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BEHAVIOUR OF CREEP STRESS AND STRAIN RATES IN A SOLID DISC MADE OF ISOTROPIC MATERIAL WITH THERMAL CONDITION

PONAŠANJE NAPONA I DEFORMACIJA USLED PUZANJA KOD ČVRSTOG DISKA OD IZOTROPNOG MATERIJALA POD UTICAJEM TEMPERATURE

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Abstract

The present research deals with the study of the behaviour of creep stress and strain rates in a solid disc of isotropic material with thermal condition by using Seth's transition theory. Results have been discussed numerically and are depicted graphically. It has been observed that the solid disc made of incompressible material requires maximal radial stress at the centre of the solid disc for the yielding condition as compared to the compressible material. Further, the values of radial, as well as circumferential stresses, also increases with increasing angular speed. With the addition of thermal effect the values of radial as well as circumferential stresses increase at the centre of the solid disc.

Izvod

Predstavljeno je istraživanje ponašanja napona i deformacija usled puzanja kod čvrstog diska od izotropnog materijala pod uticajem temperature, primenom teorije prelaznih napona Seta. Data je diskusija numeričkih dobijenih rezultata, koji su predstavljeni i grafički. Uočava se da je za pojavu tečenja u čvrstom disku od nestišljivog materijala potreban maksimalni radijalni napon u centru diska, u poređenju sa stišljivim materijalom. Osim toga, vrednosti radijalnog, kao i obimskog napona, takođe rastu sa povećanjem ugaone brzine rotacije. Uz uticaj temperature, vrednosti radijalnog i obimskog napona rastu u centru čvrstog diska.

INTRODUCTION

Creep is the third stage of deformation of solid materials which moves slowly or deforms under the action of transition stresses. It is also a type of metal deformation that occurs at stresses below yield strength and a time dependent plastic flow of material under mechanical transitional stress. The process of creep deformation of solid discs has been observed for different materials. Transitional stresses in solid discs play a very important role in the efficient design. Rotating solid discs have a wide range of applications such as rotors of rotating high speed gear engines, turbines, computer disc drives, flywheels, shrink fits, compressors and machinery, etc. There are many applications for solid discs in aerospace turbojet engines who work under low and high temperature subjected to low and high angular speeds. The analytical elastic-plastic studies of rotating solid discs can be found in many books /1-2, 5-7, 9-11/. The analysis of deformation has been analysed and presented by Swainger, /2/. The experimental and theoretical investigations of the solid discs have spread attention due to great importance in electronics and mechanical engineering. Ghose /8/ worked on the thermal effect in the transverse vibration of spinning disk of variable thickness. In this paper, it is observed that the heat-generation influences the natural frequency of vibration and the traveling waves move around the disk-circumference during the vibration. Güven et al. /12/ presented elastic plastic solid disk with non-uniform heat source subjected to external pressure by using transition theory. In addition, Sharma, et al. /14, 15, 18, 21/ also presented creep analysis of thin rotating disc under plane stress with no edge load. Creep behaviour of variable thickness rotating composite disc using approach stress established on creep guideline was investigated by Deepak et al. /16/. They displayed radial, circumferential and effective stresses in the internal and external radius. Thakur et al. /17/ worked on creep transition stresses in a thin rotating disc with shaft by finitesimal deformation under steady state temperature by using Seth's transition theory. In addition, Thakur /18/ also worked on the investigation of creep transition stresses of thick isotropic spherical shell by finitesimal deformation under steady state of temperature and internal pressure. Hosseini Kordkheili et al. /20/ analysed thermo-elastic-plastic creep behaviour of variable thickness functionally graded discs for different boundary conditions. They evaluated circumferential and radial stresses as well as strain for different thickness profiles. Further, the most recent work of Thakur et al. /26/ investigates elastic-plastic infinitesimal deformation in a solid under heat effect by using Seth's theory. For the problem considered here, the heat condition rate is given

$$\dot{q}(r) = q_0\left[1-\left(\frac{r}{r_0}\right)^s\right], \tag{1}$$

where: q_0 is the magnitude of thermal condition at $r = 0$; r is measured from the centre of solid disc; r_0 is the radius of solid disc. The generalized principal measures in Cartesian co-ordinates may be written in the form:

$$\varepsilon_{ii} = \int_0^{\varepsilon_{ii}^A} \left[1-2\varepsilon_{ii}^A\right]^{\frac{n}{2}-1} d\varepsilon_{ii}^A = \frac{1}{n}\left[1-(1-2\varepsilon_{ii}^A)^{\frac{n}{2}}\right], \quad (2)$$

where: n is strain measure coefficient; ε_{ii}^A Almansi finite strain component; and i = 1, 2, 3. For n = –2, –1, 0, 1, 2, it gives Green, Cauchy, Hencky, Swainger, and Almansi measures, respectively. The main objective of the present research is to develop a consistent analytical and mathematical model to resolve the effect of thermal condition in a solid disc by using transition theory.

