Salem Shweka[1], Radovan Petrović[2], Dragana Vasilski[1], Maja Anđelković[2], Khaled Omran[1], Mansur Jaba[2], Igor Martić[3]

STRESS AND DEFORMATION CALCULATION OF ATMOSPHERIC OIL STORAGE TANKS AT PORT TERMINALS

PRORAČUN NAPONA I DEFORMACIJA ATMOSFERSKIH REZERVOARA ZA SKLADIŠTENJE NAFTE U LUČKIM TERMINALIMA

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 22.02.2022

Adresa autora / Author's address:
1) University 'Union-Nikola Tesla', Faculty of Information Technology and Engineering, Belgrade, Serbia
2) University 'Union-Nikola Tesla', Faculty of Sustainable Development Management, Belgrade, Serbia
email: radovan4700@yahoo.com
3) University of Belgrade, Innovation Centre of the Faculty of Mechanical Engineering, Belgrade, Serbia

Keywords
- finite element method (FEM)
- deformation
- cylindrical tank
- gas pressure

Ključne reči
- metoda konačnih elemenata (MKE)
- deformacija
- cilindrični rezervoar
- pritisak gasa

Abstract

When designing storage tanks and generally pressure tanks, the strength calculation is rather significant. There are different procedures by which we can, with more or less accuracy, determine the desired sizes that define the behaviour of the structure. More significant efforts in solving this task have led to standardized procedures and recommendations and standards in the field of tank design. Standards prescribe the shape and measures, as well as conditions for development and delivery of the welded steel tanks of different volumes and purposes. They include empirical forms for the calculation of tank thickness, that contain safety coefficients with the addition of corrosion. Standards also predict material quality for the development of tanks, with special emphasis on obligatory control and examination.

Izvod

U projektovanju rezervoara i uopšte posuda pod pritiskom, proračun čvrstoće je veoma značajan. Postoje različiti postupci kojima možemo sa manjom ili većom tačnošću odrediti željene veličine koje definišu ponašanje konstrukcije. Značajniji napori u rešavanju ovog zadatka doveli su do standardizovanih procedura, preporuka i standarda u oblasti projektovanja rezervoara. Ovim standardima propisani su oblici i mere, kao i uslovi za izradu i isporuku zavarenih čeličnih rezervoara različitih zapremina i namena. Standardi sadrže empirijske obrasce za proračun debljine zida i dna rezervoara, koji sadrže značajne koeficijente počev od nivoa sigurnosti do dodataka za koroziju. Standardi takođe predviđaju kvalitet materijala za izradu rezervoara, sa posebnim naglaskom na obaveznu kontrolu i ispitivanje.

INTRODUCTION

Cylindrical tanks of large volumes (> 150 m^3) are developed as atmospheric. Tanks up to 150 m^3 are more frequently developed as tanks under pressure and can be both horizontal and vertical. In addition, they can be static or mobile.

In order to reach practical solutions, a further simplifications of the theory is required. Such simplifications are not able to give us the real image of stress and area of movement in a large number of locations on the tank.

In calculations of tanks under pressure, particularly in complex boundary conditions and loading conditions, the application of FEM offers great possibilities, /2-10/. In that case, one can choose plate or shell elements as finite elements, and in some other cases membrane elements.

The task of the paper is to determine the suitability of the application of this developed procedure and to evaluate the impact of some structural parameters on the stress state, field of displacement, and deformation field.

As an illustration of the application of developed procedures, the solutions of some derived structures are shown. Calculation of the shells relies on the following assumptions, /1/:

1. Shell thickness (δ) is small when compared to other shell dimensions.
2. Deflections are small when compared to shell thickness.
3. Points on the normal of the medium surface of the shell prior to the deformation are found normal to the deformed medium surface.
4. Normal stresses that affect the medium surface of the shell are small and can be neglected.

Figure 1 shows a volume shell element with its stress components, whose size depends on the coordinate z. The width of the lateral elements can be expressed through the coordinate z from the similarity of the triangles presented in Fig. 1, /11/.

Having in mind that for $z = 0$, the width of the cross section equals one, and from the similarity mentioned, one can determine the width of lateral surfaces of the elements as a function of coordinate z.

