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EFFICIENT TRAPEZOIDAL WEB PROFILE FOR STEEL CORRUGATED WEB PLATE I GIRDER TO RESIST LATERAL TORSIONAL BUCKLING

EFIKASNI TRAPEZOIDNI PROFIL REBRA ZA ČELIČNE TALASASTE PLOČE REBRA I-NOSAČA RADI OTPORNOSTI PREMA BOČNOM TORZIONOM IZVIJANJU

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- corrugated steel plate I girder
- lateral-torsional buckling
- angle of corrugation

Ključne reči
- efikasni talasasti profil rebra
- talasasta čelična ploča I nosača
- bočno-torziono izvijanje
- ugao talasastog profila

Abstract

Efficient web profile to resist lateral torsional buckling for steel corrugated web plate I girders is presented in this paper. The cubic equation required to obtain the most efficient corrugated web profile for any given angle and depth of corrugation is derived. The efficient web profile obtained using this cubic equation is verified numerically by analysing standard example problems. The results of all the example problems show that the presented cubic equation can be used to obtain the most efficient trapezoidal web profile for corrugated web plate I girder to resist lateral torsional buckling for any given angle and depth of corrugation.

Izvod

U radu je predstavljen efikasni profil rebra za povećanje otpornosti ka bočnom torzionom izvijanju čelične talasaste ploče rebra I nosača. Izvedena je kubna jednačina za definisanje najefikasnijeg talasastog profila rebra za bilo koji ugao i dubinu talasa. Efikasni profil rebra, dobijen prema ovoj kubnoj jednačini je verifikovan numerički, analizom standardnih primera. Rezultati svih datih primera pokazuju da se predstavljena kubna jednačina može upotrebiti za dobijanje najefikasnijeg trapezoidnog profila rebra za I nosač sa talasastom pločom rebra radi povećanja otpornosti prema bočnom torzionom uvijanju za bilo koji ugao i dubinu talasa.

INTRODUCTION

Steel I girder with flat web profile experiences lateral torsional bucking due to applied transverse loads. To minimize lateral torsional buckling and to improve shear and bending resistance, stiffeners are provided at regular intervals. This increases material and fabrication costs. In order to achieve economy in the steel I girder manufacturing process and to minimize lateral torsional buckling, many researchers use corrugated web profile in steel I girders. As corrugated web plate girders can provide more strength and stiffness than its flat web counterpart, recently the corrugated steel web plates are used by replacing stiffened flat steel web plates and box girders (Lebon /11/, Sayed Ahmed /18-19/, Hassanein & Kharoob /6/). In the references (Elgaaly et al. /15/ and Driver et al. /3/) shear strength of beams and its behaviour with corrugated webs are discussed through experimental and numerical works. Egaaly et al. /16/ discussed bending strength of corrugated web steel plate girders. Leblouba /17/ studied shear buckling through experimental work. In order to understand the behaviour of corrugated web plate girders, finite element analysis and parametric study are done by many researchers (Luo & Edlund /13-14/, Yi et al. /7/, Sadek et al. /2/, Aggarwal et al. /8/, Riahi et al. /5/). Experimental work to study shear strength of trapezoidally corrugated steel webs is carried out by Moon et al. /10/. Design aspects and parametric study is discussed on steel I girders with corrugated steel webs by Sayed-Ahmed, /4/.

Lateral torsional buckling of I girder subjected to uniform bending is studied by Moon et al. /10/. In the reference lateral torsional buckling of beams with trapezoidally corrugated webs are studied by Lindner /12/. Though a very few researchers studied lateral torsional buckling, there is a void in obtaining efficient profile for steel I girder with corrugated web profile for better resisting lateral-torsional buckling. In this paper a cubic equation has been derived to obtain the efficient trapezoidal web profile for steel I plate girder for any specified angle and depth of corrugation to resist lateral-torsional buckling, and it is validated by analysing standard example problems and computing its lateral-torsional buckling strength.

DETAILS OF TRAPEZOIDAL CORRUGATED I GIRDERS

Global x, y and z axes considered are shown in the typical trapezoidal web girder in Fig. 1. Also, the plan and side view of typical trapezoidal steel web I girder are shown in Fig. 2.

