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WAVE PROPAGATION IN THERMOELASTIC MICROELONGATED HALF-SPACE

PROSTIRANJE TALASA U TERMOELASTIČNOM MIKROIZDUŽENOM POLUPROSTORU

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- teorije LS i GL
- parametar mikroizduženja

Abstract

The research article deals with the propagation of waves in a thermoelastic microelongated solid in the context of LS and GL theories of thermoelasticity. The governing equations are solved to obtain the velocity equation. The velocity equation shows the existence of four waves in the medium. Wave velocities have been computed for a particular medium and the effect of micro-elongation parameter on the velocity components has been depicted graphically.

Izvod

U ovom radu se bavimo istraživanjem prostiranja talasa termoelastičnom mikroizduženom čvrstom telu u kontekstu LS i GL teorija termoelastičnosti. Rešavanjem izvedenih jednačina dobija se jednačina brzine. Jednačina brzine pokazuje postojanje četiri talasa u medijumu. Izračunate su brzine talasa za poseban medijum, a uticaj parametra mikroizduženja na komponente brzine je grafički prikazan.

INTRODUCTION

A group of unified particles in the form of small rigid bodies experiencing both translational and rotational motion is known as a micropolar continuum. The theory is appropriate for simulating the mechanical behaviour of material particles having rigid directors, chopped fiber composites, platelet composites, aluminium epoxy, a large class of substances like liquid crystals with rigid molecules, rigid suspensions, concrete with sand and muddy fluids, etc. The prevalent use of these types of composite materials has increased interest in this theory. The generalised theory of linear micropolar thermoelasticity is established by Boschi and Iesan /1/. Various works carried out in this field illustrate that each particle of the material may make both microrotation and volumetric elongation along with the bulk deformation. From this, we can conclude that a microstretch elastic solid has seven degrees of freedom. These may be detailed as, three degrees for translation, three degrees for rotation, and one degree for stretch. The stretching and contracting of material points of microstretched bodies are not affected by their rotation and translation. Such a generalised solid may hold more information about the microdeformation inside a material point. This information can be used as a mathematical model for different media which may not fall under the domain of micropolar elasticity.

The microstretch continuum provides a useful model in the study of different fields like, solids with micro-damages, animal bones, foams, and porous media whose pores are filled with gas or inviscid liquid, etc. Solid-liquid crystals, composite materials reinforced with chopped elastic fibers, porous media with pores filled with non-viscous fluid or gas can be categorised as a microelongated medium.

Classical theory is not sufficient to model the behaviour of materials having internal structure. Eringen and Suhubi /2-3/ developed a nonlinear theory of microelastic solids. Later on, Eringen /4–6/ developed a theory according to which material particles in solids can undergo macro-deformations as well as micro-rotations and entitled this theory as 'linear theory of micropolar elasticity'. Then he gave a theory of micropolar elastic solid with stretch in which he introduced axial stretch, /7/. Nowacki /8/, Eringen /9/, Tauchert et al. /10/, and Nowacki and Olszak /11/ explained thermal effects in the micropolar theory. The generalised theory of thermoelasticity is the first theory, whereas the second of Lord and Shulman's /12/ is the theory of temperature-rate-dependent. An entropy production inequality is presented by Muller in the review of thermodynamics of thermoelastic solids. This helped him in considering restrictions on a class of constitutive equations /13/. Green and Laws /14/ proposed a generalisation of this. Green and Lindsay developed these constitutive equations /15/ differently. These equations are discussed by Suhubi /16/ independently and explicitly which contains two relaxation time constants. This results in transforming all the equations of the coupled theory. Sherief /17/ discovered components of stress and temperature distributions in a thermoelastic medium due to a continuous source. Dhaliwal et al. /18/ examined thermoelastic interactions due to a continuous line heat source in a homogeneous isotropic unbounded solid. Thermoelastic interface due to a continuous point heat source in a homogeneous and isotropic unbounded body is investigated by Chandrasekharaiah and Srinath /19/. Sharma and Chauhan illustrated mechanical and thermal sources in a generalised thermoelastic half-space /20/. Transient disturbance in half-space due to moving internal heat source under the L-S model is studied by Sarbani and Amitava /21/ to obtain the solution for displacements in the transformed domain. The solution of a problem on a generalised thermoelastic infinite medium with a spherical cavity subjected to a moving heat source is found by Youssef /22/. Shaw and Mukhopadhyay /23/ investigated the periodically changing heat source reaction in a functionally graded microelongated medium. A thermoelastic interaction in a microelongated, isotropic, homogeneous medium in the presence of a shifting heat cause is investigated by Shaw and Mukhopadhyay /24/. Ailawalia et al. /25/ discussed the effect of internal heat source in thermoelastic microelongated solid at an interface for G-L theory.

