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WAVE PROPAGATION IN THERMO-VISCOELASTIC SEMICONDUCTING MEDIUM WITH HYDROSTATIC INITIAL STRESS

PROSTIRANJE TALASA U TERMOVISKOELASTIČNOJ POLUPROVODNIČKOJ SREDINI SA POČETNIM HIDROSTATIČKIM NAPONOM

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Keywords
- thermo-viscoelastic
- hydrostatic initial stress
- carrier density
- wave propagation
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Abstract

The present research article aims to investigate wave propagation in thermo-viscoelastic semiconducting medium with hydrostatic initial stress. The coupled wave equations in terms of displacement, temperature, and carrier density are solved analytically. The values of the penetration depth of different waves are obtained and depicted graphically against frequency to show the effect of viscosity and hydrostatic initial stress. As a special case, the authors have also discussed deformation in the medium caused due to a mechanical force along the free surface of semiconducting medium with hydrostatic initial stress. The components of displacement, mechanical stresses, carrier density, and temperature distribution in the medium are obtained and presented analytically. Graphical results of the variations of these quantities are also presented to show the effects of viscosity and hydrostatic initial stress.

Ključne reči
- termoviskoelastičnost
- hidrostatički početni napon
- gustina prenosilaca
- prostiranje talasa
- dubina penetracije

Izvod

Cilj predstavljenih istraživanja je u izučavanju prostiranja talasa u termoviskoelastičnoj poluprovodničkoj sredini sa početnim hidrostatičkim naponom. Spregnute talasne jednačine sa veličinama: pomeranje, temperatura i gustina prenosioca su rešene analitički. Dobijeni su rezultati dubine penetracije različitih talasa i predstavljeni grafički u odnosu na frekvenciju kako bi se pokazao uticaj viskoznosti i početnog hidrostatičkog napona. Kao specijalan slučaj, autori diskutuju o deformaciji sredine, izazvana mehaničkom silom duž slobodne površine poluprovodničke sredine sa početnim hidrostatičkim naponom. Komponente pomeranja, mehanički naponi, gustina prenosioca, kao i raspodela temperature u sredini su dobijeni i predstavljeni analitički. Promene ovih veličina su predstavljene i grafički kako bi se pokazali uticaji viskoznosti i početnog hidrostatičkog napona.

INTRODUCTION

Studies related to generalised thermoelasticity have dragged considerable attention during last few decades due to its application in various practical aspects of life processes such as earthquake prediction, exploration of minerals, soil dynamics, etc. Many researchers investigated wave propagation in elastic medium neglecting interaction between thermal effects and coupled plasma effects. Firstly, uncoupled classical theory of thermoelasticity which assumes infinite speed of heat propagation is replaced by Biot /1/ by considering theory of coupled thermoelasticity. Later, generalised theories of thermoelasticity were developed by Lord and Shulman /2/, Green and Lindsay /3/, which were further reviewed by Green and Nagdhi /4/, Hetnarski and Ignaczak /5/, and Ignaczak and Ostoja-Starzewski /6/.

Variation in temperature has considerable impact on mechanical and thermal properties of a material. Therefore, the effect of temperature gradient which was over-looked in various studies related to generalised theory of thermoelasticity have been taken into consideration by many researchers /7-11/. Properties of a material can not be taken as having constant values under the effect of temperature variation thereby making it essential for consideration of temperature dependence of material properties. A model showing dependence of thermal conductivity and modulus of elasticity on temperature is developed and the problem of an infinite material with spherical cavity is solved by Youssef /12/. Some other authors working in this field are listed /13-16/. Various theories of generalised thermoelasticity are being developed by researchers resulting in addition of different outer fields to equations governing motion and heat.

The study of initially stressed bodies has always been an interesting problem for researchers. Initial stresses in a medium can develop due to many factors such as slow process of crawling, variations in gravity, temperature difference, etc. Our Earth can also be considered to be under initial stresses. So significance of these initial stresses on surface wave propagation can not be overlooked. Formulation of isotropic thermoelasticity was designed by Montanaro /17/ under influence of hydrostatic initial stress. The above formulation was used for studying plane harmonic waves under generalised thermoelasticity by many authors /18-20/. Ailawalia and Budhiraja /21/ demonstrated the effect of internal heat source under influence of hydrostatic initial stress in temperature rate dependent thermoelastic medium. In the current era, wave propagation problems in semiconducting medium are gaining importance by serving the base for various fields such as plasma physics, oil extraction, mechanical engineering, etc. When light falls on a semiconducting material, a change in physical properties and temperature of material is caused by light energy. Due to this temperature gradient, elastic deformation, free carrier density appears. Gordan et al. /22/ discovered electronic deformations to photothermal spectroscopy. Photothermal methods are being applied for measuring physical quantities such as temperature, electric effects of semiconducting material /23-25/. Besides this, many researchers have explored semiconducting medium /26-31/. In above studies, relation between thermoelasticity and photothermal theory is not taken into account. In recent years, interaction between the thermal wave, elastic wave, and plasma wave motion during the photo-excitation process in dual-phase-lag thermoelastic model with moving internal heat source under effect of gravitational field was described by Lotfy /32/. A new model of two-temperature theory under photothermal theory for semiconducting elastic medium is demonstrated by Abo-Dahab and Lotfy /33/. Memory-dependent derivatives in the context of the two-temperature theory are used in generalised thermoelasticity under photothermal theory by Lotfy and Sarkar /34/. A novel mathematical model under hydrostatic initial stress is developed by Lotfy /35/ for explaining the effect of the magnetic field for polymer photothermal diffusion semiconductor medium. Photothermal waves in an unbounded semiconductor medium with cylindrical cavity are investigated by Hobiny and Abbas /36/. Lotfy /37/ investigated problem for photothermal semiconducting medium for two temperature with hydrostatic initial stress under dual phase lag model. Under the exposure of strong magnetic field, effect of Hall current of elastic semiconductor medium is studied by Lotfy et al. /38/. Lotfy along with his co-workers /39-45/ investigated different types of problems in semiconducting medium. In the recent years authors /46-50/ have explored some interesting problems for the semiconducting medium.

