the change of characteristic flow sections and changes in physical fluid characteristics, requires corresponding structure and organisation. The programme is written in Digital Visual Fortran 5.0 and realised on the measuring and controlling system ADS 2000, /5/. Principles of structural and modular programming were used. The programming consists of the main programme and a module. The more important programmes are written as complete modules, mutually connected, or with the main programme, but can be used individually as well.

On the basis of previous equations, a programme system is developed named for mathematical modelling of the flow and hydrodynamic processes for a complete time cycle of the radial piston pump with combined distribution of working fluid. The programme is modularly outlined and consists of the main program and modules.

Some programme modules consist of one or more programmes but perform an exact fixed number of operations. A simplified scheme of the programme is shown in Fig. 2.

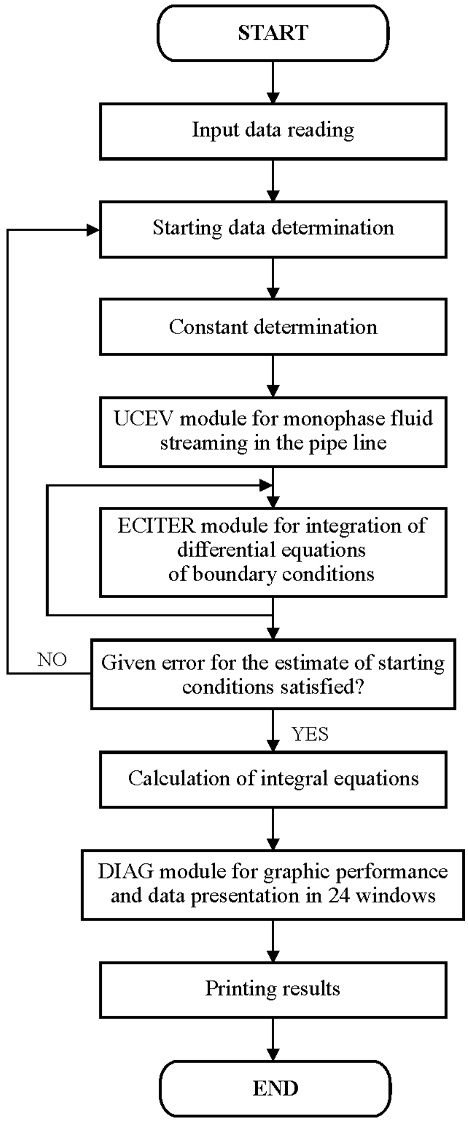

Figure 2. Structure of main AKSIP programme for the calculation of working parameters of the radial piston pump.

Input data are taken from AKSIP file .DAT, and output data are stored in output files DH1.DAT, DH2.DAT, and DH3.DAT, but with the application of DIAG module are performed as a diagram and are displayed on the monitor in 24 windows in the course of the calculation process, /6/.