The present article deals with the plane wave propagation in a thermoelastic microelongated solid. The governing equations are solved analytically to obtain the velocity equation. The wave velocities have been obtained for an aluminium epoxy-like material and the effect of microelongation parameter on the velocity components has been depicted graphically for LS and GL theories of thermoelasticity.

BASIC EQUATION

The constitutive equations for a homogeneous, isotropic, microelongated, thermoelastic solid are given by Shaw and Mukhopadhyay, /24/:

$$\sigma_{ij}=\lambda\delta_{kl}u_{r,r}+\mu(u_{k,l}+u_{l,k})-\beta_0\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T\delta_{kl}+ +\lambda_0\delta_{kl}\varphi, \tag{1}$$

$$m_k=\lambda_0\varphi_{,k}, \tag{2}$$

$$s-\sigma=\lambda_0 u_{k,k}+\mu(u_{k,l}+u_{l,k})-\beta_0\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T-\lambda_1\varphi, \tag{3}$$

$$q_k=\frac{K}{T_0}\varphi_{,k}. \tag{4}$$

The field equation of motion according to /26, 27/ and heat conduction equation according to /28/ for the displacement, microelongation, and temperature changes are

$$(\lambda+\mu)u_{j,ij}+\mu u_{i,jj}-\beta_0\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T_{,i}+\lambda_0\varphi_{,i}=\rho\ddot{u}, \tag{5}$$

$$a_0\varphi_{,ii}+\beta_1\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\lambda_1\varphi-\lambda_0 u_{j,j}=\frac{1}{2}\rho j_0\ddot{\varphi}, \tag{6}$$

$$KT_{,ii}-\rho C\left(1+t_0\frac{\partial}{\partial t}\right)\dot{T}-\beta_0T_0\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\dot{u}_{k,k}-\beta_1T_0\dot{\varphi}=0, \tag{7}$$

where: $\beta_0=(3\lambda+2\mu)\alpha_{t1}$; $\beta_1=(3\lambda+2\mu)\alpha_{t3}$; $\sigma=\sigma_{kk}$ is microelongational stress tensor; $s=s_{kk}$ is component of stress tensor; δ_{kl} is Kronecker delta; m_k is component of microstretch vector; λ and μ are Lame's elastic constants; a_0, λ_0, λ_1 are microelongational constants; C is specific heat at constant strain; K is thermal conductivity; α_{t1} and α_{t3} are coefficients of linear thermal expansion; ρ is density of microelongated medium; j_0 is microinertia; t_0, t_1 are thermal relaxation times; T is thermodynamic temperature above reference temperature T_0; φ is microelongational scalar; $\vec{u}=u_i$ is displacement vector; $k=1$ for Lord-Shulman (L-S) theory; and $k=2$ for Green-Lindsay (G-L) theory.

FORMULATION OF THE PROBLEM

We have considered two dimensional disturbances of medium parallel to xy-plane with all physical quantities depending upon (x, y, t). For this we use displacement vector $\vec{u}_l=(u_1,u_2,0)$.