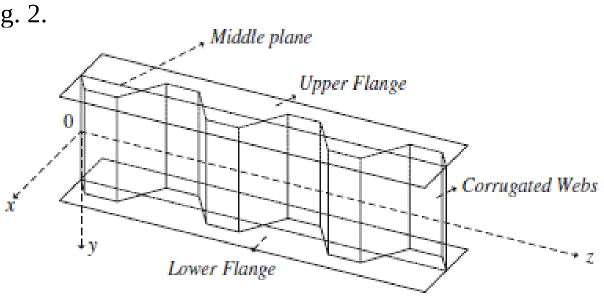
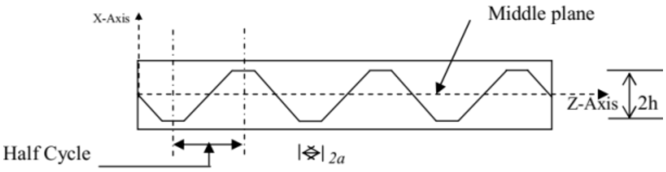


Figure 1. A typical trapezoidal steel web plate I girder.



Plan of typical trapezoidal steel web plate I girder

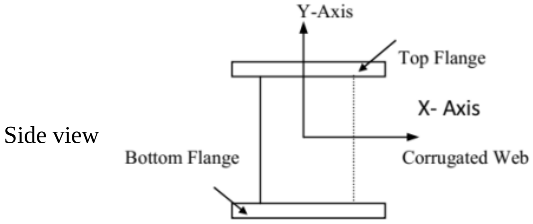


Figure 2. Plan and side view of typical trapezoidal steel web plate I girder.

FORMULATION OF CUBIC EQUATION

The portion of the plan of the trapezoidal web profile (half cycle Fig. 2) considered here is shown in Fig. 3. Parameters illustrated in Fig. 3 are: $2a$ - length of flat web portion; $2z$ - length of inclined web portion; $2b$ - projected length of inclined web portion; $2h$ - depth of corrugation; α - corrugated angle of web plate; t - thickness of web plate. X1 and Z1 are axes on the plan of web at top of web as shown in Fig. 3.

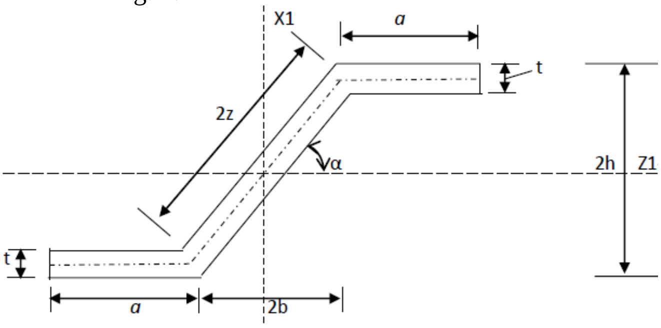


Figure 3. Plan of corrugated web plate portion (half cycle).

To avoid lateral buckling, moment of inertia I_{Z1} of the column cross section about Z1 axis is equated to moment of inertia I_{X1} of the same column about X1 axis. When the moment of inertia I_{Z1} expression is equated to moment of inertia I_{X1} expression, the cubic Eq.(1) is obtained.

This equation can be used to obtain the most efficient web profile to resist the lateral-torsional buckling for any specified angle and depth of the corrugation.

$$\left(\frac{2t}{3}\right)a^3+(2bt)a^2+\left\{2b^2t-2t(h-0.5t)^2-\frac{t^3}{6}\right\}a+$$

$$+\left\{\frac{2}{3}t_1h^3-\frac{1}{6}ht_1^3\right\}=0\,. \qquad (1)$$

It can be written in conventional notation as

$$pa^3+qa^2+ra+s=0\,, \qquad (1a)$$

where: $p = 2t/3$; $q = 2bt$; $r = 2b^2t - 2t(h - 0.5t)^2 - t^3/6$;

$s=\frac{2}{3}t_1h^3-\frac{1}{6}ht_1^3$; $t_1=\frac{t}{\sin\alpha}$.