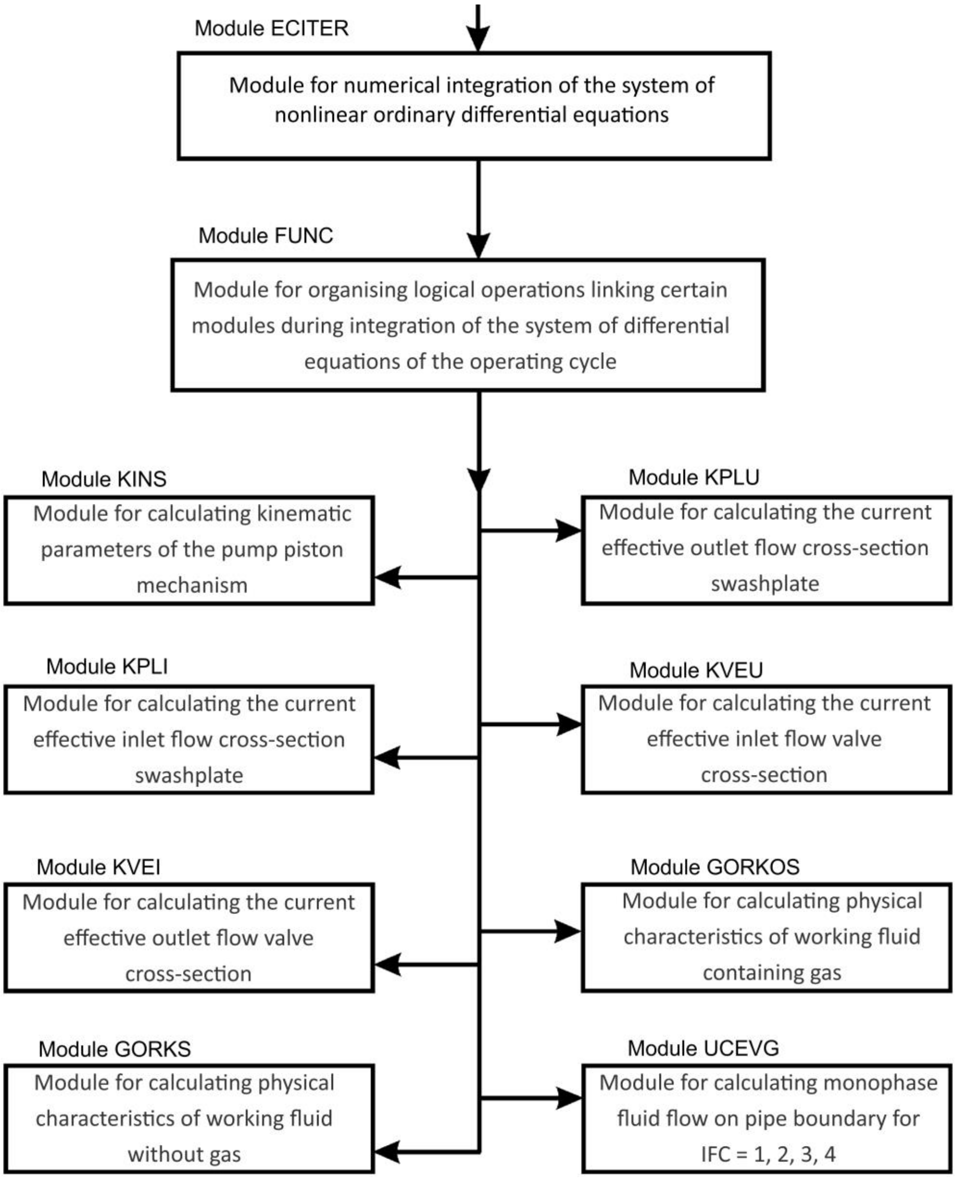

Figure 3. ECITER structure.

The second level of the programme is made of a module for integrating nonlinear ordinary differential equation of Cauchy-Euler boundary conditions with ECITER interactions. The most effective insight of the programme is given by Figs. 2 and 3, that present only the programme structure without the details of algorithms of each individual module. ECITER, CEV (UCEVD and UCEV) and DIAG modules are ST complete modules that makeup the library programme of ADS 2000 system. Since the programme system is outlined for so-called multi-modelling, the user is allowed to choose the mathematical model of some processes in the pump, as seen in the block-diagram, Fig. 3.

The choice of a required model can be done in a determined and stochastic way. If it is done in a determined way, a required model is given during the formation of a file of input data AKSIP.DAT. The stochastic choice of a model is of special interest in statistical analysis of the reliability of modelling results and the like. The programme is structured in layers at few levels. Splitting all of the important data is done by COMMON blocks taking care in the coherence of the system of units. The main programme is called AKSIP and presents the first level. During operation, the main programme invites the second level ECITER subprogramme and initiates starting values of competent quantities. Start of calculation is defined in the input file by giving the starting angle of calculation, /2-4/.

Parameters in the input file define the pump model, which among other things, conditions the way of calculating some quantities. The AKSIP programme, for now, predicts the following possibilities and corresponding identifiers during the selection of mathematical mode, /7-8/:

– variability of pressure in intake compartment IS = ? Constant IS = 0; Variability IS = 1,
– variability of pressure in a discharge compartment IT = ? Constant IT = 0; Variability IT = 1,
– choice of a discharge transfer organ IU = ? Split panel IU = 1, valve IU = 11,
– choice of a discharge transfer organ II = ? Split panel II = 1, valve II = 11,

– choice of determined way of flow coefficient IMDMI = ? Tabular IMDMI = 0 in the pressure function IMDMI = 1, as a function of the number of rotations IMDMI = 2 in the pressure function and the number of rotations IMDMI = 3,
– choice of module with and without mass inertia of working fluid IMIN = ? with IMIN = 0, without IMIN = 1.