Hence, Eqs.(5)-(7) become:

$$(\lambda+2\mu)\frac{\partial^2u_1}{\partial x^2}+(\lambda+\mu)\frac{\partial^2u_2}{\partial x\partial y}+\mu\frac{\partial^2u_1}{\partial y^2}-\beta_0\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}+ +\lambda_0\frac{\partial\varphi}{\partial x}=\rho\frac{\partial^2u_1}{\partial t^2}, \tag{8}$$

$$\mu\frac{\partial^2u_2}{\partial y^2}+(\lambda+\mu)\frac{\partial^2u_1}{\partial x\partial y}+(\lambda+2\mu)\frac{\partial^2u_2}{\partial x^2}-\beta_0\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}+ +\lambda_0\frac{\partial\varphi}{\partial y}=\rho\frac{\partial^2u_2}{\partial t^2}, \tag{9}$$

$$a_0\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\varphi+\beta_1\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T-\lambda_1\varphi-\lambda_0\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)= =\frac{1}{2}\rho j_0\frac{\partial^2\varphi}{\partial t^2}, \tag{10}$$

$$K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)T-\rho C\left(1+t_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}-\beta_0T_0\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\times \times\frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)-\beta_1T_0\frac{\partial\varphi}{\partial t}=0. \tag{11}$$

The constitutive components of microelongational stress tensor are given by

$$\sigma_{xx}=(\lambda+2\mu)\frac{\partial u_1}{\partial x}+\lambda\frac{\partial u_2}{\partial y}-\beta_1\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\lambda_0\varphi, \tag{12}$$

$$\sigma_{yy}=\lambda\frac{\partial u_1}{\partial y}+(\lambda+2\mu)\frac{\partial u_2}{\partial x}-\beta_1\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\lambda_0\varphi, \tag{13}$$

$$\sigma_{xy}=\lambda\left(\frac{\partial u_1}{\partial y}+\frac{\partial u_2}{\partial x}\right). \tag{14}$$

To simplify calculations, we use following non-dimensional variables defined by

$$x'=\frac{\omega^*}{c_1}x,\ y'=\frac{\omega^*}{c_1}y,\ u_i'=\frac{\omega^*\rho c_1}{\beta_0T_0}u_i,\ t'=\omega^*t,\ t_0'=\omega^*t_0,$$

$$t_1'=\omega^*t_1,\ \sigma_{ij}'=\frac{\sigma_{ij}}{\beta_0T_0},\ \varphi'=\frac{\lambda_0}{\beta_0T_0}\varphi,\ T'=\frac{T}{T_0},$$

where: $\omega^*=\dfrac{\rho c_1^2C}{\beta_0T_0}$; $c_1^2=\dfrac{(\lambda+2\mu)}{\rho}$.

Using the above non-dimensional variables in Eqs.(8) to (13) and after dropping superscripts we get,

$$\frac{\partial^2u_1}{\partial x^2}+l_2\frac{\partial^2u_2}{\partial x\partial y}+l_3\frac{\partial^2u_1}{\partial y^2}-\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}+\frac{\partial\varphi}{\partial x}=\frac{\partial^2u_1}{\partial t^2}, \tag{15}$$

$$l_3\frac{\partial^2 u_2}{\partial x^2}+l_2\frac{\partial^2 u_1}{\partial x\partial y}+\frac{\partial^2 u_2}{\partial y^2}-\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}+\frac{\partial\varphi}{\partial y}=\frac{\partial^2 u_2}{\partial t^2}\,, \quad (16)$$

$$\nabla^2\varphi+l_4\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T-l_5\varphi-l_6\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)=l_7\frac{\partial^2\varphi}{\partial t^2}\,, \quad (17)$$

$$\nabla^2 T-l_8\left(1+t_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}-l_9\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)-$$
$$-l_{10}\frac{\partial\varphi}{\partial t}=0\,, \quad (18)$$