In the present article, propagation of waves in thermo-viscoelastic semiconducting medium under hydrostatic initial stress are studied. The penetration depths of four waves propagating in the medium are evaluated to show the effect of viscosity and hydrostatic initial stress. A mechanical force of constant magnitude is applied along the free surface of semiconducting medium. As a special case, the components of displacement, stresses, carrier density, and temperature distribution are also evaluated, and the variations of these quantities are depicted graphically.

BASIC EQUATION

Following Mandelis et al. /29/ and Todorović /30/, the coupled plasma, thermal, and elastic transport equations can be written as

$$D_e\nabla^2 N-\frac{1}{\tau}N+\kappa T-\frac{\partial N}{\partial T}=0\,, \tag{1}$$

$$K^*\nabla^2 T-\frac{E_g}{\tau}N+\gamma^* T_0\left(n_1+n_0\tau_0\frac{\partial}{\partial t}\right)\nabla.\frac{\partial u}{\partial t}- -\rho C^*\left(n_1+n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}=0\,. \tag{2}$$

The governing equations and constitutive relations for a semiconducting thermoviscoelastic medium under hydrostatic initial stress with isotropic and homogenous properties are given

$$\left(\mu^*-\frac{P}{2}\right)\nabla^2\vec{u}+\left(\lambda^*+\mu^*-\frac{P}{2}\right)\nabla(\nabla.\vec{u})-\gamma^*\left(1+v_0\frac{\partial}{\partial t}\right)\nabla T- -\delta_n^*\nabla N=\rho\ddot{\vec{u}}\,, \tag{3}$$

$$\sigma_{ij}=-P(\delta_{ij}+\omega_{ij})+2\mu^* e_{ij}+(\lambda^* u_{k,k}-\delta_n^* N-\gamma^* T)\delta_{ij}\,, \tag{4}$$

where: $e_{ij} = (u_{j,i} + u_{i,j})/2$; $\omega_{ij} = (u_{j,i} - u_{i,j})$; $Y^* = (3\lambda^* + 2\mu^*)\alpha_t$; $\delta_n^* = (3\lambda^* + 2\mu^*)d_n$; $\lambda^* = \lambda(1 + R_1(\partial/\partial t))$; $\mu^* = \mu(1 + R_2(\partial/\partial t))$; $\gamma^* = \gamma(1 + \gamma^e(\partial/\partial t))$; $\delta_n^* = \delta_n(1 + \delta_n^e(\partial/\partial t))$; $\gamma^e = (3\lambda R_1 + 2\mu R_2)\alpha_t/\gamma$; $\delta_n^e = (3\lambda R_1 + 2\mu R_2)\delta_n/\delta_n$; and λ, μ are Lame's constants; σ_{ij} is stress tensor; ρ density; v_0 and τ_0 are thermal relaxation times; N carrier density; E_g energy gap of semiconductor; C^* specific heat at constant strain; δ difference of deformation potential of conduction and valence band; D_e carrier diffusion coefficient; K^* coefficient of thermal conductivity; α coefficient of linear thermal expansion; $\kappa = (\partial N_0/\partial t)(T/\tau)$; N_0 is equilibrium carrier concentration at temperature T; τ photogenerated carrier lifetime; T thermodynamic temperature; $\gamma = (3\lambda + 2\mu)\alpha_t$; $\delta_n = (3\lambda + 2\mu)d_n$; R_1 and R_2 are viscoelastic relaxation times; and P is hydrostatic initial stress.

FORMULATION OF THE PROBLEM

We consider a rectangular coordinate system (x,y,z) with z-axis pointing vertically downward. For a two-dimensional problem it is assumed that the waves propagate in x-z plane. Hence, in two-dimensional space i.e. x-z plane, the displacement vector in semiconducting visco-thermoelastic medium is considered as $u_1 = u_1(x,z,t)$, $u_3 = u_3(x,z,t)$, which further reduces the equations of motion and coupled generalised equations of heat conduction and carrier density Eqs.(1)-(3) and constitutive relations Eq.(4) in two-dimensional in the absence of body forces as follows,

$$(\lambda^*+2\mu^*-P)\frac{\partial^2 u_1}{\partial x^2}+\left(\lambda^*+\mu^*-\frac{P}{2}\right)\frac{\partial^2 u_3}{\partial x\partial z}+\left(\mu^*-\frac{P}{2}\right)\frac{\partial^2 u_1}{\partial z^2}- -\gamma^*\left(1+v_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}-\delta_n^*\frac{\partial N}{\partial x}=\rho\frac{\partial^2 u_1}{\partial t^2}\,, \tag{5}$$