MATHEMATICAL MODEL AND GOVERNING EQUATIONS

We describe a homogeneous solid disc with constant density and having central bore of radius r_0 as shown in Fig. 1. The solid disc rotates gradually increasing at angular speed ω around an axis perpendicular to its plane and passes through the centre. The thickness of the solid disc is assumed so small so that the solid disc is effectively in a state of plane stress, i.e. the axial stress τ_{zz} = 0.

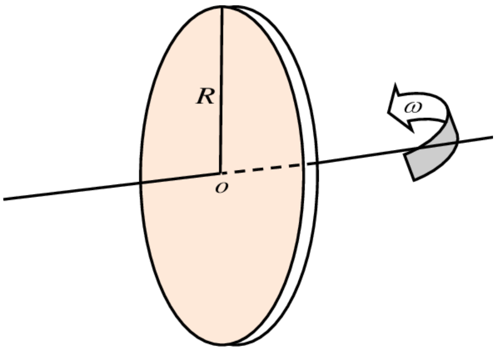

Figure 1. Geometry of solid disc.

Boundary conditions for the solid disc

The solid disc in the present study is considered with heat generation. The internal surface of the solid disc is assumed to be zero. Thus the boundary conditions of the problem are taken as:

$$u = 0, \quad r = 0; \quad \tau_{rr} = 0, \quad r = r_0 , \quad (3)$$

where: τ_{rr} and u are radial stress and displacement along the radial direction, respectively.

Displacement co-ordinates and strain measures

Since the shaft is strained symmetrically, we can take the components of displacement in cylindrical co-ordinates as:

$$u = r(1-\eta); \quad v = 0; \quad w = dz , \quad (4)$$

where: η is position function, depending on $\eta = \sqrt{(x^2 + y^2)}$ only; and d is a constant. Infinitesimal effective components of strain are given by /3, 4/ as:

$$\varepsilon_{ii}^A \equiv \frac{\partial u}{\partial r} = \left[1-(\eta+r\eta')\right], \quad \varepsilon_{\theta\theta}^A \equiv \frac{u}{r} = [-\eta+1],$$

$$\varepsilon_{zz}^A \equiv \frac{\partial w}{\partial z} = d , \quad \varepsilon_{r\theta}^A = \varepsilon_{\theta z}^A = \varepsilon_{zr}^A = 0 , \quad (5)$$

where: u, v, w are the physical components of displacement; and ε_{rr}^A, $\varepsilon_{\theta\theta}^A$, ε_{zz}^A, $\varepsilon_{r\theta}^A$, $\varepsilon_{\theta z}^A$ and ε_{zr}^A are the components of the strain tensor ε_{ij}^A; and superscript 'A' is the Almansi; and $\eta' = d\eta/dr$.

Generalized strain components

The generalized effective components of strain are given by /4/:

$$\varepsilon_{rr} = \frac{1}{n}\left[1-\left\{2(\eta+r\eta')-1\right\}^{\frac{n}{2}}\right], \quad \varepsilon_{\theta\theta} = \frac{1}{n}\left[1-\left\{2\eta-1\right\}^{\frac{n}{2}}\right],$$

$$\varepsilon_{zz} = \frac{1}{n}\left[1-(1-2d)^{\frac{n}{2}}\right], \quad \varepsilon_{r\theta} = \varepsilon_{\theta z} = \varepsilon_{zr} = 0 . \quad (6)$$

Stress-strain relations

Stress-strain relations for isotropic material are given by /1/:

$$\tau_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} - \xi\Theta\delta_{ij} , \quad (7)$$

where: τ_{ij} are stress components; e_{ij} are strain components; $I_1 = e_{kk}$ is the first strain invariant; δ_{ij} is Kronecker's delta; $\xi = \alpha(3\lambda + 2\mu)$ material parameters (constants); α being the coefficient of thermal expansion; Θ is the temperature and λ, μ are Lame's constants.