NUMERICAL TESTS AND DISCUSSIONS

The cubic equation (Eq.1) is used to obtain the efficient web profiles for the following three standard example problems of trapezoidal corrugated web plate I girder:

1. Trapezoidal corrugated web plate I girder with web inclination α = 24°, 37°, 50° and 62° thickness of web t_w = 4 mm, height of web h_w = 700 mm, depth of corrugation = 400 mm, width of flange = 500 mm, thickness of flange = 40 mm, length of corrugated web girder L = 12000 mm.
2. Trapezoidal corrugated web plate I girder with web inclination α = 24°, 37°, 50° and 62° thickness of web t_w = 2 mm, height of web h_w = 500 mm, depth of corrugation = 240 mm, width of flange = 300 mm, thickness of flange = 20 mm, length of corrugated web girder L = 10000 mm.
3. Trapezoidal corrugated web plate I girder with web inclination α = 24°, 37°, 50° and 62°, web thickness t_w = 0.2 mm, web height h_w = 150 mm, depth of corrugation = 60 mm, width of flange = 100 mm, thickness of flange = 1 mm, length of corrugated web girder L = 880 mm.

Details of the data obtained by using Eq.(1) for efficient corrugated web profiles for example problems 1, 2 and 3 are given in Tables 1, 2 and 3, respectively.

Table 1. Efficient web profiles for example problem 1.

Corrug. angle α	$2z$ (mm)	$2h$ (mm)	a (mm)
24°	983.44	400	36.52
37°	664.66	400	80.77
50°	522.16	400	153.74
62°	448.93	400	226.63

Table 2. Efficient web profiles for example problem 2.

Corrug. angle α	$2z$ (mm)	$2h$ (mm)	a (mm)
24°	590.06	240	21.93
37°	398.79	240	48.53
50°	313.30	240	98.44
62°	269.36	240	136.23

Table 3. Efficient web profiles for example problem 3.

Corrug. angle α	$2z$ (mm)	$2h$ (mm)	a (mm)
24°	147.52	60	5.50
37°	99.70	60	12.20
50°	78.32	60	23.25
62°	67.34	60	34.25

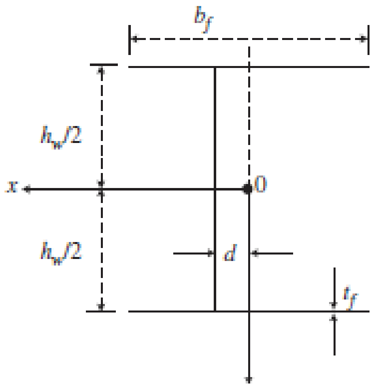
To validate the results given in Tables 1-3, the lateral-torsional buckling strength M_{cr} formula Eq.(2), given in Moon et al. /10/, is used:

$$M_{cr}=\frac{\pi}{L}\sqrt{EI_{y.co}G_{co}J_{co}}\sqrt{1+W^2}\,, \qquad (2)$$

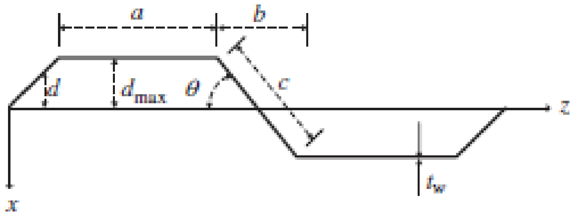
where: $W=\frac{\pi}{L}\sqrt{E\frac{C_{w.co}}{G_{co}J_{co}}}$ - warping torsional stiffness;

$J_{co}=\frac{1}{3}(2b_ft_f^3+h_wt_f^3)$ - pure torsional constant; $G_{co}=\frac{a+b}{a+c}$ - shear modulus of corrugated web; $I_{y.co}=\frac{t_fb_f^3}{6}$ - second moment of area $I_{y.co}$ for y-axis; Young's modulus E = 2×10^5 N/mm^2; shear modulus G = 84000 N/mm^2; L - length of corrugated plate girder.

Note: notations used in Eq.(2) are conformed to Fig. 4



Cross section



Plan of web profile (one cycle)

Figure 4. Cross section and Plan of web profile (one cycle) of corrugated web plate girder.

For each case of Tables 1-3, lateral-torsional buckling strengths are computed by varying the flat portion of the web ‘a’, from a/10 to 19a/10 in the interval of a/10 (9 values on either side of the optimized value ‘a’ specified in the Tables 1-3). The normalized values of lateral-torsional buckling strengths with respect to lateral-torsional buckling strength corresponding to optimised value ‘a’ specified in Tables 1-3 are given in Tables 4-15. These normalised values of lateral-torsional buckling strengths are plotted against the values of flat web portion length ‘a’ in Figs. 5-16.

