Mohamed Rida Seba[*], Said Kebdani

FREE VIBRATION ANALYSIS OF LAMINATED COMPOSITE PLATE WITH ELASTO-PLASTIC LAYERS

ANALIZA SLOBODNIH VIBRACIJA U LAMINIRANOJ KOMPOZITNOJ PLOČI SA ELASTOPLASTIČNIM SLOJEVIMA

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Adresa autora / Author's address:
Applied Mechanics Lab., Faculty of Mechanical Engineering, University of Science and Technology Oran Mohamed-boudiaf EL Mnaouar, BP 1505, Bir EL Djir 31000, Oran, Algeria, [*]email: mohamedrida.seba@univ-usto.dz

Keywords
- elastoplastic layers
- free vibration
- finite element method (FEM)
- laminated composite plate

Abstract

The present work aims to study free vibration of laminated composite plate with and without included elastoplastic layer carried out by applying the finite element method on two composite specimens [0°/45°/-45°/90°]s and [45°/-45°/-45°/45°]s. First, we validate using the results obtained from previous literature. The results obtained show a very good agreement between experimental and numerical simulations. Secondly, nonlinearity can be created by including the elastoplastic layer in the laminated composite plate, then we deal with the comparison between linear and nonlinear structure. The effects of structural parameters and boundary conditions of modular ratio on vibrations are investigated.

Ključne reči
- elastoplastični slojevi
- slobodne vibracije
- metoda konačnih elemenata (MKE)
- laminirana kompozitna ploča

Izvod

*U radu se izučavaju slobodne vibracije u laminiranoj kompozitnoj ploči sa i bez elastoplastičnog sloja, sa primenom metode konačnih elemenata na dva kompozitna uzorka [0°/45°/-45°*90°]s i [45°/-45°/-°45/45°]s. Prvo, proveravamo postupak poređenjem rezultata dobijenih iz literature. Dobijeni rezultati pokazuju vrlo dobro slaganje sa eksperimentalnim i numeričkim simulacijama. Drugo, nelinearnost se može uvesti elastoplastičnim slojem u laminiranoj kompozitnoj ploči, a zatim se upoređuju linearna i nelinearna konstrukcija. Takođe je istražen i uticaj konstrukcionih parametara i graničnih uslova modularnog koeficijenta na vibracije.*

INTRODUCTION

Laminated composite structures are made of successive layers formed from fibre reinforcements bonded with matrix, laminated structures made with composite materials, Vasiliev et al. /1/, Reid et al. /2/. In order to treat the impact behaviour of composites it is useful to consider the nature of the materials, composites consisting of glass, carbon, or polymer fibres in a polymer matrix are heterogeneous and anisotropic materials, Hashin /3/. Anisotropic unidirectional fibre materials have more stiffness and strength parameters than a particulate composite among the composite structures we have: plate and cock, they are used in several different areas of engineering mechanics, civil and aerospace.

Some the of most important research carried out so far to learn the behaviour of the free vibration analysis of laminated composite plates is performed by numerical method. Li et al. /4/, and Tran et al. /5/, preformed free vibration analysis of laminate plates applying higher-order shear. Sinha et al. /6/, study the free vibration of woven glass fibre reinforced composite laminated plates. Vescovini et al. /7/, use the Ritz method for free vibration analysis of anisotropic thin and thick plates. Fazzolari et al. /8/, analyse free vibration response of anisotropic composite plates in a thermo-mechanical pre/post-buckled state. Pingulkar /9/, perform modal analysis using finite elements for cantilevered laminated composite plates to predict modal frequencies and the effect of matrix material. Hybridization and different laminates stacking sequences is also studied by Sayyad et al. /10/, in recent research on free vibration analysis of multilayer laminated composites by finite element method for the analysis of plates.

The elastoplastic behaviour is characterised by a response of the material which gives a nonlinear behaviour and generates a vibration damping on the laminated plate. Many researchers are interested in this topic. Maheri et al. /11/ derive the calculation of the modal damping of orthotropic structural materials including anisotropic laminates under free vibration. Lalo et al. /12/, studies structures subjected to a transient dynamic response with different damping ratios and do a modification of structural stiffness under elastoplastic regime. Grzesikiewicz et al. /13/, Reisinger et al. /14/, and Dias da Silva, /15/, study model rheology for descriptive characterisation of the elastoplastic properties of materials. Starovoitov et al. /16/ examine sandwich bar using elastoplastic bending. Starovoitov et al. /17/ take into consideration thermal deformation of a three-layer elastoplastic. Thakur et al. /18/ utilize Seth's transition theory to provide a study of elastoplastic density change in a deformable disk. Bahei-El-Din et al. /19/ develop a method for describing the elastic-plastic behaviour of unidirectional composites. Thakur et al. /20/ use transition theory and investigate the elastic-plastic transition in a disk with orthotropic material. Sharma /21/, Sharma et al. /22/, and Temesgen et al. /23/, present elastic-plastic in rotating disc.

The objective of this work is to investigate the free vibration of laminated composite plate with and without including elastoplastic layers. The effects of the boundary conditions use three different support conditions in edge clamped/free (CFFF, CFCF, CCCC) and show the impact of modular ratio E_1/E_2 with lower value (4, 8, 12, 16 and 20) and finally determine the influence of variation in geometry, so there are applications with and without elastoplastic layers.