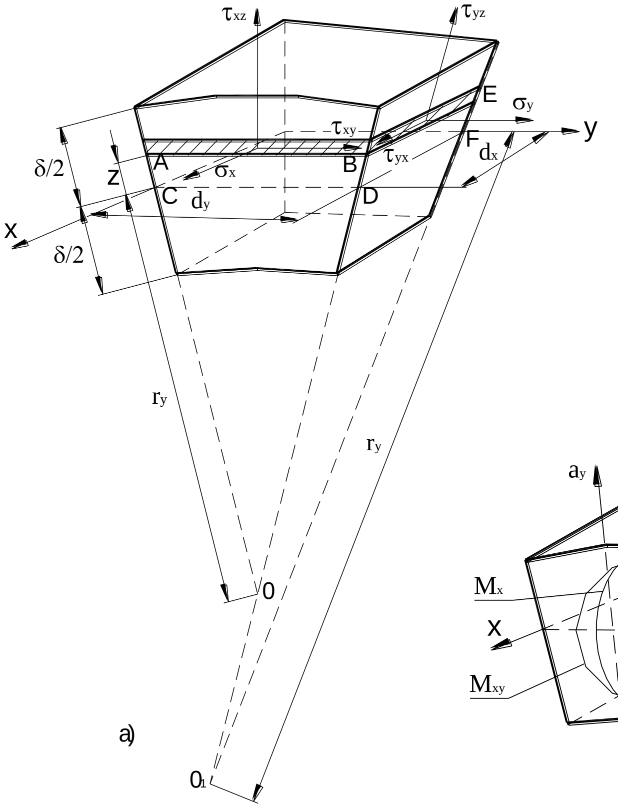
Based on Fig. 1, forces and moments per unit of shell section length will be /11/:

$$N_x = \int_{-\delta/2}^{\delta/2} \sigma_x \left(1 + \frac{z}{r_y}\right) dz\,, \quad N_y = \int_{-\delta/2}^{\delta/2} \sigma_y \left(1 + \frac{z}{r_x}\right) dz\,. \quad (1)$$

From the conditions of the normality of lateral sides of shell elements, it follows:

$$\tau_{xy} = \tau_{yx}\,. \quad (2)$$

Shear forces N_{xy} and N_{yx}, i.e. twisting moments M_{xy} and M_{yx} will be equal only in case $r_y = r_x$ (that is the case of the panel).



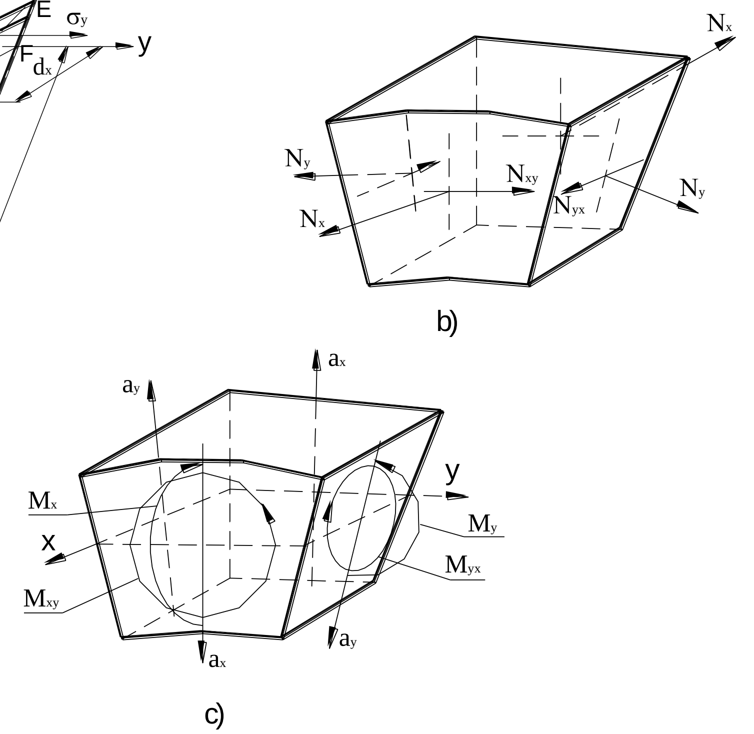


Figure 1. Forces and moments in shell sections.

FINITE ELEMENTS METHOD AND APPLICATION

For the analysis of strength and stability of bearing walls, we have chosen finite elements, as shown in Fig. 2, according to /1/, which appear to be very efficient and suitable for solving problems in general theory of shells. Here, we expose only some main assumptions of these elements, and detailed derivations can be seen in /1/.