$$\left(\mu^*-\frac{P}{2}\right)\frac{\partial^2 u_3}{\partial x^2}+\left(\lambda^*+\mu^*-\frac{P}{2}\right)\frac{\partial^2 u_1}{\partial x\partial z}+(\lambda^*+2\mu^*-P)\frac{\partial^2 u_3}{\partial z^2}- -\gamma^*\left(1+v_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}-\delta_n^*\frac{\partial N}{\partial z}=\rho\frac{\partial^2 u_3}{\partial t^2}\,, \tag{6}$$

$$D_e\nabla^2 N-\frac{1}{\tau}N+\kappa T-\frac{\partial N}{\partial T}=0\,, \tag{7}$$

$$K^*\nabla^2 T-\frac{E_g}{\tau}N+\gamma^* T_0\left(n_1+n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_3}{\partial z}\right)- -\rho C^*\left(n_1+n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}=0\,, \qquad (8)$$

$$\sigma_{xx}=(\lambda^*+2\mu^*)\frac{\partial u_1}{\partial x}+\lambda^*\frac{\partial u_3}{\partial z}-(3\lambda^*+2\mu^*)\times \times\left[\alpha_t\left(1+v_0\frac{\partial}{\partial t}\right)T+d_nN\right]-P\,, \qquad (9)$$

$$\sigma_{zz}=(\lambda^*+2\mu^*)\frac{\partial u_3}{\partial z}+\lambda^*\frac{\partial u_1}{\partial x}-(3\lambda^*+2\mu^*)\times \times\left[\alpha_t\left(1+v_0\frac{\partial}{\partial t}\right)T+d_nN\right]-P\,, \qquad (10)$$

$$\sigma_{zx}=\left(\mu^*+\frac{P}{2}\right)\frac{\partial u_1}{\partial z}+\left(\mu^*-\frac{P}{2}\right)\frac{\partial u_3}{\partial x}\,. \qquad (11)$$

For the convenience of numerical calculations, following dimensionless quantities are introduced:

$$x'=\frac{1}{c_1t^*}x\,,\quad z'=\frac{1}{c_1t^*}z\,,\quad u_1{}'=\frac{1}{c_1t^*}u_1\,,\quad u_3{}'=\frac{1}{c_1t^*}u_3\,,\quad t'=\frac{t}{t^*}\,,$$

$$P'=P\,,\quad T'=\frac{\gamma T}{(\lambda+2\mu)}\,,\quad N'=\frac{\delta_nN}{(\lambda+2\mu)}\,,\quad \sigma_{ij}{}'=\frac{\sigma_{ij}}{\mu}\,, \qquad (12)$$

where: $t^*=K^*/\rho c_1^2C^*$; $c_1^2=(\lambda+2\mu)/\rho$.

Using Eq.(12) in Eqs.(5)-(8), we get the following non-dimensional equations in the thermo-viscous semiconducting medium (after dropping the primes),

$$(\lambda^*+2\mu^*-P)\frac{\partial^2u_1}{\partial x^2}+\left(\lambda^*+\mu^*-\frac{P}{2}\right)\frac{\partial^2u_3}{\partial x\partial z}+\left(\mu^*-\frac{P}{2}\right)\frac{\partial^2u_1}{\partial z^2}- -\frac{\gamma^*}{\gamma}(\lambda+2\mu)\left(1+\frac{v_0}{t^*}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}-\frac{\delta_n^*}{\delta_n}(\lambda+2\mu)\frac{\partial N}{\partial x}=\rho c_1^2\frac{\partial^2u_1}{\partial t^2}\,, \qquad (13)$$

$$\left(\mu^*-\frac{P}{2}\right)\frac{\partial^2u_3}{\partial x^2}+\left(\lambda^*+\mu^*-\frac{P}{2}\right)\frac{\partial^2u_1}{\partial x\partial z}+(\lambda^*+2\mu^*-P)\frac{\partial^2u_3}{\partial z^2}- -\frac{\gamma^*}{\gamma}(\lambda+2\mu)\left(1+\frac{v_0}{t^*}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}-\frac{\delta_n^*}{\delta_n}(\lambda+2\mu)\frac{\partial N}{\partial z}=\rho c_1^2\frac{\partial^2u_3}{\partial t^2}\,, \qquad (14)$$

$$\nabla^2N-A_1N+\varepsilon_3T-A_2\frac{\partial N}{\partial T}=0\,, \qquad (15)$$

$$\nabla^2T-\varepsilon_2N+\varepsilon_1\left(n_1+n_0\frac{\tau_0}{t^*}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_3}{\partial z}\right)- -\left(n_1+n_0\frac{\tau_0}{t^*}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}=0\,, \qquad (16)$$

where:

$$A_1=\frac{c_1^2t^{*2}}{D_e\tau}\,,\quad A_2=\frac{K^*}{\rho D_eC^*}\,,\quad \varepsilon_3=\frac{c_1^2t^{*2}\kappa\delta_n}{D_e\gamma}\,,\quad \varepsilon_1=\frac{\gamma\gamma^*T_0t^*}{\rho K^*}\,,$$

$$\varepsilon_2=\frac{c_1^2t^{*2}E_g\gamma}{\tau K^*\delta_n}\,. \qquad (17)$$

Here, ε_1 represents the thermoelastic coupling parameter, and ε_3 is the thermoelectric coupling parameter.