$$\sigma_{xx}=\frac{\partial u_1}{\partial x}+l_{11}\frac{\partial u_2}{\partial y}-\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\varphi\,, \quad (19)$$

$$\sigma_{yy}=l_{11}\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}-\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T+\varphi\,, \quad (20)$$

$$\sigma_{xy}=l_3\left(\frac{\partial u_1}{\partial y}+\frac{\partial u_2}{\partial x}\right), \quad (21)$$

where:

$$l_2=l_3=\frac{\mu}{\rho c_1^2}\,,\ l_4=\frac{\beta_1\lambda_0 c_1^2}{a_0\omega^*\beta_0}\,,\ l_5=\frac{\lambda_1 c_1^2}{a_0\omega^*}\,,\ l_6=\frac{\lambda_0^2}{a_0\omega^*\beta_0}\,,$$
$$l_7=\frac{\rho j_0 c_1^2}{2a_0}\,,\ l_8=\frac{\rho C^* c_1^2}{K^*\omega^*}\,,\ l_9=\frac{\beta_0^2 T_0}{K^*\omega^*\rho}\,,\ l_{10}=\frac{\beta_0\beta_1 T_0 c_1^2}{K^*\omega^*\lambda_0}\,,$$
$$l_{11}=\frac{\lambda}{\rho c_1^2}\,,\ \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\,. \quad (22)$$

ANALYTICAL SOLUTION

Here, we use normal mode analysis technique to decompose the solution of the considered physical variables as

$$(u_i,T,\varphi,\sigma_{ij})=\left(u_i^*,T^*,\varphi^*,\sigma_{ij}^*\right)e^{im(x\sin q+y\cos q-vt)}\,,$$

where: v_i is phase velocity; ($\sin\theta$, $\cos\theta$) represents the projection of the wave normal onto the x-y plane; κ_i, s is the wave number; and u_i^*, T^*, φ^* and σ_{ij}^* are the amplitudes of field quantities. Using this solution from Eqs.(15) to (18) we get:

$$\left(\sin^2\theta+l_3\cos^2\theta-v^2\right)u_1+l_2\sin\theta\cos\theta u_2+\frac{1}{im}\sin\theta\varphi-$$
$$-vt_1^*\sin\theta T=0\,, \quad (23)$$

where: $t_1^*=\left(\frac{1}{imv}-t_1\delta_{2k}\right)$,

$$l_2\sin\theta\cos\theta u_1+\left(l_3\sin^2\theta+\cos^2\theta-v^2\right)u_2+\frac{1}{im}(\cos\theta)\varphi-$$
$$-vt_1^*(\cos\theta)T=0\,, \quad (24)$$

where: $t_1^*=\left(\frac{1}{imv}-t_1\delta_{2k}\right)$,

$$\frac{l_6}{(im)l_7}\sin\theta u_1+\frac{l_6}{(im)l_7}\cos\theta u_2+\left(v^2-\frac{1}{l_7}-\frac{l_5}{l_7(im)^2}\right)\varphi-$$
$$-\frac{l_4 v}{(im)l_7}t_1^* T=0\,, \quad (25)$$

where: $t_1^*=\left(\frac{1}{ikv}-t_1\delta_{2\alpha}\right)$,

$$\frac{l_9 t_0^{**}imv^2\sin\theta v^2}{l_8 t_0^*}u_1+\frac{l_9 t_0^{**}imv^2\cos\theta}{l_8 t_0^*}u_2+\frac{l_{10}v}{l_8 t_0^* im}\varphi+$$
$$+\left(\frac{1}{l_8 t_0^*}+v^2\right)T=0\,, \quad (26)$$

where: $t_0^*=\left(\frac{1}{imv}-t_0\right)$; $t_0^{**}=\left(\frac{1}{imv}-t_0\delta_{1k}\right)$.