Triangular finite elements of the shell, as can be seen in Fig. 2a, have 6 degrees of freedom per node, 3 translations and 3 rotations. The main idea in case of this element is that bending is observed separately from membrane leads and deformations.

The element provides reliable and accurate solutions for any type of geometry of the shell taking into consideration different loads and boundary conditions.

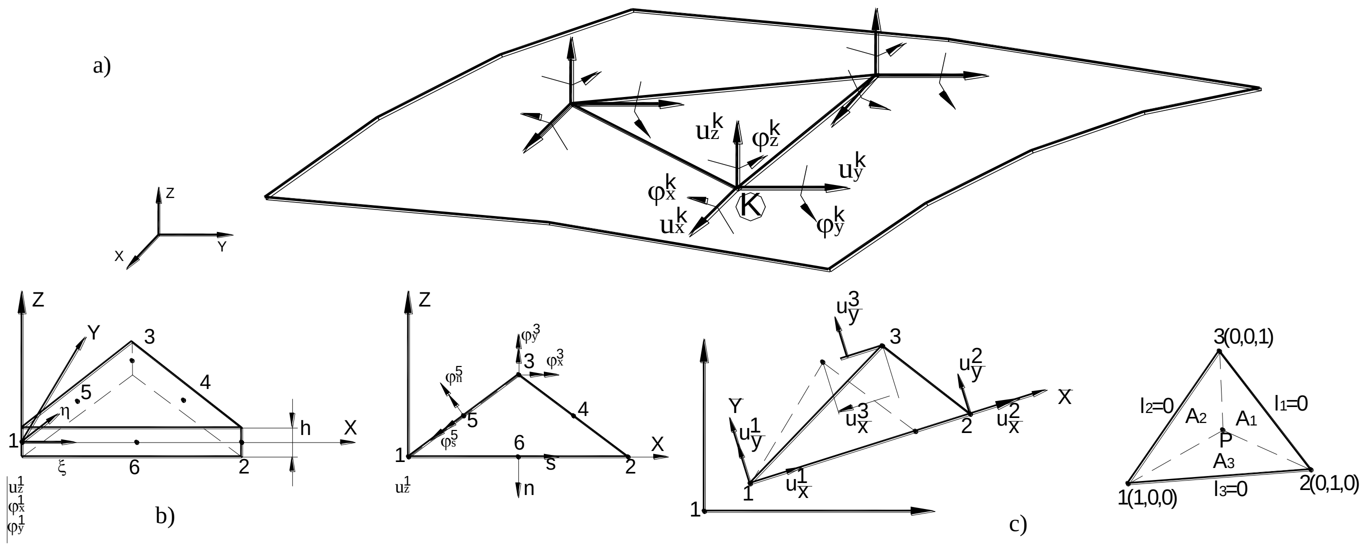


Figure 2. Triangular finite element of the shell.

In nonlinear analysis, the element is suitable for calculating large displacements, big rotations, and material nonlinearity.

The field of movement by bending is expressed in the local coordinate system *x*, *y*, *z* in the form, /1/:

$$u_x = z\beta_x(x,y),\ \ u_y = -z\beta_y(x,y),\ \ u_z = u_z(x,y), \quad (3)$$

where: β_x and β_y are rotations in the planes that contain axes *x* and *y*; and u_z is transversal movement.

Such an assumption for the movement field basically contains the physical condition for material line segments, primarily parallel to the medium plane of the shell and remain non-deformed, but they do not have to retain the direction normal to the medium surface after deformation /1/,

$$\{\bar{u}_B\}^T = \{u_z^1, \varphi_x^1, \varphi_y^1, u_z^2, \varphi_x^2, \varphi_y^2, u_z^3, \varphi_x^3, \varphi_y^3\}^T . \quad (4)$$

For thin plates, transversal shear stresses are small in comparison to bending stresses. For that reason, the energy of transverse shear is negligible in comparison to the bending energy. For calculation of the element rigidity matrix, we start from the expression /1/:

$$U = \frac{1}{2}\int_A \{K\}^T [D]_B \{K\} dA, \quad (5)$$

where: $\{K\}$ is the bending vector, /1/:

$$\{K\} = \begin{Bmatrix} \beta_{x,y} \\ -\beta_{y,y} \\ \beta_{x,y} - \beta_{y,x} \end{Bmatrix} . \quad (6)$$

