Arvind Kumar[1], Vikash Ghlawat[2], Devinder Singh[3**], Khursheed Alam[2]*

MATHEMATICAL MODEL FOR FRACTIONAL MICROSTRETCH THERMOELASTIC MEDIUM

MATEMATIČKI MODEL ZA FRAKCIONALNU MIKRORASTEGLJIVU TERMOELASTIČNU SREDINU

Originalni naučni rad / Original scientific paper
UDK /UDC:

Rad primljen / Paper received: 10.11.2021

Adresa autora / Author's address:
[1)] G.G.S.S.S. Uklana Mandi, Department of Secondary Education, Haryana, India [*]email: arvi.math@gmail.com
[2)] School of Basic Sciences and Research, Sharda Univ., G. Noida, India [**]email: despathania@yahoo.com
[3)] Department of Mathematics, GNDEC Ludhiana, India

Keywords
- microstretch thermoelastic
- integral transform
- fractional order
- inclined forces
- thermal stresses

Ključne reči
- mikrorastegljiv termoelastičan
- integralna transformacija
- frakcionalni red
- kose sile
- termički naponi

Abstract

The present investigation deals with thermal and mechanical interactions in a fractional order microstretch thermoelastic half-space subjected to inclined mechanical forces acting at the boundary of the surface of the half-space. Integral transform technique (Laplace and Fourier transform) are applied to solve the basic equations mathematically. The mathematical expressions of mechanical stresses, coupled tangential stress, microstress, and the temperature distribution are obtained numerically. Some particular results and special cases also have been derived from the present research.

Izvod

Ova istraživanja se bave termičkim i mehaničkim interakcijama unutar mikrorastegljivog termoelastičnog poluprostora frakcionalnog reda, koji je opterećen kosim mehaničkim silama, koje deluju na graničnoj površini poluprostora. Metode integralne transformacije (Laplasova i Furijeova transformacija) su primenjene za matematičko rešavanje osnovnih jednačina. Matematički izrazi za mehaničke napone, spregnuti tangencijalni napon, mikronapon, kao i za raspodelu temperature su dobijeni numerički. Pojedina partikularna rešenja i specijalni slučajevi su takođe izvedeni u ovom istraživanju.

INTRODUCTION

Eringen /1, 2/ developed the linear theory of micropolar thermoelasticity, theory of micropolar elastic solids with stretch and derived the basic equations of motion, constitutive relations and boundary conditions for these theories. The later theory was very important for a class of materials which can stretch and contract. In this theory Eringen /2/ presented a new model which explained the motion of a certain class of materials, i.e. rigid chopped fibres, many categories of composites and granular materials.

Eringen /3/ developed the theory of thermo-microstretch elastic materials including microstructural expansion and contractions. The material points of microstretch thermoelastic material are able to stretch and contract independently of their translational and relational motion. Composite substances reinforced with chopped fibres, porous materials filled with asphalt and other insertions are also categorized as microstretch thermoelastic materials. Here it is noted that the theory of microstretch thermoelastic solids is a particular case of micromorphic thermoelasticity and is the generalization of micropolar theory of thermoelasticity.