Since Eqs.(23)-(26) represent a system of homogeneous in variables u_1, u_2, φ, T, therefore, its trivial solution requires:

$$A=\begin{vmatrix}(\sin^2\theta+l_3\cos^2\theta-v^2) & l_2\sin\theta\cos\theta & \frac{1}{ik}\sin\theta & -vt_1^*\sin\theta\\ l_2\sin\theta\cos\theta & (l_3\sin^2\theta+\cos^2\theta-v^2) & \frac{1}{ik}\cos\theta & -vt_1^*\cos\theta\\ \frac{l_6}{(im)l_7}\sin\theta & \frac{l_6}{(im)l_7}\cos\theta & \left(v^2-\frac{1}{l_7}-\frac{l_5}{l_7(im)^2}\right) & -\frac{t_1^* l_4 v}{l_7(im)}\\ \frac{l_9 t_0^* imv^2\sin\theta}{l_8 t_0^*} & \frac{l_9 t_0^* imv^2\cos\theta}{l_8 t_0^*} & \frac{l_{10}v}{l_8 t_0^*(im)} & \left(\frac{1}{l_8 t_0^*}+v^2\right)\end{vmatrix}=0\,. \quad (27)$$

On solving this $A=\begin{vmatrix}D1 & D2\\ D3 & D4\end{vmatrix}=D1\cdot D4-D2\cdot D3=0\cdot$

But we can see that D2 = 0 as well as D3 = 0. Thus, we have D1·D4 = 0,

$$(w^2-K_1w+K_2)\cdot(w^2-K_3w+K_4)=0\,,$$

$$w^4-(K_1+K_3)w^3+(K_2+K_4-K_1K_3)w^2-(K_2K_3+K_1K_4)w+$$
$$+K_2K_4=0\,. \quad (28)$$

And the roots of the equation are given as follows

$$w=\frac{K_1\pm\sqrt{K_1^2-4K_2}}{2}\,;\quad w=\frac{K_3\pm\sqrt{K_3^2-4K_4}}{2}\,.$$

The four roots v_j (j = 1...4) of Eq.(25) correspond to complex values of phase velocities of the P_1, P_2, P_3, P_4 waves. If $v_i^{-1}=V_i^{-1}+\omega_i^{-1}q_i$ (i = 1…4), the phase velocity v and wave number m are complex quantities and can be written as $m=\frac{\omega}{V}+iq$, where V and q are real. If the real part $\mathrm{Re}(v)\geq 0$, then the real parts of the four roots of Eq.(28) represent the speed of propagation of P_1, P_2, P_3, P_4, and $\mathrm{Img}(v)\leq 0$ refers to the damped wave. Then clearly, V_j and q_j are the speeds of propagation and the attenuation coefficients of the coupled P_1, P_2, P_3, and P_4 waves.

NUMERICAL RESULTS AND DISCUSSION

Numerical illustrations have been performed to calculate speeds v_1, v_2, v_3, and v_4, of reflected plane waves, namely, P_1, P_2, P_3, and P_4 waves, by taking the following relevant physical constants of the medium satisfying the inequalities between the constants at T_0 = 20 °C. For numerical computations, we consider the values of constants for aluminium epoxy-like material as /24/: $\lambda = 7.59\times10^{10}$ N/m^2, $\mu = 1.89\times10^{10}$ N/m^2, $a_0 = 0.61\times10^{-10}$ N, $\rho = 2.19\times10^3$ kg/m^3, $\beta_1 = 0.05\times10^5$ N/m^{2}K, $\beta_0 = 0.05\times10^5$ Nm^{2}K, C = 966 Jkg^{-1}K^{-1}, T_0 = 293 K, $j_0 = 0.196\times10^{-4}$ m^2, $\lambda_0 = \lambda_1 = 0.37\times10^{10}$ N/m^2, t_0 = 0.3, t_1 = 0.1, K = 252 J/msK. The computations are carried out for the value of non-dimensional time t = 0.3 in the range $0 \le y \le 1.0$ and on the surface x = 1.0.