Matrice $[D]_B$ in the general case of functions has the thickness of shell δ and elastic features of particular layers of materials. In case of isotropic homogeneous plates of constant thickness, we apply the following relation /1/,

$$[D]_B = \frac{E\delta^3}{12(1-\mu^2)} \begin{bmatrix} 1 & & 0 \\ & 1 & 0 \\ sim & & \frac{1-\mu}{2} \end{bmatrix}, \quad (7)$$

where: *E* is Young's modulus of elasticity; and μ Poisson's ratio.

ATMOSPHERIC TANK UNDER PRESSURE OF VARIABLE WALL THICKNESS

Figure 3 presents a vertical atmospheric tank of variable wall thickness $\delta(x)$, loaded with fluid pressure. In section x = const., the forces and moments per section length unit are, /1/,

$$X = Y = 0, \quad Z = -\gamma(H-x). \quad (8)$$

If the tank is deformed undisturbed, a state of stress would appear in the membrane. Forces in sections can then be calculated based on the following equations, /1/:

$$N_\Theta = -ZR, \quad N_{x\Theta} = -\int\left(Y + \frac{1}{R}\frac{\partial N_\Theta}{\partial \Theta}\right)dx + C_1(\Theta),$$

$$N_x = -\int\left(X + \frac{1}{R}\frac{\partial N_{x\Theta}}{\partial \Theta}\right)dx + C_2(\Theta), \quad (9)$$

then it follows:

$$N_{\Theta_0} = \gamma R(H-x), \quad N_{x\Theta_0} = 0, \quad N_{x_0} = 0 .$$

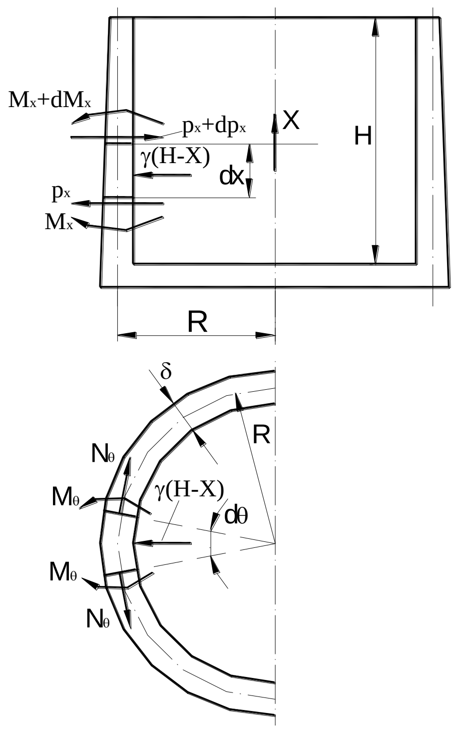


Figure 3. Vertical atmospheric tank of variable thickness.

Specific elongation towards the ring will be:

$$\varepsilon_{\Theta_0} = \frac{1}{E\delta}(N_{\Theta_0} - \mu N_{x_0}) = \frac{N_{\Theta_0}}{E\delta},$$

and thus for the radial movement we obtain:

$$w_0 = R\varepsilon_{\Theta_0} = \frac{N_{\Theta_0 r}}{E\delta} = \frac{R^2}{E\delta}\gamma(H-x) . \quad (10)$$

Based on the above-mentioned we obtain a differential equation of the tank with variable wall thickness, /1/:

$$\frac{d^2}{dx^2}\left(K\frac{d^2 w}{dx^2}\right) + E\delta\frac{w}{R^2} = \gamma(H-x) . \quad (11)$$

Determination of forces at particular nodes of the generated finite element mesh based on main data on the loads, represents a significant task in the procedure of automatic calculation considered here. The loads considered are:

– wind pressure, p_v,
– hydrostatic pressure, p_h,
– gas pressure (overpressure), p_g,
– self-weight of the structure.

Forces in the nodes are determined by projections of the force on the element on coordinate axes:

$$F_{pi} = pA_i, \quad i = 1,2,3, \quad (12)$$

where: *p* is a pressure on finite element; A_i projection of the triangle surface on *i*-coordinate axis.

