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EFFECTS OF PRE-BUCKLING IN-PLANE DEFORMATION AND CURVATURE TERMS OF LAMINATED PLATES USING REFINED THEORY

UTICAJI RAVANSKIH DEFORMACIJA I ZAKRIVLJENOSTI PRET-KRITIČNOG IZVIJANJA LAMINATNIH PLOČA PRIMENOM REVIDIRANE TEORIJE

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- composite materials
- refined theory
- buckling plate
- pre-buckling
- curvature

Abstract

The paper presents the refined theory applied for buckling loads often encountered in composite plates, starting with inclusion of pre-buckling deformation and a curvature effect. The principle of minimum total potential energy is used to derive the governing equations associated with the present theory. The refined theory involves two unknown variables without using shear correction factor. A closed form solution is obtained using trigonometric functions suggested by solution technique, that satisfies all boundary conditions. The differential equations have been solved analytically and the numerical results describe critical buckling loads for the isotropic rectangular plates and orthotropic laminates, with both subjected to various combinations of tension and compression along plate edges. This is investigated to show the correctness of the theory proposed for buckling of the laminated plates. The critical buckling load obtained is performed on the basis of present theory. The effects of pre-buckling and curvatures terms on non-dimensional critical buckling loads are investigated and compared with previously published results.

Ključne reči
- kompozitni materijali
- revidirana teorija
- izvijanje ploče
- pret-kritično izvijanje
- zakrivljenost

Izvod

U radu je predstavljena revidirana teorija primenjena kod opterećenja izvijanja koja se često pojavljuju u kompozitnim pločama, sa pojavom pret-kritičnih deformacija izvijanja i uticajem zakrivljenosti. Koristi se princip minimuma ukupne potencijalne energije za rešavanje datih jednačina shodno sadašnjoj teoriji. Revidirana teorija sadrži dve nove nepoznate promenljive, bez primene korekcionog faktora smicanja. Dobija se rešenje u zatvorenom obliku korišćenjem trigonometrijskih funkcija, shodno metodi rešavanja, koje zadovoljava sve granične uslove. Diferencijalne jednačine se rešavaju analitički, a numerički rezultati opisuju kritična opterećenja izvijanja izotropnih pravougaonih ploča i ortotropnih laminata, gde su obe konstrukcije podvrgnute raznim kombinacijama zatezanja i pritiska duž ivica ploča. Ovo se istražuje kako bi se pokazala ispravnost predložene teorije za izvijanje laminatnih ploča. Dobijeno kritično opterećenje izvijanja se bazira na sadašnjoj teoriji. Istraženi su uticaji pret-kritičnog izvijanja i zakrivljenosti na bezdimenzionalna kritična opterećenja izvijanja i dato je njihovo poređenje sa ranije objavljenim rezultatima.

INTRODUCTION

Laminated plates are increasingly employed in a wide range of industrial application. Faced with buckling can be due to geometric effects as is usually the case in instability or change in material properties, e.g. material yielding or failure from buckling occurs in a variety of structures, in most structures the displacements increase gradually with increased applied load.

First, the differential equations describing the buckling of composite plates are developed in several works. We can mention those of Reissner /1/ and Mindlin /2/ theories, known as the first-order shear deformation plate theory FSDT. Brought to light in literature, some studies have tested mechanical buckling of laminated composite plates using a higher order shear deformation theory HSDT for pre-buckling deformation and curvature effects. Reddy /3, 4/ offered approximation of the displacement field, and similarly a Third-Order Shear Deformation Theory TSDT. Teruna /5/ selected a study on the seismic behaviour of an RC building with and without buckling straps retained. Mantari et al. /6, 7/ presented buckling and free vibration analysis of laminated beams by using (HSDT). Zenkour et al. /8/ chose thermal buckling of double-layered plate. Chok et al. /9/ presented the mechanical and fatigue behaviour of woven kenaf fiber reinforced epoxy composites. Kahya /10/