We introduce potential functions q_1 and q_2 which are related to displacement components u_1 and u_3 by following relation (Helmholtz's representation),

$$u_1=\frac{\partial q_1}{\partial x}-\frac{\partial q_2}{\partial z}\,,\quad u_3=\frac{\partial q_1}{\partial z}+\frac{\partial q_2}{\partial x}\,. \qquad (18)$$

Using Eq.(18) in Eqs.(13)-(16), we obtain

$$\left[(\lambda^*+2\mu^*-P)\nabla^2-\rho c_1^2\frac{\partial^2}{\partial t^2}\right]q_1-a_1\left(1+\frac{v_0}{t^*}\frac{\partial}{\partial t}\right)T-a_2N=0\,, \qquad (19)$$

$$\left[\left(\mu^*-\frac{P}{2}\right)\nabla^2-\rho c_1^2\frac{\partial^2}{\partial t^2}\right]q_2=0\,, \qquad (20)$$

$$\nabla^2N-A_1N+\varepsilon_3T-A_2\frac{\partial N}{\partial T}=0\,, \qquad (21)$$

$$\nabla^2T-\varepsilon_2N+\varepsilon_1\left(n_1+n_0\frac{\tau_0}{t^*}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\nabla^2q_1-\left(n_1+n_0\frac{\tau_0}{t^*}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}=0\,, \qquad (22)$$

where,

$$a_1=\frac{\gamma^*(\lambda+2\mu)}{\gamma}\,,\quad a_2=\frac{\delta_n^*(\lambda+2\mu)}{\delta_n}\,. \qquad (23)$$

Equations (13)-(16) are the field equations of generalised thermoelasticity for the isotropic solid, applicable to the three different theories given by:

(i) for Coupled Theory (C-T), $v_0=0,\ \tau_0=0,\ n_0=0,\ n_1=1$

(ii) for Lord-Shulman theory (L-S), $v_0=0,\ \tau_0>0,\ n_0=1,\ n_1=1$

(iii) for Green-Lindsay theory (G-L),
$v_0>0,\ \tau_0>0,\ n_0=0,\ n_1=1$.

SOLUTION OF THE PROBLEM

Assuming the wave solution of Eqs.(19)-(22) for waves propagating in *xz* plane as,

$$[q_1,q_2,T,N]=[A,B,C,D]\exp\{\iota\xi(x+mz-ct)\}\,, \qquad (24)$$

where: $c=\omega/\xi$ is phase velocity; ω is frequency; and m is unknown parameter representing the penetration depth of the wave.

Introducing Eq.(24) in Eqs.(19)-(22), we obtain one independent wave equation and three coupled equations in terms of q_1, T, and N. On solving the coupled equations, we get the following sixth degree equation,

$$[m^6+\eta_1m^4+\eta_2m^2+\eta_3](A,C,D)=0\,. \qquad (25)$$

The values of η_1, η_2, η_3 are given in the Appendix.

The solution of Eq.(25) satisfying the radiation conditions u_1, u_3, T, $N\to 0$ as $z\to\infty$ can be expressed in the form

$$q_1=\textstyle\sum_{i=1}^3D_i\exp(-\iota\eta m_iz)\exp\{\iota\xi(x-ct)\}\,, \qquad (26)$$

$$T=\textstyle\sum_{i=1}^3D_i^*\exp(-\iota\eta m_iz)\exp\{\iota\xi(x-ct)\}\,, \qquad (27)$$

$$N=\textstyle\sum_{i=1}^3D_i^{**}\exp(-\iota\eta m_iz)\exp\{\iota\xi(x-ct)\}\,. \qquad (28)$$

The solution of independent wave Eq.(20) may be written as,

$$q_2=D_4\exp(-\iota\eta m_4z)\exp\{\iota\eta(x-ct)\}\,, \qquad (29)$$

where: the roots of Eq.(25) are obtained by Cardan's method and are represented by m_i^2 (i = 1,2,3), and

$$m_4=\sqrt{\frac{d_4}{d_3}-1}\,,\quad d_3=\left(\mu^*-\frac{P}{2}\right),\quad d_4=\frac{\rho c_1^2\omega^2}{\xi^2}\,. \qquad (30)$$

Also, the coupling constants D_j^*, D_j^* (j = 1,2,3) can be expressed in terms of D_j as,

$$D_j^*=H_{1j}D_j\,,\quad D_j^{**}=H_{2j}D_j\,, \qquad (31)$$

where: the values of H_{1j}, H_{2j} are given in the Appendix.

BOUNDARY CONDITIONS

To obtain the unknown parameters D_j (j = 1…4), the following boundary conditions are used along the free surface z = 0 considering that a mechanical force of constant magnitude F_0 is acting along the free surface:

(i) $$\sigma_{zz}=-F_0\exp\{\iota\xi(x-ct)\}\,, \qquad (32)$$

(ii) $$\sigma_{zx}=0\,, \qquad (33)$$

(iii) $$\frac{\partial T}{\partial z}=0\,, \qquad (34)$$

(iv) $$\frac{\partial N}{\partial z}-\frac{s}{D_e}N=0\,. \qquad (35)$$

Using Eq.(12) in Eqs.(32)-(35), the above boundary conditions are reduced to dimensionless form. Inserting Eq. (18), Eqs.(26)-(29) in the non-dimensional boundary conditions, we get the following non-homogeneous system of equations,

$$\textstyle\sum_{n=1}^{4}(t_nD_n)=F_1\,, \qquad (36)$$

$$\textstyle\sum_{n=1}^{4}(h_nD_n)=0\,, \qquad (37)$$

$$\textstyle\sum_{n=1}^{3}(l_rD_r)=0\,, \qquad (38)$$

$$\textstyle\sum_{n=1}^{3}(m_rH_{1r}D_r)=0\,, \qquad (39)$$

where: values of F_1, t_n, h_n, l_r (n = 1,…, 4; r = 1,2,3) are given in the Appendix.