THE THEORY

Elastoplastic model

A general elastic-plastic constitutive model usually consists of four components, using different elastic laws with an elastic modulus E, yield criterion used to separate the elastic from the elastic-plastic zone, plastic flow directions with a tangent modulus E_T, and hardening laws (K_*). These can be easily derived in form of plastic for the strain hardening hypothesis, and can be combined to develop elastic-plastic material models, Yu et al. /24/, and Bathe et al. /25/.

In the plastic zone the stress-strain relation has a linear form

$$\sigma = E\varepsilon \ . \tag{1}$$

In the plastic zone, the addition of elastic strain increment and plastic strain increment, i.e.

$$d\varepsilon = d\varepsilon^{el} + d\varepsilon^{pl} \ . \tag{2}$$

If plastic deformation is considered to be rate insensitive, the stress increment is linearly related to the elastic strain increment in the plastic region, and can be expressed as:

$$d\sigma = Ed\varepsilon^{el} = E(d\varepsilon - d\varepsilon^{pl}) \ . \tag{3}$$

The initial yield point σ_y differentiates the elastic and plastic zone. The stress in the plastic zone can be specified by a hardening rule,

$$\sigma = \sigma_y(k_*) \ . \tag{4}$$

The plastic strain ε^{pl} is usually used for the hardening parameter $k_* = \varepsilon^{pl}$, calculated by

$$\varepsilon^{pl} = \int d\varepsilon^{pl} \ . \tag{5}$$

Using the Eq.(2), we can deduce that

$$d\varepsilon^{pl} = d\varepsilon - d\varepsilon^{pl} = \frac{d\sigma}{dE_T} - \frac{d\sigma}{dE} = \left(\frac{1}{dE_T} - \frac{1}{dE}\right) d\sigma \ . \tag{6}$$

The tangent modulus E_T is defined as a function of stress and plastic strain ε^{pl}

$$E_T = E_T(\sigma, \varepsilon^{pl}) \ . \tag{7}$$

Free vibration analysis

The frequencies of the plate can be obtained easily by the resolution of the standard characteristic equation,

$$[\mathrm{K}^*]\{\Pi\} - \omega^2[\mathrm{M}^*]\{\Pi\} = \{\mathrm{F}\} \ , \tag{8}$$

where: {F} are defined forcing function; and {Π} indicates the transverse displacement vector. The suppression of {F} from the Eq.(8) results in the following equation,

$$[\mathrm{K}^*]\{\Pi\} - \omega^2[\mathrm{M}^*]\{\Pi\} = \{0\} \ , \tag{9}$$

where: K^* is stiffness matrix; and M^* represents the mass matrix, respectively. If the forcing function {F} is assumed to be zero, the above equations can be used directly to find the natural frequencies of free vibration.

MATERIALS AND NUMERICAL PROCEDURE

Materials

Elastic and elastoplastic laminated materials are studied in this research. Eight layers make up the elastic composite laminates treated by Crawley /26/. The elastoplastic laminated material includes two or four elastoplastic layers in the laminated plate on both sides. The characteristics of the elastoplastic layer and the elastic composite are shown in Tables 1 and 2, respectively.

Table 1. The elastic characteristics of the anisotropy of the laminate composite.

E_1 (GPa)	E_2 (GPa)	ν_{12}	G_{12} (GPa)	G_{13} (GPa)	density (kg/m^3)
128	11	0.25	4.48	1.53	1.5×10^3

Table 2. Characteristics of the elastoplastic layer.

Young's modulus E (GPa)	Poisson ratio ν
5.42	0.28

Finite element simulations

Results are obtained through ABAQUS®. The numerical analysis is executed by finite element method in a clamping/free configuration on six different specimens of laminate plates with and without the elastoplastic material. For determining the behaviour of the stresses and natural frequency, two specimens are studied for first type of composite and four specimens of the second type of the composite: an unidirectional laminate. They are labelled [0°/45°/-45°/90°]s and [45°/45°/-45°/45°]s in the case of elastic composites, and [ELP/45°/45°/90°]s, [ELP/-45°/-45°/45°]s, [ELP/ELP/-45°/90°]s, and [ELP/ELP/-45°/45°]s, for the elastoplastic material. Figure 1 shows the models of layers from laminates composite plate with both elastic and elastoplastic material, (ELP: elastoplastic layer).

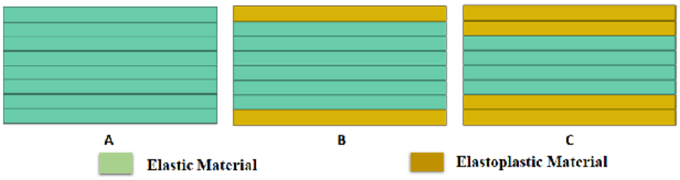


Figure 1. Models of layers from laminates composite plate.