used FE to analyse the laminated composite and sandwich beams. Aydogdu /11/ developed exponential shear deformation theory ESDT. Senthilnathan et al. /12/ developed the RPT model one variable lower than that of TSDT of Reddy, then Shimpi et al. /13-15/ based on a two-variable refined plate theory for orthotropic plate analysis. Thai /16/ RT models provide better results and yield more accurate and stable solutions. Many theories have been proposed such as Jones /17/, Whitney /18/, Iyengar /19/. Xiang et al. /20/ covered all the salient aspects of plate buckling with pre-buckling deformation effect, Liew et al. /21/ proposed Navier's solution for laminated plate buckling with pre-buckling in-plane deformation, Nayak et al. /22/ used a higher order finite element theory for buckling and vibration analysis of initially stressed composite sandwich plates, Bouazza et al. /23, 24/ developed an analytical solution of refined hyperbolic shear deformation theory to obtain the critical buckling temperature of cross-ply laminated plates with simply supported edge, Marko et al. /25/ presented the results of research on cavitation resistance of the composite based on unsaturated polyester resin and basalt powder, as reinforcement.

Li et al. /26/ proposed an analytical solution of post-buckling sandwich, Gaurav et al. /27/, Thakur et al. /28/, have analysed a homogeneous material subjected to combined effect of internal and external pressure. Rajan et al. /29/ used the refined plate theory to solve buckling analysis of laminated composite plates, Osman et al. /30/ presented a FE method to analyse thin rectangular laminated composite deck plates under biaxial action, Stevan et al. /31/ focused on stress and strength analysis of layered composite structures under mechanical and thermomechanical loads. Ifayefun /32/ chose experimental investigation into the buckling behaviour of relatively thick steel conical shells, Gašić et al. /33/ had obtained in the elements where the highest values occur in the tension zone of the end-plate.

Consequently, many variable plate theories have been developed for the analysis of plates, as in Sayyad et al. /34/, and Reddy /35/. Exact elasticity solutions for buckling analysis of laminated composite plates are given by Noor /36/ and Khdeir /37/, by using a higher-order theory for buckling and vibration.

Secondly, in this article, when the plate is configured initially (Piola-Kirchhoff principle), area change is defined with the factors of pre-buckling.

Third, using the Navier solution technique, the differential equations are solved analytically and the critical buckling loads are presented in closed-form solutions.

Finally, several examples are given to show the performance of the critical buckling loads and the correctness of the theory proposed from this development for pre-buckling deformation and curvature effects, and results obtained are compared to other published methods in literature.

Application model

Consider a simply supported rectangular plate of length *a* and width *b* subjected to different types of loading in the plane in two directions, as shown in Fig. 1.

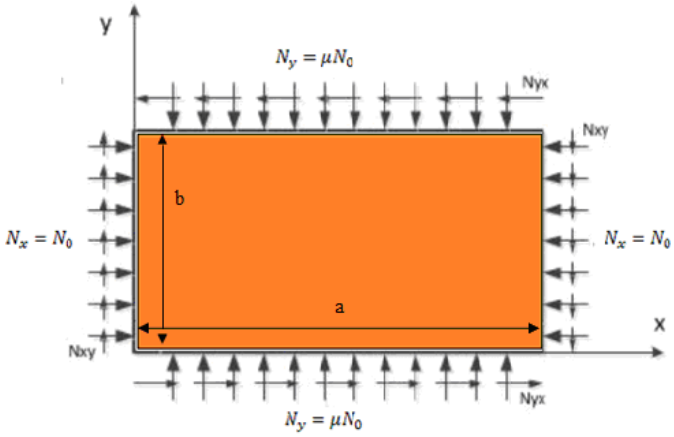

Figure 1. Composite plate subjected to various combinations of compression and tension in the *x* and *y* directions.

Refined theory

Based on analytical solution of refined hyperbolic shear deformation theory to obtain the critical buckling of laminated plates with simply supported edge, /23, 24/,

$$\Psi(z) = z - \frac{2z\sinh\left(\frac{z^2}{h^2}\right)}{2\sinh\left(\frac{1}{4}\right)+\cosh\left(\frac{1}{4}\right)}, \quad (1)$$

with
$$f(z) = z - \Psi(z), \quad (2)$$

and
$$g(z) = 1 - f'(z), \quad f'(z) = \frac{df(z)}{dz}. \quad (3)$$

Allowing for shear deformation, pre-buckling deformation and curvature effects, they require finding a ‘stationary’ value of total potential energy, $\delta\Pi = 0$.