Depending on the model, this module invites a corresponding subprogramme of third level (FUNC) with equations of boundary conditions, through which a connection is made with the rest of the subprogrammes of fourth level: a subprogramme for finding flow parameters at the ends of the pipe (UCEVG), a subprogramme for finding kinematic piston characteristics (KINS), a subprogramme for finding immediate effective input and output flow section of the split panel (KPLU, KPLI), a subprogramme for calculating immediate effective flow sections of intake and discharge valves (KAVEU, KAVEI), and a module for determining physical characteristics of fluid (GORKO, GORKOS). An illustration of the AKSIP applicability is shown in Fig. 4. Thus, the outlined programme supplies simultaneously all processes, analogously to processes in a real radial piston pump.

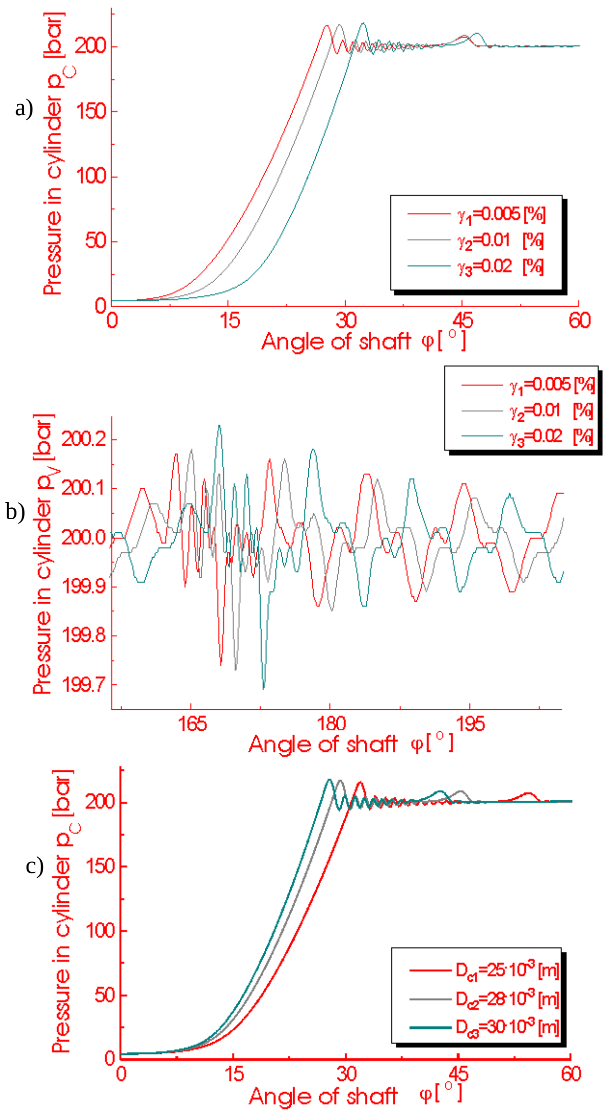

Figure 4. Cylinder pressure vs. shaft angle with a selection of: a) and b) gas content; c) cylinder diameter.

CONCLUSIONS

The basic fact radial piston pump manufacturers and users must accept is that every increase in operating pressure leads to the increase in pump noise. A much greater influence on noise level is the increase in rotational frequency of pump drive shaft rather than an increase of pressure in the pressure pipe. Since rotational frequency has a large impact on noise, it is often reduced in order to neutralise the noise caused by increasing pressure. If rotational frequency is reduced, we need to use a larger pump that is heavier and more expensive, in order for the flow to remain the same.

Cavitations that occur as a result of insufficient supplying of the pump can affect noise level. Bearings and gears also contribute to the increase of noise. It should also be noted that a selection of appropriate materials can attenuate certain vibrations that also leads to an increase of the noise.

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