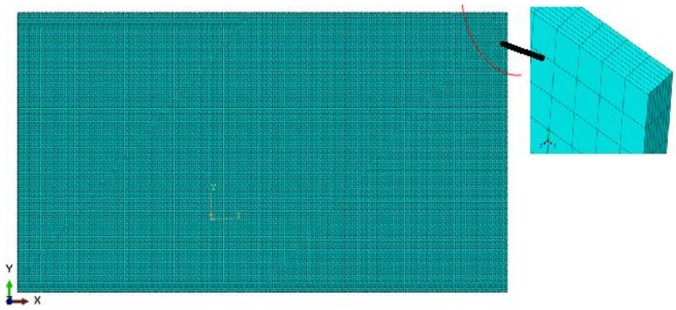


Figure 2. Finite element model of the laminates composite plate.

Convergence and validation

Convergence

As a starting point, the convergence of the current simulation method is tested by computation of the natural frequency of two first mode with different size of the mesh and presented in Table 3. From the convergence study, a size (0.75) is applied to calculate the responses for the duration of this analysis, results are extracted by finite element software. Finite element model of the laminates composite plate is shown in Fig. 2.

Table 3. Natural frequency in two first mode vs. size of mesh.

size	Natural frequency (Hz) for laminates composite plate [45°/-45°/-45°/45°]s	
	Mode 1	Mode 2
4	54.985	341.39
3	46.483	286
2	38.879	235.91
1	33.309	200.02
0.75	32.471	194.69
0.65	32.207	193.02

Validation

To ensure the validity and accuracy of finite element approaches, a parametric study is done to examine the natural frequency of the rectangular plate. As calculated using the model, the results are presented taking into account various available examples. A laminate plate under clamped/free (CFFF) boundary conditions as in Crawley /26/. For the two different laminations [0°/45°/-45°/90°]$_S$ and [45°/-45°/-45°/45°]$_S$ of glass/epoxy composite. Natural frequency values for the first five modes have been computed and are shown in Figs. 3 and 4, accordingly.

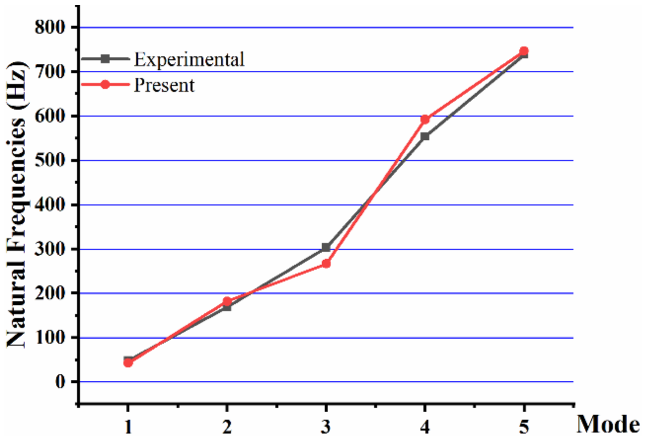


Figure 3. Natural frequency comparison for laminates plate [0°/+45°/-45°/90°]s.

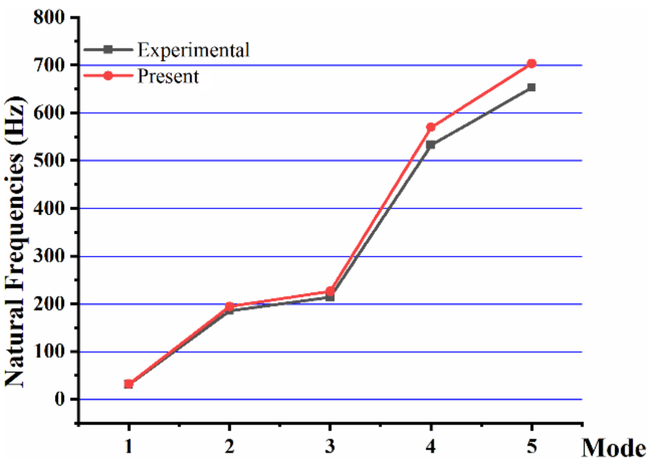


Figure 4. Natural frequency comparison for laminates plate [+45°/-45°/-45°/+45°]s.

RESULTS AND DISCUSSION

Effect of elastoplastic layers

In this section we study the impact of the number of elastoplastic layers on natural frequency of the used six different laminate rectangular plates with eight layers [0°/45°/-45°/90°]s and [4°/-45°/-45°/45°]s, [0°/45°/-45°/90°]s, and [45°/-45°/-45°/45]s in the case of elastic composites; [ELP/45°/45°/90°]s, [ELP/-45°/-45°/45°]s, [ELP/ELP/-45°/90°]s, and [ELP/ELP/-45°/45°]s for the elastoplastic material under clamped/free (CFFF) support condition in edge. A frequency response analysis is performed to find the value of natural frequencies of the first five modes and for six types of laminate rectangular plate composites.

The effect of this elastoplastic layer on the natural frequencies for the first five modes is studied. It can be seen in the six studied configurations, when compared to elastic composites, the frequency values of elastoplastic composites are significantly lower. In Figs. 5 and 6 we show a frequency comparison with and without elastoplastic layers. It also has little changes to the nature frequency in the first position, then we take important values for four other modes. Moreover, the higher number of elastoplastic layers included, decreases the value of natural frequency.