Table 4. M_{cr} for $\alpha = 24°$ (example problem 1).

Sl .No	Flat web portion length a (mm)	M_{cr}
1	3.65	0.99812
2	7.3	0.99834
3	10.96	0.99856
4	14.61	0.99877
5	18.26	0.99898
6	21.91	0.99919
7	25.56	0.9994
8	29.22	0.9996
9	32.87	0.9998
10	36.52	1
11	40.17	1.0002
12	43.82	1.00039
13	47.48	1.00058
14	51.13	1.00077
15	54.78	1.00095
16	58.43	1.00114
17	62.08	1.00132
18	65.74	1.0015
19	69.39	1.00167

Table 5. M_{cr} for $\alpha = 37°$ (example problem 1).

Sl .No	Flat web portion length a (mm)	M_{cr}
1	8.08	0.98843
2	16.15	0.98996
3	24.23	0.99141
4	32.31	0.9928
5	40.38	0.99413
6	48.46	0.99541
7	56.54	0.99663
8	64.62	0.9978
9	72.69	0.99892
10	80.77	1
11	88.85	1.00104
12	96.92	1.00204
13	105	1.003
14	113.08	1.00392
15	121.15	1.00482
16	129.23	1.00568
17	137.31	1.00651
18	145.39	1.00731
19	153.46	1.00809

Table 6. M_{cr} for $\alpha = 50°$ (example problem 1).

Sl .No	Flat web portion length a (mm)	M_{cr}
1	15.37	0.96198
2	30.75	0.96809
3	46.12	0.97355
4	61.5	0.97846
5	76.87	0.98291
6	92.24	0.98695
7	107.62	0.99064
8	122.99	0.99402
9	138.37	0.99713
10	153.74	1
11	169.11	1.00266
12	184.49	1.00512
13	199.86	1.00742
14	215.24	1.00956
15	230.61	1.01156
16	245.98	1.01344
17	261.36	1.0152
18	276.73	1.01686
19	292.11	1.01842

Table 7. M_{cr} for $\alpha = 62°$ (example problem 1).

Sl .No	Flat web portion length a (mm)	M_{cr}
1	22.66	0.9252
2	45.33	0.93962
3	67.99	0.95161
4	90.65	0.96174
5	113.32	0.97041
6	135.98	0.97792
7	158.64	0.98448
8	181.3	0.99026
9	203.97	0.9954
10	226.63	1
11	249.29	1.00414
12	271.96	1.00788
13	294.62	1.01128
14	317.28	1.01439
15	339.95	1.01723
16	362.61	1.01985
17	385.27	1.02227
18	407.93	0.02451 2
19	430.6	0.02658 2

Table 8. M_{cr} for $\alpha = 24°$ (example problem 2).

Sl .No	Flat web portion length a (mm)	M_{cr}
1	2.19	0.99808
2	4.39	0.9983
3	6.58	0.99852
4	8.77	0.99874
5	10.97	0.99896
6	13.16	0.99917

7	15.35	0.99938
8	17.54	0.99959
9	19.74	0.9998
10	21.93	1
11	24.12	1.0002
12	26.32	1.0004
13	28.51	1.00059
14	30.7	1.00079
15	32.9	1.00098
16	35.09	1.00116
17	37.28	1.00135
18	39.47	1.00153
19	41.67	1.00171

Table 9. M_{cr} for $\alpha = 37°$ (example problem 2).

Sl .No	Flat web portion length *a* (mm)	M_{cr}
1	4.85	0.98844
2	9.71	0.98996
3	14.56	0.99142
4	19.42	0.9928
5	24.27	0.99413
6	29.12	0.99541
7	33.98	0.99663
8	38.83	0.9978
9	43.69	0.99892
10	48.54	1
11	53.39	1.00104
12	58.25	1.00204
13	63.1	1.003
14	67.96	1.00393
15	72.81	1.00482
16	77.66	1.00568
17	82.52	1.00651
18	87.37	1.00732
19	92.23	1.00809

Table 10. M_{cr} for $\alpha = 50°$ (example problem 2).