The differential equations including higher-order fractional derivatives play a significant role in mathematical modelling of dynamical behaviour of some complex systems. The fractional calculus has also applications in the solution of problems of material engineering, physics, chemical engineering and bio-medical dynamics. During the last decade many researchers worked on fractional calculus. First of all, Abel presented an application of fractional calculus in formulation of the tautochrone problem. Povstenko /4/ also developed a quasi-static thermoelastic model for uncoupled equations taking the fractional time derivative in the heat conduction equation. Later Povstenko /5/ investigated the thermal stresses in an infinite medium including cylindrical holes by using the fractional heat conduction equation. Sherief et al. /6/ presented a mathematical model of fractional order theory of thermoelasticity by revising the Cattaneo law and deduced the basic equations, constitutive relations, uniqueness theorem, reciprocity theorem, and variational problems. Youssef /7/ constructed another mathematical model for fractional theory of thermoelasticity by taking different value of fractional parameter α. He also discussed an application of this theory. Ezzat /8, 9/ proposed another theory of fractional order generalized thermoelasticity using Taylor's series expansion of time-fractional order. Later Ezzat and Fayik /10/ extended this fractional order generalized thermoelasticity by including the thermo-diffusion and presented a new theory named as fractional order thermoelasticity theory with diffusion in elastic medium. They also, derived the uniqueness theory, reciprocity theorem and variational principle. Shaw and Mukhopadhyay /11/ observed the effect of two temperature and moving heat source in micropolar thermoelastic medium. Sur and Kanoria /12/ investigated a 1-dimensional problem in fractional thermoelasticity with two-temperatures in the context of LS-theory and GL-theory. Yu et al. /13/ discussed a problem in electromagnetic anisotropic medium using fractional order theory of thermoelasticity. Sumelka /14/ discussed some applications and qualitative aspects in fractional continuum mechanics. A 1-dimensional fractional thermoelastic problem with diffusion in a half-space is discussed by Povstenko and Klekot /15/. Shaw and Mukhopadhyay /16/ developed a theory of fractional ordered thermoelastic diffusion. Recently Chirila and Marin /17/ worked on dipolar thermoelastic materials with the property of double porosity and presented a generalized theory of thermoelasticity with fractional order strain. Marin et al. /18/ recently presented a mathematical model of fractional order strain in dipolar thermoelasticity.

Here we have used the fractional theory of thermoelasticity developed by Ezzat and Fayik /10/ and analysed the thermo-mechanical interactions in a fractional order microstretch thermoelastic medium. The normal stress, tangential stress, coupled tangential stress, microstress, and temperature distribution are computed using the numerical method technique involving Laplace and Fourier transform. The computed physical quantities are also depicted graphically.

BASIC EQUATIONS

Following Eringen /3/, Ezzat and Fayik /10/, the basic equations for homogeneous, isotropic microstretch generalised thermoelastic solids in the absence of body forces, body couples and stretch forces are given by:

Stress equation of motion:

$$(\lambda+\mu)\nabla(\nabla.\mathbf{u})+(\mu+K)\nabla^2\mathbf{u}+k\nabla\times\boldsymbol{\phi}+\lambda_0\nabla\phi^*-\beta_1\nabla T=\rho\ddot{\mathbf{u}}\,, \quad (1)$$

Couple stress equation of motion:

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\mathbf{u}=\rho j\ddot{\boldsymbol{\phi}}\,, \quad (2)$$

Equation of balance of stress moments:

$$(\alpha_0\nabla^2-\lambda_1)\phi^*-\lambda_0\nabla.\mathbf{u}+\upsilon_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\frac{\rho j_0}{2}\ddot{\phi}^*\,, \quad (3)$$

Fractional order equation of heat conduction:

$$K^*\nabla^2 T=\left(\frac{\partial}{\partial t}+\frac{\tau_0^{\alpha}\partial^{\alpha+1}}{\Gamma(\alpha+1)\partial t^{\alpha+1}}\right)(\rho c^* T+\upsilon_1 T_0\varphi^*)+$$
$$+\left(1+\frac{\tau_0^{\alpha}\partial^{\alpha}}{\Gamma(\alpha+1)\partial t^{\alpha}}\right)(\beta_1 T_0\nabla.\dot{\mathbf{u}}-\rho Q)\,, \quad (4)$$

here, $\beta_1=(3\lambda+2\mu+K)\alpha_{t1}$; $\upsilon_1=(3\lambda+2\mu+K)\alpha_{t2}$; α_{t1} and α_{t2} are coefficients of linear thermal expansion.

The constitutive relations are:

$$t_{ij}=(\lambda_0\phi^*+\lambda u_{r,r})\delta_{ij}+\mu(u_{ij}+u_{j,i})+K(u_{j,i}-\varepsilon_{ijk}\phi_k)-$$
$$-\beta_1\delta_{ij}T\,, \quad (5)$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\varepsilon_{mji}\phi^*_{,m}\,, \quad (6)$$

$$\lambda_i^*=\alpha_0\phi^*_{,i}+b_0\varepsilon_{ijm}\phi_{j,m}\,. \quad (7)$$

Following Sherief /6/, the Caputo fractional derivative in the heat conduction Eq.(4) can be written as:

$$\frac{d^{\alpha}f(t)}{dt^{\alpha}}=\begin{cases}\dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_0^t(t-\tau)^{n-\alpha-1}\frac{d^n f(\tau)}{d\tau^n}d\tau, & n-1<\alpha<n\\[2ex] \dfrac{d^n f(\tau)}{dt^n}, & \alpha=n\end{cases} \quad (8)$$

in which $n-1<\alpha<n$, $m\in N=\{1,2,\ldots\}$,

where: λ, μ, α, β, γ, k, λ_0, λ_1, α_0, b_0, are material constants; ρ is mass density; $\mathbf{u}=(u_1,u_2,u_3)$ is the displacement vector; and $\boldsymbol{\phi}=(\phi_1,\phi_2,\phi_3)$ is the microrotation vector; ϕ^* is the scalar microstretch function; T is temperature; and T_0 is the reference temperature of the body chosen; K^* is the coefficient of thermal conductivity; c^* is specific heat at constant strain; j is the microinertia; t_{ij} are components of stress; m_{ij} are components of coupled stress; λ_i^* is the microstress tensor; δ_{ij} is Kroneker's delta function.

FORMULATION OF THE PROBLEM

We consider an isotropic homogeneous fractional microstretch thermoelastic half-space in an intact form at uniform temperature T_0. The origin of rectangular Cartesian coordinate system is taken on the x_3 axis with x_3-axis pointing vertically downward the medium.

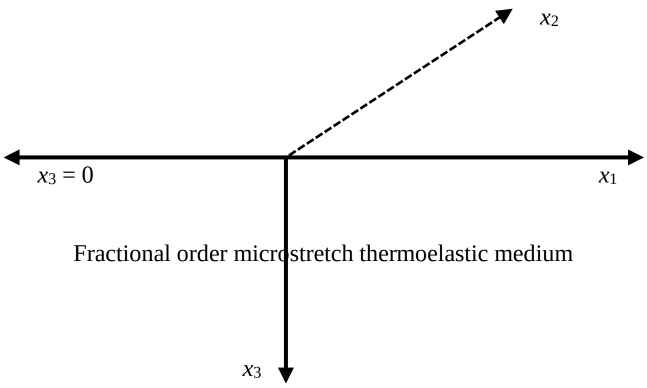

Figure 1. Geometry of the problem.

We consider plane strain problem with all the field variables depending on (x_1, x_3, t). For two-dimensional problems, we take

$$\mathbf{u}=(u_1,0,u_3)\,,\quad \boldsymbol{\phi}=(0,\phi_2,0)\,. \quad (9)$$

For further consideration, it is convenient to introduce in Eqs.(1)-(4) the dimensionless quantities defined as:

$$x_i'=\frac{\omega^*}{c_1}x_i\,,\ u_i'=\frac{\rho\omega^* c_1}{\beta_1 T_0}u_i\,,\ \phi_i'=\frac{\rho c_1^2}{\beta_1 T_0}\phi_i\,,\ \phi^{*\prime}=\frac{\rho c_1^2}{\beta_1 T_0}\phi^*\,,$$
$$T'=\frac{T}{T_0}\,,\ t'=\omega^* t\,,\ \tau_1'=\omega^*\tau_1\,,\ \tau_0'=\omega^*\tau_0\,,\ t_{ij}'=\frac{1}{\beta_1 T_0}t_{ij}\,,$$
$$\omega^*=\frac{\rho c^* c_1^2}{K^*}\,,\ c_1^2=\frac{\lambda+2\mu+k}{\rho}\,,\ m_{ij}^*=\frac{\omega^*}{c\beta_1 T_0}m_{ij}\,. \quad (10)$$

Utilizing the expressions defined by Eq.(10) in Eqs.(1)-(4) and with the help of Eq.(9), we reach the following equations:

$$a_1\frac{\partial e}{\partial x_1}+a_2\nabla^2 u_1-a_3\frac{\partial\phi_2}{\partial x_3}+a_4\frac{\partial\phi^*}{\partial x_1}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1}=\ddot{u}_1\,, \quad (11)$$

$$a_1\frac{\partial e}{\partial x_3}+a_2\nabla^2 u_3+a_3\frac{\partial\phi_2}{\partial x_1}+a_4\frac{\partial\phi^*}{\partial x_3}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3}=\ddot{u}_3\,, \quad (12)$$

$$\nabla^2\phi_2-2a_6\phi_2+a_6\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)=a_7\ddot{\phi}_2\,, \quad (13)$$

$$\nabla^2\phi^*-a_8\phi^*-a_9 e+a_{10}\left(1+\tau_1\frac{\partial}{\partial t}\right)T=a_{12}\ddot{\phi}^*\,, \quad (14)$$

$$\nabla^2 T-\left(\frac{\partial}{\partial t}+\frac{\tau_0\omega^{*\alpha-1}\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)(T-a_{13}\nabla^2\varphi+a_{14}\varphi^*)=Q_0\,, \quad (15)$$