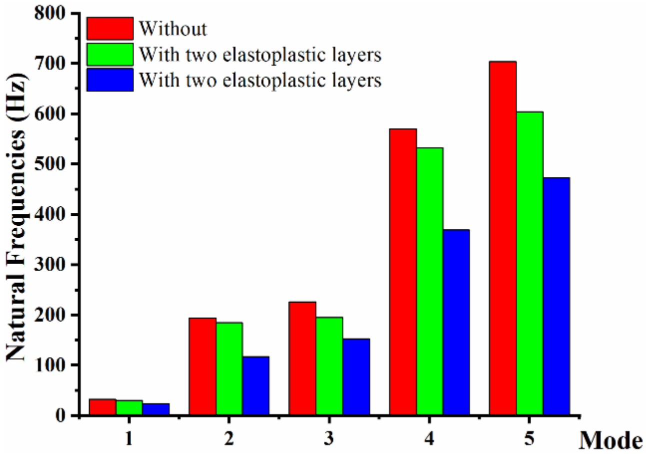


Figure 5. Comparison of elastic and elastoplastic laminates' frequency for first five modes and various specimens: [45°/-45°/-45°/45°]s, [ELP/-45°/-45°/45°]s, and [ELP/ELP/-45°/45°]s.

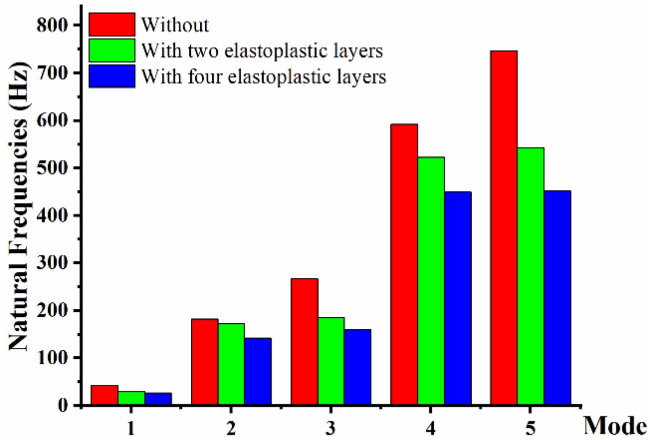


Figure 6. Comparison of elastic and elastoplastic laminates' frequency for first five modes and various specimens [0°/45°/-45°/90°], [ELP/45°/-45°/90°]s, [ELP/ELP/-45°/90°]s.