Using non-dimensional variables defined by Eq.(12) in stress components Eqs.(9)-(11), we obtain the stress components in dimensionless form. Substituting the solutions of variables q_1, q_2, T, and N from Eqs.(26)-(29) in Eq.(18) and the resulting dimensionless stress components, we obtain the components of displacement and stresses in thermoviscoelastic semiconducting medium under photothermal theory with hydrostatic initial stress as,

$$u_1=\iota\xi[\textstyle\sum_{j=1}^{3}D_j\exp\{-\iota\xi m_jz\}+m_4D_4\exp\{-\iota\xi m_4z\}]\times \times\exp\{\iota\xi(x-ct)\}\,, \qquad (40)$$

$$u_3=-\iota\xi[\textstyle\sum_{j=1}^{3}m_jD_j\exp\{-\iota\xi m_jz\}-D_4\exp\{-\iota\xi m_4z\}]\times \times\exp\{\iota\xi(x-ct)\}\,, \qquad (41)$$

$$\sigma_{zz}=[\textstyle\sum_{s=1}^{4}t_sD_s\exp\{-\iota\xi m_sz\}]\exp\{\iota\xi(x-ct)\}-P\,, \qquad (42)$$

$$\sigma_{zx}=[\textstyle\sum_{s=1}^{4}h_sD_s\exp\{-\iota\xi m_sz\}]\exp\{\iota\xi(x-ct)\}\,, \qquad (43)$$

where: $D_i=\Delta_i/\Delta$, (i = 1,2,3,4); Δ_i ; Δ are determinant of the order 4×4, whose values are given in the Appendix.

PARTICULAR CASES

Thermoviscoelastic semiconducting medium without hydrostatic initial stress

Taking P = 0, we obtain the corresponding values of penetration depth and components of displacement, stresses, carrier density and temperature distribution in thermoviscoelastic semiconducting medium without hydrostatic initial stress.

Non-viscous semiconducting medium with hydrostatic initial stress

Taking R_1 = R_2 = 0, the corresponding expressions are obtained for a non-viscous semiconducting medium with hydrostatic initial stress.

Semiconducting medium without hydrostatic initial stress

Assuming the parameters P = R_1 = R_2 = 0, the expressions for penetration depths and other quantities reduce for a semiconducting medium without hydrostatic initial stress.

NUMERICAL RESULTS

In order to verify the analytical results obtained in the previous section, we present a numerical example by taking silicon as a semiconducting material. Physical constants for the material are given by Song et al. /28/: λ = 3.64×10^{10} N/m^2, μ = 5.46×10^{10} N/m^2, ρ = 2330 kg/m^3, T_0 = 800 K, τ = 5×10^{-5} s, D_e = 2.5×10^{-3} m^2/s, δ_n = (2μ + 3λ)β, E_g = 1.11 V, α_t = 4.14×10^{-6} 1/K, K^* = 150 W/mK, d_n = –9×10^{-31} m^3, C^* = 695 J/kgK, s = 2 m/s, b = 0.6, F_0 = 1.0.

Numerical computations are carried out on the surface z = 1.0 at t = 1.0 for ε_3 = 0.3, ξ = 2.0 and viscoelastic relaxation times R_1 = 0.04, R_2 = 0.06. Graphical results of penetration depth m_n are shown in Figs. 1-4 for three values of P (5.0; 2.0; 0) against frequency ω.

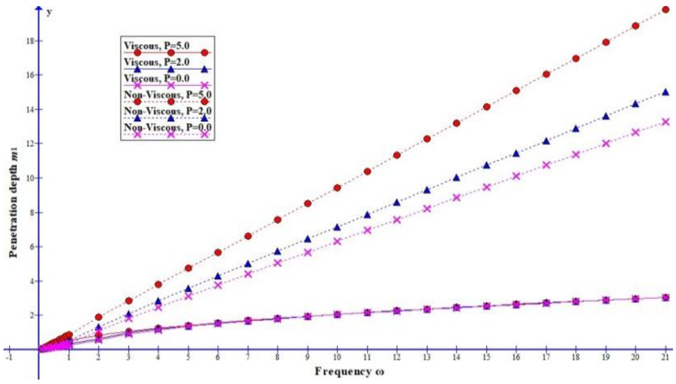

Figure 1. Penetration depth m_1 vs. frequency ω (CT theory).