To observe the effects of microelogation parameters λ_1 on the velocity of these waves, authors have solved Eq. (25) numerically and calculated the absolute value of the speed of propagation of P_1, P_2, P_3, and P_4 waves.

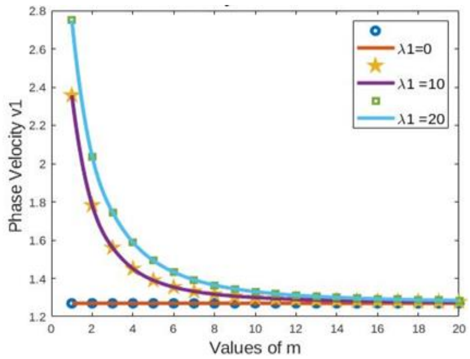

Figure 1. Variation of phase velocity v_1 against wave number.

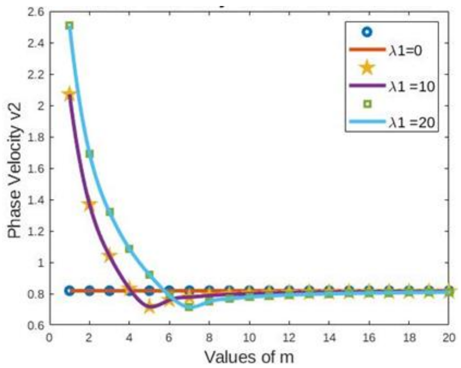

Figure 2. Variation of phase velocity v_2 against wave number.

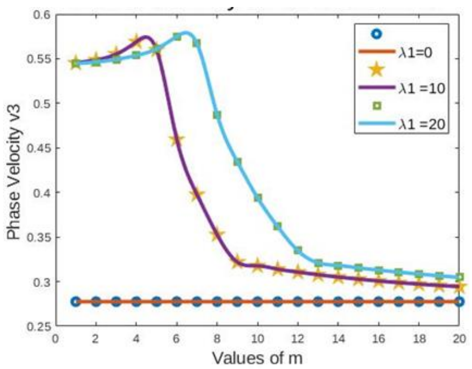

Figure 3. Variation of phase velocity v_3 against wave number.

The speeds of P_1, P_2, P_3, and P_4 waves are plotted against the wave number for different values of microelogation parameters λ_1 = 0, 10, 20. These variations of velocities of P_1, P_2, P_3, and P_4 waves are shown in Figs. 1-4, in respect.

The values of phase velocity v_1 decrease sharply in the range $0 \le k \le 12.0$ and the degree of sharpness increase with increase in value of microelongation parameter λ_1. The effect of microelongation parameter λ_1 reduces with increase in wave number k. The variations of phase velocity v_1 against wave number k are shown in Fig. 1. The variations of phase velocity v_2 are similar in nature to the variations of phase velocity v_1 with the difference being that the values decrease in the range $0 \le k \le 5.0$ for λ_1 = 10.0 and $0 \le k \le 7.0$ for λ_1 = 20.0. These variations of v_2 are shown in Fig. 2.

Contrary to the variations of phase velocities v_1 and v_2, the value of phase velocity v_3 first increases and then decreases sharply for λ_1 = 10.0, 20.0. Values of phase velocities are very less and are almost constant in magnitude in the absence of microelongation parameter λ_1. Values of phase velocity v_4 for λ_1 = 10.0 and 20.0 are identical to each other in the range $0 \le k \le 2.2$. However, with increase in wave number, the difference between the values of phase velocity v_4 becomes more and more significant. These variations of v_3 and v_4 are shown in Figs. 3 and 4, respectively.