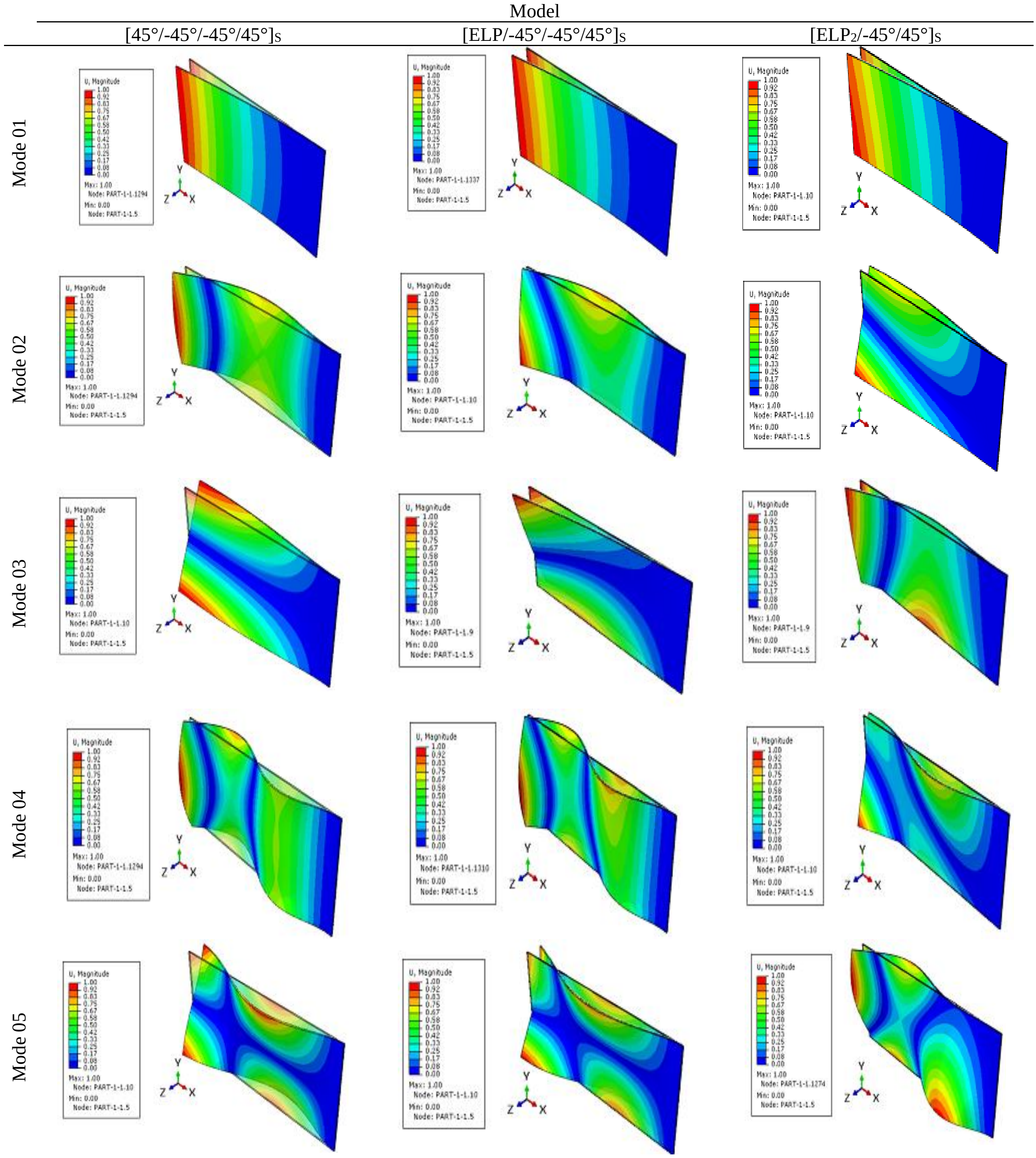


Figure 7. Typical modal shapes of laminate plates with and without elastoplastic layers.

Mode shapes obtained for [+45°/-45°/-45°/+45°]s plates with and without elastoplastic layers are presented Fig. 7.

EFFECT OF BOUNDARY CONDITION

This analysis is carried out under three different boundary conditions. Clamped/free (CFFF, CFCF, CCCC), shown in Fig. 2, for a laminate plate of length a = 152 mm, width b = 76 mm, thickness 1.04 mm, is carried out in six laminates of composites mentioned in the previous section. All elastic properties are given in Tables 1 and 2.

The effects of boundary conditions with and without an elastoplastic layer are studied. The variation of the von Mises stress with the natural frequency for different boundary conditions CFFF, CFCF and CCCC are imposed at the edges of the plate, as shown in Fig. 9, respectively.

It can be seen in Fig. 9(1) that the von Mises stress increases as frequency increases. In Figs. 9(2) and 9(3) von Mises stress decreases in frequency value (1334.5, 935.29, 900.88, and 617.53), and (3113.1 and 2735.7), respectively. This can be explained by mode shape. Figure 8 presents the evolution of the von Mises stress with different natural frequency in three cases of boundary conditions of laminate plates with and without elastoplastic layers (a: [0°/45°/-45°/90°] and b: [45°/-45°/-45°/45°]s).

Whatever the difference in cases of laminates plate, the plate with a support condition (CCCC) has the highest frequency level, and the plate with support condition (CFFF) has the lowest frequency level, whereas in the case of the CFCF plate, they are located between these two levels.

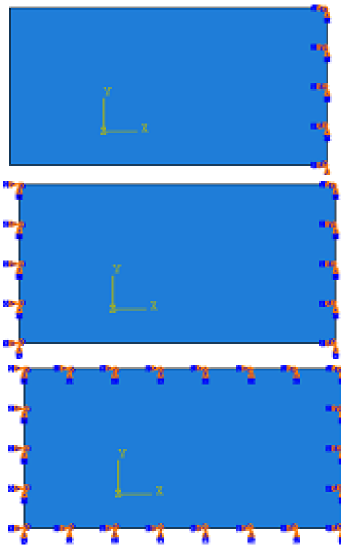
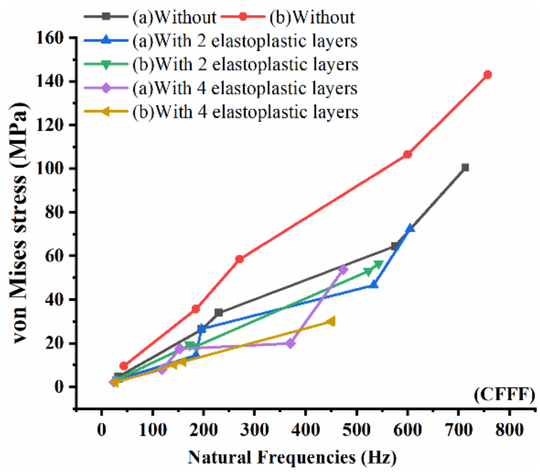
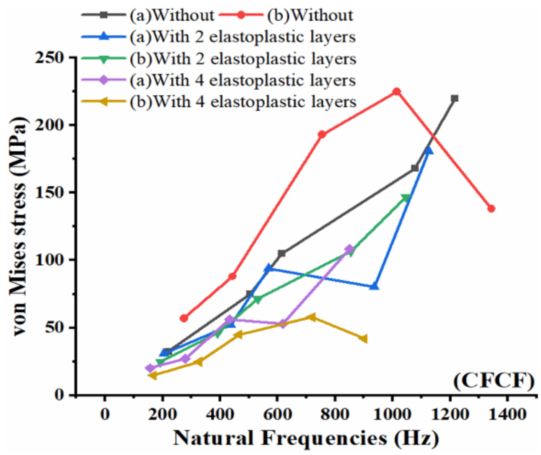


Figure 8. Boundary conditions considered in the study: (C) clamped and (F) free.