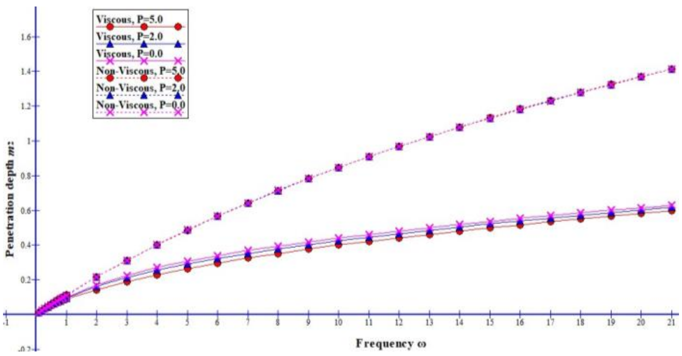

Figure 2. Penetration depth m_2 vs. frequency ω (CT theory).

In Figs. 5-8, the variations of penetration depths are shown against frequency for a fixed value of hydrostatic initial stress parameter P = 5.0 and three theories of thermoelasticity (C-T, L-S, and G-L).

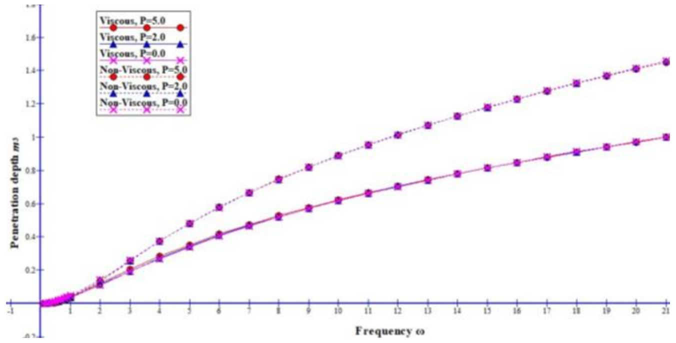

Figure 3. Penetration depth m_3 vs. frequency ω (CT theory).

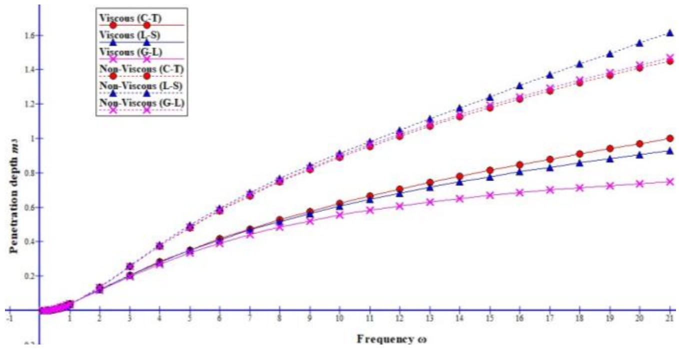

Figure 7. Penetration depth m_3 vs. frequency ω (P = 5.0).

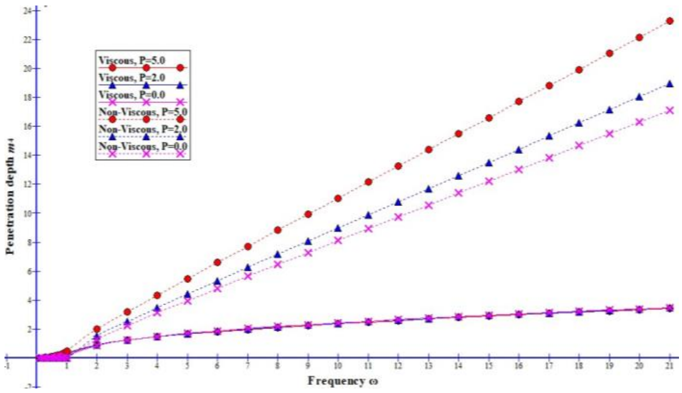

Figure 4. Penetration depth m_4 vs. frequency ω (CT theory).

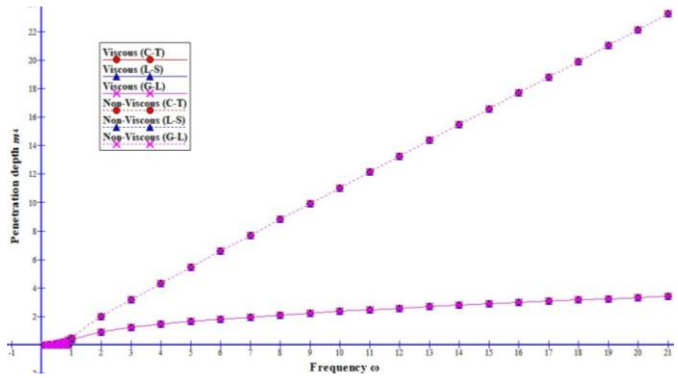

Figure 8. Penetration depth m_4 vs. frequency ω (P = 5.0).

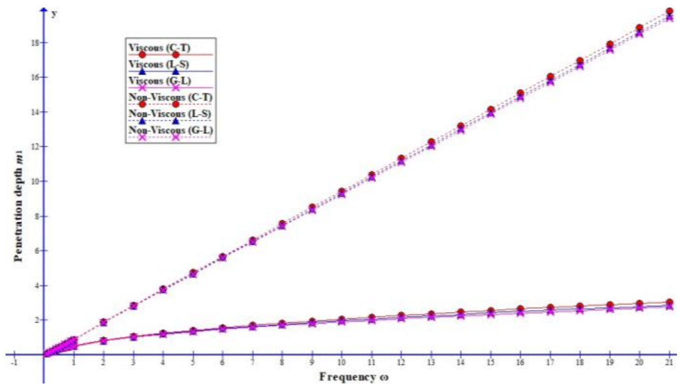

Figure 5. Penetration depth m_1 vs. frequency ω (P = 5.0).