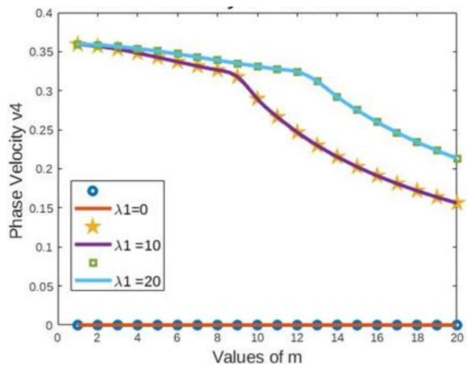

Figure 4. Variation of phase velocity v_4 against wave number.

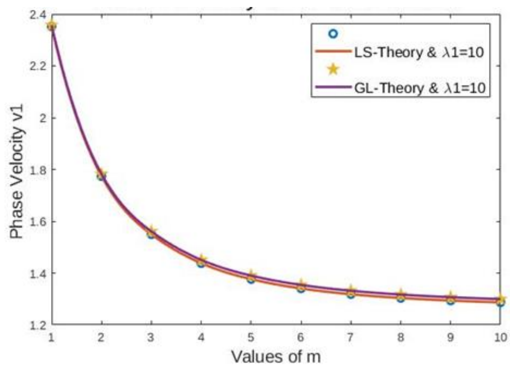

Figure 5. Variation of phase velocity v_1 against wave number for L-S and G-L theories.

The velocities of P_1, P_2, P_3, and P_4 waves are plotted against the wave number to show the comparison between GL theory (–o–) and LS theory(–*–) of thermoelasticity for a fixed value of microelongation parameter λ_1 = 1.0. These variations of velocities of P_1, P_2, P_3, and P_4 waves are shown in Figs. 5-8, respectively. These variations of velocities show similar behaviour for both theories of thermoelasicity. The values of phase velcocity v_1 decrease sharply with increase in wave number and converge to a constant value as shown in Fig. 5.

Values of phase velocity v_2 decrease sharply in the range $0 \leq k \leq 4.8$ and then increase in the remaining range. These values of v_2 are quite close to each other for both LS and GL theories of thermoelasticity. The difference in values of v_2 among the two theories is visible in the range $3.0 \leq k \leq 7.0$. These variations of v_2 are shown in Fig. 6. It is observed from Fig. 7 that the values of phase velocity v_3 increase in the range $0 \leq k \leq 5.0$ and further decrease in the range $5.0 < k \leq 10.0$. Unlike the values of v_1 and v_2, the values of phase velocity v_3 for LS and GL theories are significant in magnitude in the range $0 \leq k \leq 9.0$. The variations of phase velocity v_4 follows a downward trend for both LS and GL theories of thermoelasticity. This downward pattern becomes more sharp in the region $9.0 \leq k \leq 10.0$. These variations of phase velocities v_3 and v_4 are shown in Figs. 7 and 8, respectively.

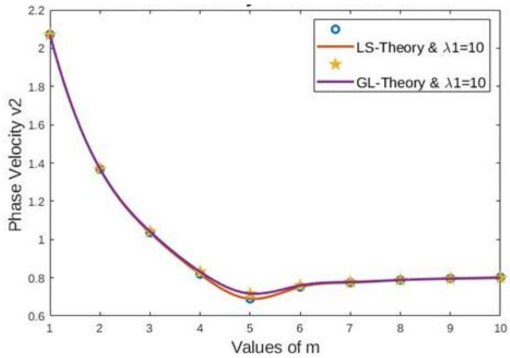

Figure 6. Variation of phase velocity v_2 against wave number for L-S and G-L theories.

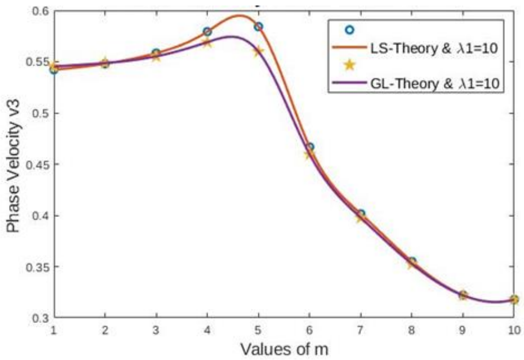

Figure 7. Variation of phase velocity v_3 against wave number for L-S and G-L theories.