(1)



(2)



(3)

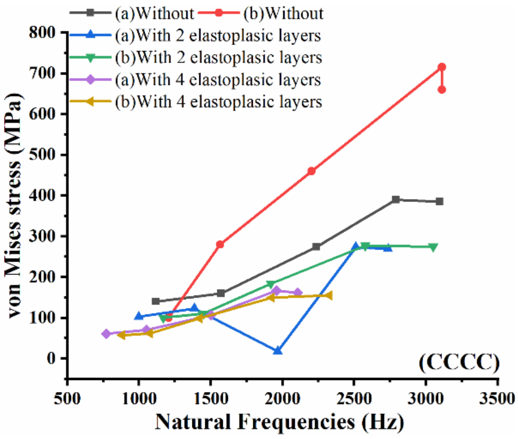


Figure 9. Evolution of von Mises stress with different natural frequency in three cases of boundary conditions of laminate plate with and without elastoplastic layers.

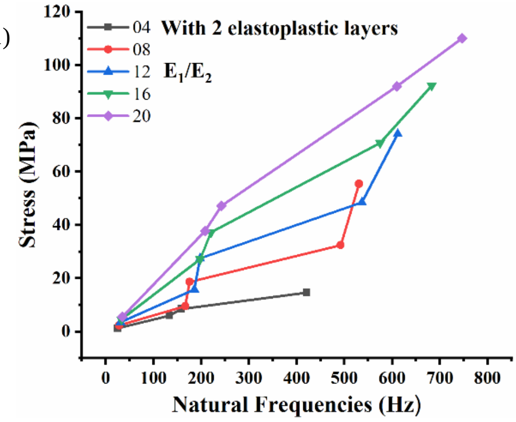
Influence of modular ratio

The influence of modular ratio on natural frequency with and without an elastoplastic layer clamped laminates composite plate [45°/-45°/-45°/45°]$_S$ has been studied. A change in stress with the natural frequency value is determined for respective configurations: E_1/E_2 = 4, 8, 12, 16 and 20.

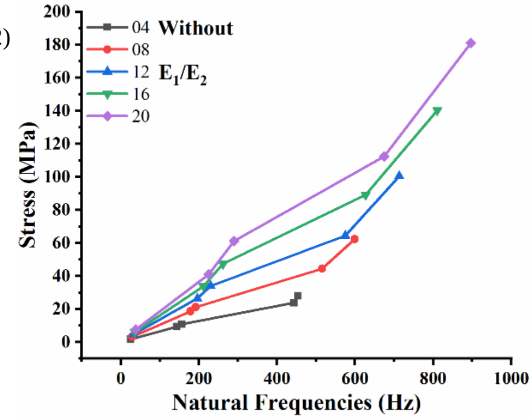
It can be remarked in Fig. 10 that the stress increases as value of frequency increases. It is interesting to note that the first natural frequencies for configurations of elastic composites are very close to the elastoplastic martial frequency.

It can be clearly observed from three figures, the higher value of modular ratio increases value of natural frequency and then it results in an increase in the value of stress in all cases (with and without elastoplastic layers).

(1)



(2)



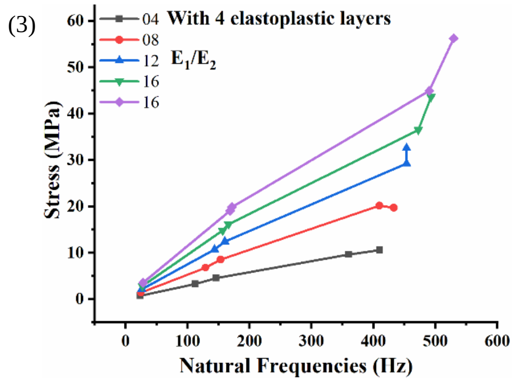


Figure 10. Evolution of stress with natural frequency in different modular ratio E_1/E_2 of laminate plate with and without elastoplastic layers.

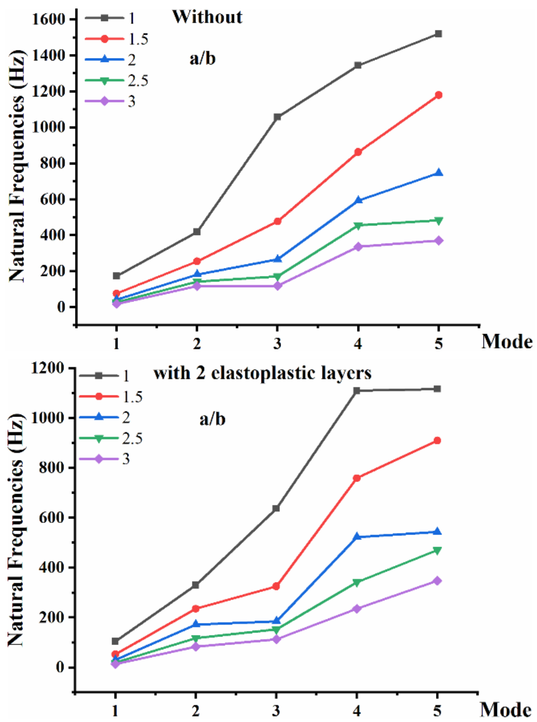
Note that the lowest value of the natural frequency with two and four elastoplastic layers and without them is 25.438 Hz, 23.452 Hz, and 25.598 Hz, respectively. In addition, the highest values are 746.38, 529.51 and 896.84 Hz.