The mathematical formulation for thermal condition by the temperature field satisfying Fourier's heat equation:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\Theta}{dr}\right) + \frac{\dot{q}(r)}{k} = 0, \quad 0 < r < r_0 , \quad (8)$$

where: k is thermal conductivity. The temperature field satisfies Eq.(8) and $d\Theta/dr = 0$ at $r = 0$, $\Theta = 0$ at $r = r_0$. Using Eq. (1) and these boundary conditions, the effect of thermal condition, and temperature distribution is obtained as we get:

$$\Theta(r) = \frac{q_0 r_0^2}{4k}\left[1-\left(\frac{r}{r_0}\right)^2 - \frac{4}{(s+2)^2}\left\{1-\left(\frac{r}{r_0}\right)^{s+2}\right\}\right]. \quad (9)$$

Equation (7) for this problem becomes

$$\tau_{rr} = \frac{2\lambda\mu}{\lambda+2\mu}[\varepsilon_{rr}+\varepsilon_{\theta\theta}] + 2\mu\varepsilon_{rr} - \frac{2\mu\xi\Theta}{(\lambda+2\mu)},$$

$$\tau_{\theta\theta} = \frac{2\lambda\mu}{\lambda+2\mu}[\varepsilon_{rr}+\varepsilon_{\theta\theta}] + 2\mu\varepsilon_{\theta\theta} - \frac{2\mu\xi\Theta}{(\lambda+2\mu)},$$

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = \tau_{zz} = 0 . \quad (10)$$

From Eq.(7) the effective strain components in terms of stresses are obtained as

$$\varepsilon_{rr} \equiv \frac{\partial u}{\partial r} = \frac{1}{E}(\tau_{rr} - \nu\tau_{\theta\theta}) + \alpha\Theta , \quad (11)$$

$$\varepsilon_{\theta\theta} \equiv \frac{u}{r} = \frac{1}{E}(\tau_{\theta\theta} - \nu\tau_{rr}) + \alpha\Theta , \quad (12)$$

$$\varepsilon_{zz} \equiv \frac{\partial w}{\partial z} = -\frac{\nu}{E}(\tau_{rr} - \tau_{\theta\theta}) + \alpha\Theta , \quad (13)$$

$$\varepsilon_{r\theta} = \varepsilon_{\theta z} = \varepsilon_{zr} = 0 ,$$

where: $E = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$; $\nu = \frac{\lambda}{2(\lambda+\mu)}$.

Substituting Eq.(6) in Eq.(10), the stresses are obtained

$$\tau_{rr}=\frac{2\mu}{n}\left[(3-2c)-(2-c)\left\{2\eta(T+1)-1\right\}^{\frac{n}{2}}-(1-c)(2\eta-1)^{\frac{n}{2}}-\frac{nc\xi\Theta}{2\mu}\right],$$

$$\tau_{\theta\theta}=\frac{2\mu}{n}\left[(3-2c)-(1-c)\left\{2\eta(T+1)-1\right\}^{\frac{n}{2}}-(2-c)(2\eta-1)^{\frac{n}{2}}-\frac{nc\xi\Theta}{2\mu}\right],$$

$$\tau_{r\theta}=\tau_{\theta z}=\tau_{zr}=\tau_{zz}=0\,, \qquad (14)$$

where: $r\eta'=T\eta$; and $c=2\mu/(\lambda+2\mu)$.

Equations of equilibrium for stress-strain are all satisfied except:

$$\frac{d}{dr}(\tau_{rr})+\frac{(\tau_{rr}-\tau_{\theta\theta})}{r}+\rho\omega^2 r=0\,, \qquad (15)$$

where: ρ is the constant density of the solid disc.

Asymptotic solution at transition points

Using Eqs.(14) and (8) into Eq.(15), we get a nonlinear differential equation in η as:

$$(2-c)n\eta^2 T\left\{2\eta(T+1)-1\right\}^{\frac{n}{2}-1}\frac{dT}{d\eta}=\frac{n\rho\omega^2 r^2}{2\mu}-\left\{2\eta(T+1)-1\right\}^{\frac{n}{2}}\times$$

$$\times\left\{1+\frac{n\eta T(T+1)(2-c)}{\left\{2\eta(T+1)-1\right\}}\right\}+\left\{2\eta-1\right\}^{\frac{n}{2}}\left\{1-\frac{n\eta T(2-c)}{2\eta-1}\right\}-\frac{nc\xi q_o r^2}{8\mu k}\times$$

$$\times\left\{\frac{4}{(s+2)}\left(\frac{r}{r_o}\right)^s-\frac{2}{r_o}\right\}, \qquad (16)$$

where: T (T is function of η and η is function of r only).

From Eq.(16) the transitional points of η are $T=-1$ and $T\to\pm\infty$.