Here,

$$a_2=\frac{\mu+k}{\rho c_1^2};\ a_3=\frac{k}{\rho c_1^2};\ a_4=\frac{\lambda_0}{\delta c_1^2};\ a_6=\frac{2kc_1^2}{\gamma\omega^{*2}};\ a_7=\frac{\delta jc_1^2}{\gamma},$$

$$a_8=\frac{\lambda_1 c_1^2}{\alpha_0\omega^{*2}};\ a_9=\frac{\lambda_0 c_1^2}{\alpha_0\omega^{*2}};\ a_{10}=\frac{v_1\delta c_1^4}{\beta_1\alpha_0\omega^{*2}};\ a_{12}=\frac{\delta c_1^2 j_0}{2\alpha_0};$$

$$a_{13}=-\frac{\beta_1^2 T_0}{k^*\omega^*};\quad a_{14}=\frac{v_1\beta_1 T_0}{\rho\omega^* k^*};\quad a_{15}=s+\tau_0\omega^{*\alpha-1}s^{\alpha+1};$$

$\nabla^2=\dfrac{\partial^2}{\partial x_1^2}+\dfrac{\partial^2}{\partial x_3^2}$ is the Laplacian operator.

Making use of Helmholtz's decomposition theorem i.e. representation of a vector into scalar and vector potentials, the displacement components u_1 and u_3 are related to non-dimensional potential functions ϕ and ψ as:

$$u_1=\frac{\partial\phi}{\partial x_1}-\frac{\partial\psi}{\partial x_3}\,,\quad u_3=\frac{\partial\phi}{\partial x_3}+\frac{\partial\psi}{\partial x_1}\,. \quad (16)$$

Substituting the values of u_1 and u_3 from Eq.(16) in Eqs. (11)-(15), we obtain:

$$\nabla^2\phi-\ddot{\phi}+a_4\phi^*-\left(1+\tau_1\frac{\partial}{\partial t}\right)T=0\,, \quad (17)$$

$$\left(\nabla^2-a_8-a_{12}\frac{\partial^2}{\partial t^2}\right)\phi^*-a_9\nabla^2\phi+a_{10}\left(1+\tau_1\frac{\partial}{\partial t}\right)T=0\,, \quad (18)$$

$$\nabla^2 T-\left(\frac{\partial}{\partial t}+\frac{\tau_0\omega^{*\alpha-1}\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)(T-a_{13}\nabla^2\phi+a_{14}\phi^*)=Q_0\,, \quad (19)$$

$$a_2\nabla^2\psi-\ddot{\psi}+a_3\phi_2=0\,, \quad (20)$$

$$\nabla^2\phi_2-2a_6\phi_2-a_6\nabla^2\psi=a_7\ddot{\phi}_2\,. \quad (21)$$

SOLUTION OF THE PROBLEM

We define the Laplace and Fourier transforms, respectively as:

$$\bar{f}(s,x_1,x_3)=\int_0^\infty f(t,x_1,x_3)e^{-st}dt\,, \quad (22)$$

$$\hat{f}(x_3,\xi,s)=\int_{-\infty}^{\infty}\bar{f}(s,x_1,x_3)e^{\iota\xi x_1}dx_1\,. \quad (23)$$