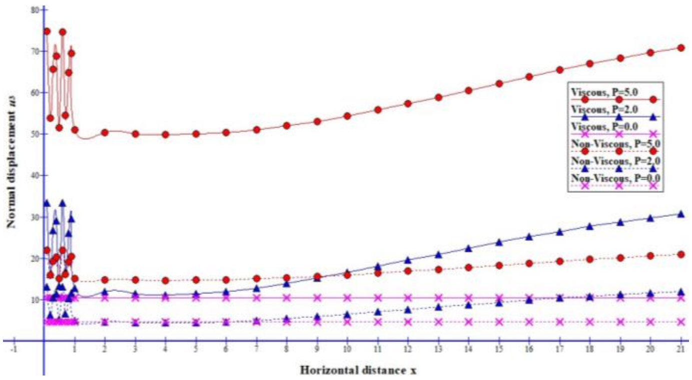

Figure 9. Normal displacement u_3 vs. horizontal distance x (CT theory).

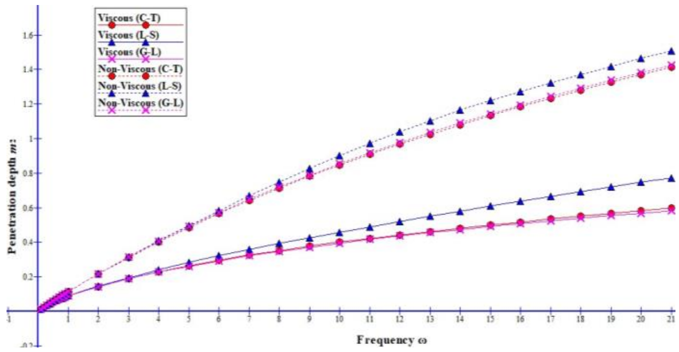

Figure 6. Penetration depth m_2 vs. frequency ω (P = 5.0).

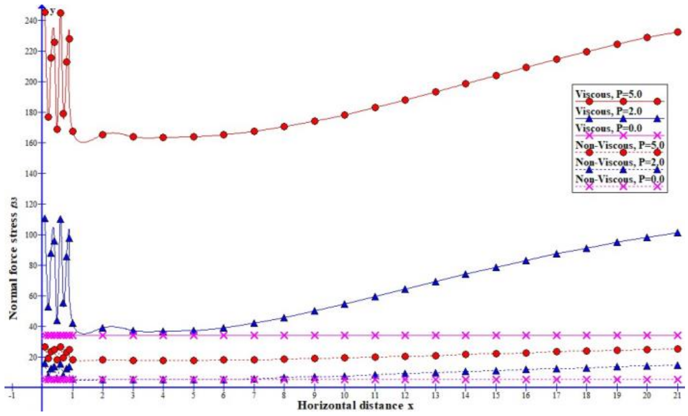

Figure 10. Normal displacement t_{33} vs. horizontal distance x (CT theory).

DISCUSSION

The values of penetration depth m_1 and m_4 are quite close to each other for viscoelastic semiconducting medium, depicting that the effect of hydrostatic initial stress is less significant. On the contrary, hydrostatic initial stress has a relevant effect on the values of penetration depths m_1 and m_4 for a non-viscous semiconducting medium. The variations of penetration depth m_1 and m_4 for a non-viscous semiconducting medium are linear in nature and the slope of the linear curve increases with increase in value of hydrostatic

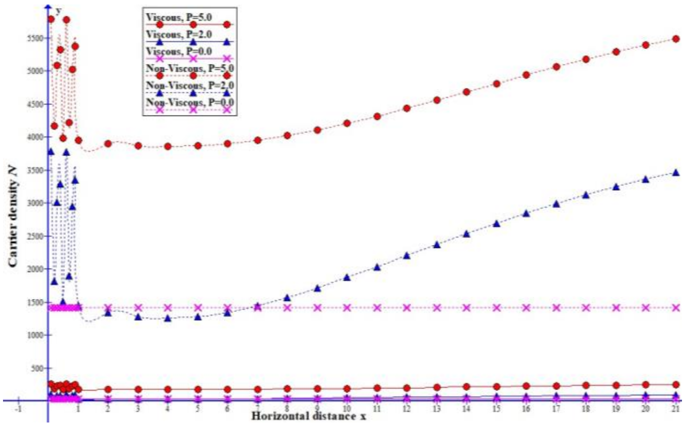

Figure 11. Carrier density N vs. horizontal distance x (CT theory).