Sl .No	Flat web portion length *a* (mm)	M_{cr}
1	9.24	0.96242
2	18.49	0.96845
3	27.73	0.97385
4	36.98	0.97871
5	46.22	0.98311
6	55.46	0.98711
7	64.71	0.99075
8	73.95	0.99409
9	83.2	0.99717
10	92.44	1
11	101.68	1.00262
12	110.93	1.00506
13	120.17	1.00732
14	129.42	1.00944
15	138.66	1.01142
16	147.9	1.01327
17	157.15	1.01501
18	166.39	1.01664
19	175.64	1.01819

Table 11. M_{cr} for $\alpha = 62°$ (example problem 2).

Sl .No	Flat web portion length *a* (mm)	M_{cr}
1	13.62	0.92661
2	27.25	0.94079
3	40.87	0.95257
4	54.49	0.96251
5	68.11	0.97101
6	81.74	0.97837
7	95.36	0.9848
8	108.98	0.99047
9	122.61	0.9955
10	136.23	1
11	149.85	1.00405
12	163.48	1.00771
13	177.1	1.01104
14	190.72	1.01407
15	204.34	1.01686
16	217.97	1.01942
17	231.59	1.02178
18	245.21	1.02397
19	258.84	1.026

Table 12. M_{cr} for $\alpha = 24°$ (example problem 3).

Sl .No	Flat web portion length *a* (mm)	M_{cr}
1	0.55	0.99906
2	1.1	0.99917
3	1.65	0.99928
4	2.2	0.99938
5	2.75	0.99949
6	3.3	0.99959
7	3.85	0.9997
8	4.4	0.9998
9	4.95	0.9999
10	5.5	1
11	6.05	1.0001
12	6.6	1.0002
13	7.15	1.0003
14	7.7	1.00039
15	8.25	1.00049
16	8.8	1.00059
17	9.35	1.00068
18	9.9	1.00077
19	10.45	1.00087

Table 13. M_{cr} for $\alpha = 37°$ (example problem 3).

Sl .No	Flat web portion length *a* (mm)	M_{cr}
1	1.22	0.9969
2	2.44	0.99729
3	3.66	0.99767
4	4.88	0.99803
5	6.1	0.99839
6	7.32	0.99873
7	8.54	0.99907
8	9.76	0.99939
9	10.98	0.9997
10	12.2	1
11	13.42	1.00029
12	14.64	1.00057
13	15.86	1.00085
14	17.08	1.00111
15	18.3	1.00137
16	19.52	1.00162
17	20.74	1.00186
18	21.96	1.00209
19	23.18	1.00232

Table 14. M_{cr} for $\alpha = 50°$ (example problem 3).

Sl .No	Flat web portion length *a* (mm)	M_{cr}
1	2.33	0.99322
2	4.65	0.99429
3	6.98	0.99526
4	9.3	0.99614
5	11.63	0.99694
6	13.95	0.99766
7	16.27	0.99832
8	18.6	0.99893
9	20.92	0.99949
10	23.25	1
11	25.57	1.00047
12	27.9	1.00091

13	30.22	1.00132
14	32.55	1.0017
15	34.88	1.00205
16	37.2	1.00238
17	39.53	1.00269
18	41.85	1.00298
19	44.18	1.00325

Table 15. M_{cr} for $\alpha = 62°$ (example problem 3).

Sl .No	Flat web portion length a (mm)	M_{cr}
1	3.42	0.99034
2	6.85	0.99239
3	10.27	0.99404
4	13.7	0.99537
5	17.13	0.99648
6	20.55	0.99741
7	23.98	0.99821
8	27.4	0.99889
9	30.83	0.99948
10	34.25	1
11	37.67	1.00046
12	41.1	1.00087
13	44.52	1.00123
14	47.95	1.00156
15	51.37	1.00186
16	54.8	1.00213
17	58.22	1.00238
18	61.65	1.00261
19	65.07	1.00282

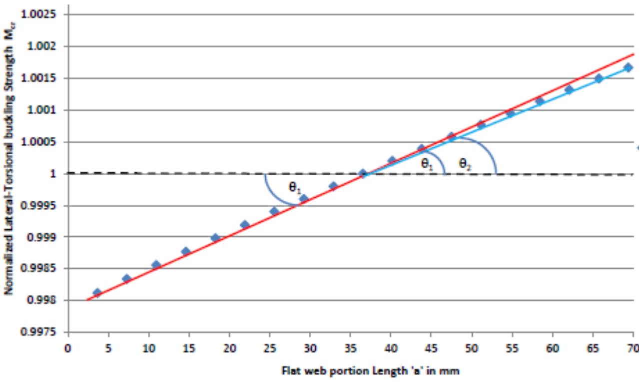


Figure 5. M_{cr} for $\alpha = 24°$ (example problem 1).