Wind pressure forces on the tank shell

Wind pressure can be taken into consideration in case of vertical tanks. The disposition of wind pressure is shown in Fig. 4 and is the same for all points in one generating function of the shell, and that the function is only of the angle ∅, /1/.

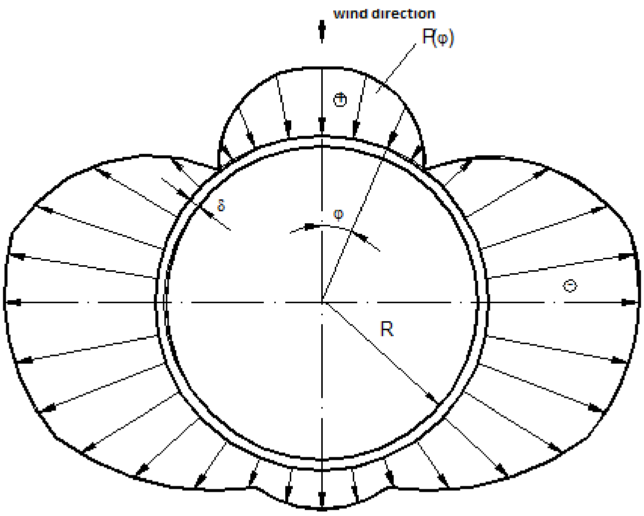


Figure 4. Distribution of wind pressure.

In input data for loads we set the value of wind pressure p (based on which we determine the coefficients p_n). If we use primarily five members of the order, we take the coefficients p_n obtained by measurement of wind pressure on one gas tank with smooth walls /1/: $p_0 = 0.526p$; $p_1 = 0.253p$; $p_2 = 0.950p$; $p_3 = 0.462p$; and $p_4 = 0.189p$.

Hydrostatic pressure forces on tank shell

Figure 5 shows the effect the hydrostatic pressure force on differential element of the surface. In addition, the surface element, and the projection of that element to the plane y-z (A_x) are also shown.

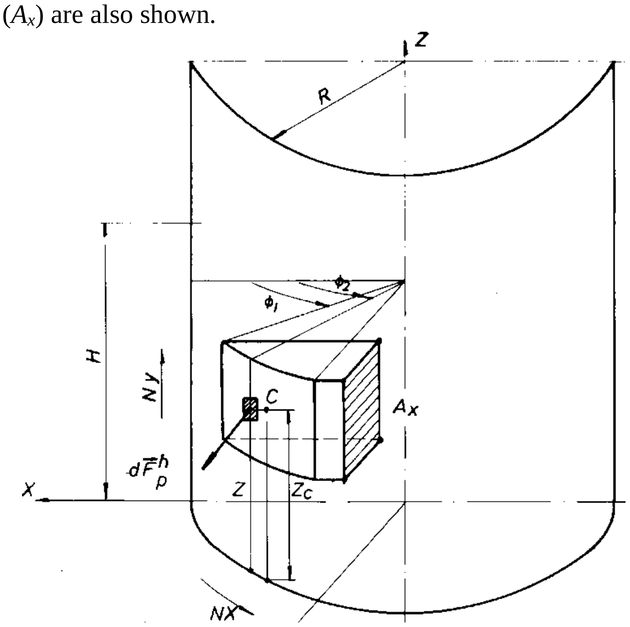


Figure 5. Hydrostatic pressure force.

Differential size of hydrostatic pressure force, according to Fig. 7:

$$d\vec{F}_p^h d = \gamma (H - z) d\vec{A}, \qquad (13)$$

and its projections:

$$F_{px}^h = \gamma (H - z_c) A_x = \gamma hR(H - z_c)(\sin\alpha_2 - \sin\alpha_1),$$

$$F_{py}^h = \gamma (H - z_c) A_y = \gamma hR(H - z_c)(\cos\alpha_1 - \cos\alpha_2), \qquad (14)$$

If only one part of the element is submerged in the liquid, then in the above equations, instead of $(H - z_c)$ it will be $(H - z'_c)$, where z'_c is the coordinate of the bulk submerged part of the element.