ANALYTICAL CREEP SOLUTION OF THE PROBLEM

Thakur et al. /13, 17, 19, 22-49/ have shown the asymptotic solution which leads to creep state at transition point $T\to-1$ via principal stress differences. We define the transitional function ψ to find the creep stresses as:

$$\psi=\tau_{rr}-\tau_{\theta\theta}\equiv\frac{2\mu}{n}\left[(2\eta-1)^{\frac{n}{2}}-\left\{2\eta(T+1)-1\right\}^{\frac{n}{2}}\right]. \qquad (17)$$

By taking the logarithmic differentiation of Eq.(17) with respect to r, we get

$$\frac{d}{dr}(\ln\psi)=\left(\frac{nT\eta}{r}\right)\frac{1}{(2\eta-1)^{\frac{n}{2}}-\left\{2\eta(T+1)-1\right\}^{\frac{n}{2}}}$$

$$\times\left[(2\eta-1)^{\frac{n}{2}-1}-\left\{2\eta(T+1)-1\right\}^{\frac{n}{2}-1}\left\{\eta\frac{dT}{d\eta}+(T+1)\right\}\right]. \qquad (18)$$

By substituting the value of $dT/d\eta$ from Eq.(16) into Eq.(18) and by taking the asymptotic value $T\to-1$, we get

$$\frac{d}{dr}(\ln\psi)=-\frac{1}{r(2-c)}\,. \qquad (19)$$

By integrating Eq.(19) with respect to r, we get

$$\psi=A_1 r^{-\frac{1}{(2-c)}}\,, \qquad (20)$$

where: A_1 is a constant of integration. Substituting Eq.(20) into Eq.(17), we get

$$\tau_{rr}=A_1(2-c)r^{-\frac{1}{2-c}}-\frac{\rho\omega^2 r^2}{2}+B_1\,, \qquad (21)$$

where: B_1 is a constant of integration.

Now substituting Eq. (21) into Eq. (17), we get

$$\tau_{\theta\theta}=A_1(1-c)r^{-\frac{1}{2-c}}-\frac{\rho\omega^2 r^2}{2}+B_1\,, \qquad (22)$$

$$\frac{\partial u}{\partial r}=\frac{1}{E}\left[\frac{1}{(2-c)}B_1+\left(\frac{3-2c}{2-c}\right)A_1 r^{-\frac{1}{2-C}}-\frac{1}{(2-c)}\frac{\rho\omega^2 r^2}{2}+\frac{\alpha E q_0 r_0^2}{4k}\left[1-\left(\frac{r}{r_0}\right)^2-\frac{4}{(s+2)^2}\left\{1-\left(\frac{r}{r_0}\right)^{s+2}\right\}\right]\right], \qquad (23)$$

$$\frac{u}{r}=\frac{1}{E}\left[\frac{1}{(2-c})B_1-\frac{1}{(2-c)}\frac{\rho\omega^2 r^2}{2}+\frac{\alpha E q_0 r_0^2}{4k}\left[1-\left(\frac{r}{r_0}\right)^2-\frac{4}{(s+2)^2}\left\{1-\left(\frac{r}{r_0}\right)^{s+2}\right\}\right]\right], \qquad (24)$$

where: $E=2\mu(3-2c)/(2-c)$ is the Young's modulus; and $\nu=(1-c)/(2-c)$. Integrating Eq.(23) with respect to r, we get

$$u=\frac{1}{E}\left[\frac{1}{(2-c)}B_1 r-\frac{1}{(2-c)}\frac{\rho\omega^2 r^3}{6}+\left(\frac{3-2c}{1-c}\right)A_1 r^{\frac{1-c}{2-c}}+\frac{\alpha E q_0 r_0^2}{4k}\left[r-r_0\left(\frac{r}{r_0}\right)^3-\frac{4}{(s+2)^2}\left\{r-\frac{r_0}{(s+3)}\left(\frac{r}{r_0}\right)^{s+3}\right\}\right]\right]+D\,, \qquad (25)$$

where: D is a constant of integration which can be determined. Comparing Eq.(24) and Eq.(25), we get

$$\left(\frac{3-2c}{1-c}\right)A_1 r^{\frac{1-c}{2-c}}=-\frac{1}{(2-c)}\frac{\rho\omega^2 r^3}{3}+\frac{\alpha E q_0 r_0^2}{4k}\left[-\frac{2r_0}{3}\left(\frac{r}{r_0}\right)^3+\frac{4r_0}{(s+2)(s+3)}\frac{r_0}{(s+3)}\left(\frac{r}{r_0}\right)^{s+3}\right]-DE\,. \qquad (26)$$