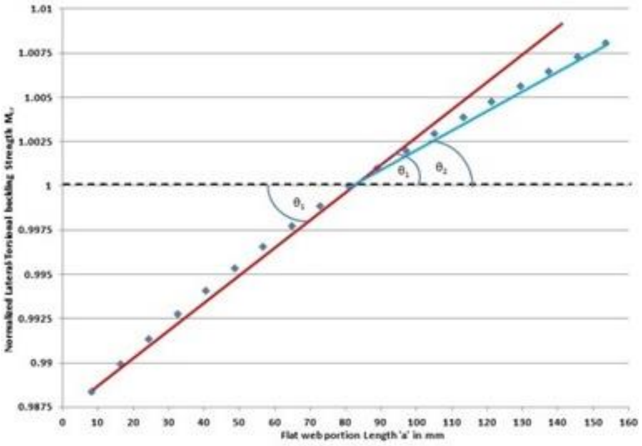


Figure 6. M_{cr} for $\alpha = 37°$ (example problem 1).

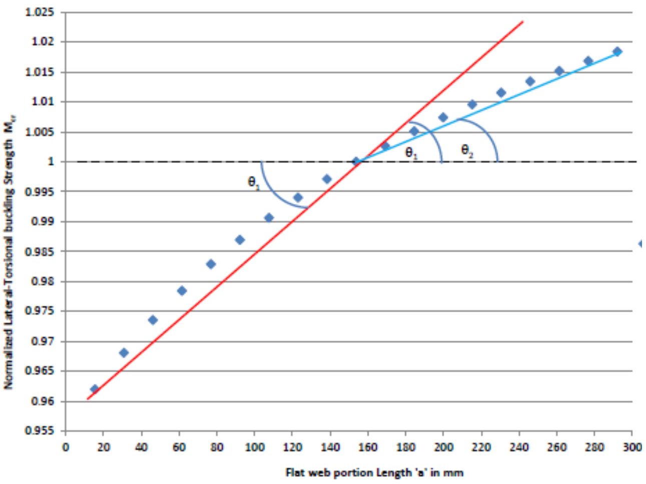


Figure 7. M_{cr} for $\alpha = 50°$ (example problem 1).

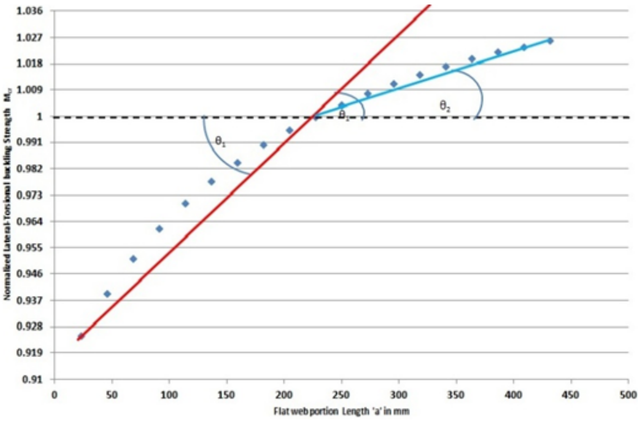


Figure 8. M_{cr} for $\alpha = 62°$ (example problem 1).

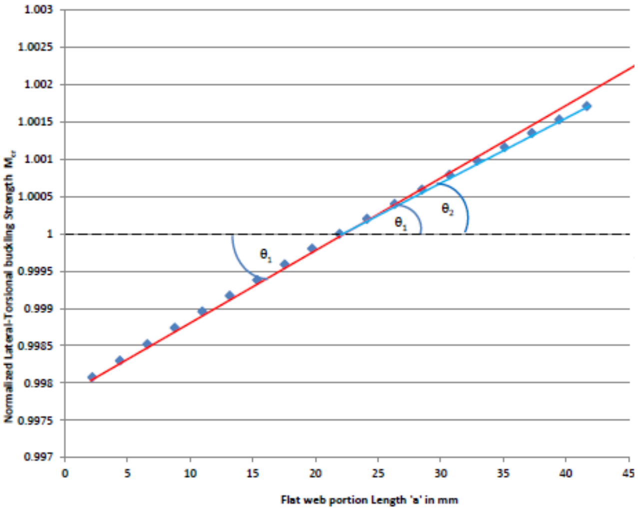


Figure 9. M_{cr} for $\alpha = 24°$ (example problem 2).