Hydrostatic pressure forces for ultimate nodes will be:

$$F_1^h = \sqrt{(F_x^h)^2 + (F_{py}^h)^2},\ F_{px}^h = F_R^h \cos\alpha,\ F_{py}^h = F_R^h \sin\alpha\,. \quad (15)$$

Example: Vertical atmospheric tank under fluid pressure /1/

Data: R = 800 cm tank radius; H = 1447 cm height of filling with fluid; H_1 = 199 cm first height; δ = 0.9 cm shell sheet thickness; t = 1.0 cm bottom sheet thickness; γ = 0.0014 daN/cm^2 specific weight of fluid; E = 210000 MPa; μ = 0.3.

Calculation of forces in individual sections N_θ (daN/cm)

Total force by section unit length equals the sum of forces from the impact of hydrostatic pressure, edge force P and edge moment M:

$$N_\Theta = N_{\Theta_0} + N_{\Theta_P} + N_{\Theta_M}\,,$$

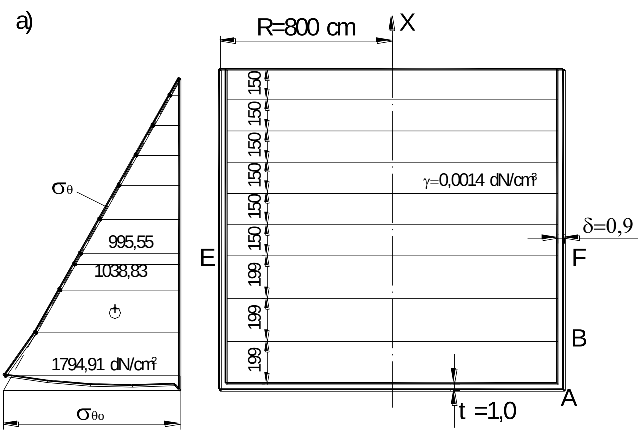
$$N_\Theta = \gamma R(H - x) + \frac{E\delta e^{-\lambda x}}{2RK\lambda^3}\{M\lambda[\cos\lambda x - \sin\lambda x] - P\cos\lambda x\}\,. \quad (16)$$

Based on Eq.(16) for particular sections and starting from the bottom towards the top of tank, we calculate the value of forces N_θ (Table 1).

Table 1. Calculated forces N_θ.

x (cm)	N_Θ (daN/cm)	$\sigma_\Theta = N_\Theta/\delta$ (daN/cm^2)
0	22.823	25.359
66	1615.421	1794.912
265	1323.834	1470.928
464	1100.96	1223.289
647	896	995.555
797	728	808.889
947	560	622.222
1097	392	435.555
1247	224	248.889
1397	56	62.222
1447	0	0

Based on the obtained values in Table 1, a diagram of circular stress σ_θ is given in sections presented in Fig. 6a. From the figure we see that the largest value of circular stress is somewhat above bottom-shell junction (section x = 66 cm) and not in the junction itself. The explanation lies in the disorder of membrane state of stress at the junction location that prevents shell extension. In case there is no bottom, the tank shell can go through undisturbed extension under the impact of hydrostatic pressure, and we would have a membrane state of stress presented by an interrupted line in Fig. 6a.



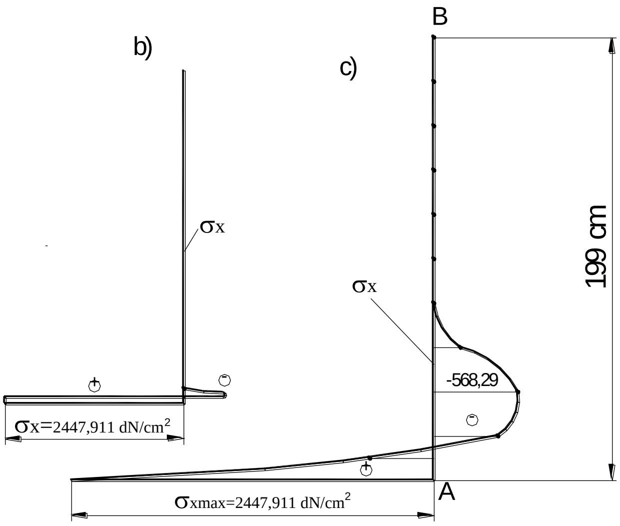


Figure 6. Circular and axial stress diagram for vertical atmospheric tank $V = 3000$ m^3.