In general, we shall work to obtain approximate solutions expressed in terms of a collection of unknown parameters and the energy of laminated plate under mechanical properties of the used material, where parameters: D_{ij}, D_{ij}^s, H_{ij}^s, A_{ij}^s, F_i^D, F_i^s, and F_{i1}^H are determined by:

$$(A_{ij}, D_{ij}, D_{ij}^s, H_{ij}^s) = \int_{-h/2}^{h/2} \bar{Q}_{ij}(1, z^2, zf(z), (f(z))^2)dz, \quad (4)$$
$$(i, j = 1,2,6),$$

$$(F_i^D, F_i^s, F_i^H) = \int_{-h/2}^{h/2} \sigma_i (z^2, zf(z), (f(z))^2)dz, \ (i = 1,2), \quad (5)$$

$$A_{ij}^s = \int_{-h/2}^{h/2} \bar{Q}_{ij}(g(z))^2 dz \quad (i, j = 4,5), \quad (6)$$

where: $\bar{Q}_{ij}$ is the transformed reduced stiffness coefficient matrix.

The transverse displacement *w* comprises two elements, w_b of bending and w_s of shear. These two components are functions of *x* and *y*. For simply supported symmetric cross-ply laminated rectangular plates, the following boundary conditions are to be satisfied.

On the sides parallel to the *x* axis,
$$w_b = M_y = w_s = 0. \quad (7)$$

On the sides parallel to the *y* axis,
$$w_b = M_x = w_s = 0, \quad (8)$$

where: M_x and M_y are bending moments along x and y directions, respectively.

The following trigonometric functions satisfy all the boundary conditions,

$$w_b(x,y) = A_{mn}\sin(\lambda x)\sin(\beta y)\,, \tag{9}$$

$$w_s(x,y) = B_{mn}\sin(\lambda x)\sin(\beta y)\,, \tag{10}$$

where: A_{mn}, B_{mn} are unknown coefficients; and m, n vary from 1 to infinity; with:

$$\lambda = \frac{m\pi}{a\Lambda_x},\quad \beta = \frac{n\pi}{b\Lambda_y}\,. \tag{11}$$

When the plate is configured initially (Piola-Kirchhoff principle), the area change with the factors of pre-buckling is

$$A = a\Lambda_x b\Lambda_y\,, \tag{12}$$

where: Λ_x and Λ_y are pre-buckling factors in-plane deformation in x and y directions, respectively.

Factors of pre-buckling deformation in the plane Λ_x and Λ_y can be determined by:

$$\Lambda_x = 1 - T_p\hat{\varepsilon}_x\,, \tag{13}$$

$$\Lambda_y = 1 - T_p\hat{\varepsilon}_y\,, \tag{14}$$

where: $\hat{\varepsilon}_x$ and $\hat{\varepsilon}_y$ are strains in different laminates under plane loads N_x and N_y, respectively.

The scalar indicators are given by:
T_p = 0 if pre-buckling in plane deformation is neglected,
T_p = 1 if pre-buckling in plane deformation is included.

The minimization of the energy functional with respect to w_b and w_s using the calculus of variations yields two homogeneous equations which can be written as,

$$\begin{bmatrix} (P_1 - N_0(T_c b_1 + c_1)) & (P_2 - N_0(T_c b_2 + c_1)) \\ (P_2 - N_0(T_c b_2 + c_1)) & (P_3 - N_0(T_c b_3 + c_1)) \end{bmatrix} \begin{bmatrix} A_{mn} \\ B_{mn} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{15}$$

with

T_c = 0, if the curvature terms are neglected,
T_c = 1, if the curvature terms are included,

where the elements of the matrix are given by

$$P_1 = D_{11}\lambda^4 + D_{22}\beta^4 + (2D_{12} + 4D_{66})\lambda^2\beta^2\,,$$

$$P_2 = D^s_{11}\lambda^4 + D^s_{22}\beta^4 + (2D^s_{12} + 4D^s_{66})\lambda^2\beta^2\,,$$

$$P_3 = H^s_{11}\lambda^4 + H^s_{22}\beta^4 + (2H^s_{12} + 4H^s_{66})\lambda^2\beta^2 + A^s_{55}\lambda^2 + A^s_{44}\beta^2\,,$$

$$b_1 = \mu\Big((F_{11} - F_{12})\lambda^4 + ((F_{11} + F_{12}) - (F_{12} + F_{22}))\lambda^2\beta^2 + +(F_{12} - \xi F_{22})\beta^4\Big)\,,$$