Applying Laplace transform defined by Eq.(22) on Eqs. (17)-(21), and then applying Fourier transforms defined by Eq.(23) on the resulting quantities, we obtain:

$$\left(\frac{d^2}{dx^2}-\xi_1\right)\hat{\phi}+a_4\hat{\phi}^*-\tau_{11}\hat{T}=0\,, \quad (24)$$

$$-a_9\left(\frac{d^2}{dx^2}-\xi^2\right)\hat{\phi}+\left(\frac{d^2}{dx^2}-a_{20}\right)\hat{\phi}^*+a_{21}\hat{T}=0\,, \quad (25)$$

$$(D^2-\xi^2)\hat{T}-(s+z_0\omega^{*\alpha-1}s^{\alpha+1})\hat{T}-a_{13}(D^2-\xi^2)\hat{\phi}+ +a_{14}\hat{\phi}^*=Q_1\,, \quad (26)$$

$$\left(\frac{d^2}{dx_3^2}-\xi^2-s^2\right)\hat{\psi}-a_3\hat{\phi}_2=0\,, \quad (27)$$

$$\left(\frac{d^2}{dx_3^2}-a_{30}\right)\hat{\phi}_2+a_6\left(\frac{d^2}{dx_3^2}-\xi^2\right)\hat{\psi}=0\,. \quad (28)$$

Eliminating $\hat{\phi}^*$ and $\hat{T}$, $\hat{\phi}$ and $\hat{T}$, and $\hat{\phi}$ and $\hat{\phi}^*$, respectively from Eqs.(24)-(26), we obtain:

$$[D^6-AD^4-BD^2+C]\hat{\phi}=f_1\,, \quad (29a)$$

$$[D^6-AD^4-BD^2+C]\hat{\phi}^*=f_2\,, \quad (29b)$$

$$[D^6-AD^4-BD^2+C]\hat{T}=f_3\,. \quad (29c)$$

Also eliminating $\hat{\phi}_2$ from Eqs.(27)-(28) yields:

$$[D^4+ED^2+F]\hat{\psi}=0\,, \quad (30)$$

where: $\xi_{11}=\xi^2+s^2$; $a_{16}=\xi^2+a_8+a_{12}s^2$; $a_{17}=\xi^2+a_{15}$; $a_{18}=a_{13}a_{15}$; $a_{19}=a_{14}a_{15}$; $a_{20}=a_2\xi^2-s^2$; $a_{21}=\xi^2+2a_6+a_7s^2$; $f_1=-Q_1(\tau_{11}\gamma^{*2}+a_{25})$; $f_2=-Q_1(a_{26}\gamma^{*2}+a_{27})$; $f_3=-Q_1(\gamma^{*4}-a_{28}\gamma^{*2}+a_{29})$; $f_4=(\gamma^{*6}-A\gamma^{*4}-B\gamma^{*2}+C)$; $A=-a_{16}-a_{17}-\xi_{11}+a_4a_9+a_{18}$; $B=a_{10}a_{19}+\xi_{11}a_{16}+\xi_{11}a_{17}-a_4a_9\xi^2-a_4a_9a_{17}-a_{10}a_{18}\xi^2$; $C=-\xi_{11}a_{16}a_{17}-\xi_{11}a_{10}a_{19}-a_4a_{17}\xi^2+a_{10}a_{18}\xi^2+a_{16}a_{18}\xi^2$; $B_2=(a_3a_6-a_2a_{21}-a_{20})/a_2$; $B_3=(a_{20}a_{21}-a_3a_6\xi^2)/a_2$.

The mathematical solutions of Eqs.(29)-(30) satisfying the radiation conditions that ($\hat{\phi}$, $\hat{\phi}^*$, $\hat{T}$, $\hat{\phi}_2$, $\hat{\psi}$) $\to 0$ as $x_3\to\infty$ are given by:

$$\hat{\phi}=B_1e^{-m_1x_3}+B_2e^{-m_2x_3}+B_3e^{-m_3x_3}+L_1\,, \quad (31)$$

$$\hat{\phi}^*=d_1B_1e^{-m_1x_3}+d_2B_2e^{-m_2x_3}+d_3B_3e^{-m_3x_3}+L_2\,, \quad (32)$$

$$\hat{T}=e_1B_1e^{-m_1x_3}+e_2B_2e^{-m_2x_3}+e_3B_3e^{-m_3x_3}+L_3\,, \quad (33)$$

$$\hat{\psi}=B_4e^{-m_4x_3}+B_5e^{-m_5x_3}\,, \quad (34)$$

$$\hat{\phi}_2=h_4B_4e^{-m_4x_3}+h_5B_5e^{-m_5x_3}\,, \quad (35)$$

where:

$$d_i=\frac{a_9m_i^4-(a_9a_{17}+a_9\xi^2-a_{10}a_{18})m_i^2+a_9\xi^2a_{17}-a_{10}\xi^2a_{18}}{m_i^4-(a_{16}+a_{17})m_i^2+a_{16}a_{17}+a_{10}a_{19}};$$

$$e_i=\frac{-a_{18}m_i^4-(a_9a_{19}+a_{18}\xi^2+a_{16}a_{18})m_i^2-a_9\xi^2a_{19}-a_{16}\xi^2a_{18}}{m_i^4-(a_{16}+a_{17})m_i^2+a_{16}a_{17}+a_{10}a_{19}},$$

$i=1,2,3$; $h_l=\dfrac{a_2(m_l^2-\xi_1)}{a_3}$, $l=4,5$; and m_i^2 ($i=1,2,3$) are the roots of the characteristic equation given by Eq.(29a); and m_l^2 ($l=4,5$) are the roots of the characteristic equation of Eq.(30).

BOUNDARY CONDITIONS

We consider concentrated normal force and concentrated thermal source at the boundary surface $x_3=0$, mathematically, these can be written as:

$$t_{33}=-F_1\delta(x_1)\delta(t)\,,\ t_{31}=-F_2\delta(x_1)\delta(t)\,,\ m_{32}=0\,,$$
$$\lambda_3^*=0\,,\quad \frac{\partial T}{\partial x_3}=0\,, \quad (36)$$

where: F_1, F_2 are the magnitudes of the applied forces.

Substituting the values of $\hat{\phi}$, $\hat{\phi}^*$, $\hat{T}$, $\hat{\phi}_2$, $\hat{\psi}$ from Eqs. (31)-(35) in boundary condition Eq.(36) and using Eqs.(5)-

(7), (9)-(10), (22)-(23) and solving the resulting equations, we obtain:

$$\hat{t}_{33} = \sum_{i=1}^{5} G_{1i} e^{-m_i x_3} + M_1 e^{-\gamma^* x_3} , \tag{37}$$

$$\hat{t}_{31} = \sum_{i=1}^{5} G_{2i} e^{-m_i x_3} + M_2 e^{-\gamma^* x_3} , \tag{38}$$

$$\hat{m}_{32} = \sum_{i=1}^{5} G_{3i} e^{-m_i x_3} + M_3 e^{-\gamma^* x_3} , \tag{39}$$

$$\hat{\lambda}_3^* = \sum_{i=1}^{5} G_{4i} e^{-m_i x_3} + M_4 e^{-\gamma^* x_3} , \tag{40}$$

$$\hat{T} = \sum_{i=1}^{5} G_{5i} e^{-m_i x_3} + M_5 e^{-\gamma^* x_3} . \tag{41}$$

Here, $b_1 = \dfrac{\lambda_0}{\rho c_1^2}$; $b_2 = \dfrac{\lambda}{\rho c_1^2}$; $b_3 = \dfrac{2\mu + K}{\rho c_1^2}$; $b_5 = \dfrac{\mu + K}{\rho c_1^2}$; $b_6 = \dfrac{\mu}{\rho c_1^2}$; $b_7 = \dfrac{K}{\rho c_1^2}$; $b_8 = \dfrac{\omega^{*2}\gamma}{\rho c_1^4}$; $b_9 = \dfrac{\omega^{*2} b_0}{\rho c_1^4}$; $b_{10} = \dfrac{\omega^{*2}}{\rho c_1^4}$;

$G_{mi} = g_{mi} C_i$; $C_i = \dfrac{\Delta_i}{\Delta_0}$, i = 1,2,…,5.

Special case:

Micropolar thermoelastic solid

In absence of microstretch effect in Eqs.(37)-(41), we obtain the corresponding expressions of stresses, displacements, and temperature for micropolar generalised thermoelastic half space.

Inversion of the transform

The transformed displacements, stresses, and temperature changes are functions of the parameters of Laplace and Fourier transforms s and ξ, respectively, and hence these are of the form $f(s, \xi, z)$. To obtain the solution of the problem in the physical domain, we must invert the Laplace and Fourier transform by using the method applied by Kumar /19/.

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