Calculating the moments in particular sections M_x

Total moment per section length unit will be equal to the sum of the moments from edge force and edge moment:

$$M_x = M_{xp} + M_{xM}\,.$$

Total moment in the section M_x will be, /1/:

$$M_x = Me^{-\lambda x}(\cos\lambda x + \sin\lambda x) - \frac{P}{\lambda}e^{-\lambda x}\sin\lambda x\,. \quad (17)$$

In Table 2 given are values of moment M_x for particular sections, as well as stress values in axial direction. Based on the values in Table 2, the diagrams of stress in axial direction are shown in Fig. 6b-c. From Fig. 6b one can see that the highest value of this stress is found at the support.

In addition, we observe the change of the sign of this stress and its abrupt drop. The characteristic of this bending stress is that in the first ring of the tank it is muted and that its impact further towards the top of the tank can be neglected. Therefore, at a sufficient distance from the tank bottom, we can apply the membrane theory.

Table 2. Values of moment M_x in particular sections.

x (cm)	M_x (daNcm/cm)	$\sigma_x = 6M_x/\delta^2$ (daN/cm^2)
0	330.468	2447.911
10	57.338	424.722
20	-63.629	-471.324
40	-76.719	-568.29
60	-24.456	-181.152
80	0.484	3.582
100	8.87	65.704
120	1.614	11.952
140	0.15	1.11
160	-0.171	-1.266
180	-0.0977	-0.726
200	-0.0179	-0.132

CONCLUSIONS

The method of discretization, the choice of the shape of elements, as well as the total number of elements, depend on the nature of the problem to be solved and the required accuracy of the required solution.

In the example shown according to stress state analysis, optimisations are possible in the direction of reducing the sheets thickness to the allowable stress, or the use of other modern building materials such as polymers and composite materials.

ACKNOWLEDGEMENTS

Some of these studies are supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia, grants TR 32036 and TR 35038.

REFERENCES

1. Petrović, R., Kojić, M., Đorđević, D., Design of Tanks for Storage and Transport of Fluids (in Serbian: Projektovanje rezervoara za skladištenje i transport fluida), University of Kragujevac, Faculty of Mechanical Engineering Kraljevo, 2005. ISBN 86-82631-21-0
2. He, Z., Yuan, S., Wang, Z.R. (2008), *Deformation of revolving closed-shell with multi-curvature profile under inner pressure*, J Mater. Process. Technol. 206(1-3): 400-404. doi: 10.1016/j.jmatprotec.2007.12.085
3. Portela, G, Godoy, L.A. (2005), *Wind pressures and buckling of cylindrical steel tanks with a dome roof*, J Constr. Steel Res. 61(6): 808-824. doi: 10.1016/j.jcsr.2004.11.001
4. Meløysund, V., Lisø, K.R., Hygen, H.O., et al. (2007), *Effects of wind exposure on roof snow loads*, Build. Environ. 42(10): 3726-3736. doi: 10.1016/j.buildenv.2006.09.005
5. Yang, Z.R., Shou, B.N., Sun, L., Wang, J.J. (2011), *Earthquake response analysis of spherical tanks with seismic isolation*, Procedia Eng. 14: 1879-1886. doi: 10.1016/j.proeng.2011.07.236
6. Virella, J.C., Godoy, L.A., Suarez, L.E. (2006), *Dynamic buckling of anchored steel tanks subjected to horizontal earthquake excitation*, J Constr. Steel Res. 62(6): 521-531. doi: 10.1016/j.jcsr.2005.10.001
7. Milovanović, A., Sedmak, A., Tomić, R., et al. (2016), *Calculation of local loads in torispherical end of vertical vessel with skirt according to EN 13445-3:2014*, Struct. Integ. Life, 16(1): 53-58.
8. Milovanović, A.M., Mijatović, T., Diković, Lj., et al. (2021), *Structural integrity analysis of a cracked pressure vessel*, Struct. Integ. Life, 21(3): 285-289.
9. Medjo, B., Arsić, M., Mladenović, M., et al. (2020), *Influence of defects on limit loads and integrity of the pipeline at Hydropower Plant 'Pirot'*, Struct. Integ. Life, 20(1): 82-86.
10. Milovanović, A.M., Martić, I., Trumbulović, Lj., et al. (2021), *Finite element analysis of spherical storage tank stress state*, Struct. Integ. Life, 21(3): 273-278.
11. Timoshenko, S., Woinowsky-Krieger, S., Theory of Plates and Shells, 2nd Ed., McGraw-Hill Book Co., 1987.