$$b_2 = \mu\Big((F^s_{11} - F^s_{12})\lambda^4 + ((F^s_{11} + F^s_{12}) - (F^s_{12} + F^s_{22}))\lambda^2\beta^2 + +(F^s_{12} - F^s_{22})\beta^4\Big)\,,$$

$$b_3 = \mu\Big((F^H_{11} - F^H_{12})\lambda^4 + ((F^H_{11} + F^H_{12}) - (F^H_{12} + F^H_{22}))\lambda^2\beta^2 + +(F^H_{12} - F^H_{22})\beta^4\Big)\,,$$

$$c_1 = (-\lambda^2 + \mu\beta^2)\,, \tag{16}$$

and

$$N_x = N_0\,,\quad N_y = \mu N_0\,, \tag{17}$$

In which the buckling load N_0, corresponding solution of the determinant of the matrix in Eq.(15) must vanish, is given by

1- for the case of curvature T_c equal (to zero) and the pre-buckling T_p is equal (to zero or one),

$$N_0 = \frac{P_2^2 - P_1P_3}{c_1(P_1 + P_3 - (2P_2))}\,. \tag{18}$$

2- for the case of curvature T_c equal (to one) and the pre-buckling T_p is equal (to zero or one), we find the two-order equation,

$$AN_0^2 + BN_0 + C\,, \tag{19}$$

with
$$A = [T_c^2(b_1b_3 - b_2^2) - T_c c_1(b_1 + b_3 - 2b_2)]\,,$$
$$B = [T_c(P_1b_3 + P_2b_1 - 2P_3b_2) - c_1(P_1 + P_2 - 2P_3)]\,, \tag{20}$$
$$C = [P_1P_2 - P_3^2]\,,$$

and

$$\Delta = A^2 - 4BC\,. \tag{21}$$

The roots of this equation are given by:

$$N_0 = -\frac{1}{2}\left(\frac{A \pm \sqrt{\Delta}}{B}\right). \tag{22}$$

The critical buckling load is the value of m and n which gives the lowest value of N_0.

Comparative analysis and discussion

As a first step, based on the mathematical formulations, a computer program is developed by using several different algorithms under MATLAB and the interpretation of the obtained results helped to understand the influence of geometric imperfections on the behaviours of simply supported plates using the refined theory with the inclusion of pre-buckling and curvature terms, as shown in Table 1, for the used material, /20/: E_1/E_2 = 1; $G_{12} = E_2/2(1+\nu_{12})$; $G_{13} = G_{23} = G_{12}$; $\nu_{12} = \nu_{21}$ = 0.3.

Table 1. Non-dimensional buckling load, $\overline{N} = N_0a^2/\pi^2D_{11}$.

	h/b					
	a/b = 1			*a/b* = 2		
	0.05	0.1	0.15	0.05	0.1	0.15
$T_p = T_c = 0$, $N_x = N_0$, $N_y = 0$						
	M			*N*		
present	3.9444	3.7866	3.5502	3.9444	3.7866	3.5502
Xiang	3.9437	3.7839	3.5446	3.9437	3.7839	3.5446
$T_p = T_c = 1$, $N_x = N_0$, $N_y = 0$						
	M			*N*		
present	3.9394	3.7692	3.5168	3.9394	3.7692	3.5168
Xiang	3.9074	3.6569	3.3110	3.9074	3.6569	3.3110
$T_p = T_c = 0$, $N_x = N_0$, $N_y = N_0$						
	M					
present	1.9722	1.8933	1.7751M	1.2391	1.2075	1.1582
Xiang	1.9719	1.8920	1.7723	1.2389	1.2069	1.1571
$T_p = T_c = 1$, $N_x = N_0$, $N_y = N_0$						
	M					
present	1.9722	1.8933	1.7751	1.2391	1.2075	1.1582
Xiang	1.9719	1.8920	1.7723	1.2389	1.2069	1.1571
$T_p = T_c = 0$, $N_x = N_0$, $N_y = -N_0$						
	M			*V*		
present	8.0462	7.2923	6.3074	8.0462	7.2923	6.3074
Xiang	8.0497	7.3051	6.3321	8.0497	7.3051	6.3321
$T_p = T_c = 1$, $N_x = N_0$, $N_y = -N_0$						
	M			*V*		
present	7.8193	7.7207	5.5606	7.6138	6.1778	4.9578
Xiang	7.9749	7.2055	6.3279	7.6719	6.2800	5.2355
Present: mode *M* ($m = n = 1$), *N* ($m = 2$, $n = 1$), *V* ($m = 3{:}4$, $n = 1$)						

Results obtained of non-dimensional critical load buckling for isotropic rectangular plates simply supported with consideration of pre-buckling effects and curvature terms are very close in agreement with the authors' comparative studies, giving logical procedures that confirm analytical solutions that yield an exact validity and precision to study other parametric influences.