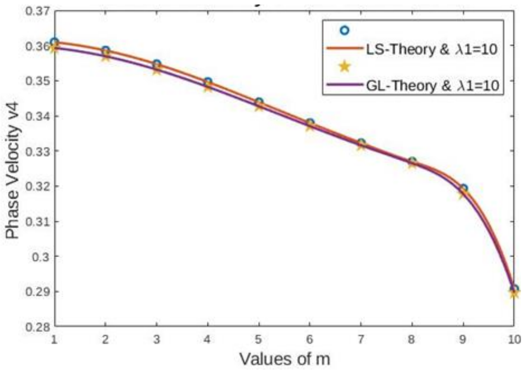

Figure 8. Variation of phase velocity v_4 against wave number for L-S and G-L theories.

CONCLUSION

Coupled partial differential equations for a homogeneous, isotropic, microelongated, thermoelastic solid are formulated, and speeds of waves, namely, P_1, P_2, P_3, and P_4 waves, are obtained for a particular material. The effect of microelongation parameters on phase velocities is studied. It is found that the microelongation parameter effects the speeds of plane waves significantly. In the absence of microelongation parameter the values of phase velocities are very less in magnitude. Also, the values of phase velocities for both the theories of thermoelasticity are quite close to each other.

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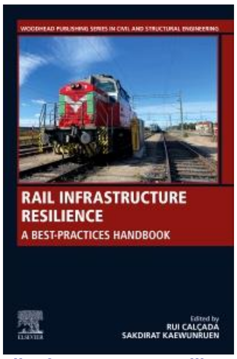

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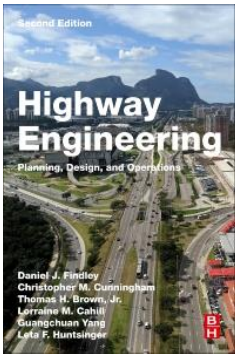

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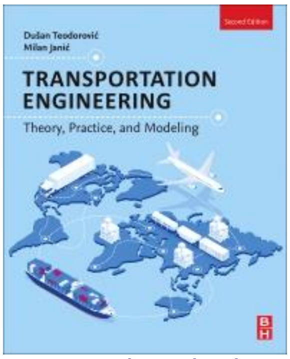

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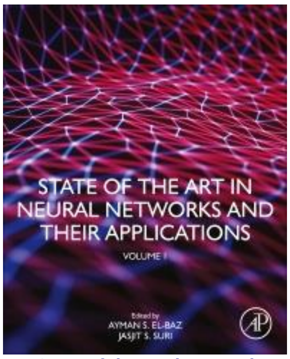

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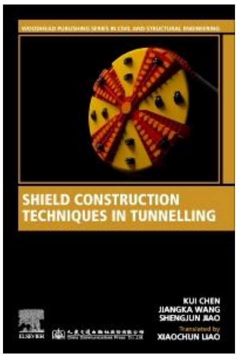

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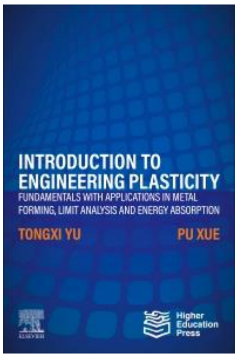

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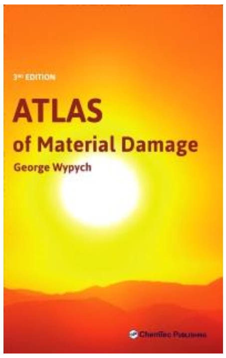

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