Using boundary condition Eq.(3) in Eq.(25), we get $D=0$. Also using $\tau_{rr}=0$ at $r=r_0$ in Eq.(21) by making use of Eq.(26), we get

$$A_1=-\frac{(1-c)}{(2-c)(3-2c)}r_0^{\frac{1}{2-c}}\left[\frac{\rho\omega^2 r_0^2}{3}+\frac{\alpha E q_0 r_0^2(2-c)s(s+5)}{6k(s+2)(s+3)}\right],\quad B_1=\frac{1}{6(3-2c)}\left[(11-8c)\rho\omega^2 r_0^2+\frac{\alpha E q_0 r_0^2(1-c)(2-c)s(s+5)}{k(s+2)(s+3)}\right].$$

Putting the values of A_1 and B_1 in Eqs.(21), (22), and (24), we get stresses and displacement of the solid disc:

$$\tau_{rr}=\left(\frac{11-8c}{3-2c}\right)\frac{\rho\omega^2 r_0^2}{6}+\frac{\alpha E q_0 r_0^2(1-c)(2-c)s(s+5)}{6k(3-2c)(s+2)(s+3)}-\frac{\rho\omega^2 r^2}{2}-\frac{(1-c)}{(3-2c)}\left(\frac{r}{r_0}\right)^{-\frac{1}{2-c}}\left[\frac{\rho\omega^2 r_0^2}{3}+\frac{\alpha E q_0 r_0^2(2-c)s(s+5)}{6k(s+2)(s+3)}\right], \quad (27)$$

$$\tau_{\theta\theta}=\left(\frac{11-8c}{3-2c}\right)\frac{\rho\omega^2 r_0^2}{6}+\frac{\alpha E q_0 r_0^2(1-c)(2-c)s(s+5)}{6k(3-2c)(s+2)(s+3)}-\frac{\rho\omega^2 r^2}{2}-\left\{\frac{(1-c)^2}{(2-c)(3-2c)}\right\}\left(\frac{r}{r_i}\right)^{-\frac{1}{2-c}}\left[\frac{\rho\omega^2 r_0^2}{3}+\frac{\alpha E q_0 r_0^2(2-c)s(s+5)}{6k(s+2)(s+3)}\right], \quad (28)$$

$$u=\frac{r}{E}\left[\frac{11-8c}{(2-c)(3-2c)}\frac{\rho\omega^2 r_0^2}{6}+\frac{\alpha E q_0 r_0^2(1-c)(2-c)s(s+5)}{6k(3-2c)(s+2)(s+3)}-\left(\frac{1}{2-c}\right)\frac{\rho\omega^2 r^2}{2}+\frac{\alpha E q_0 r_0^2}{4k}\left[1-\left(\frac{r}{r_0}\right)^2-\frac{4}{(s+2)^2}\left\{1-\left(\frac{r}{r_0}\right)^{s+2}\right\}\right]\right]. \quad (29)$$

Non-dimensional quantities

The dimensionless parameters are introduced as: $R = r/r_0$, $\sigma_r = \tau_{rr}/E$, $\sigma_\theta = \tau_{\theta\theta}/E$, $U = u/r$, $\Omega^2 = \rho\omega^2 r_0^2/E$, $\beta = \alpha q_0 r_0^2/k$. The creep stresses and displacement of the solid disc are attained in dimensionless quantities as:

$$\sigma_r=\left[\left(\frac{11-8c}{6(3-2c)}\right)-\frac{R^2}{2}-\frac{(1-c)}{3(3-2c)}R^{-\frac{1}{2-c}}\right]\Omega^2+\frac{\beta(1-c)(2-c)s(s+5)}{6(3-2c)(s+2)(s+3)}\left\{1-R^{-\frac{1}{2-c}}\right\}, \quad (30)$$

$$\sigma_\theta=\left[\left(\frac{11-8c}{6(3-2c)}\right)-\frac{R^2}{2}-\frac{(1-c)^2}{3(2-c)(3-2c)}R^{-\frac{1}{2-c}}\right]\Omega^2+\frac{\beta(1-c)s(s+5)}{6(3-2c)(s+2)(s+3)}\left\{(2-c)-(1-c)R^{-\frac{1}{2-c}}\right\}, \quad (31)$$

$$U=R\left[\left(\frac{11-8c}{6(2-c)(3-2c)}-\frac{R^2}{2(2-c)}\right)\Omega^2+\frac{\beta(1-c)(2-c)s(s+5)}{6(3-2c)(s+2)(s+3)}+\frac{\beta}{4}\left[1-R^2-\frac{4}{(s+2)^2}\{1-R^{s+2}\}\right]\right]. \quad (32)$$