The second step verifies the theory by comparing results with solutions available in open literature, shown in Figs. 2 to 5. The following material properties are used in the numerical study: $E_1/E_2 =$ open; $G_{12} = G_{13} = 0.6E_2$; $G_{23} = 0.5E_2$; $\nu_{12} = 0.25$; $\nu_{21} = \nu_{12}(E_2/E_1)$.

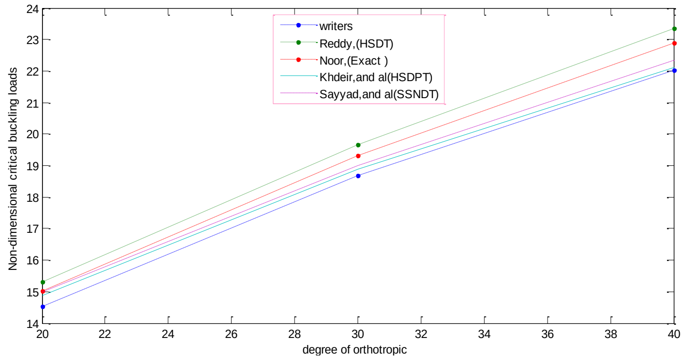

Figure 2. Comparison of non-dimen. critical buckling load for simply supported laminated composite square plates under uniaxial and biaxial compression $b = a$, $a/h = 10$, $N_x = N_0$, $N_y = 0$, 0°, 90°, 0°.

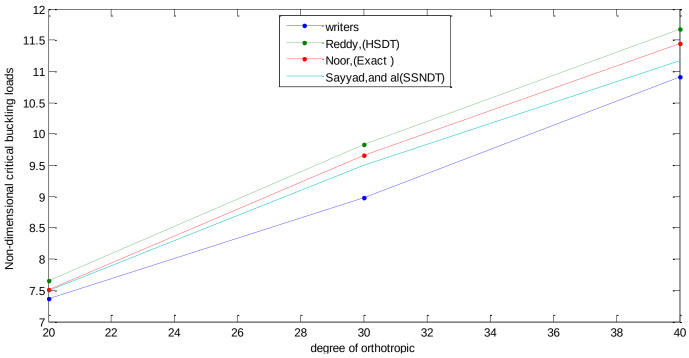

Figure 3. Comparison of non-dimen. critical buckling load for simply supported laminated composite square plates under uniaxial and biaxial compression $b = a$, $a/h = 10$, $N_x = N_0$, $N_y = N_0$, 0°, 90°, 0°.

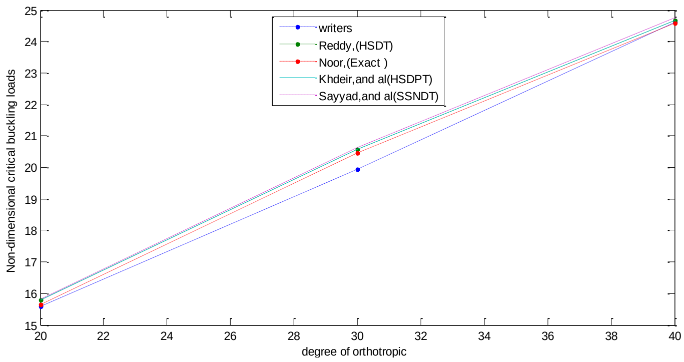

Figure 4. Comparison of non-dimen. critical buckling load for simply supported laminated composite square plates under uniaxial and biaxial compression $b = a$, $a/h = 10$, $N_x = N_0$, $N_y = 0$, 0°, 90°, 0°, 90°, 0°.