Effect of geometrical parameters on naturel frequency

Effect of aspect ratio

For the computation, panel geometry is considered to be a = 152 mm, h = 1.04 mm constant, and b is modified so that a/b = 1, 1.5, 2, 2.5, 3. In this section is the influence of aspect ratio a/b treated on natural frequency. Used rectangle laminate plates with and without elastoplastic support condition (CFFF) are taken into consideration.

The plate's stiffness is affected by changing the aspect ratio leading to a change in natural frequency value of the plate. The natural frequency in all cases generally decreases as the aspect ratio increases; a/b = 1 case causing a maximal natural frequency through five first modes which can be seen in Figs. 11 and 12. It should also be noted that a decrease in natural frequency appears with the increase in the number of elastoplastic layers.



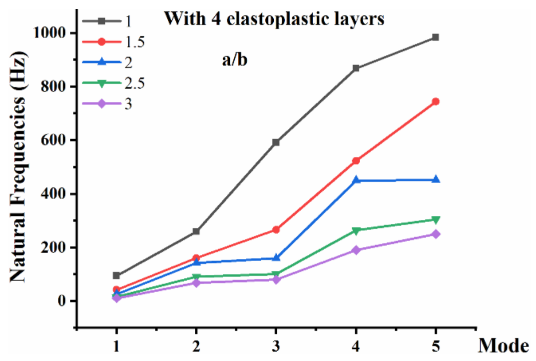


Figure 11. Frequency variation with aspect ratio a/b relative to the case of laminated plates [0°/+45°/-45°/90°]s.

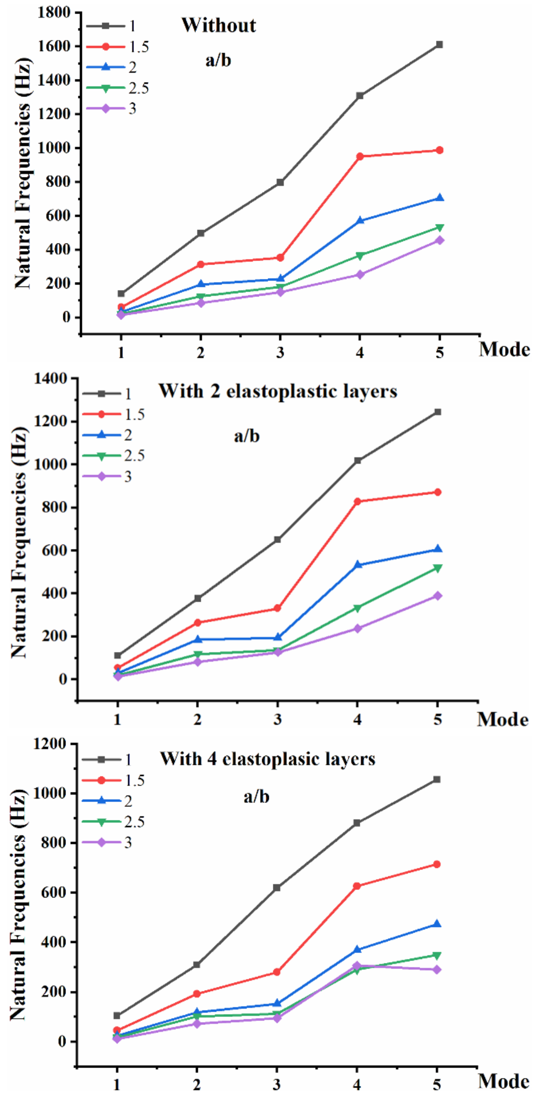


Figure 12. Frequency variation with aspect ratio a/b relative to the case of laminated plates [45°/-45°/-45°/45°]s.

Effect of thickness ratio

The study of the impact of the thickness ratio on natural frequency behaviour of the laminate rectangular plate with and without elastoplastic layers the size of the plate is chosen as follows: a = 152 mm, b = 76mm and h is replaced. So that a/h = 20, 40, 50, 80 and 100.

The variation of natural frequency with five first modes for different values of thickness ratio with and without elastoplastic layers is shown in figure 13 and 14; it can be observed that the natural frequency value increases as the thickness ratio decreases in all cases. For the thinnest plate (a/h=100), the natural frequency value is higher than for other plates as a result of reduced rigidity. Also, it should be noted that the normal frequency closes some cases in the fourth mode.

It is worth noting that The lowest value of natural frequency Hz with (two elastoplastic layers, four elastoplastic layers) and without respectively is (40.469, 41.530),(34.113, 36.779) and (62.733, 46.765) , Moreover .it The highest value is (3473.7, 3433.3).(2437, 2959.9)and (3759.1, 3584.2).