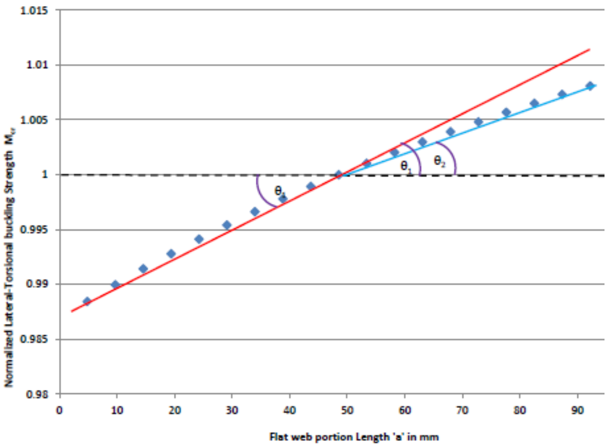


Figure 10. M_{cr} for $\alpha = 37°$ (example problem 2).

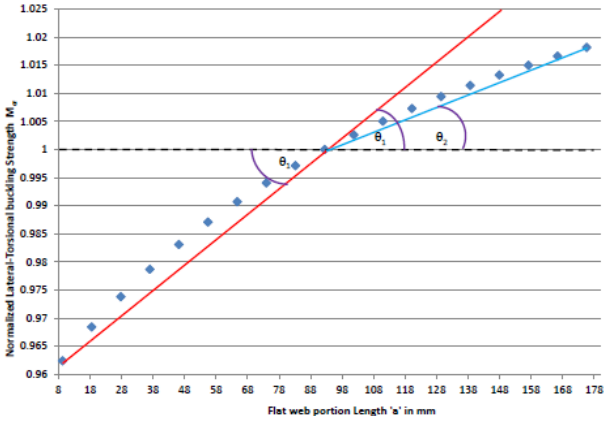


Figure 11. M_{cr} for $\alpha = 50°$ (example problem 2).

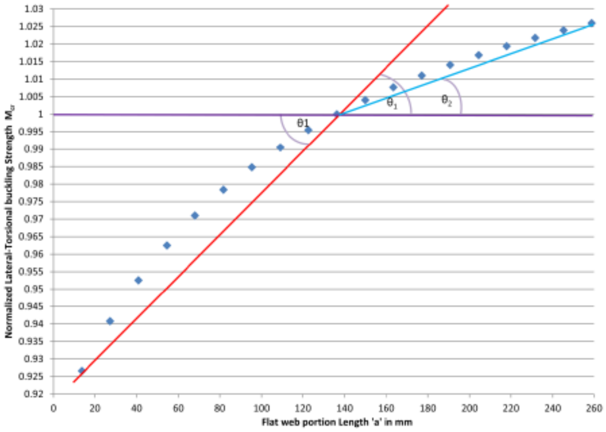


Figure 12. M_{cr} for $\alpha = 62°$ (example problem 2).

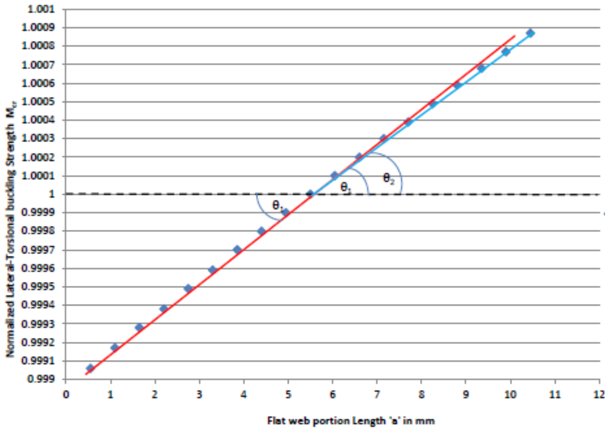


Figure 13. M_{cr} for $\alpha = 24°$ (example problem 3).