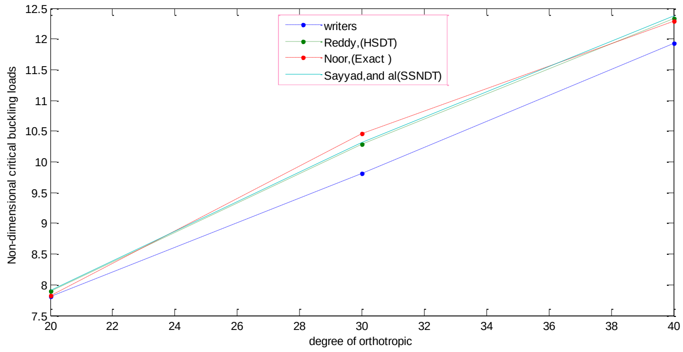

Figure 5. Comparison of non-dimen. critical buckling load for simply supported laminated composite square plates under uniaxial and biaxial compression $b = a$, $a/h = 10$, $N_x = N_0$, $N_y = N_0$, 0°, 90°, 0°, 90°, 0°.

From a pre-estimate of the current theory for three and five layer symmetrical cross-laminated composite square plates subjected to uniaxial and biaxial compression for different modular ratios, we observe that the current results are in superb agreement with the solutions of mentioned authors.

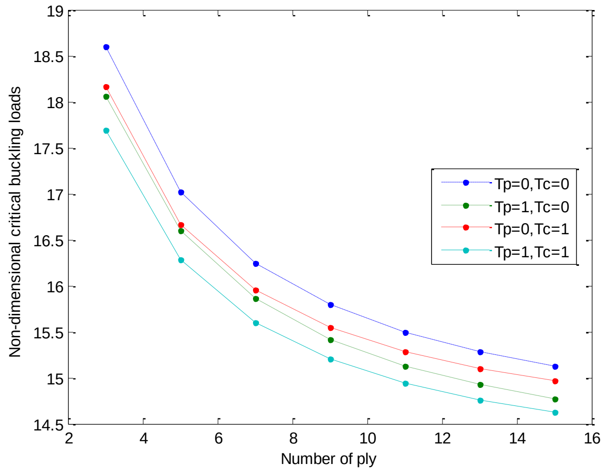

Figure 6. Variation of non-dimensional critical buckling loads vs. number of layers.

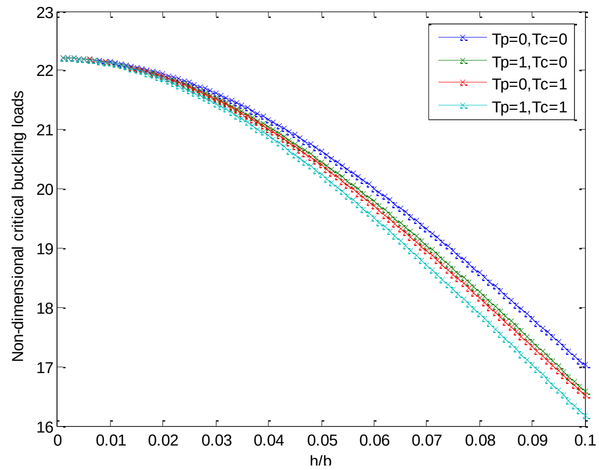

Figure 7. Variation of the non-dimensional critical buckling loads vs. thickness ratio h/b.

In the third step we are interested in the comparison of results of laminated plates of five layers of the same thickness and orientation at 0°, 90°, 0°, 90°, 0° obtained by the refined theory to study different orthotropic ratios of the material and geometric ratio. Plates are subjected to bi-axial compressive-tensile loads with consideration of pre-buckling effects and curvature, shown in Figs. 6 to 9.

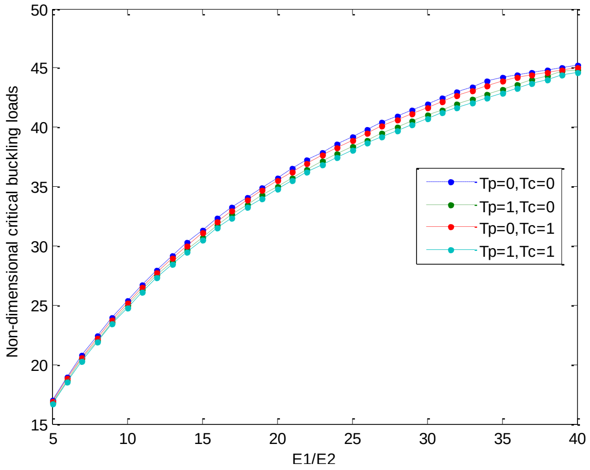

Figure 8. Variation of the non-dimensional critical buckling loads vs. degree of orthotropic E_1/E_2.