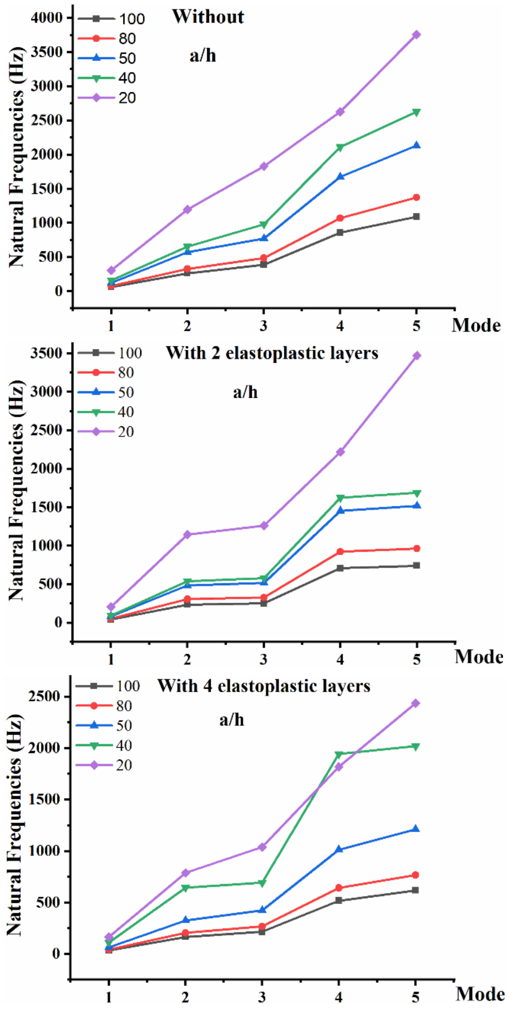
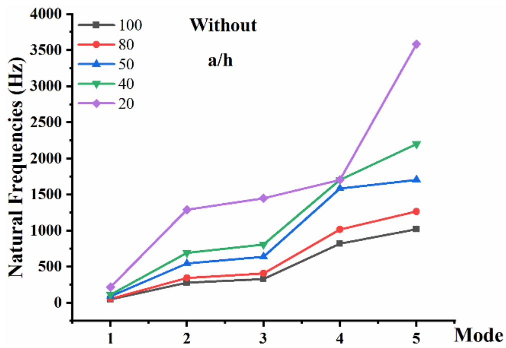


Figure 13. Frequency variation with thickness ratio *a/h* relative to the case of laminated plates [0/+45/-45/90]s.



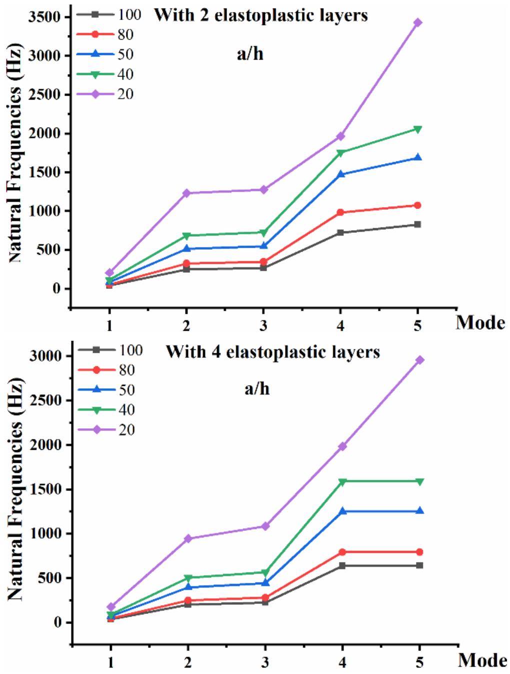


Figure 14. Frequency variation with thickness ratio *a/h* relative to the case of laminated plates [45/-45/-45/45]s.

CONCLUSION

In this research work, free vibration analysis is performed using finite element software for cantilevered laminated composite plates to forecast natural frequency values and mode shapes. In this study, included are two and four elastic layers in the composite laminates and the results are compared between the composite laminated plates with and without elastoplastic layers. The second part of this work is focused on the study of the effects of boundary conditions, modular ratio and geometrical parameters (aspect and thickness ratios) on the behaviour of the plates with and without elastoplastic layers. Conclusions can be stated as follows:

- Numerical examples show that decrease in natural frequency value by including and increasing the number of elastoplastic layers can be explained by vibration damping generated by the nonlinear physics of the structure.
- A lower level of von Mises stress is accompanied by the increase in number of elastoplastic layers in the laminates composite plate.
- The plate with support condition (CCCC) has the highest frequency level and plate with support condition (CFFF) has the lowest frequency level, but in the case of CFCF plate, they are positioned between these two levels.
- The natural frequency value for laminates plates with and without elastoplastic layers increases by increasing aspect ratio E_1/E_2 and is accompanied by an increase in stress.
- The natural frequency value for laminates plates with and without elastoplastic layers decreases by increasing aspect ratio *a/b*.
- The natural frequency value for laminates plates with and without elastoplastic layers decreases by increasing the thickness ratio *a/h*.
- In addition, laminated composite plates [0°/45°/-45°/90°]s have higher natural frequencies than laminated composite plates [45°/-45°/-45°/45]s.

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APPENDIX

Notation

a .... length of plate (mm)
b .... width of plate (mm)
h .... height of plate (mm)
E .... Young’s modulus (GPa)
G .... shear modulus (GPa)
ν .... Poisson ratio
ELP ... elastoplastic