Creep strain rates

The creep strain rates can be computed versus transitional stresses and effective strain rates. Hence, the effective creep strain rates are given by /18/:

$$\dot{\varepsilon}_{rr}=\left[\frac{n(3-2c)(\sigma_r-\sigma_\theta)}{(2-c)}\right]^{\frac{1}{n}-1}\left[\sigma_r-\nu\sigma_\theta+\frac{\beta}{4}\left\{1-R^2-\frac{4}{(s+2)^2}(1-R^{s+2})\right\}\right], \quad (33)$$

$$\dot{\varepsilon}_{\theta\theta}=\left[\frac{n(3-2c)(\sigma_r-\sigma_\theta)}{(2-c)}\right]^{\frac{1}{n}-1}\left[\sigma_\theta-\nu\sigma_r+\frac{\beta}{4}\left\{1-R^2-\frac{4}{(s+2)^2}(1-R^{s+2})\right\}\right], \quad (34)$$

$$\dot{\varepsilon}_{zz}=-\left[\frac{n(3-2c)(\sigma_r-\sigma_\theta)}{(2-c)}\right]^{\frac{1}{n}-1}\left[\nu(\sigma_r+\sigma_\theta)+\frac{\beta}{4}\left\{1-R^2-\frac{4}{(s+2)^2}(1-R^{s+2})\right\}\right]. \quad (35)$$

For the incompressible material ($c \to 0$, $\nu \to 1/2$), Eqs.(33)-(35) become:

$$\dot{\varepsilon}_{rr}=\left[\frac{3n(\sigma_r-\sigma_\theta)}{2}\right]^{\frac{1}{n}-1}\left[\sigma_r-\frac{1}{2}\sigma_\theta+\frac{\beta}{4}\left\{1-R^2-\frac{4}{(s+2)^2}(1-R^{s+2})\right\}\right], \quad (36)$$

$$\dot{\varepsilon}_{\theta\theta}=\left[\frac{3n(\sigma_r-\sigma_\theta)}{2}\right]^{\frac{1}{n}-1}\left[\sigma_\theta-\frac{1}{2}\sigma_r+\frac{\beta}{4}\left\{1-R^2-\frac{4}{(s+2)^2}(1-R^{s+2})\right\}\right], \quad (37)$$

$$\dot{\varepsilon}_{zz}=-\left[\frac{3n(\sigma_r-\sigma_\theta)}{2}\right]^{\frac{1}{n}-1}\left[\frac{1}{2}(\sigma_r+\sigma_\theta)+\frac{\beta}{4}\left\{1-R^2-\frac{4}{(s+2)^2}(1-R^{s+2})\right\}\right]. \quad (38)$$

NUMERICAL RESULTS AND DISCUSSION

For calculating the stress distribution, strain rates, and displacement, based on the below analysis, the following numerical values are taken: $\Omega^2 = \rho\omega^2 r_0^2/E = 15, 50$; the creep stress distribution along the radii ratio $R = r/r_0$ at $\Omega^2 = 15, 50$ and temperature $\beta = 50, 100$; and $n = 1/3, 1/5$ (i.e. $N = 3$).

Figure 2 is portrayed in order to demonstrate the behaviour of creep stress distribution under the effect of temperature $\beta = 50, 100$ for compressible and incompressible materials at angular speed $\Omega^2 = 15, 50$. It is observed that solid disc made of incompressible material requires maximal radial stress at the centre as compared to the compressible material.

Further, the value of radial, as well as circumferential stresses, increases with increasing angular speed $\Omega^2 = 50$. With the addition of thermal effect, the value of radial, as well as circumferential stresses, increases at the centre.

Figure 3 are made to discuss the creep rates versus radii ratio $R = r/r_0$ for the compressible/incompressible material at angular speed $\Omega^2 = 15$ and temperature $\beta = 50$, 100. It is shown that solid disc of compressible material requires maximal strain rates $\dot{\varepsilon}_{rr}$ and $\dot{\varepsilon}_{\theta\theta}$ for the centre and intermediate surface of the disc for measure $n = 1/3$, respectively. With the introduction of thermal effect, the values of strain rates further increase.