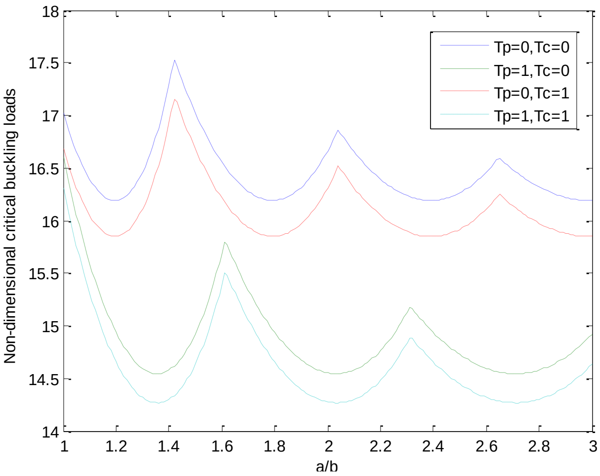

Figure 9. Variation of the non-dimensional critical buckling vs. aspect ratio *a/b*.

The non-dimensional critical buckling loads, using the refined theory (RT) with different combinations vs. number of layers for simply supported rectangular laminates, E_1/E_2 = 5, *h/b* = 0.1, *a/b* = 1 subject to compression along the *x*-axis and tension along the *y*-axis, $N_x = N_0$, $N_y = -N_0$, increases with the effects of curvature more than inclusion of pre-buckling.

A case study of the buckling behaviour of rectangular laminate plate simply supported with varying thickness ratio undergoes at compression-tension ($N_x = N_0$, $N_y = -N_0$), E_1/E_2 = 5, 0°, 90°, 0°, 90°, 0°, *a/b* = 1.

When a plate is thin, the effects of pre-buckling in-plane deformation and curvature terms on the non-dimensional critical buckling loads are insignificant; these effects become more pronounced as the plate thickness ratio *h/b* increases. A second observation of the non-dimensional critical buckling load increases with inclusion of pre-buckling more than due to curvature effects.

The relationship between the non-dimensional critical load and degree of orthotropic E_1/E_2 for square plates, 0°, 90°, 0°, 90°, 0°, *h/b* = 0.1, subject to compression along the *x*-axis and tension along the *y*-axis ($N_x = N_0$, $N_y = -N_0$), using RT with different combinations of the effects of pre-buckling in-plane deformation and curvature terms, changes marginally with increasing ratio E_1/E_2, and the results are close for the four cases with inclusion or not for the pre-buckling and curvature effects.

Non-dimensional critical buckling loads using RT with different combinations of the pre-buckling deformation and the effects of curvature terms in against aspect ratio *a/b* for rectangular laminates simply supported E_1/E_2 = 5, *h/b* = 0.1, 0°, 90°, 0°, 90°, 0°, subjected to compression-tension ($N_x = N_0$, $N_y = -N_0$), the critical buckling loads are very important for this ratio if the pre-buckling is included, but the ratio is small if the effect of curvature is included.

Conclusions

The refined theory for buckling analysis of laminated plates with various boundary conditions has been derived. The correctness of RT development used in this paper is verified for simply supported isotropic rectangular plates with consideration of the effects of pre-buckling in-plane deformation and curvature terms. Very close agreement has been achieved between results obtained by authors' results.

From the assessment of the present theory for a three- and a five-layered symmetric cross-ply laminated composite square plate subjected to uniaxial and biaxial compression for various modular ratios, it is observed that the present results are in excellent agreement with the solutions of Reddy, Noor, Khdeir et al., and Sayyad et al.

These comparison studies confirm the validity and accuracy of the proposed solution method. Using the RT, the critical buckling solutions have been generated for plates with various loading conditions, aspect ratios *a/b*, thickness ratios *h/b*, degrees of orthotropic E_1/E_2 and numbers of layers.

Depending on the choice of conditions that we have posed, pre-buckling in-plane deformation and curvature effects may increase or decrease the non-dimensional critical load buckling. It can be concluded that the refined theory is effective in predicting critical load of laminated plate.

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