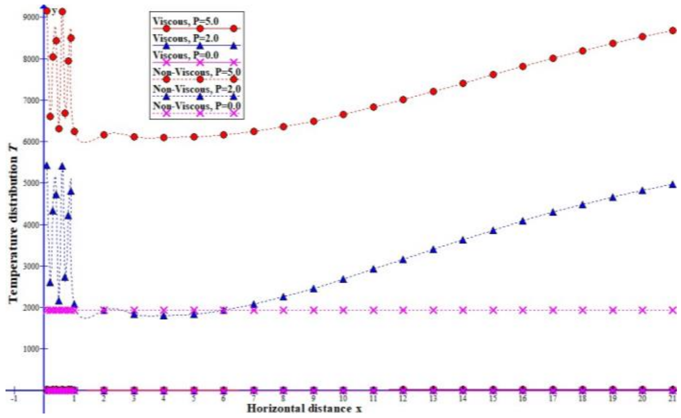

Figure 12. Temperature distribution T vs. horiz. distance x (CT theory).

initial stress parameter. The hydrostatic initial stress parameter has negligible effect on the penetration depth m_2 and m_3 for a non-viscous semiconducting medium but this effect is visible if the medium is viscoelastic in nature. The variations of penetration depth m_i (I = 1,2,3) are shown in Figs. 1-4, respectively. Figures 5-8 show the variations of penetration depth of the waves for different theories of thermoelasticity (P = 5.0). It is observed that the values of penetration depths m_i (1,2,3) for CT, L-S, and G-L theories are very close to each other in the initial range. However, with increase in value of frequency ω, the difference between values of penetration depths for the three theories of thermoelasticity are observed. It is interesting to see that the different theories of thermoelasticity do not effect the penetration depth of transverse displacement wave.

Variations of normal displacement, normal force stress, carrier density, and temperature are shown in Figs. 9-12, respectively.

CONCLUSION

The penetration depth of displacement, thermal, and plasma waves propagating in the medium and components of displacement, stress, moisture concentration, and temperature are obtained, and the results conclude that:
– the viscosity in the medium has appreciable effect on the penetration depths of waves propagating in the medium;
– penetration depths m_1 and m_4 are not affected by the hydrostatic initial stress parameter in viscoelastic semiconducting medium, whereas this parameter has significant effect on the penetration depth m_2.
– the generalised theories of thermoelasticity do not effect the penetration depth of transverse displacement wave;
– at a particular frequency, the values of displacement, stress, carrier density, and temperature increase with hydrostatic initial stress parameter.

APPENDIX

$$\eta_1=\frac{3f_4f_7-f_4f_6+f_2f_9-f_3f_7}{f_4f_7},$$

$$\eta_2=\frac{f_7(3f_4-2f_3)+f_9(2f_2-f_1)+f_6(f_3-2f_4)+f_4f_5+f_2f_8}{f_4f_7},$$

$$\eta_3=\frac{(f_4-f_3)(f_7-f_6+f_5)-(f_1-f_2)(f_8+f_9)}{f_4f_7},$$

$$f_1=\varepsilon_2d_1,\ f_2=\varepsilon_2d_2+d_7a_2,\ f_3=a_1^*\varepsilon_2+a_2d_8,\ f_4=a_2d_6,$$

$$f_5=d_1(d_5-A_1),\ f_6=d_1d_6+d_2(d_5-A_1),\ f_7=d_2d_6,$$

$$a_1^*=a_1\left[1-\frac{\iota\xi cv_0}{t^*}\right],\ d_1=\rho c_1^2\omega^2,\ d_2=\xi^2(\lambda^*+\mu^*-P),$$

$$d_3=\mu^*-\frac{P}{2},\ d_4=\frac{d_1}{d_6},\ d_5=\iota\omega,\ d_6=\xi^2,\ d_7=\xi^2\varepsilon_1d_8,$$

$$d_8=\iota\omega\varepsilon_1\left(n_1-\frac{\iota n_0\tau_0\omega}{t^*}\right),\ H_{1j}=\frac{f_1-f_2(m_j^2+1)}{f_3-f_4(m_j^2+1)},$$

$$H_{2j}=\frac{a_1^*H_{1j}-d_1+d_2(m_j^2+1)}{a_2}\quad (j=1,2,3),$$

$$t_j=\frac{\lambda+2\mu}{\mu}\left[\frac{\gamma^*}{\gamma}\left(1-\frac{\iota\xi cv_0}{t^*}\right)H_{1j}+\frac{\delta_n^*}{\delta_n}H_{2j}\right]-\xi^2\left[\frac{(\lambda^*+2\mu^*)}{\mu}m_j^2+\frac{\lambda^*}{\mu}\right],$$

$$t_4=\frac{2\mu^*\xi^2m_4}{\mu},\ h_j=\frac{m_j}{\mu}\left[\left(\mu^*+\frac{P}{2}\right)\xi^2+\left(\mu^*-\frac{P}{2}\right)\right],$$

$$h_4=\frac{\xi^2}{\mu}\left[\left(\mu^*+\frac{P}{2}\right)m_4^2+\left(\mu^*-\frac{P}{2}\right)\right],\ l_j=\left[\frac{s}{D_e}-\iota\xi m_j\right]H_{2j},$$

$$\Delta_1=F_1h_4(m_3l_2H_{13}-m_2l_3H_{12}),\ \Delta_2=-F_1h_4(m_3l_1H_{13}-m_1l_3H_{11}),$$

$$\Delta_3=F_1h_4(m_2l_1H_{12}-m_1l_2H_{11}),$$

$$\Delta_4=-F_1\left[h_1(m_3l_2H_{13}-m_2l_3H_{12})-h_2(m_3l_1H_{13}-m_1l_3H_{11})+\right.$$

$$\left.+h_3(m_2l_1H_{12}-m_1l_2H_{11})\right],\ \Delta=\frac{1}{F_1}\sum_{j=1}^{3}(t_j\Delta_j),$$

$$F_1=P\exp\{\iota\xi(x-ct)\}-F_0.$$

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