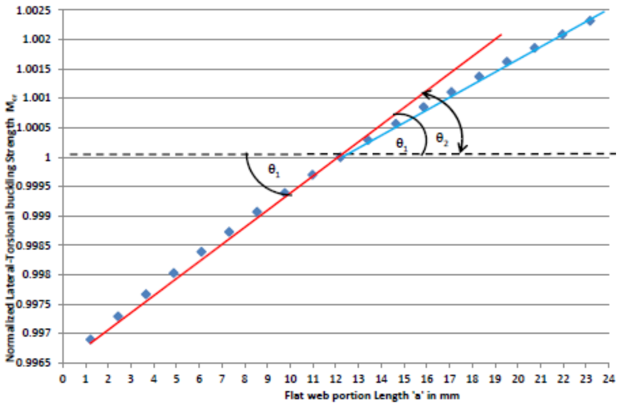


Figure 14. M_{cr} for $\alpha = 37°$ (example problem 3).

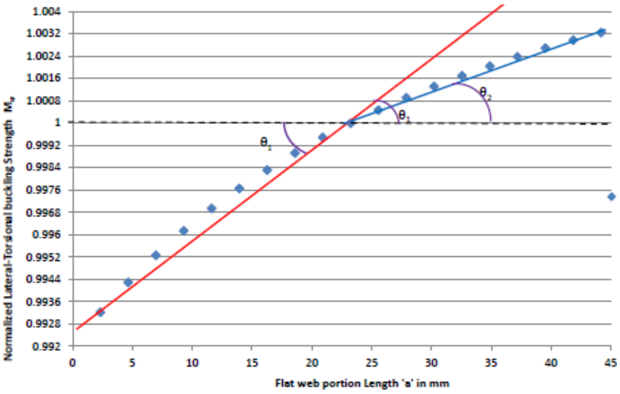


Figure 15. M_{cr} for $\alpha = 50°$ (example problem 3).

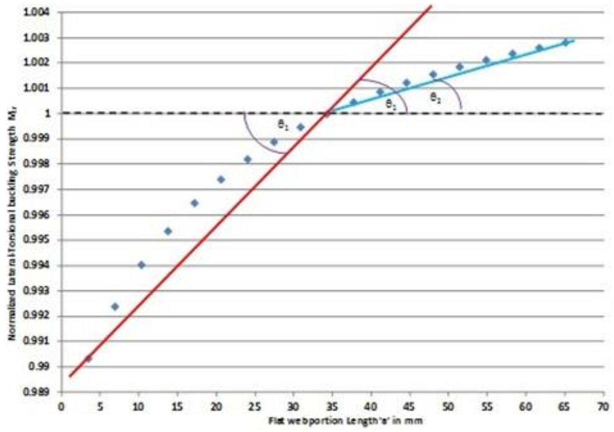


Figure 16. M_{cr} for $\alpha = 62°$ (example problem 3).

In all the Tables 4-15 and in corresponding Figs. 5-16, it can be seen that, though value of flat web portion length a, increases with constant increment $a/10$, the lateral-torsional buckling strength increases at a higher rate up to the optimal value of a as specified in Tables 1-3. Further increase in value of a, at the same rate, after the optimal values, lateral-torsional buckling strength increases but at a slower rate. It can be seen in all Figs. 5-16 (as $\theta_1 > \theta_2$). Hence, it can be concluded that the value of a corresponding to serial number 10 in all Tables 4-15 and 10th point in all Figs. 5-16 is the optimum value for flat web portion length to resist the lateral-torsional buckling. Therefore, the cubic equation presented in this paper can be used to get the efficient trapezoidal corrugated web profile for I girder to resist lateral-torsional buckling.

CONCLUSIONS

The cubic equation required to get the most efficient trapezoidal corrugated web profile for any specified angle and depth of corrugation is presented in this paper. To validate the web profile obtained, various standard example problems are analysed numerically to get lateral-torsional bucking strengths. In all the example problems, the results show that trapezoidal corrugated web profiles obtained using the cubic equation presented in this paper are efficient profiles. Therefore, the cubic equation can be used to obtain the most efficient trapezoidal corrugated web profile to resist lateral torsional buckling for any specified angle and depth of corrugation.

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