CONCLUSIONS

In this work, behaviour of creep stress distribution under the thermal condition by using the transition theory is investigated. Solutions of the stress distribution are obtained for compressible and incompressible materials and evaluated numerically and depicted graphically. The main findings can be concluded as follows:
- solid disc of incompressible material requires maximal radial stress at the centre of the solid disc as compared to the compressible material at angular speed $\Omega^2 = 15$, 50;
- with the introduction of thermal condition, the value of radial and circumference stresses increases at the centre of the solid disc;
- the value of angular speed also increases with increasing thermal condition.

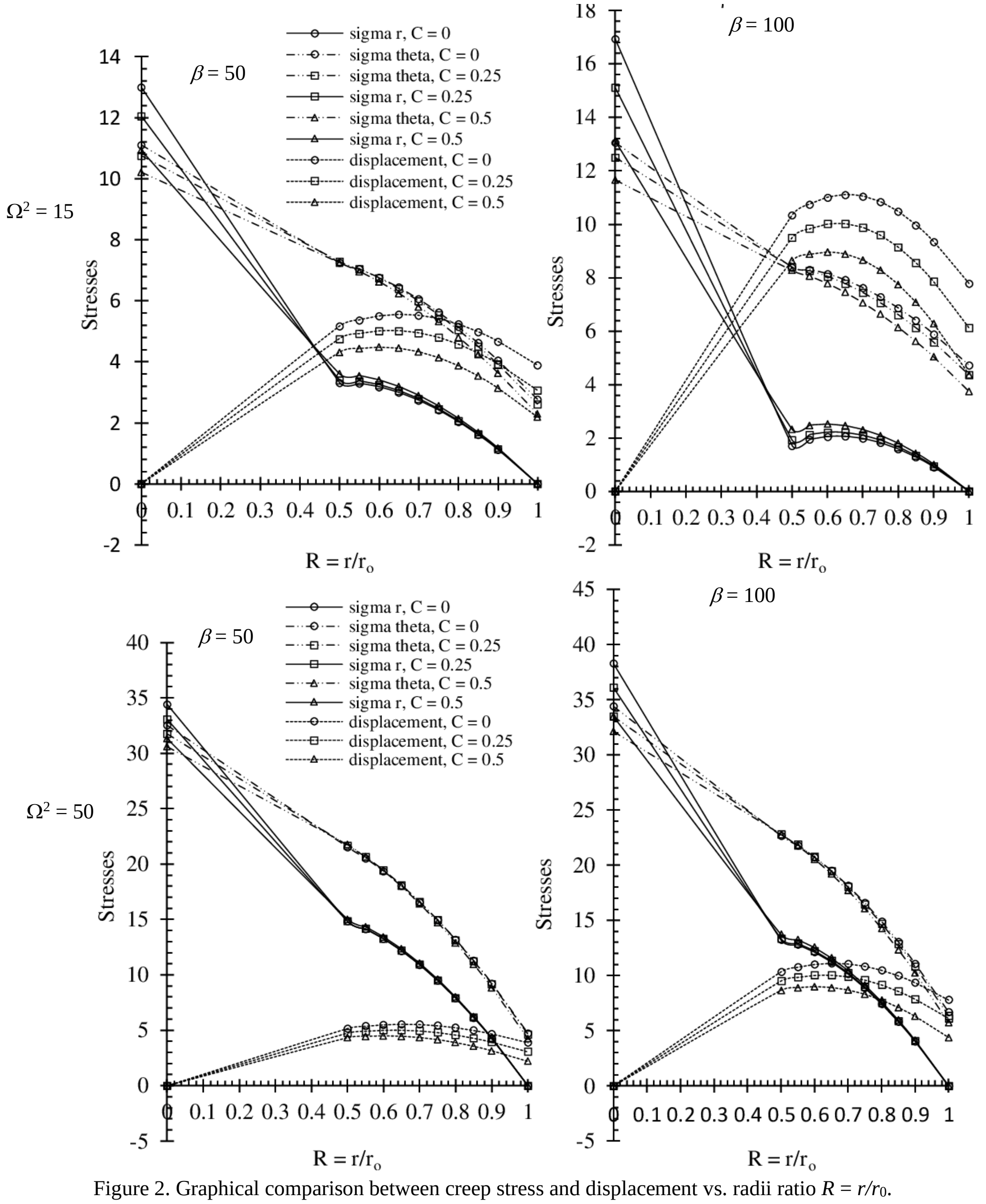

Figure 2. Graphical comparison between creep stress and displacement vs. radii ratio $R = r/r_0$.

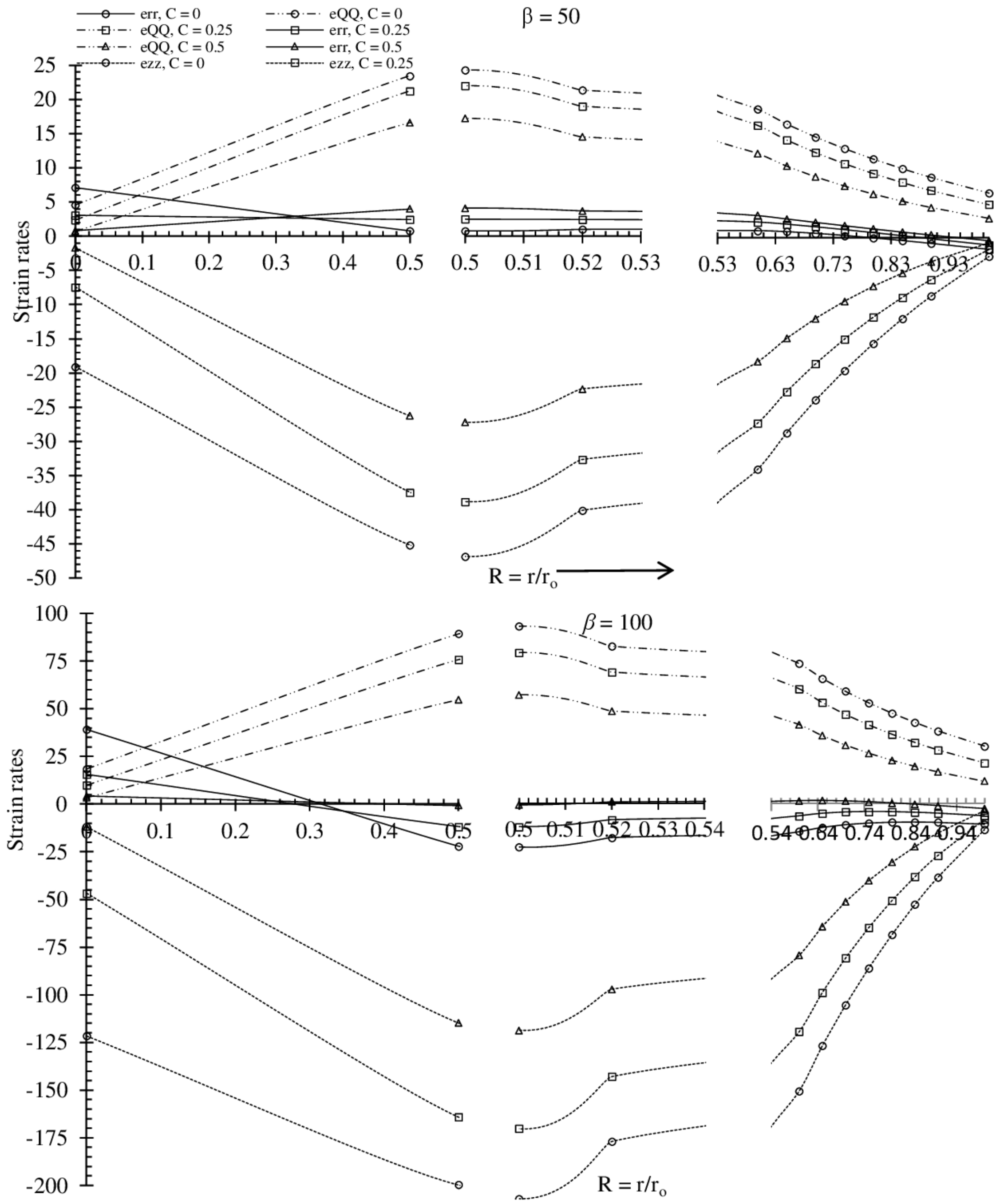

Figure 3. Graphical comparison between creep strain rates vs. radii ratio $R = r/r_0$ ($\Omega^2 = 15$, $n = 1/3$).

Abbreviations

r_0	radius
ε_{ij}	strain tensor
c	compressibility
u, v, w	displacement components
U	displacement
n	strain measure coefficients
A_1, B_1, D, k	constants of integration
d	constant
Ω^2	angular speed
α	thermal expansion coefficient
η	function of r only
ρ	density
τ_{ij}	stress tensor
Θ	temperature
ψ	transition function
ω	angular velocity
E	Young’s modulus
λ, μ	Lame’s constants
ν	Poisson’s ratio
$\dot{\varepsilon}_{ij}$	strain rate tensors
σ_r	radial stress component
σ_θ	circumferential stress component

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